

Edexcel - Pure Year 2 exam questions

NOTE:

Please be aware that in the Year 13 collections, you will miss questions from the Year 13 papers. However, these questions are intentionally included because they align with Year 12 content and topics.

Happy studying!

Ch. 1: Algebraic Methods	6
June 2024 Question 15 Paper 1	6
June 2022 Question 7 Paper 1	7
November 2021 Question 1 Paper 1	9
November 2021 Question 15 Paper 1	10
November 2020 Question 16 Paper 1	11
Ch. 2: Functions and Graphs	12
June 2024 Question 8 Paper 1	12
June 2024 Question 6 Paper 1	14
June 2023 Question 12 Paper 2	16
June 2023 Question 7 Paper 1	18
June 2022 Question 10 Paper 2	19
June 2022 Question 1 Paper 2	21
June 2022 Question 1 Paper 1	22
November 2021 Question 2 Paper 1	23
November 2021 Question 2 Paper 2	24
November 2021 Question 11 Paper 2	25
November 2020 Question 4 Paper 1	27
November 2020 Question 11 Paper 2	28
June 2019 Question 5 Paper 1	30
June 2019 Question 6 Paper 2	32
June 2018 Question 1 Paper 2	34
June 2018 Question 3 Paper 2	36
June 2018 Question 6 Paper 2	38
June 2018 Question 2 Core 3	39
Ch. 3: Sequences and Series	40
June 2024 Question 4 Paper 2	40
June 2024 Question 2 Paper 2	42
June 2024 Question 9 Paper 1	44
June 2023 Question 2 Paper 2	45
June 2023 Question 9 Paper 1	46
June 2022 Question 15 Paper 2	47
June 2022 Question 13 Paper 1	49
November 2021 Question 3 Paper 1	51
November 2021 Question 5 Paper 1	52
November 2021 Question 1 Paper 2	54

November 2021 Question 9 Paper 2	55
November 2020 Question 5 Paper 1	56
November 2020 Question 13 Paper 1	57
November 2020 Question 15 Paper 2	59
June 2019 Question 11 Paper 1	61
June 2019 Question 8 Paper 2	63
June 2018 Question 4 Paper 2	65
June 2018 Question 4 Core 1	67
June 2018 Question 6 Core 1	69
June 2018 Question 6 Core 2	71
Ch. 4: Binomial Expansion	72
June 2022 Question 7 Paper 2	73
November 2021 Question 9 Paper 1	73
November 2020 Question 1 Paper 1	76
June 2019 Question 4 Paper 1	77
June 2018 Question 11 Paper 1	79
Ch. 5: Radians	81
June 2024 Question 5 Paper 5	81
June 2024 Question 11 Paper 1	82
June 2023 Question 4 Paper 1	84
June 2023 Question 8 Paper 1	86
November 2021 question 4 Paper 2	88
November 2021 Question 6 Paper 2	89
November 2020 Question 11 Paper 1	91
June 2019 Question 2 Paper 1	93
June 2019 Question 3 Paper 2	95
June 2018 Question 1 Paper 1	97
June 2018 Question 3 Paper 1	98
June 2018 Question 4 Core 2	99
Ch. 6: Trigonometric Functions	101
June 2024 Question 8 Paper 2	101
November 2021 Question 10 Paper 1	103
November 2020 Question 12 Paper 1	105
November 2020 Question 10 Paper 2	106
June 2019 Question 12 Paper 2	107
June 2018 Question 7 Paper 2	109
June 2018 Question 12 Paper 2	112
Ch. 7: Trigonometry and Modelling	114
June 2024 Question 12 Paper 1	114
June 2023 Question 15 Paper 2	116
June 2023 Question 14 Paper 2	117
June 2023 Question 8 Paper 2	118
June 2023 Question 13 Paper 1	119

June 2022 Question 9 Paper 2	121
June 2022 Question 14 Paper 1	123
November 2021 Question 15 Paper 2	125
November 2020 Question 6 Paper 1	127
June 2019 Question 6 Paper 1	129
June 2019 Question 12 Paper 1	130
June 2018 Question 8 Paper 1	132
Ch. 8: Parametric Equations	133
June 2024 Question 10 Paper 2	133
June 2023 Question 9 Paper 2	135
November 2021 Question 13 Paper 1	137
June 2019 Question 4 Paper 2	138
June 2018 Question 14 Paper 1	140
Ch. 9: Differentiation	143
June 2024 Question 5 Paper 1	143
June 2023 Question 7 Paper 2	144
June 2023 Question 1 Paper 2	146
June 2023 Question 12 Paper 1	147
June 2022 Question 16 Paper 2	148
June 2022 Question 12 Paper 2	151
November 2021 Question 14 Paper 1	152
November 2021 Question 8 Paper 2	153
November 2021 Question 13 Paper 2	155
November 2020 Question 9 Paper 1	156
November 2020 Question 15 Paper 1	158
November 2020 Question 13 Paper 2	160
June 2019 Question 3 Paper 1	162
June 2019 Question 14 Paper 1	163
June 2018 Question 5 Paper 1	165
June 2018 Question 9 Paper 1	166
June 2018 Question 9 Paper 2	168
June 2018 Question 14 Paper 2	169
June 2018 Question 1 Core 3	171
Ch. 10: Numerical Methods	172
June 2024 Question 6 Paper 2	172
June 2024 Question 3 Paper 1	174
June 2023 Question 15 Paper 1	175
June 2023 Question 5 Paper 1	178
June 2022 Question 6 Paper 2	180
November 2021 Question 4 Paper 1	182
November 2020 Question 7 Paper 2	184
June 2019 Question 11 Paper 2	186
June 2018 Question 4 Paper 1	188

