Mark Scheme (Results)

Summer 2025

Pearson Edexcel GCE
In Mathematics (9MA0)

Paper 02 Pure Mathematics

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 100.

2. The Edexcel Mathematics mark schemes use the following types of marks:
- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
   If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question	Scheme	Marks	AOs
1	$2x\left(x^2-12x+20\right)$ or $x\left(2x^2-24x+40\right)$	B1	1.1b
	$x^2-12x+20=(x-2)(x-10)$ or e.g. $2x^2-24x+40=(2x-4)(x-10)$	M1	1.1b
	$2x(x-2)(x-10)$	A1	1.1b
		(3)	
			(3 marks)

Notes

B1: Takes a factor of $2x$ or x out of the given cubic **correctly**.
Dividing by 2 and then achieving $x\left(x^2-12x+20\right)$ does not score this mark unless recovered.

M1: Attempts to factorise their **quadratic**. Invisible brackets may be implied by later work.
Score for $(x\pm b)(x\pm d)$ where $|bd|=20$ coming from $x^2\pm12x\pm20$
or for $(ax\pm b)(cx\pm d)$ where $|ac|=2$ and $|bd|=40$ coming from $2x^2\pm24x\pm40$
May be scored if they have divided by x but not from e.g. $2x^3-24x^2+40x\to(2x-4)(x-10)$ without clear indication that they have divided by or taken out a factor of x or $2x$.
There may be incorrect intermediate steps such as $(2x-20)(2x-4)$ which do not score the mark on their own but may be ignored if they return to e.g. $(2x-20)(x-2)$.

A1: $2x(x-2)(x-10)$ or e.g. $2(x-10)(x-2)x$. Do not accept e.g. $x(2x-4)(x-10)$. Ignore = 0
ISW after a fully correct factorisation e.g. $2x(x-2)(x-10)$ that becomes $x(x-2)(x-10)$ and ignore any attempt to find roots (before or after factorisation). Allow $(2x)(x-2)(x-10)$

Alternative:

B1: Takes a factor of $(x-2)$ or $(x-10)$ out **correctly** i.e. $(x-2)\left(2x^2-20x\right)$ or $(x-10)\left(2x^2-4x\right)$. Must be seen as a product of factors and not just in a division attempt but may be implied by later work.

M1: Attempts to factorise their **quadratic**. Invisible brackets may be implied by later work.
Score for $ax(bx\pm c)$ where $|ab|="2"$ and $|ac|=\text{their }"20"$ or $"4"$ coming from Ax^2+Bx

A1: As main scheme.

Note: Solutions that solve the cubic = 0 to achieve x = 0, 2 and 10 and then arrive at e.g. $x(x-2)(x-10)$ score no marks without a prior line of working such as $x\left(x^2-12x+20\right)$ (would score M1) or $x\left(2x^2-24x+40\right)$ (would score B1).

Some examples:

- $2x^3-24x^2+40x\to x(x-2)(x-10)$ scores B0M0A0
- $2x^3-24x^2+40x\to(2x-4)(x-10)$ scores B0M0A0
- $x^2-12x+20\to(x+2)(x-10)$ scores B0M1A0
- $x^3-12x^2+20x\to x(x-2)(x-10)$ scores B0M1A0
- $x\left(2x^2-24x+40\right)\to x(x-2)(x-10)$ scores B1M0A0
- $x\left(2x^2-24x+40\right)\to\left\{x(2x+2)(2x+40)\to\right\}\ x(2x+2)(x+20)$ scores B1M1A0
- $2x\left(x^2-12x+20\right)\to 2x(x-1)(x+20)$ scores B1M1A0
- $2x(x-2)(x-10)$ on its own scores B1M1A1

Question	Scheme	Marks	AOs
2	$x^4 \to \dots x^5$ or $x^{\frac{1}{2}} \to \dots x^{\frac{3}{2}}$ or $-3 \to \dots x$	M1	1.1a
	Any of $\frac{1}{5}x^5$ or $-\frac{6}{\left(\frac{3}{2}\right)}x^{\frac{3}{2}}$ or $-3x$	A1	1.1b
	Any two of $\frac{1}{5}x^5$ or $-\frac{6}{\left(\frac{3}{2}\right)}x^{\frac{3}{2}}$ or $-3x$	A1	1.1b
	$\frac{1}{5}x^5 - 4x^{\frac{3}{2}} - 3x + c$	A1	1.1b
		(4)	
			(4 marks)

Notes

M1: For increasing any power by one. Score for $x^n \to x^{n+1}$ in any term, including, $-3 \to \dots x$ where … is a constant, but not for + c. Allow the indices to be unprocessed, e.g., x^{4+1}

A1: One correct term which may be unsimplified and indices may be unprocessed.

Condone e.g. $-3x^1$ or $-\frac{6}{\left(\frac{1}{2}+1\right)}x^{\frac{1}{2}+1}$ for this mark. Not scored for + c

A1: Two correct terms which may be unsimplified but indices must be processed.
Condone $-3x^1$ for this mark. Not scored for + c

A1: cao Requires all terms simplified and + c
Ignore the LHS i.e. ignore what they call their integral.

Allow $0.2x^5$ for $\frac{1}{5}x^5$ and $-4\sqrt{x^3}$ or $-4\sqrt{x}^3$ or $-4x\sqrt{x}$ or $-4x^{1.5}$ for $-4x^{\frac{3}{2}}$

Do not allow $-3x^1$ for this mark.

Condone spurious integral signs e.g. $\int \frac{1}{5}x^5 - 4x^{\frac{3}{2}} - 3x + c$ or dx left in their answer

ISW after a correct expression seen e.g. if they multiply through by 5 or e.g. try to solve = 0

Do not allow e.g. $\frac{1}{5}x^5 + -4x^{\frac{3}{2}} + -3x + c$

Question	Scheme	Marks	AOs
3	$3^x = 7^y \rightarrow x\log 3 = y\log 7$	M1	3.1a
	e.g. $\left(\frac{x}{y} =\right) \frac{\log 7}{\log 3}$	A1	1.1b
		(2)	
			(2 marks)

Notes

Note: Condone absence of reference to the value of $\frac{x}{y}$ being undefined when $x = y = 0$

M1: For the key step in attempting to take logarithms with the same base of both sides **and** apply the power law to both sides, e.g., $x = y\log_3 7$ or $y = x\log_7 3$ or $x\ln 3 = y\ln 7$

Condone e.g. $x = \log_3 7y$ but not $x = \log_3(7y)$

Alternatively, takes a root of both sides (either x or y), takes logs with the same base of both sides and applies the power law e.g., $3^x = 7^y \rightarrow 3^{\frac{x}{y}} = 7 \rightarrow \frac{x}{y}\log 3 = \log 7$

May be implied by a correct answer.

A1: $\left(\frac{x}{y} =\right) \frac{\log 7}{\log 3}$ or equivalent, e.g, $\frac{\ln 7}{\ln 3}$ or $\log_3 7$ or $\frac{1}{\log_7 3}$ Condone e.g. $\frac{\ln|7|}{\ln|3|}$

Correct answer only scores both marks provided there is no incorrect log work.

Any base k may be used for e.g. $\frac{\log_k 7}{\log_k 3}$ provided it is the same base in both logs, although watch out for e.g. $\left(\frac{\log_3 7}{\log_7 3}\right)^{\frac{1}{2}}$ or $\frac{\log_3 k}{\log_7 k}$ for constant $k > 0$ which are exceptions and correct.

There is no need to see $\frac{x}{y} =$ but it should be clear what their answer is.

Do not ISW if they incorrectly apply log laws e.g. $\frac{\log 7}{\log 3} = \log\frac{7}{3}$ or $= \log 7 - \log 3$ or $= \log(7-3)$ all of which score M1A0.

You may ISW after a correct answer if they go on to provide a decimal approximation.

A decimal approximation with no correct log work seen (1.7712…) scores no marks.

Question	Scheme	Marks	AOs
4(a)	e.g. $6.4^2 = 8^2 + 13^2 - 2(8)(13)\cos ABC$	M1	1.1b
	$(\text{angle } ABC =)\quad \cos^{-1}\left(\dfrac{8^2+13^2-6.4^2}{2\times 8\times 13}\right) = 0.394 \text{ (radians)*}$	A1*	1.1b
		(2)	
(b)	e.g. $\left(\text{Area}(ABC) =\right)\quad \dfrac{1}{2}(8)^2(0.394)\quad (= \text{awrt } 12.6)$ **or** $\left(\text{Area}(ABD) =\right)\quad \dfrac{1}{2}(8)(13)\sin 0.394\quad (= \text{awrt } 20.0)$	M1	1.1b
	$\left(\text{Area}(R) =\right)\quad \dfrac{1}{2}(8)(13)\sin 0.394 - \dfrac{1}{2}(8)^2(0.394) = \ldots$	dM1	3.1a
	$\left(\text{Area}(R) =\right)\quad$ awrt 7.34 to 7.37	A1	1.1b
		(3)	
			(5 marks)

Notes

It is acceptable in this question to work in degrees and convert if necessary.
NB Angle ABC in degrees is 22.591…°

(a)

M1: Attempts to use the cosine rule with values correctly placed. May be implied by, e.g.,

$$\cos\theta = \frac{8^2+13^2-6.4^2}{2\times 8\times 13} \quad\text{or}\quad \cos^{-1}\left(\frac{6.4^2-8^2-13^2}{-2\times 8\times 13}\right)$$

Condone slips in substitution if a correct cosine formula is seen. The placement of the values should be correct. So, condone e.g. a missing 2 once a correct cosine formula is seen.

Alt 1: attempts the cosine rule and finds angle ADB $(= \text{awrt } 0.5 \text{ or } 29°)$ **or** angle BAD $(= \text{awrt } 2.2 \text{ or } 2.3 \text{ or } 129°)$ **and** uses the sine rule correctly with angle ABC involved.

Alt 2: attempts the cosine rule and finds angle ADB $(= \text{awrt } 0.5 \text{ or } 29°)$ **and** angle BAD $(= \text{awrt } 2.2 \text{ or } 2.3 \text{ or } 129°)$ **and** sums angles ADB, BAD, and ABC to π (or 180°)

Alt 3: using Pythagoras **and** simultaneous equations to find the length of AN or BN (see diagram on the next page) **and** uses e.g. $\cos ABC = \dfrac{BN}{8}$ or $\sin ABC = \dfrac{AN}{8}$

A1*: cso Correct proof.

If starting with $6.4^2 = 8^2 + 13^2 - 2(8)(13)\cos ABC$ then there must be an intermediate line such as

- $\cos ABC = \dfrac{8^2+13^2-6.4^2}{2\times 8\times 13}$ or angle $ABC = \cos^{-1}\left(\dfrac{8^2+13^2-6.4^2}{2\times 8\times 13}\right)$
- $\cos ABC = \text{awrt } 0.92$ or $\dfrac{4801}{5200}$
- angle ABC = awrt 0.3943

The minimum required is e.g. $\cos^{-1}\left(\dfrac{8^2+13^2-6.4^2}{2\times 8\times 13}\right)$ or $\cos\theta = \dfrac{8^2+13^2-6.4^2}{2\times 8\times 13} \rightarrow \theta = 0.394$

both of which score M1A1*.

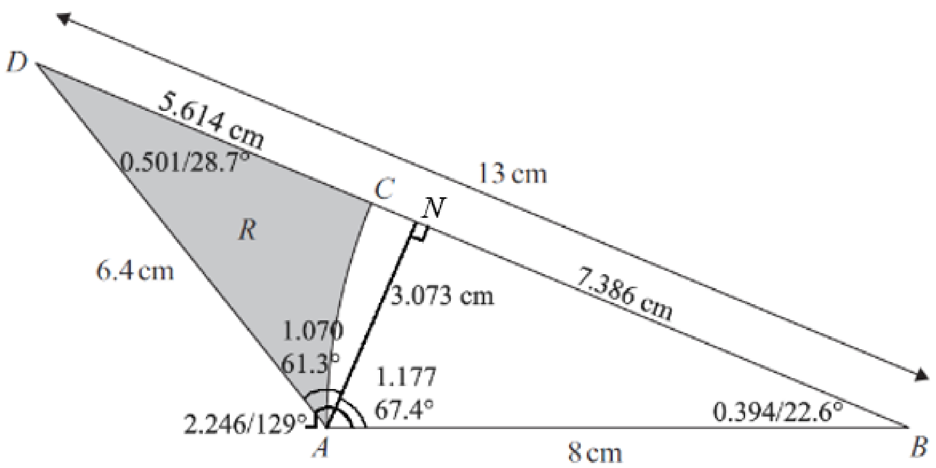
The final value must be 0.394 and not e.g. 0.3943. There should be no obvious incorrect statements in the proof e.g. $\cos(0.92) = 0.394$ and no obvious incorrect work.

Allow the use of, e.g., θ, x, or even e.g. A throughout. Mention of radians is not required.