June 2018 Question 5 Paper 2	190
June 2018 Question 4 Core 3	191
Ch. 11: Integration	193
June 2024 Question 12 Paper 2	193
June 2024 Question 11 Paper 2	195
June 2024 Question 14 Paper 1	197
June 2024 Question 13 Paper 1	199
June 2024 Question 7 Paper 1	201
June 2023 Question 13 Paper 2	203
June 2023 Question 11 Paper 2	205
June 2023 Question 10 Paper 2	207
June 2023 Question 5 Paper 2	208
June 2023 Question 1 Paper 1	209
June 2022 Question 14 Paper 2	210
June 2022 Question 5 Paper 2	212
June 2022 Question 16 Paper 1	214
June 2022 Question 12 Paper 1	216
June 2022 Question 4 Paper 1	217
November 2021 Question 11 Paper 1	218
November 2021 Question 12 Paper 2	220
November 2021 Question 14 Paper 2	222
November 2020 Question 8 Paper 1	225
November 2020 Question 10 Paper 1	226
November 2020 Question 14 Paper 1	228
November 2020 Question 1 Paper 2	230
November 2020 Question	1
November 2020 Question 12 Paper 2	1
June 2019 Question 13 Paper 1	1
June 2019 Question 5 Paper 2	1
June 2019 Question 14 Paper 2	1
June 2018 Question 7 Paper 1	1
June 2018 Question 10 Paper 1	1
June 2018 Question 13 Paper 1	1
June 2018 Question 10 Paper 2	1
June 2018 Question 13 Paper 2	1
June 2018 Question 1 Core 2	1
Ch. 12: Vectors	1
June 2024 Question 7 Paper 2	1
June 2023 Question 6 Paper 2	1
June 2023 Question 3 Paper 1	1
June 2022 Question 13 Paper 2	1
June 2022 Question 9 Paper 1	1
November 2021 Question 6 Paper 1	1



Ch. 1: Algebraic Methods

June 2024 Question 15 Paper 1

15. (i) Show that $k^2 - 4k + 5$ is positive for all real values of k .

(2)

(ii) A student was asked to prove by contradiction that

“There are no positive integers x and y such that $(3x + 2y)(2x - 5y) = 28$ ”

The start of the student’s proof is shown below.

Assume that positive integers x and y exist such that

$$(3x + 2y)(2x - 5y) = 28$$

If $3x + 2y = 14$ and $2x - 5y = 2$

$$\left. \begin{array}{l} 3x + 2y = 14 \\ 2x - 5y = 2 \end{array} \right\} \Rightarrow x = \frac{74}{19}, y = \frac{22}{19} \text{ Not integers}$$

Show the calculations and statements needed to complete the proof.

(4)

ANSWER

Question	Scheme	Marks	AOs
15(i)	$k^2 - 4k + 5 = (k - 2)^2 + \dots$	M1	2.1
	$k^2 - 4k + 5 = (k - 2)^2 + 1$ so $k^2 - 4k + 5 \dots 1$ so $k^2 - 4k + 5$ is always positive*	A1*	2.4
		(2)	

June 2022 Question 7 Paper 1

7. (i) Given that p and q are integers such that

$$pq \text{ is even}$$

use algebra to prove by contradiction that at least one of p or q is even.

(3)

(ii) Given that x and y are integers such that

- $x < 0$
- $(x + y)^2 < 9x^2 + y^2$

show that $y > 4x$

(2)

ANSWER

Question	Scheme	Marks	AOs
7 (i)	For setting up the contradiction: There exists integers p and q such that pq is even and both p and q are odd	B1	2.5
	For example, sets $p = 2m + 1$ and $q = 2n + 1$ and then attempts $pq = (2m + 1)(2n + 1) = \dots$	M1	1.1b
	Obtains $pq = (2m + 1)(2n + 1) = 4mn + 2m + 2n + 1$ $= 2(2mn + m + n) + 1$ States that this is odd, giving a contradiction so "if pq is even, then at least one of p and q is even" *	A1*	2.1
		(3)	
(ii)			
	$(x + y)^2 < 9x^2 + y^2 \Rightarrow 2xy < 8x^2$	M1	2.2a
	States that as $x < 0 \Rightarrow 2y > 8x$ $\Rightarrow y > 4x$ *	A1*	2.1
		(2)	
(5 marks)			

November 2021 Question 1 Paper 1

1. $f(x) = ax^3 + 10x^2 - 3ax - 4$

Given that $(x - 1)$ is a factor of $f(x)$, find the value of the constant a .

You must make your method clear.

(3)

ANSWER

Question	Scheme	Marks	AOs
1	$f(1) = a(1)^3 + 10(1)^2 - 3a(1) - 4 = 0$	M1	3.1a
	$6 - 2a = 0 \Rightarrow a = \dots$	M1	1.1b
	$a = 3$	A1	1.1b
		(3)	
(3 marks)			
Notes			

November 2021 Question 15 Paper 1

15. (i) Use proof by exhaustion to show that for $n \in \mathbb{N}, n \leq 4$

$$(n+1)^3 > 3^n \quad (2)$$

(ii) Given that $m^3 + 5$ is odd, use proof by contradiction to show, using algebra, that m is even.

(4)

ANSWER

Question	Scheme	Marks	AOs
15(i)	$n = 1, 2^3 = 8, 3^1 = 3, (8 > 3)$ $n = 2, 3^3 = 27, 3^2 = 9, (27 > 9)$ $n = 3, 4^3 = 64, 3^3 = 27, (64 > 27)$ $n = 4, 5^3 = 125, 3^4 = 81, (125 > 81)$	M1	2.1
	So if $n \leq 4, n \in \mathbb{N}$ then $(n+1)^3 > 3^n$	A1	2.4
		(2)	
(ii)	Begins the proof by negating the statement. "Let m be odd " or "Assume m is not even"	M1	2.4
	Set $m = (2p \pm 1)$ and attempt $m^3 + 5 = (2p \pm 1)^3 + 5 = \dots$	M1	2.1
	$= 8p^3 + 12p^2 + 6p + 6$ AND deduces even	A1	2.2a
	Completes proof which requires reason and conclusion - reason for $8p^3 + 12p^2 + 6p + 6$ being even - acceptable statement such as "this is a contradiction so if $m^3 + 5$ is odd then m must be even"	A1	2.4
		(4)	
(6 marks)			

November 2020 Question 16 Paper 1

16. Prove by contradiction that there are no positive integers p and q such that

$$4p^2 - q^2 = 25$$

(4)

ANSWER

Question	Scheme	Marks	AOs
16	Sets up the contradiction and factorises: There are positive integers p and q such that $(2p+q)(2p-q) = 25$	M1	2.1
	If true then $\begin{array}{ccc} 2p+q=25 & & 2p+q=5 \\ 2p-q=1 & \text{or} & 2p-q=5 \end{array}$ Award for deducing either of the above statements	M1	2.2a
	Solutions are $p = 6.5, q = 12$ or $p = 2.5, q = 0$ Award for one of these	A1	1.1b
	This is a contradiction as there are no integer solutions hence there are no positive integers p and q such that $4p^2 - q^2 = 25$	A1	2.1
		(4)	
	(4 marks)		

Video solution:

<https://youtu.be/gl3zrmAFyJU>

Ch. 2: Functions and Graphs

June 2024 Question 8 Paper 1

8. The functions f and g are defined by

$$f(x) = 4 - 3x^2 \quad x \in \mathbb{R}$$

$$g(x) = \frac{5}{2x - 9} \quad x \in \mathbb{R}, x \neq \frac{9}{2}$$

(a) Find $fg(2)$

(2)

(b) Find g^{-1}

(3)

(c) (i) Find $gf(x)$, giving your answer as a simplified fraction.

(ii) Deduce the range of $gf(x)$.

(3)

The function h is defined by

$$h(x) = 2x^2 - 6x + k \quad x \in \mathbb{R}$$

where k is a constant.

(d) Find the range of values of k for which the equation

$$f(x) = h(x)$$

has no real solutions.

(3)

ANSWER

Question	Scheme	Marks	AOs
8(a)	$fg(2) = 4 - 3\left(\frac{5}{2(2)-9}\right)^2 = \dots$	M1	1.1b
	$fg(2) = 1$	A1	1.1b
		(2)	
(b)	$y = \frac{5}{2x-9} \Rightarrow 2xy - 9y = 5 \Rightarrow 2xy = 5 + 9y$	M1	1.1b
	$2xy = 5 + 9y \Rightarrow x = \frac{5+9y}{2y}$	A1	2.1
	$g^{-1}(x) = \frac{5+9x}{2x} \quad x \neq 0 \{x \in \mathbb{R}\}$	A1	2.5
		(3)	
(c)(i)	$\{gf(x)\} = \frac{5}{2(4-3x^2)-9}$	M1	1.1b
	$= \frac{5}{-1-6x^2} \text{ or } \frac{-5}{1+6x^2}$	A1	1.1b
(ii)	$-5 \leq gf(x) < 0$	B1	2.2a
		(3)	
(d)	$f(x) = h(x) \Rightarrow 4 - 3x^2 = 2x^2 - 6x + k$ $\Rightarrow 5x^2 - 6x + k - 4 = 0$	M1	1.1b
	$b^2 - 4ac < 0 \Rightarrow 36 - 4(5)(k-4) < 0 \Rightarrow k > \dots$	dM1	3.1a
	$k > 5.8 \text{ o.e.}$	A1	2.2a
		(3)	
(11 marks)			

June 2024 Question 6 Paper 1

6.

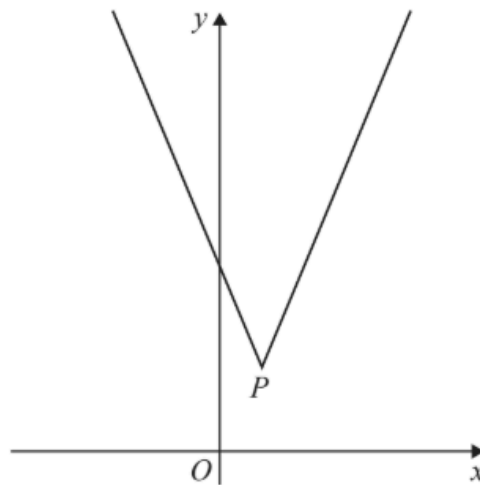


Figure 1

Figure 1 shows a sketch of the graph with equation

$$y = 3|x - 2| + 5$$

The vertex of the graph is at the point P , shown in Figure 1.

(a) Find the coordinates of P .

(2)

(b) Solve the equation

$$16 - 4x = 3|x - 2| + 5$$

(2)

A line l has equation $y = kx + 4$ where k is a constant.

Given that l intersects $y = 3|x - 2| + 5$ at 2 distinct points,

(c) find the range of values of k .

(2)

ANSWER

Question	Scheme	Marks	AOs
6(a)	$x = 2 \text{ or } y = 5$	B1	1.1b
	$P(2, 5)$	B1	2.2a
		(2)	
(b)	$16 - 4x = 3(x - 2) + 5 \Rightarrow x = \dots$	M1	1.1b
	$x = \frac{17}{7}$	A1	2.1
		(2)	
(c)	$k_{\max} = 3 \text{ or } k_{\min} = \frac{"5"-4}{"2"}$	M1	3.1a
	$\frac{1}{2} < k < 3$	A1	2.5
		(2)	
(6 marks)			

June 2023 Question 12 Paper 2

12.

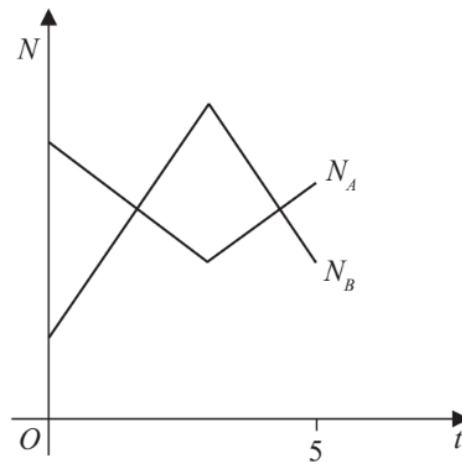


Figure 2

The number of subscribers to two different music streaming companies is being monitored.

The number of subscribers, N_A , in thousands, to **company A** is modelled by the equation

$$N_A = |t - 3| + 4 \quad t \geq 0$$

where t is the time in years since monitoring began.

The number of subscribers, N_B , in thousands, to **company B** is modelled by the equation

$$N_B = 8 - |2t - 6| \quad t \geq 0$$

where t is the time in years since monitoring began.

Figure 2 shows a sketch of the graph of N_A and the graph of N_B over a 5-year period.

Use the equations of the models to answer parts (a), (b), (c) and (d).

- (a) Find the initial difference between the number of subscribers to **company A** and the number of subscribers to **company B**.

(2)

When $t = T$ **company A** reduced its subscription prices and the number of subscribers increased.

- (b) Suggest a value for T , giving a reason for your answer.