Those working in degrees and achieve 22.6 can convert to 0.394 without calculation for M1A1

	Attempts via verification e.g. $AD^2 = 8^2 + 13^2 - 2(8)(13)\cos 0.394 \to AD =$ awrt 6.4 score maximum M1A0*.
(b)	**Note: you may need to check the diagram for working.**
M1:	Attempts the area of the sector **or** the area of the triangle *ABD* via a **correct** method. May be implied by a full attempt at the area of *R*. The sector area may be implied by $\frac{1576}{125}$ or by $\frac{0.394}{2\pi}\times\pi\times 8^2$ or $\frac{22.6}{360}\times\pi\times 8^2$ The angle *ABC* is given in the question as 0.394 and should be used (or a more accurate value). Do not allow use of a different value for angle *ABC* (other than the angle in degrees). There are many acceptable alternative approaches to find the area of triangle *ABD*, e.g. using their angle *ADB* **or** their angle *BAD* **or** using right-angled triangles e.g. $\frac{1}{2}(13)(\text{"3.07"})$ **or** $\frac{1}{2}(\text{"7.39"})(\text{"3.07"})+\frac{1}{2}(\text{"5.614"})(\text{"3.07"})$ and could be implied by e.g. 11.3 + 8.6 **Alternatively**, attempts the area of the triangle *ACD* $\left(\text{e.g. } \frac{1}{2}(5)(6.4)\sin\text{"0.501"} = \text{awrt } 7.68\right)$ **or** the area of the segment (in sector *ABC*) $\left(\frac{1}{2}(8^2)(0.394)-\frac{1}{2}(8^2)\sin 0.394 = \text{awrt } 0.324\right)$ via a **correct** method. The area of the triangle *ABC* alone is insufficient for this mark. Note that $\sin 0.394$ might be seen as $\sqrt{1-\left(\frac{4801}{5200}\right)^2}$ For values, see the diagram below.
dM1:	Complete and correct method for the area of *R*. Requires correct attempts at both the area of the sector **and** the area of the triangle *ABD* **and** subtraction of the two values (or expressions). Allow sector – triangle *ABD* if this is then recovered by making the area positive. Alternatively, correct attempts at both the area of the triangle *ACD* **and** the area of the segment (in sector *ABC*) **and** subtraction of the two values (or expressions). Condone slips in substitution provided a correct formula is seen. May be implied by a correct answer.
A1:	awrt 7.34 to 7.37 no units required but penalise incorrect units. ISW after an acceptable answer is seen.

Helpful Diagram:



Question	Scheme	Marks	AOs
5(a)	$-3=\dfrac{t-1}{2}\Rightarrow t=-5\Rightarrow y=5\left("-5"+2\right)^4$	M1	1.1b
	$(y=)\,405$	A1	1.1b
		(2)	
(b)	$x=\dfrac{t-1}{2}\Rightarrow t=2x+1\Rightarrow y=5\left("2x+1"+2\right)^4$	M1	1.1b
	$y=5\left(2x+3\right)^4$	A1	1.1b
		(2)	
(c)	$\left("2x+3"\right)^n\rightarrow ...\left("2x+3"\right)^{n-1}$	M1	1.1b
	$\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}=\right)40\left(2\left(-3\right)+3\right)^3$	dM1	1.1b
	$\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}=\right)-1080$	A1	1.1b
		(3)	
		(7 marks)	

Notes

(a) **Mark parts (a) and (b) together.**

M1: Substitutes $x=-3$ into $x=\dfrac{t-1}{2}$, attempts to find *t*, and substitutes their *t* into $y=5\left(t+2\right)^4$

Condone slips. May be implied by substitution of $t=-5$ into *y* or by 405 or by $y=5\left(-3\right)^4$

Alternatively, they may substitute $x=-3$ into their $y=\mathrm{f}\left(x\right)$

A1: cao Answer only scores full marks and ISW after seeing 405 e.g. (–5, 405)
Accept (–3, 405) or e.g. $P=405$

(b) **Mark parts (a) and (b) together.**

M1: Attempts to make *t* the subject of $x=\dfrac{t-1}{2}$ using the correct **order** of operations and substitutes into $y=5\left(t+2\right)^4$ Condone slips e.g. dividing by 2 first: $t=\dfrac{x}{2}+1$ or missing the +2 in $y=5\left(t+2\right)^4$

Alt 1: Writes $t=\left(\dfrac{y}{5}\right)^{\frac{1}{4}}-2$, substitutes into $x=\dfrac{t-1}{2}$ and attempts to make *y* the subject using the correct **order** of operations, i.e., $2x=\left(\dfrac{y}{5}\right)^{\frac{1}{4}}-3\Rightarrow\left(2x+3\right)^4=\dfrac{y}{5}\Rightarrow y=5\left(2x+3\right)^4$

Alt 2: Makes *t* the subject and then substitutes into an expanded $5\left(t+2\right)^4$

Do not be too concerned about their expansion, but it must contain t^4 and a constant term.

A1: cao and ISW after a correct answer seen. May be scored for $y=5\left(2x+1+2\right)^4$

Allow e.g. $y=80x^4+480x^3+1080x^2+1080x+405$ or e.g.

$y=5\left(2x+1\right)^4+40\left(2x+1\right)^3+120\left(2x+1\right)^2+160\left(2x+1\right)+80$

Their RHS may be unsimplified but do not allow if e.g. binomial coefficients are still present.
Do not accept f (*x*) = … It must be *y* = …

(c)	**Note: Differentiation seen in (a) or (b) can score marks if used in (c)**
M1:	Reduces the power of their $("2x+3")^n$ by one to $Q("2x+3")^{n-1}$ where Q is a constant and could be 1. There should be no other terms using this method. Alternatively, attempts to expand their $("2x+3")^n$ (may have been expanded in (b)) and reduces the power of x by one in at least one term. They may use parametric differentiation, i.e., $\frac{\mathrm{d}y}{\mathrm{d}t}=a(t+2)^3$ and $\frac{\mathrm{d}x}{\mathrm{d}t}=b$ where a and b are constants, leading to $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{a(t+2)^3}{b}$ i.e. they must divide the correct way round. Condone attempts at the chain rule that reach e.g. $y'=\ldots u^3$ but make a slip when substituting back in for x or t, e.g., $\frac{\mathrm{d}y}{\mathrm{d}x}=(5x+3)^3$ or $\frac{\mathrm{d}y}{\mathrm{d}t}=(t+2)^2$, provided the intention is clear.
dM1:	Substitutes $x=-3$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x which may be implied by their answer. Using parametric differentiation, substitutes their t (found from an attempt at substituting $x=-3$ into $x=\frac{t-1}{2}$ which may have been seen in (a)) into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. Note for reference, if correct, the parametric differentiation is $\frac{\mathrm{d}y}{\mathrm{d}t}=20(t+2)^3$ and $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{1}{2}$ leading to $\frac{\mathrm{d}y}{\mathrm{d}x}=40(t+2)^3$ They may substitute their value of t into $\frac{\mathrm{d}y}{\mathrm{d}t}$ first before using the chain rule to reach $\frac{\mathrm{d}y}{\mathrm{d}x}$ which is acceptable and implies the first M mark.
A1:	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)-1080$ Correct answer only scores full marks. May be seen labelled as m or something else.

Question	Scheme	Marks	AOs
6(a)	Attempts $\cos^2(2x) \approx \left(1-\frac{(2x)^2}{2}\right)^2 = ...\left(1-4x^2+4x^4\right)$	M1	1.1b
	$1-\cos^2(2x) \approx 1-\left(1-4x^2+4x^4\right) = 4x^2-4x^4$ *	A1*	1.1b
		(2)	
(b)	$\dfrac{1-\cos^2(2x)}{\sin\left(\frac{x}{3}\right)\tan\left(\frac{x}{2}\right)} \approx \dfrac{4x^2-4x^4}{\left(\frac{x}{3}\right)\left(\frac{x}{2}\right)}$	M1	1.1b
	$= \frac{6}{x^2}\left(4x^2-4x^4\right) = 24-24x^2$	A1	2.1
		(2)	
(c)	24	B1ft	2.2a
	If x is (very) small, then any terms in x^2 are negligible.	dB1ft	2.4
		(2)	
			(6 marks)

Notes

(a)

M1: Attempts to use $\cos\theta \approx 1-\frac{\theta^2}{2}$ with θ, 2θ, x or $2x$ and squares the resulting expression.

Condone poor squaring, e.g., $\left(1-2x^2\right)^2 = 1-4x^4$ or missing brackets $\left(1-\frac{2x^2}{2}\right)^2 = \left(1-x^2\right)^2 = ...$

but not e.g. $\cos 2x \approx 2\left(1-\frac{x^2}{2}\right)$

May be implied by e.g. $1-\left(1-2x^2\right)^2 = 2x^2\left(2-2x^2\right)$ from difference of two squares.

A1*: Correct proof with an intermediate line such as $1-\left(1-4x^2+4x^4\right)$

Do not be concerned if the LHS does not appear and ignore any spurious = 0 in their work.

There should be no obvious incorrect statements in the proof e.g. $1-\left(1+4x^2-4x^4\right)$

Do not condone incorrect work including invisible brackets such as $1-1-4x^2+4x^4$ or $\left(1-\frac{2x^2}{2}\right)$ but condone a missing trailing bracket e.g. $1-(1-4x^2+4x^4$

Condone an attempt that starts in stages, which may not deal with the full expression e.g., $1-\left(1-\frac{4x^2}{2}\right) \to 1-\left(1-2x^2\right)^2 = ...$ has a missing square on the first bracket but is recovered in the next stage/step.

The final line should be in terms of x but condone a slip to e.g. θ in the workings of the proof.

If they complete the proof using θ they must revert to x to score this mark.

Special Cases: In each of these cases below, please send to review.

You may see attempts that use e.g.

- $\cos 2x = \pm 2\cos^2 x \pm 1$ or $\cos^2 2x = \pm\frac{1}{2} \pm \frac{1}{2}\cos 4x$
- $\cos 2x = \pm 1 \pm 2\sin^2 x$ followed by the small angle approximation for $\sin x$
- Maclaurin expansions

(b)

M1: Uses the **given answer** to part (a) and writes $\sin\left(\frac{x}{3}\right)$ **and** $\tan\left(\frac{x}{2}\right)$ using correct small angle approximations. Only allow misreads that are clearly misreads e.g. they cannot replace $\tan\left(\frac{x}{2}\right)$ with $\frac{x}{3}$ unless $\tan\left(\frac{x}{3}\right)$ is seen first. In such cases they will lose the A mark in (b) but both of the marks in (c) are available. Condone mixed variables for this mark.

A1: $24-24x^2$ but condone e.g. $24+-24x^2$ or e.g. $a = 24$ and $b = -24$
Condone recovery of missing/invisible brackets but the work must otherwise be correct.
Do not condone mixed variables being recovered unless explicitly replaced with x before cancelling x^2.

No marks are scored in (b) for using the Maclaurin expansions for $\sin\left(\frac{x}{3}\right)$ and/or $\tan\left(\frac{x}{2}\right)$

(c)

B1ft: 24 but this must follow from the non-zero constant term in their answer to (b).
Do not allow e.g. $x = 24$
Allow follow through on their non-zero constant term from a polynomial in x.

dB1ft: Suitable reason given but it should refer to their x^2 term in some way (and any additional terms if they have any). Ignore spurious remarks e.g. "if $x < 1$" unless contradictory.
Dependent on the previous B1ft mark.
Some acceptable examples:

- $\text{"}24\text{"}x^2 \to 0$ or $x^2 \to 0$
- "We can ignore the x^2 term"
- "x^2 is much smaller than 24"
- As $x \to 0$ their $a + bx^2 \to a$ (condone as $x \to 0$, bx^2 "becomes" 0)
- $\lim\limits_{x\to 0} a+bx^2 = \left(a+b(0)^2\right) = a$
- Since x is very small, $24 - 24(0)^2 = 24$ (and condone if the squared is missing).

They cannot *just* substitute in 0 or a very small value for x to score the mark for the reason.
A reason such as "the answer rounds to 24" is not acceptable.
There must be some justification, either "as $x \to 0$" or "since x is very small"
so e.g. "$24 - 24(0)^2 = 24$ on its own scores B1ft dB0ft
This mark may be scored from any polynomial in x that includes at least a constant term and an x^2 term.

They cannot go back to e.g. $4x^4$ or $\dfrac{4x^4}{\frac{1}{6}x^2}$ and say this is negligible or $\to 0$ as they haven't dealt with the x in the denominator.

Question	Scheme	Marks	AOs
7	Any one of $A=3,\ B=-2,\ C=4,\ D=-5$ correct from correct work	B1	2.2a
	Finds the value of B, C or D using a valid method e.g. $3x^3-8x^2-6x-11=(Ax+B)(x+1)(x-3)+C(x-3)+D(x+1)$ **and** substitutes $x=-1\Rightarrow\ldots=\ldots C\Rightarrow C=\ldots$ **or** substitutes $x=3\Rightarrow\ldots=\ldots D\Rightarrow D=\ldots$	M1	1.1b
	Valid method for finding all four constants. e.g. Deduces A = 3, substitutes x = 3 leading to D = … **and** substitutes x = –1 leading to C = … **and** substitutes e.g. x = 0 leading to B = …	M1 (A1 on EPEN)	1.1b
	$A=3$, $B=-2$, $C=4$ and $D=-5$	A1	2.1
		(4)	
			(4 marks)

Notes

Note: Condone different uses of 'A', 'B' etc for the M marks provided the intention is clear but values must be correctly attributed to each letter or implied (if embedded in a final expression) for the A mark.

B1: Any one of $A=3,\ B=-2,\ C=4,\ D=-5$ correct (usually A = 3 is deduced) from correct work. May be implied by $3x$ seen. Condone $A = 3x$ for this mark.

M1: This mark is for finding the value of B, C or D, using a valid method, which may be in one of the following ways:

- Attempts algebraic division of $(3x^3-8x^2-6x-11)$ by their expanded $(x+1)(x-3)=("x^2-2x-3")$ **or** separately by both $(x+1)$ **and** $(x-3)$ as far as $\mathbf{3x\pm 2}$ Ignore any remainder for this mark. For reference, the correct remainder is $-x-17$

- Algebraic division might be seen as $3x(x^2-2x-3)\pm 2(x^2-2x-3)+"9x-4x-6-6x-11"$ o.e. and scores the B1 for the $3x$ in the correct place and M1 for both the $3x$ and the ± 2 in the correct places.

  In the next two bullets, they should achieve $3x^3-8x^2-6x-11=(Ax+B)(x+1)(x-3)+C(x-3)+D(x+1)$ o.e. but condone small slips e.g. sign errors or a slip on a constant. The bracketed expressions should be in the correct places and Ax and B should both be multiplied by $(x+1)(x-3)$.
  Note that $3x^3-8x^2-6x-11=C(x-3)+D(x+1)$ without sight of the identity above (condoning the previously mentioned slips) is an incorrect method and scores M0.

- Attempts to multiply up by $(x+1)(x-3)$ and substitutes one of $x=-1$ **or** $x=3$ leading to $C=\ldots$ **or** $D=\ldots$ (you do not need to check the accuracy of their substitution)

- **or** attempts to multiply up by $(x+1)(x-3)$, expands and compares coefficients leading to a value for B, C or D. Do not be concerned about how they expand the brackets.

- Attempts at the cover up method may be seen. If unsure then send to review.

M1: For finding values of all four constants via a valid method.
Not dependent on the previous method mark so B1M0M1A0 is possible.

- If attempting algebraic division, they must have a quotient of $3x \pm k$ for constant k ($3x \pm 2$ is **not** required for this mark) **and** they must go on to deal with their remainder using a correct method, i.e., $"-x-17" = C(x-3) + D(x+1)$ followed by either comparing coefficients or substitution of appropriate values. You do not need to check the accuracy of their substitution.

- For those attempting repeated algebraic division, the values of C and D must come from a correct method and so cannot be equal to e.g. the remainders from successive division by $(x+1)$ and $(x-3)$ or vice versa. If, e.g., they divide by $(x+1)$ (to get remainder R_1) and then $(x-3)$ (to get remainder R_2) they should get $3x \pm k + \frac{R_1}{(x+1)(x-3)} + \frac{R_2}{(x-3)}$. They can then either combine the two fraction terms before using partial fractions or they can use partial fractions on $\frac{R_1}{(x+1)(x-3)}$ before combining their two terms in $\frac{1}{(x-3)}$.

- If using $3x^3 - 8x^2 - 6x - 11 = (Ax+B)(x+1)(x-3) + C(x-3) + D(x+1)$ o.e. they may use a combination of substitution and comparing coefficients, but the method must be valid. Do not be concerned about how they expand the brackets, and you do not need to check the accuracy of their substitution.

- If using substitution only they are likely to use $x = -1$, 3 and 0 but the final value for x could be anything (and in fact they could use three/four different values for x). You do not need to check the accuracy of their substitution.

- If attempting to compare coefficients, do not be concerned about how they expand the brackets.

A1: Requires:

- all of $A = 3$, $B = -2$, $C = 4$ and $D = -5$ which may be stated separately in their work ($A = 3x$ that is not recovered would score final A0).
- **or** $3x - 2 + \frac{4}{x+1} - \frac{5}{x-3}$ seen. Condone $3x + -2 + \frac{4}{x+1} + \frac{-5}{x-3}$

There is no requirement to write out the form given in the question once correct values for A, B, C and D have been seen.

ISW after the four correct values (e.g. if they swap C and D in the fractions so $A = 3,\ B = -2,\ C = 4,\ D = -5$ followed by $3x - 2 - \frac{5}{x+1} + \frac{4}{x-3}$ scores full marks).

Repeated Algebraic Division

An example for repeated algebraic division. Note that the repeated division could be carried out in the other order – i.e. dividing by $(x - 3)$ first followed by $(x + 1)$.

$$\begin{array}{r|l}
 & 3x^2 - 11x + 5 \\
\hline
x+1 & 3x^3 - 8x^2 - 6x - 11 \\
 & 3x^3 + 3x^2 \\
\hline
 & -11x^2 - 6x \\
 & -11x^2 - 11x \\
\hline
 & 5x - 11 \\
 & 5x + 5 \\
\hline
 & -16
\end{array}$$

Followed by

$$\begin{array}{r|l}
 & 3x - 2 \\
\hline
x-3 & 3x^2 - 11x + 5 \\
 & 3x^2 - 9x \\
\hline
 & -2x + 5 \\
 & -2x + 6 \\
\hline
 & -1
\end{array}$$

B1 Scored for either '3' **or** '–2' embedded in the quotient.

1[st] **M1** Scored for achieving $3x \pm 2$

$$\frac{3x^3 - 8x^2 - 6x - 11}{(x+1)(x-3)} = 3x - 2 + \frac{-16}{(x+1)(x-3)} + \frac{-1}{(x-3)}$$

Correctly deals with both remainders from the repeated division.

$$\frac{3x^3 - 8x^2 - 6x - 11}{(x+1)(x-3)} = 3x - 2 + \frac{-16}{(x+1)(x-3)} + \frac{-(x+1)}{(x+1)(x-3)}$$

$$\frac{3x^3 - 8x^2 - 6x - 11}{(x+1)(x-3)} = 3x - 2 + \frac{-x-17}{(x+1)(x-3)}$$

$-x - 17 = C(x - 3) + D(x + 1)$

Sets up an identity in x and C and D using partial fractions.

$x = 3 \Rightarrow D = -5$ and $x = -1 \Rightarrow C = 4$

2[nd] **M1** Finds the values of all four constants via a correct method (in this case by substitution of appropriate values).

$\boxed{A = 3, B = -2, C = 4. D = -5}$

A1 $A = 3, B = -2. C = 4$ and $D = -5$

$$\frac{3x^3 - 8x^2 - 6x - 11}{(x+1)(x-3)} = 3x - 2 + \frac{F}{(x+1)} + \frac{G}{(x-3)} + \frac{-1}{(x-3)}$$

$-16 = G(x + 1) + F(x - 3)$

Alternatively, splits the first fraction using partial fractions and sets this equal to their remainder divided by $(x + 1)(x - 3)$ and multiplies up correctly.

$x = -1 \Rightarrow F = 4,\ x = 3 \Rightarrow G = -4$

2[nd] **M1** Finds the values of all four constants via a valid method.

$$\frac{3x^3 - 8x^2 - 6x - 11}{(x+1)(x-3)} = 3x - 2 + \frac{4}{(x+1)} + \frac{-4}{(x-3)} + \frac{-1}{(x-3)}$$

$$\boxed{\frac{3x^3 - 8x^2 - 6x - 11}{(x+1)(x-3)} = 3x - 2 + \frac{4}{(x+1)} - \frac{5}{(x-3)}}$$

A1 $A = 3, B = -2$, $C = 4$ and $D = -5$ seen embedded in a final expression.

Question	Scheme	Marks	AOs
8(a)	$x^4 \to \ldots x^3$ or $\ldots x^3 \to \ldots x^2$ or $\ldots x^2 \to \ldots x$ or $ax \to a$	M1	1.1b
	$\left(f'(x)=\right)\ 4x^3+x^2-16x+a$	A1	1.1b
	$\left(f'\left(-\frac{1}{4}\right)=\right)\ 4\left(-\frac{1}{4}\right)^3+\left(\pm\frac{1}{4}\right)^2-16\left(-\frac{1}{4}\right)+a=0$ $\Rightarrow a=\ldots$	dM1	3.1a
	$a=-4$ *	A1*	2.1
		(4)	
(b)	$\frac{4735}{768}$	B1	1.1b
		(1)	

Notes

(a) The first two marks are the same for all approaches.

M1: Reduces the power by one for any term in $f(x)$. Look for $x^n \to x^{n-1}$ and allow for $ax \to a$ including $ax^1 \to ax^0$ or $-4x \to -4$

A1: Correct differentiation $\left(f'(x)=\right)\ 4x^3+x^2-16x+a$ or $4x^3+x^2-16x-4$
Ignore the absence of the LHS. May be in terms of *a* or have –4 substituted at this point as above.

Main Scheme:

dM1: Substitutes $x=-\frac{1}{4}$ into their $f'(x)$, sets = 0 (= 0 may be implied) and solves for *a*.
Condone $4+a=0$ as evidence of substitution for the dM1 only.
This mark is not available if their $f'(x)$ does not have a term with *a* in it using this approach.

A1*: cso Requires:

- a correct derivative
- correct substitution of $x=-\frac{1}{4}$ into $f'(x)$ and set = 0 (now the = 0 must be seen somewhere for this mark). Each term does not need to be evaluated.
- achieves $a = -4$

For the A1*, evidence of correct substitution might be $-\frac{1}{16}+\frac{1}{16}+4+a=0$ but not $4+a=0$

Condone invisible brackets around $-\frac{1}{4}$ provided this is recovered before the given answer e.g. $-\frac{1}{4}^2$ recovered to $+\frac{1}{16}$

Alt 1: Verification

dM1: Substitutes $x=-\frac{1}{4}$ and $a = -4$ into their $f'(x)$, and finds a value.
The $a = -4$ may already have been substituted into f (x) as above.

A1*: cso Requires:

- a correct derivative

- correct substitution of $x=-\frac{1}{4}$ into $\mathrm{f}'(x)$ and use of a = –4 (at some stage) to achieve $\mathrm{f}'\left(-\frac{1}{4}\right)=...=0$. The substitution must be embedded in $\mathrm{f}'(x)$ **or** evaluated in an intermediate step e.g. $-\frac{1}{16}+\frac{1}{16}+4-4$ before the = 0
- a minimal conclusion e.g. "hence proven" or "∴ a = –4" or e.g. a tick.
  but **not** e.g. "max = $-\frac{1}{4}$" or just "0 = 0"

Alt 2: Algebraic division

dM1: Attempts to divide their $\mathrm{f}'(x)$ by $(4x+1)$ $\left(\text{or}\left(x+\frac{1}{4}\right)\right)$, achieves a linear remainder in a only, sets = 0 and achieves a value for a. Condone slips in their calculations.
As a minimum, expect to see $x^2+\lambda x+\mu$ $\left(\text{or } 4x^2+\lambda x+\mu\right)$ as their quotient (with $\mu\neq 0$) leading to a linear remainder in a only set = 0 leading to a value for a.

A1*: cso Requires:
- a correct derivative
- correct quotient x^2-4 $\left(\text{or}\left(4x^2-16\right)\right)$ and correct remainder $a+4$ set = 0
- achieves a = –4

(b)

B1: $\frac{4735}{768}$ cao Allow exact equivalents e.g. $6\frac{127}{768}$ or the recurring decimal $6.16536458\dot{3}$
There is no need to see a calculation. Decimal approximations e.g. $6.1653645833...$ score B0.

Question	Scheme	Marks	AOs
8(c)	$(f'(x)=)\ 4x^3+x^2-16x-4=(4x+1)(x^2+...)$	M1	3.1a
	$=(4x+1)(x^2-4)$ **or** e.g. $\dfrac{4x^3+x^2-16x-4}{4x+1}=x^2-4$	A1	1.1b
	$f("-2")=...(-5)$ $\Rightarrow "-5"<k<"\dfrac{4735}{768}"$	M1	2.1
	$\left\{k\in\mathbb{R}:-5<k<"\dfrac{4735}{768}"\right\}$	A1ft	2.5
		(4)	
		(9 marks)	

Notes

(c) **Note: Candidates who do not score the first M mark may score maximum M0A0M1A1ft via the special case at the end of the mark scheme for part (c).**

M1: For the key step in using the factor theorem to take $(4x+1)$ or $\left(x+\frac{1}{4}\right)$ out as a factor of their $f'(x)$, **not** $f(x)$. Look for $(4x+1)(x^2+\lambda x-4)$ or $(4x+1)(x^2\pm\mu)$ or $\left(x+\frac{1}{4}\right)(4x^2\pm\mu)$ or $\left(x+\frac{1}{4}\right)(4x^2\pm\lambda x-16)$ (where the λ or μ is a constant) either by inspection or division.

If they attempt division in (a) then they must use their quotient in (c) to score the marks.
They may "spot" that one of $f'(\pm2)=0$ and factor out $(x\pm2)(4x^2+\lambda x\pm2)$ without evidence that $f'(\pm2)=0$
There must be some factorisation or algebraic division present – not just roots stated from a calculator.

A1: Correct factorisation of the cubic to a linear and quadratic product, so $(4x+1)(x^2-4)$ or $\left(x+\frac{1}{4}\right)(4x^2-16)$ or $(x-2)(4x^2+9x+2)$ or $(x+2)(4x^2-7x-2)$
or for the correct quadratic seen e.g. x^2-4 or $4x^2-16$ (which may be the quotient in their algebraic division and may come from their work in (a) provided it is used in (c)).
There is no need to complete the long division if the appropriate quotient is found.
Note that proceeding directly to $(4x+1)(x-2)(x+2)$ does not score either mark without sight of the e.g. $(4x+1)(x^2-4)$ but they can score the final 2 marks via the special case.
Similarly, proceeding directly to $\left(x+\frac{1}{4}\right)(2x-4)(2x+4)$ or $4\left(x+\frac{1}{4}\right)(x-2)(x+2)$ does not score either mark without sight of $\left(x+\frac{1}{4}\right)(4x^2-16)$ but they can score marks via the SC.

M1: Solves their quadratic factor = 0, substitutes one of these solutions (not $x=-\frac{1}{4}$) into $f(x)$ to find a value **and** selects the inside region between this and their answer to (b).
If their quadratic is of the form Ax^2-B then they can write down their solution for x but if it is a 3TQ then they must find their value of x using the usual non-calculator rules.
Condone "incorrect" factorisation following the correct quadratic leading to $x=\pm2$

e.g. $\left(x+\frac{1}{4}\right)\left(4x^2-16\right)\to\left(x+\frac{1}{4}\right)(x-2)(x+2)=0\to x=\pm 2$

y (or k) = –5 (or $-\frac{47}{3}$) with nothing else can imply substitution of x = –2 (or 2)

Note that if they "spot" that f'(±2) = 0 earlier then they can go straight to substitution.