(2)

- (c) Find the range of values of t for which $N_A > N_B$ giving your answer in set notation. (5)
- (d) State a limitation of the model used for **company B**. (1)

ANSWER

Question	Scheme	Marks	AOs
12(a)	$N_A - N_B = (3+4) - (8-6) = \dots$	M1	3.4
	5000 (subscribers)	A1	3.2a
		(2)	
(b)	$(T=)3$	B1	3.4
	This was the point when company A had the lowest number of subscribers	B1	2.4
		(2)	
(c)			
	$-t+7=2t+2$ o.e. or $t+1=14-2t$ o.e.	B1	3.1a
	$-t+7=2t+2$ o.e. $\Rightarrow t=\dots$ or $t+1=14-2t$ o.e. $\Rightarrow t=\dots$	M1	3.4
	One of the two critical values $t=\frac{5}{3}$ or $t=\frac{13}{3}$	A1	1.1b
	Chooses the outside region for their two values of t Both of $t < \frac{5}{3}$, $t > \frac{13}{3}$	A1ft	2.2a
	$\left\{t \in \mathbb{R} : t < \frac{5}{3}\right\} \cup \left\{t \in \mathbb{R} : t > \frac{13}{3}\right\}$	A1	2.5
		(5)	
(d)	The number of subscribers will become negative (when $t > 7$)	B1	3.5b
		(1)	
(10 marks)			

June 2023 Question 7 Paper 1

7. The function f is defined by

$$f(x) = 3 + \sqrt{x-2} \quad x \in \mathbb{R} \quad x > 2$$

(a) State the range of f

(1)

(b) Find f^{-1}

(3)

The function g is defined by

$$g(x) = \frac{15}{x-3} \quad x \in \mathbb{R} \quad x \neq 3$$

(c) Find $gf(6)$

(2)

ANSWER

Question	Scheme	Marks	AOs
7(a)	$f(x) > 3$	B1	1.1b
		(1)	
(b)	$y = 3 + \sqrt{x-2} \Rightarrow x = \dots$	M1	1.1b
	$f^{-1}(x) = (x-3)^2 + 2$	A1	1.1b
	$x > 3$	B1ft	2.2a
		(3)	
(c)	$f(6) = 3 + \sqrt{6-2} = 5 \Rightarrow g("5") = \frac{15}{"5"-3} = \dots$	M1	1.1b
	$= \frac{15}{2}$	A1	1.1b
		(2)	
(d)	$3 + \sqrt{a^2 + 2 - 2} = \frac{15}{a-3} \Rightarrow "a^2 - 9 = 15"$	M1	1.1b
	$a = 2\sqrt{6}$	A1	2.2a
		(2)	
(8 marks)			

June 2022 Question 10 Paper 2

10. The function f is defined by

$$f(x) = \frac{8x + 5}{2x + 3} \quad x > -\frac{3}{2}$$

(a) Find $f^{-1}\left(\frac{3}{2}\right)$

(2)

(b) Show that

$$f(x) = A + \frac{B}{2x + 3}$$

where A and B are constants to be found.

(2)

The function g is defined by

$$g(x) = 16 - x^2 \quad 0 \leq x \leq 4$$

(c) State the range of g^{-1}

(1)

(d) Find the range of $f \circ g^{-1}$

(3)

ANSWER

Question	Scheme	Marks	AOs
10(a)	Attempts to solve $\frac{3}{2} = \frac{8x+5}{2x+3} \Rightarrow x = \dots$ Or substitutes $x = \frac{3}{2}$ into $\frac{5-3x}{2x-8}$	M1	3.1a
	$\left(f^{-1}\left(\frac{3}{2}\right) = \right) - \frac{1}{10}$	A1	1.1b
		(2)	
(b)	$\left(\frac{8x+5}{2x+3} = \right) 4 \pm \frac{\dots}{2x+3}$	M1	1.1b
	$\left(\frac{8x+5}{2x+3} = \right) 4 - \frac{7}{2x+3}$	A1	2.1
		(2)	
(c)	$0 \leq g^{-1}(x) \leq 4$	B1	2.2a
		(1)	
(d)	Attempts either boundary $f(0) = \frac{8 \times 0 + 5}{2 \times 0 + 3}$ or $f(4) = \frac{8 \times 4 + 5}{2 \times 4 + 3}$	M1	3.1a
	Attempts both boundaries $f(0) = \frac{8 \times 0 + 5}{2 \times 0 + 3}$ and $f(4) = \frac{8 \times 4 + 5}{2 \times 4 + 3}$	dM1	1.1b
	Range $\frac{5}{3} \leq fg^{-1}(x) \leq \frac{37}{11}$	A1	2.1
		(3)	

Video solution:

<https://youtu.be/EhjZKTGzTwM>

June 2022 Question 1 Paper 2

1. In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

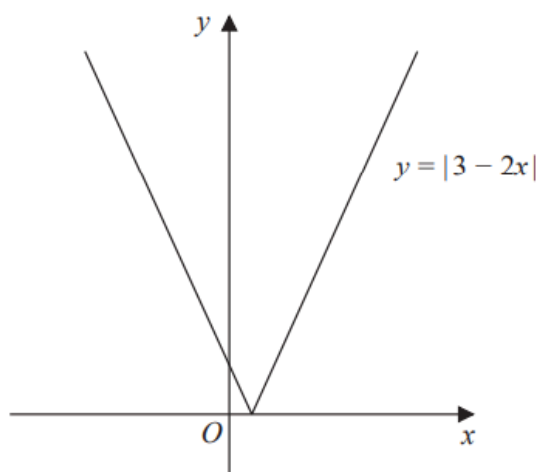


Figure 1

Figure 1 shows a sketch of the graph with equation $y = |3 - 2x|$

Solve

$$|3 - 2x| = 7 + x$$

(4)

ANSWER

Question	Scheme	Marks	AOs
1	For an attempt to solve Either $3 - 2x = 7 + x \Rightarrow x = \dots$ or $2x - 3 = 7 + x \Rightarrow x = \dots$	M1	1.1b
	Either $x = -\frac{4}{3}$ or $x = 10$	A1	1.1b
	For an attempt to solve Both $3 - 2x = 7 + x \Rightarrow x = \dots$ and $2x - 3 = 7 + x \Rightarrow x = \dots$	dM1	1.1b
	For both $x = -\frac{4}{3}$ and $x = 10$ with no extra solutions	A1	1.1b
		(4)	

June 2022 Question 1 Paper 1

1. The point $P(-2, -5)$ lies on the curve with equation $y = f(x)$, $x \in \mathbb{R}$

Find the point to which P is mapped, when the curve with equation $y = f(x)$ is transformed to the curve with equation

- (a) $y = f(x) + 2$ (1)
- (b) $y = |f(x)|$ (1)
- (c) $y = 3f(x - 2) + 2$ (2)

ANSWER

Question	Scheme	Marks	AOs
1 (a)	$(-2, -3)$	B1	1.1b
		(1)	
(b)	$(-2, 5)$	B1	1.1b
		(1)	
(c)	Either $x = 0$ or $y = -13$	M1	1.1b
	$(0, -13)$	A1	1.1b
		(2)	
(4 marks)			
Notes:			

November 2021 Question 2 Paper 1

2. Given that

$$f(x) = x^2 - 4x + 5 \quad x \in \mathbb{R}$$

(a) express $f(x)$ in the form $(x + a)^2 + b$ where a and b are integers to be found.