Condone the use of y but not x in their region. Condone e.g. $k>-5, k<"\frac{4735}{768}"$ for this mark.

Condone the use of ≤ in place of < for this mark. Note that $\left(2,-\frac{47}{3}\right)$ is the other minimum.

A1ft: $\left\{k:-5<k<"\frac{4735}{768}"\right\}$ Requires correct use of set notation but condone absence of $\in\mathbb{R}$

Follow through on their upper limit from (b) which may be a decimal but must be positive.
Must be strict inequalities i.e. not include the end points and must now be in terms of k.

Other acceptable notation includes: $k\in\left(-5,"\frac{4735}{768}"\right)$, $\{k\,,k>-5\}\cap\left\{k\,,k<"\frac{4735}{768}"\right\}$

Condone $\left\{k:k>-5\cap k<"\frac{4735}{768}"\right\}$ or $\left\{-5<k<"\frac{4735}{768}"\right\}$

Do not allow e.g. $\{k:k>-5\}\cup\left\{k:k<"\frac{4735}{768}"\right\}$ or just $-5<k<"\frac{4735}{768}"$

SC: Candidates that have not scored the first M1 may go on to score the final two marks by finding an acceptable range between –5 and $"\frac{4735}{768}"$.

Use of their "–5" or "$-\frac{47}{3}$", coming from x = ±2, (or –5 or $-\frac{47}{3}$ directly from a calculator) with $"\frac{4735}{768}"$ will score M1 if the inside region is selected. A correct $\left\{k:-5<k<"\frac{4735}{768}"\right\}$ will score A1ft

There is no need to see any working to achieve the –5 (or $-\frac{47}{3}$).

All the relevant guidance regarding the region in the M1 and A1ft notes above continues to apply. e.g. for the M1 we will condone the use of y but not x in their region while for the A1ft we require use of k.

For example:

- $\left\{k:-5<k<"\frac{4735}{768}"\right\}$ without scoring the first M scores M0A0M1A1ft
- $"-5"<k<"\frac{4735}{768}"$ without scoring the first M scores M0A0M1A0ft
- $\left\{k:"-\frac{47}{3}"<k<"\frac{4735}{768}"\right\}$ without scoring the first M scores M0A0M1A0ft

Question	Scheme	Marks	AOs
9(a)	$5000-5000\mathrm{e}^{-0.075\times 3}=\ldots$	M1	3.4
	1007	A1	1.1b
		(2)	
(b)	$5000-5000\mathrm{e}^{-0.075T}=3000\rightarrow 5000\mathrm{e}^{-0.075T}=2000$	M1	1.1b
	$\Rightarrow T=\dfrac{1}{-0.075}\ln\dfrac{2000}{5000}$	dM1	1.1b
	$\Rightarrow T=12.22$	A1	1.1b
		(3)	
(c)	$\left(\dfrac{\mathrm{d}N}{\mathrm{d}t}=\right)\ -5000\times -0.075\mathrm{e}^{-0.075\times 3}=\ldots$	M1	3.4
	299 (car sales per month)	A1	1.1b
		(2)	
(d)	Change the constant 5000 to 6500.	B1	3.5c
		(1)	
		(8 marks)	

Notes

(a)

M1: Substitutes $t=3$ into the given model and proceeds to a value.
Implied by awrt 1007, 1008 or 1010
Condone copying slips e.g. –0.75 or –0.0075 in place of –0.075 or 500 in place of 5000 – in such cases you may need to check their calculation if the substitution isn't shown.

A1: 1007 but allow 1008 or 1010 (3sf). Must be a whole number.
Correct answer only scores both marks. A0 for e.g. 1007.4
Do not ISW if they go on to sum their values of N from $t=1$ to 3

(b)

M1: Sets $5000-5000\mathrm{e}^{-0.075T}$ equal to 3000 and proceeds to either $A\mathrm{e}^{-0.075T}=B$ or $\mathrm{e}^{-0.075T}=k$ where A, B, k are constants with no restrictions for this mark.
May use t instead of T. Condone slips.
Condone copying slips e.g. –0.75 or –0.0075 in place of –0.075 or 500 in place of 5000.

dM1: Uses the correct order of operations and correct log work from an equation of the form $A\mathrm{e}^{-0.075T}=B$ with $AB>0$ **or** $\mathrm{e}^{-0.075T}=k$ with $k>0$ to proceed to a value for T or t which may be a numerical expression.

e.g. $A\mathrm{e}^{-0.075T}=B \Rightarrow \ln A-0.075T=\ln B \Rightarrow T=\dfrac{\ln A-\ln B}{0.075}$ or $=\dfrac{\ln\left(A/B\right)}{0.075}$

Condone log being used in place of ln unless there is clear evidence that they have used an inconsistent base.

May not be scored if "recovered" from e.g. $\mathrm{e}^{-0.075T}=-\dfrac{2}{5}$

Condone copying slips e.g. –0.75 or –0.0075 in place of –0.075 or 500 in place of 5000.

A1: 12.22 cao but must see a correct equation following use of logs, e.g., $-0.075T=\ln\dfrac{2}{5}$ or

$T=\dfrac{1}{-0.075}\times -0.916$ which may have intermediate rounding. Ignore any units given.
This mark is available following the occasional slip in writing –0.075 as –0.75 or –0.0075 provided it is not consistently used.

(c)	
M1:	Differentiates once to the form $\lambda e^{-0.075t}$, λ a constant (even ±5000), and substitutes in $t = 3$ Award for an expression of the form $ke^{-0.075\times 3}$ with constant k if no incorrect working is seen. The substitution of $t = 3$ may be implied by a correct value for their derivative of the required form, provided the derivative is seen. Do not be concerned with what they call their derivative.
A1:	awrt 299 (car sales per month). Answer only scores no marks. Condone 300 following 299.4 Requires a correct derivative to be seen, e.g., $375e^{-0.075t}$ o.e., with or without the substitution of $t = 3$ present. Units may be omitted, but score A0 if an incorrect time frame is given e.g. cars per week
(d)	
B1:	Acceptable refinement. Some examples: • Change the 5000 to 6500 (condone ambiguity about which 5000) • Change the first 5000 to 6500 • Change both 5000s to 6500s • Multiply the model by 1.3 (or e.g. $\frac{65}{50}$ or $\frac{13}{10}$) • Add 1500 to the model • Change the constant to 6500 The statement of an acceptable refined model e.g. $(N =)\ 6500 - 6500e^{-0.075t}$ or e.g. $(N =)\ 6500 - 5000e^{-0.075t}$ scores B1. Ignore any extra unnecessary refinements such as increase/decrease the 0.075. The following score B0: • Change 5000 in the model (not specific enough – they haven't said to 6500) • Change the second 5000 to 6500 (incorrect)

<table>
<tr><th>Question</th><th colspan="2">Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="3">10(a)</td><td colspan="2">$\frac{\mathrm{d}V}{\mathrm{d}t} = 0.45$ or $\frac{\mathrm{d}V}{\mathrm{d}t} = \pm 0.3V$</td><td>M1</td><td>3.1b</td></tr>
<tr><td colspan="2">$\frac{\mathrm{d}V}{\mathrm{d}t} = 0.45 - \frac{3}{10}V$
$20\frac{\mathrm{d}V}{\mathrm{d}t} = 9 - 6V$ *</td><td>A1*</td><td>2.1</td></tr>
<tr><td colspan="2"></td><td>(2)</td><td></td></tr>
<tr><td rowspan="6">(b)</td><td colspan="2">e.g. $\frac{1}{9-6V}\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{1}{20} \rightarrow \int \frac{1}{9-6V}\mathrm{d}V = \int \frac{1}{20}\mathrm{d}t$</td><td>B1</td><td>1.1b</td></tr>
<tr><td colspan="2">$\frac{1}{9-6V} \rightarrow \ldots \ln\lvert 9-6V \rvert$</td><td>M1</td><td>1.1b</td></tr>
<tr><td colspan="2">$-\frac{1}{6}\ln\lvert 9-6V \rvert = \frac{t}{20} \quad (+c)$</td><td>A1</td><td>1.1b</td></tr>
<tr><td>$-\frac{1}{6}\ln\lvert 9-6V \rvert = \frac{t}{20} + c$
$9 - 6V = A\mathrm{e}^{-\frac{3t}{10}}$
$t = 0,\ V = 0.25 \Rightarrow A = (7.5)$</td><td>$-\frac{1}{6}\ln\lvert 9-6V \rvert = \frac{t}{20} + c$
$t = 0,\ V = 0.25 \Rightarrow c = \left(-\frac{1}{6}\ln 7.5\right)$</td><td>dM1</td><td>3.1a</td></tr>
<tr><td>$9 - 6V = 7.5\mathrm{e}^{-\frac{3t}{10}}$
$V = \frac{3}{2} - \frac{5}{4}\mathrm{e}^{-\frac{3t}{10}}$</td><td>$\frac{1}{6}\ln 7.5 - \frac{1}{6}\ln\lvert 9-6V \rvert = \frac{t}{20}$
$\ln\frac{7.5}{9-6V} = 0.3t$
$9 - 6V = 7.5\mathrm{e}^{-0.3t}$
$V = \frac{3}{2} - \frac{5}{4}\mathrm{e}^{-0.3t}$</td><td>A1</td><td>2.1</td></tr>
<tr><td colspan="2"></td><td>(5)</td><td></td></tr>
</table>

Notes

(a) **Marks for part (a) may not be scored in part (b)**

M1: Either $\frac{\mathrm{d}V}{\mathrm{d}t} = 0.45$ or $\frac{\mathrm{d}V}{\mathrm{d}t} = \pm 0.3V$ o.e. e.g. $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{9}{20}$ or $\frac{\mathrm{d}V}{\mathrm{d}t} = \pm\frac{3}{10}V$ seen or implied by e.g. $\frac{\mathrm{d}V}{\mathrm{d}t} = 0.45 - \frac{3}{10}V$ (but not implied by just stating the given answer). Condone use of $\dot{V}$

It may be seen as part of their $\frac{\mathrm{d}V}{\mathrm{d}t}$ e.g. $\frac{\mathrm{d}V}{\mathrm{d}t} = 0.45 + V + 0.3V$ scores M1A0*

Condone e.g. change in volume = (inflow – outflow =) 0.45 – 0.3V for this mark.

A1*: Achieves $20\frac{\mathrm{d}V}{\mathrm{d}t} = 9 - 6V$ with no errors, following $\frac{\mathrm{d}V}{\mathrm{d}t} = 0.45 - \frac{3}{10}V$ o.e. (including the $\frac{\mathrm{d}V}{\mathrm{d}t}$ or $\dot{V}$ but note that it must be $\frac{\mathrm{d}V}{\mathrm{d}t}$ in the final line and not $\dot{V}$).

change in volume = 0.45 – 0.3V $\rightarrow 20\frac{\mathrm{d}V}{\mathrm{d}t} = 9 - 6V$ scores M1A0*.

Ignore any units used in their working for both marks.

(b)

B1: Separates the variables correctly, e.g., $\int \frac{1}{9-6V}\mathrm{d}V = \int \frac{1}{20}\mathrm{d}t$ or $\int \frac{20}{9-6V}\mathrm{d}V = \int \{1\}\,\mathrm{d}t$ o.e.

The integral symbol and/or dV and/or dt may be implied if they go on to integrate **both** sides to the correct form $\ldots\ln\lvert \alpha(9-6V) \rvert = \ldots t \quad (+c)$ with or without the modulus brackets.

M1:	Attempts to integrate the reciprocal term $\frac{\beta}{9-6V} \to \ldots \ln\lvert 9-6V\rvert$ or $\to \ldots \ln\lvert\alpha(6V-9)\rvert$ for some constant β (and α if used). Condone e.g. $\frac{20}{9-6V} \to \ldots \ln 9-6V$ or $\to \ldots \ln 6V-9$
A1:	Correct integration for both sides. They do not need the + c for this mark. Note scoring this mark implies the earlier B1 (unless it is a verification attempt – see SC). Note that e.g. $-\frac{1}{6}\ln\lvert 3-2V\rvert = \frac{t}{20} \quad (+c)$ or $-\frac{10}{3}\ln\lvert 2V-3\rvert = t \quad (+c)$ are also correct. $-\frac{1}{6}\ln(9-6V) = \frac{t}{20} \quad (+c)$ is also correct. Condone log being used in place of ln.
dM1:	Requires constant of integration now. Substitutes (or states) $t = 0$ and $V = 0.25$ and finds a value for c, which may be "A" = e^c if they rearrange first to eliminate ln terms. Dependent on the previous method mark. Do not be concerned about their processing to find c or "A" = e^c and does not need to be exact.
A1:	Achieves the required form e.g. $V = \frac{3}{2} - \frac{5}{4}e^{-\frac{3t}{10}}$ with no errors and clear working. Allow equivalent fractions or decimals e.g. $V = 1.5 - 1.25e^{-\frac{6}{20}t}$
SC:	**Attempts by verification may score maximum B0M1A1dM1A0 – see below.**
Alt:	**Use of an integrating factor – see below.**
(b)	**Special Case: Attempts by verification may score maximum B0M1A1dM1A0**
B0:	This mark may not be scored via this approach.
M1:	Differentiates $V = P - Qe^{-kt}$ to the form $\frac{dV}{dt} = \alpha e^{-kt}$ where α is a constant (note it should be $\frac{dV}{dt} = Qke^{-kt}$) **and** substitutes both this and $V = P - Qe^{-kt}$ into $20\frac{dV}{dt} = 9 - 6V$ **and** deduces a value for P **or** k by comparing coefficients.
A1:	Correct values for both P **and** k.
dM1:	Substitutes (or states) $t = 0$ and $V = 0.25$ and finds a value for Q. Requires a value for P to have been found using the above approach.
A0:	This mark may not be scored via this approach.
	Alternative: Using Integrating Factor (Further Maths)
B1:	Deduces the correct integrating factor for the equation, $e^{0.3t}$ This should come from $\frac{dV}{dt} + 0.3V = 0.45 \Rightarrow \text{I.F.} = e^{\int 0.3\,dt} = e^{0.3t}$ May be implied by sight of $\frac{d\left(Ve^{0.3t}\right)}{dt} = \ldots$
M1:	Fully multiplies through by their integrating factor and integrates both sides. Score for $Ve^{kt} = \int \ldots e^{kt}\,dt = \ldots e^{kt}$ Condone missing dt
A1:	Correct integration $Ve^{0.3t} = \int 0.45e^{0.3t}\,dt = \frac{3}{2}e^{0.3t} \; (+c)$
dM1:	As main scheme.
A1:	As main scheme.

Question	Scheme	Marks	AOs
10(c)	**Examples:** (1) $\frac{dV}{dt}=0 \Rightarrow V=(1.5)$ (or e.g. max V is 1.5) (2) As $t \to \infty$, $e^{-"0.3"t} \to 0$ (or $V \to$ "1.5") (3) Flow in = flow out at max V so $0.3V = 0.45 \Rightarrow V = 1.5$ (4) As $e^{-"0.3"t} > 0$, $V < "1.5"$ (5) When $V > 1.5$, $\frac{dV}{dt} < 0$ (6) $V = 2 \Rightarrow \frac{dV}{dt} = -0.15$ **or** compares $\frac{dV_{out}}{dt}$ $(=0.6)$ against $\frac{dV_{in}}{dt}$ $(=0.45)$ at $V = 2$ (7) $V = 2 \Rightarrow "1.5" - "1.25" e^{-"0.3"t} = 2 \Rightarrow e^{-"0.3"t} < 0$ (8) $"1.5" - "1.25" e^{-"0.3"t} = 2 \Rightarrow$ ln(–0.4) is undefined (condone e.g. gives a maths error)	M1	3.2a
	• The (upper) limit for V is 1.5 (m^3) so no (the container will not become full) (first 4 bullets) • If V = 2 (or V > 1.5), it would be emptying so no (it can never be full) (bullets 5, 6) • No, as the equation cannot be solved (or is not true/doesn't work) when V = 2 (bullets 7, 8)	A1ft	2.4
		(2)	
			(9 marks)

Notes

(c)

M1: See main scheme. If using the answer to part (b) it must be of the form $V = P - Qe^{-kt}$ but there is no limitation on the values of their P, Q or k.
Substitution of a large value for t may score this mark but it is unlikely to be recovered to score the A1 unless they reference e.g. V_{max} being "1.5".
Reference to an (upper) limit of "1.5" or their P can imply the method mark.
If setting V = 2 in their equation they must reach either ln(–ve) **or** solve the equation to reach a value for t to score this mark.

A1ft: Must conclude "no" or equivalent e.g. "the container will not become full".
Makes a correct interpretation for their method (see bullets 1-8) with a clear conclusion e.g. "no".
To score this mark through ft, their V must be of the form $V = P - Qe^{-kt}$ with $k > 0$, $Q > 0$ **and** $0 < P < 2$ if used (but note that they can still use the answer to part (a) to score both marks via bullets 1, 3, 5 or 6). Allow "it" in place of the "container"/"tank".
Just stating "the equation cannot be solved when V = 2" without any evidence is M0A0.
There must be no incorrect working if solving their equation or contradictory statements such as "t cannot be negative" but condone notational errors provided the intention is clear.

Question	Scheme	Marks	AOs
11(a)	$\frac{300-1500}{50-30}=(-60)$	M1	3.1b
	$y-1500="-60"(x-30)$	dM1	1.1b
	$y=3300-60x$	A1	3.3
		(3)	
(b)	Either $x("3300-60x")$ or $-10("3300-60x")$ seen	M1	1.1b
	$(P=)\ \frac{x("3300-60x")-10("3300-60x")-8000}{1000}$	dM1	3.3
	$P=3.3x-0.06x^2-33+0.6x-8$ $P=-0.06x^2+3.9x-41$ *	A1*	2.1
		(3)	
(c)	$\frac{-3.9\pm\sqrt{3.9^2-4(-0.06)(-41)}}{2\times-0.06}$	M1	3.1b
	$13.18<x<51.82$ (or $13.19\le x\le 51.81$)	A1	3.2a
		(2)	
(d)	£32.50	B1ft	3.4
		(1)	
(e)	$(P=)\ -0.06(35)^2+3.9(35)-41=...$	M1	3.4
	£22 000 which is close to £21 750 so the model is suitable.	A1	3.5a
		(2)	
			(11 marks)

Notes

(a)

M1: Attempts the gradient of the straight line either way round.

Look for $\frac{300-1500}{50-30}=(-60)$ o.e. or $\frac{50-30}{300-1500}=\left(-\frac{1}{60}\right)$ o.e.

Condone 1 sign/copying slip only if a correct formula is seen or implied e.g. $\frac{y_2-y_1}{x_2-x_1}$

Alternatively, sets up simultaneous equations $1500=30a+b$ and $300=50a+b$ (or $30=1500a+b$ and $50=300a+b$) and attempts to solve, leading to a value for *a* and/or *b*. If using simultaneous equations, condone slips.

dM1: Attempts the equation relating *x* and *y* with either coordinate (50, 300) or (30, 1500) used correctly. Look for $y-1500="-60"(x-30)$ or $x-30="-\frac{1}{60}"(y-1500)$ or using (50, 300). Note that e.g. $y-30="-60"(x-1500)$ scores dM0 as the values have been used incorrectly. If using $y=mx+c$ they must proceed to $c=\ldots$

If using simultaneous equations, they must proceed to values for *a* **and** *b*

(Note for reference *x* in terms of *y* is $x=-\frac{1}{60}y+55$)

If they use e.g. $\frac{y-300}{1500-300}=\frac{x-50}{30-50}$ both M's can be scored together.

Condone 1 sign/copying slip only if a correct formula is seen or implied.

A1: cao Either $y=3300-60x$ or $y=-60x+3300$ or equivalent with *y* in terms of *x*

The correct equation with no working scores full marks.

(b)

M1: Attempts either $\pm x("3300-60x")$ or $\pm 10("3300-60x")$ with or without dividing by 1000.

May see both in one step, i.e., $\pm(x\pm 10)("3300-60x")$

Alternatively, allow $(P=)\ \pm xy\pm 10y\pm 8000$ with or without dividing by 1000.

Allow their model for y to be any expression in x (i.e. it does not need to be linear) for M1.

dM1: Full attempt to find P in terms of x only. There is no need to see a LHS for this mark.

$(P=)\ \dfrac{x("3300-60x")-10("3300-60x")-8000}{1000}$ or $(P=)\ \dfrac{(x-10)("3300-60x")-8000}{1000}$

Condone the consistent absence of division by 1000 e.g.

$(P=)\ x("3300-60x")-10("3300-60x")-8000$ or $(P=)\ (x-10)("3300-60x")-8000$

Condone invisible brackets if recovered. Dependent on the previous method mark.

Their model for y must now be a linear expression in x for dM1.

A1*: Achieves the given answer through rigorous argument with all elements pulled together clearly. Requires P = (not just "profit = ") at some point which may be on a previous line. Any brackets seen must have been expanded in an intermediate line before the given answer.

Allow otherwise correct work leading to $P=-60x^2+3900x-41000$ followed by $P=-0.06x^2+3.9x-41$ (without justifying the division by 1000).

Allow recovery if they write e.g. 800 but it is recovered before the given answer.

Note: Attempts by verification are unlikely to score any marks. If unsure, send to review.

e.g. $x=10\rightarrow P=-0.06(10)^2+3.9(10)-41=-8$ without an accompanying argument involving the (x, y) values (50, 300) and (30, 1500) scores M0dM0A0*.

(c)

M1: Attempts to find either end point of the interval which may come from a calculator.

The exact values are $\dfrac{195\pm 5\sqrt{537}}{6}$ but may just be seen in the quadratic formula, e.g.,

$\dfrac{-3.9\pm\sqrt{3.9^2-4(-0.06)(-41)}}{2\times -0.06}$ or using completing the square – see general marking principles.

Accept awrt 13.2 or awrt 51.8 for this mark.

A1: Either $13.18<x<51.82$ or $13.19\le x\le 51.81$ No units are required but 2d.p. are required.

Correct answer scores both marks. Allow e.g. $13.18<x\le 51.81$ or $13.19\le x<51.82$

Allow e.g. "$x>13.18$ and $x<51.82$" or "$x>13.18\cap x<51.82$" but not use of exact values and not "$x>13.18$, $x<51.82$" or "$x>13.18\cup x<51.82$" or "$x>13.18$ or $x<51.82$"

Must be x or e.g. "selling price" for the range, not y or P.

(d)

B1ft: £32.50. Units and 2dp required. Not £32.50p. Ignore attempts to find maximum profit.

Allow FT on the mean of their end points from (c) (their end points must be > 0, their answer requires £ and 2dp).

May come directly from the given answer to (b) using $x=-\dfrac{b}{2a}=-\dfrac{3.9}{2\times -0.06}=£32.50$

or from $\dfrac{\mathrm{d}P}{\mathrm{d}x}=-0.12x+3.9=0$ or directly from a calculator. No working required.

Note: Following use of 13.2 and/or 51.8 (not e.g. 51.80) with either type of inequality in part (c), £32.5 will score B1ft so that the missing decimal place(s) is only penalised once.

(e)

M1: Attempts to substitute 35 (and not any other value) into the given equation for P to find a value for P.
May be implied by 22 or 22000. Condone slips.
Alternatively, substitutes $P = 21.75$ into the model, and solves via any acceptable method, including by calculator, to find a value for x. If using 21750 it must be substituted into their $(P =)\ -60x^2 + 3900x - 41000$. Note $x = 35.73$ or 29.27

A1: Requires:

- A correct value. Usually (£)22 000 but accept, e.g., 22 thousand (pound). If using 22 they must compare with 21.75. The alternative requires x = awrt 35.7 to be compared to 35.
- A correct comparison. Some examples below:
  - "Close to (£)21 750" o.e. e.g. "the model predicts 22000 and they make close to this amount"
  - "Agree to 2sf". Do not condone "it rounds to 22 000" without mention of the degree of accuracy.
  - 1.1% error (accept approx. 1%) Do not be concerned about the mechanics of any percentage error calculation seen.
  - "The difference is small"
  - Condone e.g. $22 \approx 21.75$ o.e.
  - "**Only** £250 off". However, just stating "£250 off" without suggesting this is a (relatively) small difference is not sufficient for this component. Similarly, "£350 off is a small difference" is incorrect (requires correct £250).
- A conclusion. e.g. Concludes it (the model) is suitable / reliable / good / fairly accurate / accurate.

Question	Scheme	Marks	AOs
12(a)	$4\|1-3\|-5=...$ or $4\|-1-3\|-5=...$	M1	1.1b
	3 and 11	A1	1.1b
		(2)	
(b)	$2(-5)+17=...$	M1	1.1b
	$\mathrm{gf}(x)\geq 7$	A1	2.2a
		(2)	
(c)	$k\ldots-4$	B1	2.2a
	Minimum at $(3,-5)$ so $k\ldots-\dfrac{5}{3}$	M1	3.1a
	$k<"-\dfrac{5}{3}"$	A1ft	1.1b
		A1	2.5
		(4)	
			(8 marks)

Notes

(a)

M1: Attempts $4|1-3|-5=...$ or $4|-1-3|-5=...$ proceeding to a value.
Either correct value (3 or 11) that is clearly their answer can imply the M mark.
Alternatively, attempts any of $-4(-1-3)-5$ or $-4(1-3)-5$ or $-4(-1)+7$ or $-4(1)+7$ i.e. without the modulus signs using the 'left' branch of the function.

A1: Both 3 and 11 found and no other values that are clearly meant to be their answers (i.e. ignore reference to $a=1$ or $a=-1$). Accept "3, 11" or "3 or 11" or "3 and 11" or {3, 11} but not (3, 11). Condone $y=3$, 11 but not $x=3$, 11.

(b)

M1: Attempts $2(-5)+17$ which may be implied by sight of 7 used in their range, including in an incorrect range e.g. $\mathrm{f}(x)>7$
Alternatively, attempts $\mathrm{gf}(x)=2(4|x-3|-5)+17$ $(=8|x-3|+"7")$ **and** replaces $|x-3|$ with 0 or x with 3 or simply deduces the minimum value of their "7" in this case.
Must be an attempt at gf and not fg.

A1: Deduces the range $\mathrm{gf}(x)\geq 7$ using correct notation.
Condone $y\geq 7$ but **not** e.g. $x\geq 7$ or $\mathrm{g}(x)\geq 7$ or $\mathrm{f}(x)\geq 7$ or $\mathrm{fg}(x)\geq 7$
Other acceptable notation includes: $\mathrm{gf}(x)\in[7,\infty)$
Do **not** accept $\mathrm{gf}(x)\in[7,\infty]$ or $\mathrm{gf}(x)\in(7,\infty)$

(c)

B1: Deduces $k\ldots-4$ allow any equality or inequality here. Condone $k\ldots\pm 4$ (but not just k … 4) and condone e.g. "gradient … – 4"
May be seen coming from e.g. $kx=-4x+7\rightarrow(k+4)x=7$ or $x=\dfrac{7}{k+4}$ which is acceptable but see the note below. Should not be x or y for this mark but may be m.
Note: Finding the 'discriminant' of a linear equation e.g. $(k+4)x-7=0$ in order to obtain k … – 4 is invalid and cannot earn either the B1 or the final A1 mark unless there is an alternative valid reason given.