(2)

The curve with equation $y = f(x)$

- meets the y -axis at the point P
- has a minimum turning point at the point Q

(b) Write down

- (i) the coordinates of P
- (ii) the coordinates of Q

(2)

ANSWER

Question	Scheme	Marks	AOs
2(a)	$f(x) = (x - 2)^2 \pm \dots$	M1	1.2
	$f(x) = (x - 2)^2 + 1$	A1	1.1b
		(2)	
(b)(i)	$P = (0, 5)$	B1	1.1b
(b)(ii)	$Q = (2, 1)$	B1ft	1.1b
		(2)	
(4 marks)			

November 2021 Question 2 Paper 2

2. The functions f and g are defined by

$$f(x) = 7 - 2x^2 \quad x \in \mathbb{R}$$

$$g(x) = \frac{3x}{5x-1} \quad x \in \mathbb{R} \quad x \neq \frac{1}{5}$$

- (a) State the range of f (1)
- (b) Find $gf(1.8)$ (2)
- (c) Find $g^{-1}(x)$ (2)

ANSWER

Question	Scheme	Marks	AOs
2(a)	$y \leq 7$	B1	2.5
		(1)	
(b)	$f(1.8) = 7 - 2 \times 1.8^2 = 0.52 \Rightarrow gf(1.8) = g(0.52) = \frac{3 \times 0.52}{5 \times 0.52 - 1} = \dots$	M1	1.1b
	$gf(1.8) = 0.975$ oe e.g. $\frac{39}{40}$	A1	1.1b
		(2)	
(c)	$y = \frac{3x}{5x-1} \Rightarrow 5xy - y = 3x \Rightarrow x(5y-3) = y$	M1	1.1b
	$(g^{-1}(x) =) \frac{x}{5x-3}$	A1	2.2a
		(2)	
(5 marks)			

November 2021 Question 11 Paper 2

11.

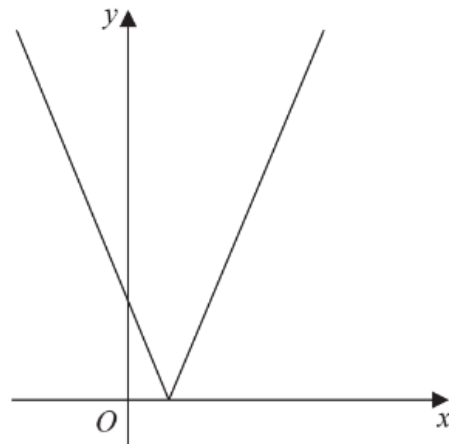


Figure 4

Figure 4 shows a sketch of the graph with equation

$$y = |2x - 3k|$$

where k is a positive constant.

(a) Sketch the graph with equation $y = f(x)$ where

$$f(x) = k - |2x - 3k|$$

stating

- the coordinates of the maximum point
- the coordinates of any points where the graph cuts the coordinate axes

(4)

(b) Find, in terms of k , the set of values of x for which

$$k - |2x - 3k| > x - k$$

giving your answer in set notation.

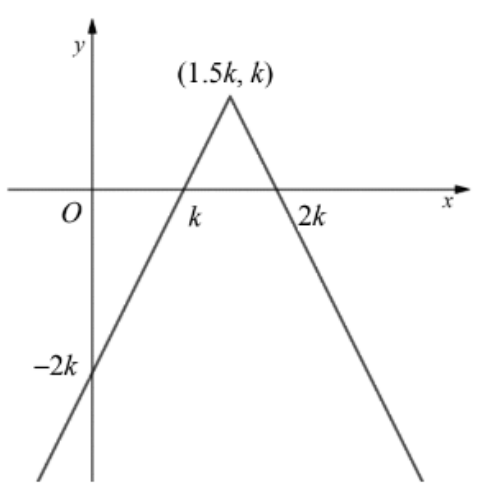
(4)

(c) Find, in terms of k , the coordinates of the minimum point of the graph with equation

$$y = 3 - 5f\left(\frac{1}{2}x\right)$$

(2)

ANSWER

Question	Scheme	Marks	AOs
11(a)			
	Δ shape in any position	B1	1.1b
	Correct x -intercepts or coordinates	B1	1.1b
	Correct y -intercept or coordinates	B1	1.1b
	Correct coordinates for the vertex of a Δ shape	B1	1.1b
		(4)	
(b)	$x = k$	B1	2.2a
	$k - (2x - 3k) = x - k \Rightarrow x = \dots$	M1	3.1a
	$x = \frac{5k}{3}$	A1	1.1b
	Set notation is required here for this mark $\left\{x : x < \frac{5k}{3}\right\} \cap \{x : x > k\}$	A1	2.5
		(4)	
(c)	$x = 3k$ or $y = 3 - 5k$	B1ft	2.2a
	$x = 3k$ and $y = 3 - 5k$	B1ft	2.2a
		(2)	
(10 marks)			

Video solution:

<https://youtu.be/1jKajcJ28-8>

November 2020 Question 4 Paper 1

4. The function f is defined by

$$f(x) = \frac{3x-7}{x-2} \quad x \in \mathbb{R}, x \neq 2$$

(a) Find $f^{-1}(7)$

(2)

(b) Show that $ff(x) = \frac{ax+b}{x-3}$ where a and b are integers to be found.

(3)

ANSWER

Question	Scheme	Marks	AOs
4 (a)	Either attempts $\frac{3x-7}{x-2} = 7 \Rightarrow x = \dots$ Or attempts $f^{-1}(x)$ and substitutes in $x = 7$	M1	3.1a
	$\frac{7}{4}$ oe	A1	1.1b
		(2)	
(b)	Attempts $ff(x) = \frac{3 \times \left(\frac{3x-7}{x-2} \right) - 7}{\left(\frac{3x-7}{x-2} \right) - 2} = \frac{3 \times (3x-7) - 7(x-2)}{3x-7-2(x-2)}$	M1, dM1	1.1b 1.1b
	$= \frac{2x-7}{x-3}$	A1	2.1
		(3)	
(5 marks)			

November 2020 Question 11 Paper 2

11.