M1: Attempts to use the vertex to find the (upper) limit for k. Look for $\dfrac{\pm 5}{\pm 3}$ but allow this mark to be scored for $\dfrac{\pm B}{\pm A}$ if there is clear evidence that they think the vertex is at (A, B)

Allow use of m, x, y or another variable for this mark.

Alt 1 via squaring and the discriminant:

$$kx = 4|x-3|-5 \Rightarrow kx+5 = 4|x-3|$$
$$\Rightarrow k^2x^2+10kx+25 = 16\left(x^2-6x+9\right)$$
$$\Rightarrow \left(k^2-16\right)x^2+\left(10k+96\right)x-119=0$$
$$\Rightarrow \left(10k+96\right)^2-4\left(k^2-16\right)\left(-119\right)=0$$
$$\Rightarrow 576k^2+1920k+1600=0 \Rightarrow k=-\frac{5}{3}$$

Scores M1 for setting $kx = 4|x-3|-5$, isolating $|x-3|$ (or $4|x-3|$), squaring both sides, using $b^2-4ac\ldots 0$ where … is any equality or inequality, and solving the resulting 3TQ using the usual rules and may be by calculator, leading to a value for k.
Condone slips in expanding the brackets.

Alt 2 via solving simultaneous equations:

$$\text{e.g. } kx = 4(x-3)-5 \Rightarrow x = \frac{-17}{k-4}$$
$$kx = 4(3-x)-5 \Rightarrow (k+4)x = 7$$
$$\Rightarrow (k+4)\left(\frac{-17}{k-4}\right)=7 \Rightarrow k=-\frac{5}{3}$$

Scores M1 for setting $kx = 4(x-3)-5$ and $kx = 4(3-x)-5$, eliminating x, and solving for k

A1ft: Uses their value of k, found using one of the above methods, as the upper end of the range for k, i.e., $k < "-\frac{5}{3}"$ which may be seen as part of their range. Condone $k \le "-\frac{5}{3}"$ for this mark.
Score once seen as an upper limit and do not withhold if they then incorrectly combine as e.g. $-\frac{5}{3} < k < -4$
Condone the use of m for this mark, but not x or y.

A1: $-4 \le k < -\frac{5}{3}$ o.e. and no other solutions seen. Their range must use k and not e.g. x, y or m.

Other acceptable notation includes: $k \in \left[-4, -\frac{5}{3}\right)$, "$k < -\frac{5}{3}$ and $k \ge -4$", $k < -\frac{5}{3} \cap k \ge -4$

Do not accept e.g. "$k < -\frac{5}{3}$, $k \ge -4$" or "$k < -\frac{5}{3}$ or $k \ge -4$" or "$k < -\frac{5}{3} \cup k \ge -4$"

Allow $-1.\dot{6}$ but **not** -1.6 or $-1.6\ldots$for $-\frac{5}{3}$

Question	Scheme	Marks	AOs
13(i)	$(n =)\ 3$	B1	2.2a
		(1)	
(ii)	30 solutions **or** e.g. each interval of length π has 6 solutions	B1	2.2a
	e.g., Each interval of length π has 6 solutions, and there are 5 intervals of length π, so $5 \times 6 = 30$ solutions.	dB1	2.4
		(2)	
		(3 marks)	

Notes

(i)

B1: cao $(n =)\ 3$

(ii)

B1: Either:

- Deduces 30 solutions
- **or** deduces the correct **total** number of solutions for $\sin(nx) = k$ **and** $\sin(nx) = -k$ (or $\sin^2(nx) = k^2$) in any relevant interval $(0 \text{ to } \pi, 2\pi, 4\pi \text{ or } 5\pi)$
- **or** deduces the correct number of solutions in 0 to 5π for either $\sin(nx) = k$ or $\sin(nx) = -k$
- **or** deduces the correct **total** number of solutions for $\sin(x) = k$ **and** $\sin(x) = -k$ in any relevant interval $(0 \text{ to } \pi, 2\pi, 4\pi \text{ or } 5\pi)$

dB1: Requires 30 (solutions) and correct justification comprising all the elements in **one** of the bullet points below. See the tables on the next page for the correct values.

- Deduces the correct **total** number of solutions for $\sin(nx) = k$ **and** $\sin(nx) = -k$ (or $\sin^2(nx) = k^2$) in any relevant interval $(0 \text{ to } \pi, 2\pi \text{ or } 4\pi)$ **and** scales the interval to 0 to 5π
- **or** deduces the correct number of solutions in 0 to 5π for $\sin(nx) = k$ **and** for $\sin(nx) = -k$ and then adds
- **or** finds the **total** correct number of solutions for $\sin(x) = k$ **and** $\sin(x) = -k$ in any relevant interval $(0 \text{ to } \pi, 2\pi, 4\pi \text{ or } 5\pi)$ **and** then scales the interval to 0 to 5π **and** multiplies by 3

Just stating e.g. $2 \times 6 \times 2.5$ is insufficient for the dB1 mark without further justification. There are many acceptable variations, and some examples are below and on the next page.

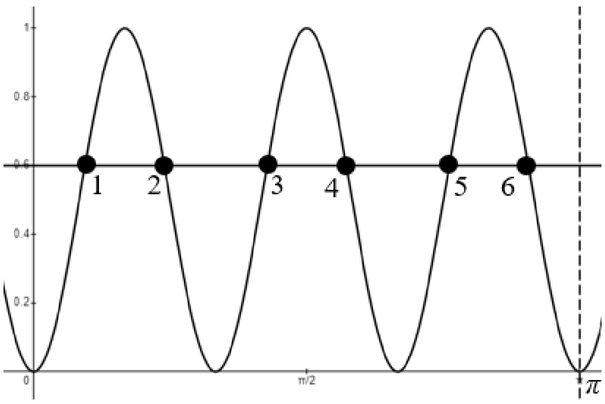
Some examples:

- 6 solutions in each interval of length π (B1) **and** 5 intervals of length π so 30 (dB1)
- 12 solutions in each interval of length 2π (B1) **and** 2.5 intervals of length 2π so 30 (dB1)
- 2 solutions for $\sin x = k$ up to 2π, so for $\sin^2 x$ there are 4 per 2π (B1) **and** since $0 \leqslant x < 5\pi$ then $0 \leqslant 3x < 15\pi$ so $\frac{15}{2} \times 4 = 30$ (dB1)
- Double the 6 solutions since $\sin(nx) = \pm k$ (or $\sin^2(nx) = k^2$) so 12 (B1) **and** so 24 in 4π, hence 30 in 5π (dB1)
- 16 solutions for $\sin(nx) = k$ (B1) **and** 14 (solutions) for $\sin(nx) = -k \rightarrow 30$ (dB1)
- $\sin^2(3x) = \frac{1 - \cos(6x)}{2}$ and as $\cos x = k$ has 5 solutions in 0 to 5π (B1) **and** so $\cos 6x$ has $5 \times 6 = 30$ (dB1)

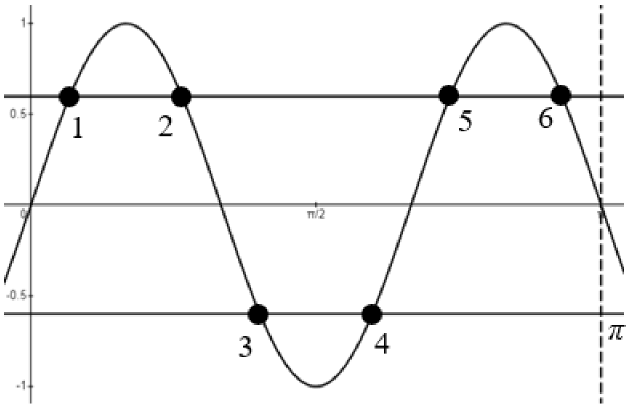
Graphical approaches may be used but these must be convincing, clearly showing the correct number of solutions in one of the relevant intervals. Be lenient with the shape of the curve.

e.g. 30 solutions in $0 \leq x < 5\pi$ [with 6 solutions on, for example, either sketch below]

$y=\sin^2(3x)$ $0 \leq x < \pi$



$y=\sin(3x)$ $0 \leq x < \pi$



Note that a suitable graph of $\sin x$ showing, e.g. 10 solutions up to 5π, followed by 30 (calculation of ×3 clearly implied) would also be eligible to score the dB1.

The following are examples of an incorrect justification but scores B1dB0 for reaching 30:

- $\sin(nx) = k$ has 3×6 solutions in the interval 0 to 5π, $\sin(nx) = -k$ has 3×4 solutions (in the interval 0 to 5π) so 18 + 12 = 30 solutions in total.
- $\sin(nx) = k$ has 15 solutions in the interval 0 to 5π, so for $\sin^2(nx)$ has 15×2 = 30 solutions in total.

For reference, the tables below show the total number of solutions for each branch of $\sin^2(nx)=k^2$ with $n = 3$ and the bold values score the first B1 **provided they are in the correct interval**.

	$0 \leq x < \pi$	$0 \leq x < 2\pi$	$0 \leq x < 4\pi$	$0 \leq x < 5\pi$
$\sin(nx)=k$	4	6	12	**16**
$\sin(nx)=-k$	2	6	12	**14**
Total solutions	**6**	**12**	**24**	**30**

and $n = 1$ for those that deal with nx $(3x)$ last.

	$0 \leq x < \pi$	$0 \leq x < 2\pi$	$0 \leq x < 4\pi$	$0 \leq x < 5\pi$
$\sin(x)=k$	2	2	4	6
$\sin(x)=-k$	0	2	4	4
Total solutions	**2**	**4**	**8**	**10**

Question	Scheme	Marks	AOs
14(a)	$\left(\overrightarrow{AD}=\overrightarrow{AB}-\overrightarrow{DB}=\right)\ 2\mathbf{a}+3\mathbf{b}-\left(-4\mathbf{a}+k\mathbf{b}\right)\ \ \left(=6\mathbf{a}+\left(3-k\right)\mathbf{b}\right)$	M1	1.1b
	$\frac{15}{6}=2.5\Rightarrow\frac{-5}{3-k}=2.5\to k=\ldots$	dM1	1.1b
	$k=5$ *	A1*	2.1
		(3)	

Notes

(a) **Note: Condone the use of column vectors throughout this question.**
There may be working on the diagram that can be awarded marks.

M1: Attempts either $\left(\overrightarrow{AD}=\overrightarrow{AB}-\overrightarrow{DB}=\right)\ 2\mathbf{a}+3\mathbf{b}-\left(-4\mathbf{a}+k\mathbf{b}\right)$ or $\left(\overrightarrow{DA}=\right)\ -4\mathbf{a}+k\mathbf{b}-\left(2\mathbf{a}+3\mathbf{b}\right)$

or $\left(\overrightarrow{DB}=\overrightarrow{DA}+\overrightarrow{AB}=\right)\ \alpha\left(15\mathbf{a}-5\mathbf{b}\right)+2\mathbf{a}+3\mathbf{b}$

Allow subtraction either way round and may be implied by one correct component or by e.g. $2\mathbf{a}+3\mathbf{b}=\overrightarrow{AD}-4\mathbf{a}+k\mathbf{b}$

For reference: $\overrightarrow{AD}=6\mathbf{a}+\left(3-k\right)\mathbf{b}$ and $\overrightarrow{DA}=-6\mathbf{a}+\left(k-3\right)\mathbf{b}$

Allow e.g. $\left(\overrightarrow{AD}=\right)\ \begin{pmatrix}6\mathbf{a}\\ \left(3-k\right)\mathbf{b}\end{pmatrix}$ or $\left(\overrightarrow{AD}=\right)\ \begin{pmatrix}6\\ 3-k\end{pmatrix}$ including without the brackets $\begin{matrix}6\\ 3-k\end{matrix}$

Condone the use of gradients or ratios for $\overrightarrow{AD}$ e.g. $\frac{6}{3-k}$ or "6 : 3 – k" to imply this mark (either way round).

There are alternatives using e.g. $\overrightarrow{DC}$ but, in these cases, we require two expressions for the same vector, of which one expression must use $\overrightarrow{DB}$.

dM1: A full method to solve the problem. Some possible approaches:

- attempts to find a scale factor and uses it to find k
- sets up equivalent fractions e.g. $\frac{15}{6}=-\frac{5}{3-k}$ or e.g. $\frac{2--4}{3-k}=\frac{15}{-5}$ and solves for k
- sets up equivalent ratios e.g. 6 : 15 = 3 – k : –5 and solves for k
- sets up simultaneous equations and solves for k e.g. $2\mathbf{a}+3\mathbf{b}+4\mathbf{a}-k\mathbf{b}=\alpha\left(15\mathbf{a}-5\mathbf{b}\right)$ o.e. **or** $\left(\overrightarrow{DB}=\overrightarrow{DA}+\overrightarrow{AB}=\right)\ \alpha\left(-15\mathbf{a}+5\mathbf{b}\right)+2\mathbf{a}+3\mathbf{b}=-4\mathbf{a}+k\mathbf{b}$ leading to $6=15\alpha$ $\left(\alpha=\frac{2}{5}\right)$ and $3-k=-5\alpha$ hence $k=\ldots$

Note that their coefficient might be the reciprocal, from e.g. $\beta\left(2\mathbf{a}+3\mathbf{b}+4\mathbf{a}-k\mathbf{b}\right)=15\mathbf{a}-5\mathbf{b}$ $\left(\beta=2.5\right)$ and directions might be reversed in which case $\alpha=-0.4$ or $\beta=-2.5$ can be used.

A1*: Arrives at $k=5$ via a correct method. Usually this will be following:

- A correct expression for $\overrightarrow{AD}$ (e.g. $2\mathbf{a}+3\mathbf{b}-\left(-4\mathbf{a}+k\mathbf{b}\right)$) or $\overrightarrow{DA}$ or $\overrightarrow{BD}$ or $\overrightarrow{DB}$ which may be mislabelled.
- Correct scale factor stated (±2.5 or ±0.4) or implied (e.g., by $\frac{-5}{3-k}=\frac{15}{6}$ or $3\left(3-k\right)=-6$)
- A correct intermediate equation that leads to $k=5$

An example minimal response might look like:

e.g. $\alpha\left(15\mathbf{a}-5\mathbf{b}\right)=6\mathbf{a}+\left(3-k\right)\mathbf{b}\to 6=15\alpha\to\alpha=\frac{2}{5}\to-5\left(\frac{2}{5}\right)=3-k\to k=5$

or e.g. $6\mathbf{a}+\left(3-k\right)\mathbf{b}\to\frac{15}{6}=-\frac{5}{3-k}\to k=5$

Condone missing/invisible brackets if recovered.

Alternative by verification:

M1: Sets $k = 5$, substitutes into $\overrightarrow{DB}$ and attempts $\left(\overrightarrow{AD} = \overrightarrow{AB} - \overrightarrow{DB} =\right)\ 2\mathbf{a} + 3\mathbf{b} - \left(-4\mathbf{a} + 5\mathbf{b}\right)$ o.e.

dM1: Attempts to compare $\overrightarrow{AD}$ o.e. with $\overrightarrow{BC}$ (usually $6\mathbf{a} - 2\mathbf{b} = \alpha\left(15\mathbf{a} - 5\mathbf{b}\right)$) and finds α

A1*: Requires:

- Correct $\overrightarrow{AD}$ o.e. e.g. $\overrightarrow{DA}$
- Correct scale factor $\alpha = \pm\frac{2}{5}$ or $\beta = \pm 2.5$ (sign dependent on their approach)
- Conclusion referencing the lines being parallel e.g. "hence $\overrightarrow{AD}$ and $\overrightarrow{BC}$ are parallel."

Question	Scheme	Marks	AOs
14(b)	e.g., $\left(\overrightarrow{BN}=\right)\ \frac{1}{5}\overrightarrow{BC}\ \left(=3\mathbf{a}-\mathbf{b}\right)$	B1	2.2a
	e.g., $\left(\overrightarrow{BX}=\lambda\overrightarrow{BD}=\right)\ \lambda\left(4\mathbf{a}-5\mathbf{b}\right)$	M1	2.1
	e.g., $\left(\overrightarrow{BX}=\lambda\overrightarrow{BD}=\right)\ \lambda\left(4\mathbf{a}-5\mathbf{b}\right)$ **and** e.g. $\left(\overrightarrow{BX}=\overrightarrow{BA}+\mu\overrightarrow{AN}=\right)\ \left(-2\mathbf{a}-3\mathbf{b}\right)+\mu\left(2\mathbf{a}+3\mathbf{b}+"3\mathbf{a}-\mathbf{b}"\right)$	dM1	3.1a
	$\begin{array}{l}4\lambda=-2+5\mu\\-5\lambda=-3+2\mu\end{array}\quad\Rightarrow\lambda=\ldots\left(\frac{1}{3}\right)\text{ or }\mu=\ldots\left(\frac{2}{3}\right)$	ddM1	1.1b
	$1:2$	A1	2.2a
		(5)	
			(8 marks)

Notes

(b) **Note: Condone the use of column vectors throughout this question.**
There may be working on the diagram that can be awarded marks.

B1: Deduces a correct interpretation of the ratio $BN:NC=1:4$ that enables a start to a solution.
i.e., progresses to a correct statement that is at least as far as $\left(\overrightarrow{BN}=\right)\ \frac{1}{5}\overrightarrow{BC}$ (or e.g. $3\mathbf{a}-\mathbf{b}$)
or $\left(\overrightarrow{CN}=\right)\ \frac{4}{5}\overrightarrow{CB}$ (or e.g. $4\mathbf{b}-12\mathbf{a}$). May be embedded in e.g. $\left(\overrightarrow{AN}=\right)\ 2\mathbf{a}+3\mathbf{b}+\frac{1}{5}\overrightarrow{BC}$
Allow e.g. $\frac{1}{5}\begin{pmatrix}15\\-5\end{pmatrix}$ or $\begin{pmatrix}3\\-1\end{pmatrix}$ or $\begin{pmatrix}3a\\-b\end{pmatrix}$ for this mark.

M1: For the key step in attempting a valid expression for $\overrightarrow{BX}$ (**or** $\overrightarrow{AX}$ **or** $\overrightarrow{DX}$ **or** $\overrightarrow{CX}$ **or** $\overrightarrow{NX}$) in terms of **a** and **b**. See diagram/notes on the next page for helpful vectors.
Condone slips provided their intention is clear.
Note that they might be using λ and μ the other way round or alternative variables.

Note: May be seen as a single expression such as $\overrightarrow{AB}=\overrightarrow{AX}+\overrightarrow{XB}$ $\left(\text{i.e. }\overrightarrow{AB}=p\overrightarrow{AN}+q\overrightarrow{DB}\right)$
or $\overrightarrow{BN}=\overrightarrow{BX}+\overrightarrow{XN}$ $\left(\text{i.e. }\overrightarrow{BN}=p\overrightarrow{BD}+q\overrightarrow{AN}\right)$ either of which scores M1dM1 simultaneously.

dM1: For the key step in attempting a **second** valid expression for their $\overrightarrow{BX}$ (**or** $\overrightarrow{AX}$ **or** $\overrightarrow{DX}$ **or** $\overrightarrow{CX}$ **or** $\overrightarrow{NX}$) in terms of **a** and **b** which enables the problem to be solved, i.e., it must not be parallel in approach to the first. See diagram/notes on the next page for helpful vectors.
One expression should involve $"\lambda"\left(4\mathbf{a}-5\mathbf{b}\right)$ and the other should involve $"\mu"\left("5\mathbf{a}+2\mathbf{b}"\right)$
Condone slips provided their intention is clear.
They must use different parameters in their approaches, e.g., λ and μ.
If using e.g. $\overrightarrow{DX}=-6\mathbf{a}+2\mathbf{b}+\mu\left(5\mathbf{a}+2\mathbf{b}\right)$ and $\overrightarrow{XB}=\lambda\left(-4\mathbf{a}+5\mathbf{b}\right)$ this mark is not scored until they set $\overrightarrow{DX}+\overrightarrow{XB}=\overrightarrow{DB}$ as $-6\mathbf{a}+2\mathbf{b}+\mu\left(5\mathbf{a}+2\mathbf{b}\right)+\lambda\left(-4\mathbf{a}+5\mathbf{b}\right)=-4\mathbf{a}+5\mathbf{b}$
Dependent on the previous method mark.

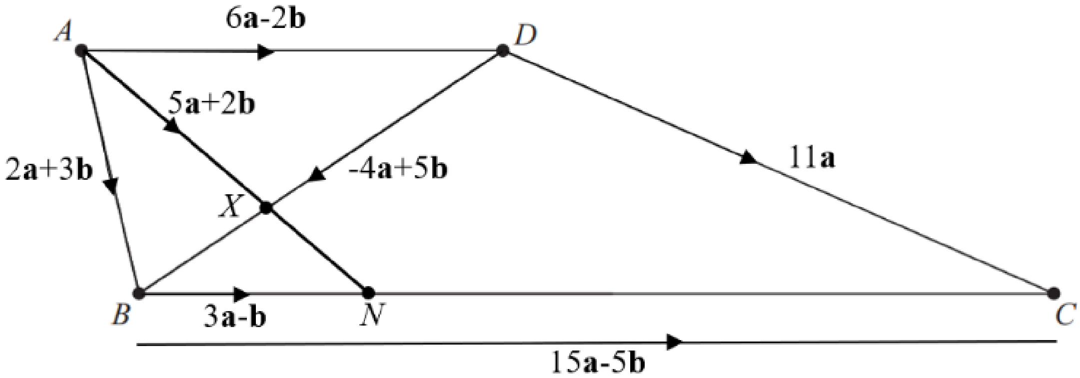
ddM1: Compares coefficients of **a** and **b** to create simultaneous equations in their parameters and attempts to solve (which may be by calculator) leading to a value for one of their parameters.
Condone slips provided the intention is clear.
This mark may be implied by a **correct** value for e.g. λ following their two **correct** expressions for e.g. $\overrightarrow{BX}$
Dependent on both previous method marks.

A1: $1:2$ o.e. Must follow a correct value for their parameter.
The correct ratio seen does **not** imply full marks. Candidates must show detailed reasoning.

Allow equivalent ratios e.g. $\frac{1}{3}:\frac{2}{3}$ and ISW (e.g. $1:3$) but they must be the correct way round.

There may be attempts using similar triangles. Send to review.

Helpful Diagram:



Note: Some examples of valid expressions for the M and dM marks for part (b) are:
In each expression they may use different parameters and e.g. $1-\lambda$ might just be e.g. ϕ.

- $\overrightarrow{BX}=\lambda\overrightarrow{BD}=\lambda(4\mathbf{a}-5\mathbf{b})$
- $\overrightarrow{BX}=\overrightarrow{BA}+\mu\overrightarrow{AN}=(-2\mathbf{a}-3\mathbf{b})+\mu(2\mathbf{a}+3\mathbf{b}+\text{"}3\mathbf{a}-\mathbf{b}\text{"})$
- $\overrightarrow{BX}=\overrightarrow{BN}+(1-\mu)\overrightarrow{NA}=(\text{"}3\mathbf{a}-\mathbf{b}\text{"})+(1-\mu)(-2\mathbf{a}-3\mathbf{b}+\text{"}-3\mathbf{a}+\mathbf{b}\text{"})$
- $\overrightarrow{DX}=(1-\lambda)\overrightarrow{DB}=(1-\lambda)(-4\mathbf{a}+5\mathbf{b})$
- $\overrightarrow{DX}=\overrightarrow{DA}+\mu\overrightarrow{AN}=(\text{"}-6\mathbf{a}+2\mathbf{b}\text{"})+\mu(2\mathbf{a}+3\mathbf{b}+\text{"}3\mathbf{a}-\mathbf{b}\text{"})$
- $\overrightarrow{DX}=\overrightarrow{DN}+(1-\mu)\overrightarrow{NA}=(\text{"}-\mathbf{a}+4\mathbf{b}\text{"})+(1-\mu)(-2\mathbf{a}-3\mathbf{b}+\text{"}-3\mathbf{a}+\mathbf{b}\text{"})$
- $\overrightarrow{AX}=\mu\overrightarrow{AN}=\mu(2\mathbf{a}+3\mathbf{b}+\text{"}3\mathbf{a}-\mathbf{b}\text{"})$
- $\overrightarrow{AX}=\overrightarrow{AB}+\lambda\overrightarrow{BD}=(2\mathbf{a}+3\mathbf{b})+\lambda(4\mathbf{a}-5\mathbf{b})$
- $\overrightarrow{AX}=\overrightarrow{AD}+(1-\lambda)\overrightarrow{DB}=(\text{"}6\mathbf{a}-2\mathbf{b}\text{"})+(1-\lambda)(-4\mathbf{a}+5\mathbf{b})$
- $\overrightarrow{XN}=\mu\overrightarrow{AN}=\mu(2\mathbf{a}+3\mathbf{b}+\text{"}3\mathbf{a}-\mathbf{b}\text{"})$
- $\overrightarrow{XN}=\lambda\overrightarrow{DB}+\overrightarrow{BN}=\lambda(-4\mathbf{a}+5\mathbf{b})+(\text{"}3\mathbf{a}-\mathbf{b}\text{"})$
- $\overrightarrow{XN}=(1-\lambda)\overrightarrow{BD}+\overrightarrow{DN}=(1-\lambda)(4\mathbf{a}-5\mathbf{b})+(\text{"}-\mathbf{a}+4\mathbf{b}\text{"})$

or alternatives using C or N as starting points, but these are unlikely.

Question	Scheme		Marks	AOs
15(a)	$\left(\mathrm{f}'(x)=\right)\ \dfrac{\lambda x\left(1+x^2\right)^2-\mu x\left(1-x^2\right)\left(1+x^2\right)}{\left(1+x^2\right)^4}$		M1	1.1b
	$=\dfrac{-2x\left(1+x^2\right)^2-4x\left(1-x^2\right)\left(1+x^2\right)}{\left(1+x^2\right)^4}$		A1	1.1b
	e.g. $\dfrac{-2x\left(1+x^2\right)-4x\left(1-x^2\right)}{\left(1+x^2\right)^3}=0$ $\Rightarrow\left(1+x^2\right)+2\left(1-x^2\right)=0$ $\Rightarrow 3-x^2=0$	or e.g. $=\dfrac{-2x\left(1+x^2\right)\left[\left(1+x^2\right)+2\left(1-x^2\right)\right]}{\left(1+x^2\right)^4}$ $=\dfrac{-2x\left(1+x^2\right)\left(3-x^2\right)}{\left(1+x^2\right)^4}$	dM1	3.1a
	e.g. $3-x^2=0$ or $2x\left(1+x^2\right)\left(x^2-3\right)=0$ $\Rightarrow x=\ldots\Rightarrow y=\ldots$		ddM1	2.1
	$\left(-\sqrt{3},\ -\dfrac{1}{8}\right)$		A1	2.3
			(5)	

Notes

(a) **There may be other valid ways to differentiate or to solve the resulting equation.**

M1: Attempts the quotient rule to achieve $\dfrac{\pm\lambda x\left(1+x^2\right)^2\pm\mu x\left(1-x^2\right)\left(1+x^2\right)}{\left(1+x^2\right)^4}$

but condone $\dfrac{\pm\lambda x\left(1+x^2\right)^2\pm\mu x\left(1-x^2\right)\left(1+x^2\right)}{\left(1+x^2\right)^2}$ provided an incorrect quotient rule is not seen.