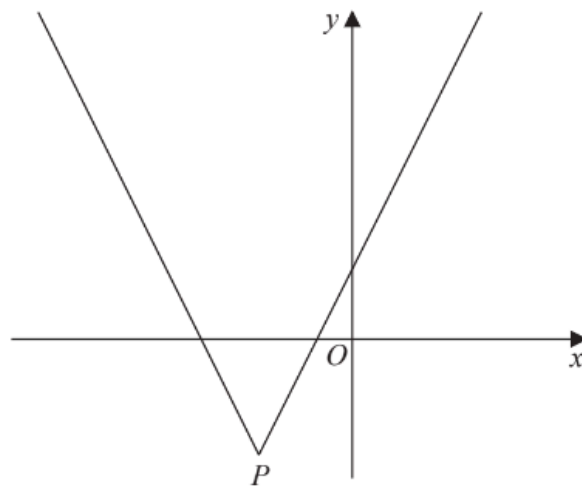


Figure 2

Figure 2 shows a sketch of the graph with equation

$$y = 2|x + 4| - 5$$

The vertex of the graph is at the point P , shown in Figure 2.

(a) Find the coordinates of P .

(2)

(b) Solve the equation

$$3x + 40 = 2|x + 4| - 5$$

(2)

A line l has equation $y = ax$, where a is a constant.

Given that l intersects $y = 2|x + 4| - 5$ at least once,

(c) find the range of possible values of a , writing your answer in set notation.

(3)

ANSWER

Question	Scheme	Marks	AOs
11(a)	$x = -4$ or $y = -5$	B1	1.1b
	$P(-4, -5)$	B1	2.2a
		(2)	
(b)	$3x + 40 = -2(x + 4) - 5 \Rightarrow x = \dots$	M1	1.1b
	$x = -10.6$	A1	2.1
		(2)	
(c)	$a > 2$	B1	2.2a
	$y = ax \Rightarrow -5 = -4a \Rightarrow a = \frac{5}{4}$	M1	3.1a
	$\{a : a \leq 1.25\} \cup \{a : a > 2\}$	A1	2.5
		(3)	
(7 marks)			

Video solution:

<https://youtu.be/6hXXcBz7V7w>

June 2019 Question 5 Paper 1

5. $f(x) = 2x^2 + 4x + 9 \quad x \in \mathbb{R}$

(a) Write $f(x)$ in the form $a(x + b)^2 + c$, where a , b and c are integers to be found. (3)

(b) Sketch the curve with equation $y = f(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point. (3)

(c) (i) Describe fully the transformation that maps the curve with equation $y = f(x)$ onto the curve with equation $y = g(x)$ where

$$g(x) = 2(x - 2)^2 + 4x - 3 \quad x \in \mathbb{R}$$

(ii) Find the range of the function

$$h(x) = \frac{21}{2x^2 + 4x + 9} \quad x \in \mathbb{R} \quad (4)$$

ANSWER

5 (a)	$2x^2 + 4x + 9 = 2(x \pm k)^2 \pm \dots$	$a = 2$	B1	1.1b
	Full method $2x^2 + 4x + 9 = 2(x+1)^2 \pm \dots$	$a = 2$ & $b = 1$	M1	1.1b
	$2x^2 + 4x + 9 = 2(x+1)^2 + 7$		A1	1.1b
			(3)	
(b)		U shaped curve any position but not through (0,0)	B1	1.2
		y - intercept at (0,9)	B1	1.1b
		Minimum at (-1,7)	B1ft	2.2a
			(3)	
(c)	(i) Deduces translation with one correct aspect.		M1	3.1a
	Translate $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$		A1	2.2a
	(ii) $h(x) = \frac{21}{2(x+1)^2 + 7} \Rightarrow$ (maximum) value $\frac{21}{7} (= 3)$		M1	3.1a
	$0 < h(x) \leq 3$		A1ft	1.1b
			(4)	
(10 marks)				

June 2019 Question 6 Paper 2

6.

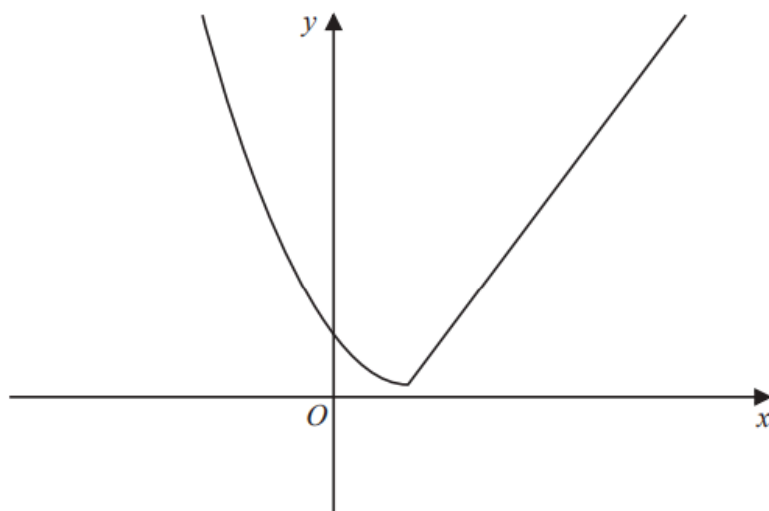


Figure 4

Figure 4 shows a sketch of the graph of $y = g(x)$, where

$$g(x) = \begin{cases} (x-2)^2 + 1 & x \leq 2 \\ 4x - 7 & x > 2 \end{cases}$$

(a) Find the value of $gg(0)$.

(2)

(b) Find all values of x for which

$$g(x) > 28$$

(4)

The function h is defined by

$$h(x) = (x-2)^2 + 1 \quad x \leq 2$$

(c) Explain why h has an inverse but g does not.

(1)

(d) Solve the equation

$$h^{-1}(x) = -\frac{1}{2}$$

(3)

ANSWER

Question	Scheme	Marks	AOs
6 (a)	$gg(0) = g((0-2)^2+1) = g(5) = 4(5) - 7 = 13$	M1	2.1
		A1	1.1b
		(2)	
(b)	Solves either $(x-2)^2 + 1 = 28 \Rightarrow x = \dots$ or $4x - 7 = 28 \Rightarrow x = \dots$	M1	1.1b
	At least one critical value $x = 2 - 3\sqrt{3}$ or $x = \frac{35}{4}$ is correct	A1	1.1b
	Solves both $(x-2)^2 + 1 = 28 \Rightarrow x = \dots$ and $4x - 7 = 28 \Rightarrow x = \dots$	M1	1.1b
	Correct final answer of ' $x < 2 - 3\sqrt{3}$, $x > \frac{35}{4}$ '	A1	2.1
	Note: Writing awrt -3.20 or a truncated -3.19 or a truncated -3.2 in place of $2 - 3\sqrt{3}$ is accepted for any of the A marks	(4)	
(c)	<u>h</u> is a <u>one-one</u> {function (or mapping) so has an inverse} <u>g</u> is a <u>many-one</u> {function (or mapping) so does not have an inverse}	B1	2.4
		(1)	
(d) Way 1	$\left\{ h^{-1}(x) = -\frac{1}{2} \Rightarrow \right\} x = h\left(-\frac{1}{2}\right)$	M1 B1 on open	1.1b
	$x = \left(-\frac{1}{2} - 2\right)^2 + 1$ Note: Condone $x = \left(\frac{1}{2} - 2\right)^2 + 1$	M1	1.1b
	$\Rightarrow x = 7.25$ only cs0	A1	2.2a
		(3)	
(d) Way 2	$\{ \text{their } h^{-1}(x) \} = \pm 2 \pm \sqrt{x \pm 1}$	M1	1.1b
	Attempts to solve $\pm 2 \pm \sqrt{x \pm 1} = -\frac{1}{2} \Rightarrow \pm \sqrt{x \pm 1} = \dots$	M1	1.1b
	$\Rightarrow x = 7.25$ only cs0	A1	2.2a
		(3)	

(10 marks)

June 2018 Question 1 Paper 2

1.

$$g(x) = \frac{2x + 5}{x - 3} \quad x \geq 5$$

(a) Find $gg(5)$.

(2)

(b) State the range of g .

(1)

(c) Find $g^{-1}(x)$, stating its domain.

(3)

ANSWER

Question	Scheme	Marks	AOs
1	$g(x) = \frac{2x+5}{x-3}, x \geq 5$		
(a) Way 1	$g(5) = \frac{2(5)+5}{5-3} = 7.5 \Rightarrow gg(5) = \frac{2("7.5")+5}{"7.5"-3}$	M1	1.1b
	$gg(5) = \frac{40}{9} \left(\text{or } 4\frac{4}{9} \text{ or } 4.\dot{4} \right)$	A1	1.1b
		(2)	
(a) Way 2	$gg(x) = \frac{2\left(\frac{2x+5}{x-3}\right)+5}{\left(\frac{2x+5}{x-3}\right)-3} \Rightarrow gg(5) = \frac{2\left(\frac{2(5)+5}{(5)-3}\right)+5}{\left(\frac{2(5)+5}{(5)-3}\right)-3}$	M1	1.1b
	$gg(5) = \frac{40}{9} \left(\text{or } 4\frac{4}{9} \text{ or } 4.\dot{4} \right)$	A1	1.1b
		(2)	
(b)	{Range:} $2 < y \leq \frac{15}{2}$	B1	1.1b
		(1)	
(c) Way 1	$y = \frac{2x+5}{x-3} \Rightarrow yx-3y=2x+5 \Rightarrow yx-2x=3y+5$	M1	1.1b
	$x(y-2)=3y+5 \Rightarrow x = \frac{3y+5}{y-2} \left\{ \text{or } y = \frac{3x+5}{x-2} \right\}$	M1	2.1
	$g^{-1}(x) = \frac{3x+5}{x-2}, 2 < x \leq \frac{15}{2}$	A1ft	2.5
		(3)	
(c) Way 2	$y = \frac{2x-6+11}{x-3} \Rightarrow y = 2 + \frac{11}{x-3} \Rightarrow y-2 = \frac{11}{x-3}$	M1	1.1b
	$x-3 = \frac{11}{y-2} \Rightarrow x = \frac{11}{y-2} + 3 \left\{ \text{or } y = \frac{11}{x-2} + 3 \right\}$	M1	2.1
	$g^{-1}(x) = \frac{11}{x-2} + 3, 2 < x \leq \frac{15}{2}$	A1ft	2.5
		(3)	
(6 marks)			

June 2018 Question 3 Paper 2

3. (a) “If m and n are irrational numbers, where $m \neq n$, then mn is also irrational.”

Disprove this statement by means of a counter example.

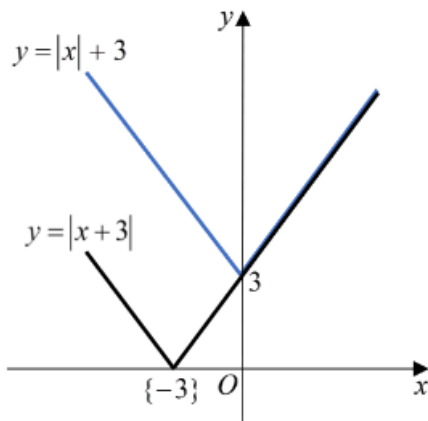
(2)

- (b) (i) Sketch the graph of $y = |x| + 3$

- (ii) Explain why $|x| + 3 \geq |x + 3|$ for all real values of x .

(3)

ANSWER

Question	Scheme	Marks	AOs	
3	Statement: “If m and n are irrational numbers, where $m \neq n$, then mn is also irrational.”			
(a)	E.g. $m = \sqrt{3}, n = \sqrt{12}$	M1	1.1b	
	$\{mn = \} \quad (\sqrt{3})(\sqrt{12}) = 6$ $\Rightarrow$ statement untrue or 6 is not irrational or 6 is rational	A1	2.4	
		(2)		
(b)(i), (ii) Way 1		V shaped graph {reasonably} symmetrical about the y -axis with vertical intercept (0, 3) or 3 stated or marked on the positive y -axis	B1	1.1b
		Superimposes the graph of $y = x + 3 $ on top of the graph of $y = x + 3$	M1	3.1a
	the graph of $y = x + 3$ is either the same or above the graph of $y = x + 3 $ {for corresponding values of x } or when $x \geq 0$, both graphs are equal (or the same) when $x < 0$, the graph of $y = x + 3$ is above the graph of $y = x + 3 $	A1	2.4	
		(3)		
(b)(ii) Way 2	<u>Reason 1</u> When $x \geq 0, x + 3 = x + 3 $	Any one of Reason 1 or Reason 2	M1	3.1a
	<u>Reason 2</u> When $x < 0, x + 3 > x + 3 $	Both Reason 1 and Reason 2	A1	2.4
(5 marks)				

June 2018 Question 6 Paper 2

6.

$$f(x) = -3x^3 + 8x^2 - 9x + 10, \quad x \in \mathbb{R}$$

(a) (i) Calculate $f(2)$

(ii) Write $f(x)$ as a product of two algebraic factors.