Alternatively, attempts the product rule on $\mathrm{f}(x)=\left(1-x^2\right)\left(1+x^2\right)^{-2}$ to achieve

$\pm\lambda x\left(1+x^2\right)^{-2}\pm\mu x\left(1-x^2\right)\left(1+x^2\right)^{-3}$

There may be attempts at splitting the numerator to $\mathrm{f}(x)=\left(1+x^2\right)^{-2}-x^2\left(1+x^2\right)^{-2}$ which should differentiate to $\pm\lambda x\left(1+x^2\right)^{-3}\pm\mu x\left(1+x^2\right)^{-2}\pm\gamma x^3\left(1+x^2\right)^{-3}$

In **all** cases, do not penalise signs of λ, μ and/or γ unless an incorrect quotient rule or an incorrect product rule is stated.

Any occurrences of $\left(1+x^2\right)^2$ may be replaced with $1+2x^2+x^4$

There is no need for a LHS e.g. $\mathrm{f}'(x)=$ to be present so ignore an incorrect LHS.

Invisible brackets may be recovered/implied by later work.

A1: Correct differentiation. May be unsimplified. Ignore absence of (or incorrect) LHS.

The correct derivative using the quotient rule is $=\dfrac{-2x\left(1+x^2\right)^2-4x\left(1-x^2\right)\left(1+x^2\right)}{\left(1+x^2\right)^4}$ o.e.

using the product rule is $-2x\left(1+x^2\right)^{-2}-4x\left(1-x^2\right)\left(1+x^2\right)^{-3}$ o.e.

and splitting the numerator is $-4x\left(1+x^2\right)^{-3}-2x\left(1+x^2\right)^{-2}+4x^3\left(1+x^2\right)^{-3}$ o.e.

You may need to check carefully for equivalent derivatives and ISW after a correct expression is seen. Note that some candidates may set = 0 at the start and may omit the denominator as a result – send to review if you are unsure in such cases.

dM1: Attempts to reduce the expression to a suitable form so that the roots can be found, either by cancelling a factor of $\left(1+x^2\right)$ and simplifying the other brackets

e.g. $\dfrac{-2x\left(1+x^2\right)-4x\left(1-x^2\right)}{\left(1+x^2\right)^3} \rightarrow$ e.g. $\dfrac{-6x+2x^3}{\left(1+x^2\right)^3}$ or $\dfrac{2x\left(x^2-3\right)}{\left(1+x^2\right)^3}$ or $2x\left(x^2-3\right)\ (=0)$

or by attempting to take x, $\left(1+x^2\right)$ or $\pm 2x\left(1+x^2\right)$ out as a factor and simplify the other brackets. If they only take out a factor of x then they must have $x\left(\pm Ax^4 \pm Bx^2 \pm C\right)$

e.g. $\dfrac{-2x\left(1+x^2\right)\left[\left(1+x^2\right)+2\left(1-x^2\right)\right]}{\left(1+x^2\right)^4} \rightarrow$ e.g. $\dfrac{-2x\left(1+x^2\right)\left(3-x^2\right)}{\left(1+x^2\right)^4}$ or $\dfrac{x\left(2x^4-4x^2-6\right)}{\left(1+x^2\right)^4}$

Depends on the first M mark.

Note: they might attempt to expand the brackets first, or set = 0 and multiply through by $\left(1+x^2\right)^k$ and/or divide through by x or $2x$ at any stage.

Do not be concerned if any factors of $\left(1+x^2\right)$, x or 2 go "missing" during their work.

The = 0 does not explicitly need to be seen.

Look for $\pm\dfrac{2x\left(1+x^2\right)\left(Ax^2+B\right)}{\left(1+x^2\right)^k}$ or $\pm\dfrac{\left(1+x^2\right)\left(Ax^3+Bx\right)}{\left(1+x^2\right)^k}$ or $\pm\dfrac{x\left(Ax^4+Bx^2+C\right)}{\left(1+x^2\right)^k}$ where

- k may be 0, 1, 2, 3 or 4
- A and B (and C if present) are non-zero
- any of $2x$ or $\left(1+x^2\right)$ in the numerator and/or $\left(1+x^2\right)^k$ in the denominator may be absent

ddM1: Attempts to solve their numerator set = 0 using a valid non-calculator method **and** uses a solution for x to find a corresponding value for y.

The substitution may be implied by their value of y but, if not, the substitution must be seen.

- For either Ax^2+B or Ax^3+Bx, we require $A \neq B$ and $A \times B < 0$ From either expression they can write down their $x = \pm\sqrt{\dfrac{-B}{A}}$ without working.
- For Ax^4+Bx^2+C they must show a valid non-calculator method for solving the quartic, treating it as a quadratic in x^2 and using the usual rules for solving a quadratic algebraically, e.g. $a = x^2 \rightarrow 2a^2-4a-6\,(=0) \rightarrow (a+1)(2a-6)\,(=0) \rightarrow x = \pm\sqrt{3}$. They must reach a value for x (and not just x^2) as well as finding a value for y.

Dependent on both previous method marks. They must use a value for x that is not –1, 0 or 1.

A1: Deduces the correct **exact** coordinates for P $\left(-\sqrt{3}, -\dfrac{1}{8}\right)$ o.e. $\left(-\sqrt{3}, -0.125\right)$

Requires all the previous marks to have been scored.

Condone $x = -\sqrt{3}$, $y = -0.125$ or e.g. $-\sqrt{3}, -\dfrac{1}{8}$

If there is more than one pair of coordinates given, then the correct coordinates must be clearly selected or any others clearly rejected.

Ignore any mistakes that occur if they multiply out the denominator $\left(1+x^2\right)^4$ after differentiation.

Question	Scheme		Marks	AOs
15(b)	$x=-1\Rightarrow\alpha=-\frac{\pi}{4}$ and $x=1\Rightarrow\beta=\frac{\pi}{4}$		B1	2.2a
	$\frac{\mathrm{d}x}{\mathrm{d}\theta}=\sec^2\theta$		B1	1.1b
	$\int\frac{1-x^2}{\left(1+x^2\right)^2}\mathrm{d}x=\int\frac{1-\tan^2\theta}{\left(1+\tan^2\theta\right)^2}\text{"}\sec^2\theta\text{"}\,\mathrm{d}\theta$ or $=\int\frac{1-\tan^2\theta}{\sec^4\theta}\text{"}\sec^2\theta\text{"}\,\mathrm{d}\theta$		M1	1.1b
	$=\int\left(1-\tan^2\theta\right)\cos^2\theta\,\mathrm{d}\theta$ $=\int\left(1-\frac{\sin^2\theta}{\cos^2\theta}\right)\cos^2\theta\,\mathrm{d}\theta$ $=\int\left(\cos^2\theta-\sin^2\theta\right)\mathrm{d}\theta$	$=\int\frac{2-1-\tan^2\theta}{\sec^2\theta}\mathrm{d}\theta$ $=\int\left(\frac{2}{\sec^2\theta}-\frac{1+\tan^2\theta}{\sec^2\theta}\right)\mathrm{d}\theta$ $=\int\left(2\cos^2\theta-1\right)\mathrm{d}\theta$	dM1	3.1a
	$=\int\cos 2\theta\,\mathrm{d}\theta$ *		A1*	2.1
			(5)	

Notes

(b) **Mark parts (b) and (c) together.**

B1: Deduces the correct limits for the integral in θ which may be seen separately as side working or within their integral work. This mark cannot be scored working in degrees.
Allow $\frac{3\pi}{4}$ instead of $-\frac{\pi}{4}$ but not decimal approximations.

B1: $\frac{\mathrm{d}x}{\mathrm{d}\theta}=\sec^2\theta$ or equivalent e.g. $\frac{\mathrm{d}x}{\mathrm{d}\theta}=1+x^2$ or $\frac{\cos^2\theta--\sin^2\theta}{\cos^2\theta}$ (coming from $x=\frac{\sin\theta}{\cos\theta}$) but must be correct, so not e.g. $\frac{\mathrm{d}x}{\mathrm{d}\theta}=\sec^2 x$ unless recovered.

M1: Makes a complete attempt at using the substitution $x=\tan\theta$
Requires:

- An attempt at using $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ to replace dx with $\mathrm{d}\theta$ either way round, so if $\frac{\mathrm{d}x}{\mathrm{d}\theta}=\mathrm{g}(\theta)$ allow either $\mathrm{d}x=\mathrm{g}(\theta)\{\mathrm{d}\theta\}$ **or** $\mathrm{d}x=\frac{1}{\mathrm{g}(\theta)}\{\mathrm{d}\theta\}$. $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ must be a function of θ and not a constant.
- All terms in x replaced with $\tan\theta$ (or $1+x^2$ with $\sec^2\theta$)

Condone the absence of $\mathrm{d}\theta$ but dx must no longer be present.
Condone if they fail to square the denominator or e.g. a slip in missing a θ
Use of notation such as $\mathrm{d}(\tan\theta)$ is correct but does not score the M1 until replaced with $\sec^2\theta\{\mathrm{d}\theta\}$ or their derivative of $\tan\theta$ (which must not be a constant multiple of $\tan\theta$).

dM1: Uses trigonometric identities e.g. $\pm1\pm\tan^2\theta=\pm\sec^2\theta$, $\tan\theta=\frac{\sin\theta}{\cos\theta}$, $\sec\theta=\frac{1}{\cos\theta}$, $\pm\cos^2\theta\pm\sin^2\theta=\pm1$ to simplify the integral to an expression of the form $\pm a\sin^2\theta\pm b\cos^2\theta\pm c$ where one of a, b or c may be 0.
The algebra should essentially be correct but condone e.g. sign slips or errors collecting terms.

Alternatively, e.g. $\int \frac{1-\tan^2\theta}{1+\tan^2\theta}\,\mathrm{d}\theta = \int \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}\,\mathrm{d}\theta = \int \frac{\cos^2\theta - \sin^2\theta}{1}\,\mathrm{d}\theta$

They must have replaced dx with $\mathrm{d}\theta$ the correct way round for this mark, so if $\frac{\mathrm{d}x}{\mathrm{d}\theta} = \mathrm{g}(\theta)$ then they must have $\mathrm{d}x = \mathrm{g}(\theta)\{\mathrm{d}\theta\}$ and **not** $\mathrm{d}x = \frac{1}{\mathrm{g}(\theta)}\{\mathrm{d}\theta\}$

It is acceptable to replace e.g. $\tan^2\theta\cos^2\theta$ with $\sin^2\theta$
Dependent on the previous method mark.

A1*: cso Arrives at $\int \cos 2\theta\,\mathrm{d}\theta$ with sufficient working shown and no incorrect work ignoring limits. The final line must be fully correct, including the integral sign and $\mathrm{d}\theta$ (ignoring limits). Note that this may be seen in part (c) and may score the mark.
All trigonometric identities must be fully correct.
Condone one or two slips in a missing θ but not frequent omissions.
Condone missing integral signs in their intermediate work, but it must be present on the final line.
dx must be replaced with $\mathrm{d}\theta$ at some stage before the final line but does not need to be present in every intermediate line of working.
Condone notational errors throughout e.g. $\sin\theta^2$ provided they are recovered.
This mark is independent from any work to do with limits, i.e., B0B1M1dM1A1* is possible.

Question	Scheme	Marks	AOs
15(c)	$\int \cos 2\theta \, d\theta = \frac{1}{2}\sin 2\theta$	M1	1.1b
	$\left[\frac{1}{2}\sin 2\theta\right]_{"-\frac{\pi}{4}"}^{"\frac{\pi}{4}"} = \frac{1}{2}\sin\left(2\times\frac{\pi}{4}\right) - \frac{1}{2}\sin\left(2\times -\frac{\pi}{4}\right)$	dM1	1.1b
	$=1$	A1	2.1
		(3)	
			(13 marks)

Notes

(c) **Mark parts (b) and (c) together.**

M1: $\int \cos 2\theta \, d\theta \to \pm\frac{1}{2}\sin 2\theta \; \{+c\}$

dM1: Substitutes their changed limits (not 1 and -1) into $\pm\frac{1}{2}\sin 2\theta$ and subtracts either way round.

May be implied e.g. $\frac{1}{2}+\frac{1}{2}$ provided they have the correct limits.

Dependent on the previous method mark.

A1: Area of 1 found following correct work. This mark requires clear substitution of the correct limits into $\frac{1}{2}\sin 2\theta$ the correct way round. If they have the limits the wrong way round and achieve an answer of -1 they cannot just make the answer positive for this mark.

They may use limits from 0 to $\frac{\pi}{4}$ and multiply their result by 2.

$\frac{3\pi}{4}$ may be used in place of $-\frac{\pi}{4}$ as the lower limit which is acceptable.

Condone any spurious integral symbol accompanying $\frac{1}{2}\sin 2\theta$

The substitution may be implied by e.g. $\frac{1}{2}--\frac{1}{2}$ or $\frac{1}{2}\sin\left(\frac{\pi}{2}\right)-\frac{1}{2}\sin\left(-\frac{\pi}{2}\right)$ but must follow sight of $\frac{1}{2}\sin 2\theta$

Condone use of limits –45 and 45 (degrees) the correct way round for full marks in part (c).
Note that use of a calculator on the original integral will give the correct answer. An answer of 1 scores no marks without evidence of scoring both method marks above including the substitution.

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