(3)

Using the answer to (a)(ii),

(b) prove that there are exactly two real solutions to the equation

$$-3y^6 + 8y^4 - 9y^2 + 10 = 0$$

(2)

(c) deduce the number of real solutions, for $7\pi \leq \theta < 10\pi$, to the equation

$$3 \tan^3 \theta - 8 \tan^2 \theta + 9 \tan \theta - 10 = 0$$

(1)

ANSWER

Question	Scheme	Marks	AOs
6	(a) $f(x) = -3x^3 + 8x^2 - 9x + 10, \quad x \in \mathbb{R}$		
(a)	(i) $\{f(2) = -24 + 32 - 18 + 10 \Rightarrow\} f(2) = 0$	B1	1.1b
	(ii) $\{f(x) =\} (x-2)(-3x^2 + 2x - 5) \text{ or } (2-x)(3x^2 - 2x + 5)$	M1	2.2a
		A1	1.1b
		(3)	
(b)	$-3y^6 + 8y^4 - 9y^2 + 10 = 0 \Rightarrow (y^2 - 2)(-3y^4 + 2y^2 - 5) = 0$		
	Gives a partial explanation by - explaining that $-3y^4 + 2y^2 - 5 = 0$ has no {real} solutions with a reason, e.g. $b^2 - 4ac = (2)^2 - 4(-3)(-5) = -56 < 0$ - or stating that $y^2 = 2$ has 2 {real} solutions or $y = \pm\sqrt{2}$ {only}	M1	2.4
	Complete proof that the given equation has exactly two {real} solutions	A1	2.1
		(2)	
(c)	$3 \tan^3 \theta - 8 \tan^2 \theta + 9 \tan \theta - 10 = 0; \quad 7\pi \leq \theta < 10\pi$		
	{Deduces that} there are 3 solutions	B1	2.2a
		(1)	
(6 marks)			

June 2018 Question 2 Core 3

2. The function f is defined by

$$f(x) = \frac{6}{2x+5} + \frac{2}{2x-5} + \frac{60}{4x^2-25}, \quad x > 4$$

(a) Show that $f(x) = \frac{A}{Bx+C}$ where A , B and C are constants to be found. (4)

(b) Find $f^{-1}(x)$ and state its domain. (3)

ANSWER

Question Number	Scheme	Marks
2(a)	$4x^2 - 25 \rightarrow (2x+5)(2x-5)$ $\frac{6}{2x+5} + \frac{2}{2x-5} + \frac{60}{4x^2-25} = \frac{6(2x-5) + 2(2x+5) + 60}{(2x+5)(2x-5)}$ $= \frac{16x+40}{(2x+5)(2x-5)}$ $= \frac{8(2x+5)}{(2x+5)(2x-5)} = \frac{8}{(2x-5)}$	B1 M1 A1 A1 (4)
(b)	$f(x) = \frac{8}{2x-5} \Rightarrow y = \frac{8}{2x-5} \Rightarrow 2xy - 5y = 8 \Rightarrow x = \frac{8+5y}{2y}$ $\Rightarrow f^{-1}(x) = \frac{8+5x}{2x} \text{ oe}$ $0 < x < \frac{8}{3}$	M1 A1 B1ft (3) (7 marks)

Ch. 3: Sequences and Series

June 2024 Question 4 Paper 2

4. A sequence $u_1, u_2, u_3, \dots$ is defined by

$$u_{n+1} = ku_n - 5$$

$$u_1 = 6$$

where k is a positive constant.

Given that $u_3 = -1$

- (a) show that

$$6k^2 - 5k - 4 = 0 \quad (2)$$

- (b) Hence

- (i) find the value of k ,

- (ii) find the value of $\sum_{r=1}^3 u_r$ (3)

ANSWER

Question	Scheme	Marks	AOs
4(a)	$u_1 = 6 \Rightarrow u_2 = 6k - 5$ $u_2 = 6k - 5 \Rightarrow u_3 = k(6k - 5) - 5$ $\Rightarrow k(6k - 5) - 5 = -1$	M1	1.1b
	$\Rightarrow 6k^2 - 5k - 4 = 0^*$	A1*	2.1
		(2)	
Alternative:			
	$u_3 = -1 \Rightarrow -1 = ku_2 - 5 \Rightarrow u_2 = \frac{4}{k}$ $u_1 = 6 \Rightarrow u_2 = 6k - 5 \Rightarrow \frac{4}{k} = 6k - 5$	M1	1.1b
	$\Rightarrow 6k^2 - 5k - 4 = 0^*$	A1*	2.1
(b)(i)	$k = \frac{4}{3}$	B1	2.2a
(ii)	$k = \frac{4}{3} \Rightarrow u_2 = \frac{4}{3} \times 6 - 5 \Rightarrow \sum_{r=1}^3 u_r = 6 + \frac{4}{3} \times 6 - 5 - 1$	M1	1.1b
	$\sum_{r=1}^3 u_r = 8$	A1	1.1b
		(3)	
(5 marks)			

June 2024 Question 2 Paper 2

2. Jamie takes out an interest-free loan of £8100

Jamie makes a payment every month to pay back the loan.

Jamie repays £400 in month 1, £390 in month 2, £380 in month 3, and so on, so that the amounts repaid each month form an arithmetic sequence.

- (a) Show that Jamie repays £290 in month 12

(1)

After Jamie's N th payment, the loan is completely paid back.

- (b) Show that $N^2 - 81N + 1620 = 0$

(2)

- (c) Hence find the value of N .

(2)

ANSWER

Question	Scheme	Marks	AOs
2(a)	$(u_{12} =) 400 + 11 \times -10 = 290^*$ or e.g. $(u_{12} =) 400 - 110 = 290^*$ or e.g. $(u_{12} =) 400 + (12 - 1) \times -10 = 290^*$ or e.g. $(u_{12} =) 410 + 12 \times -10 = 290^*$	B1*	1.1b
		(1)	
(b)	$8100 = \frac{1}{2} N (2 \times 400 + (N - 1) \times -10)$ or e.g. $8100 = \frac{1}{2} N (400 + 400 + (N - 1) \times -10)$	M1	1.1b
	$8100 = \frac{1}{2} N (2 \times 400 + (N - 1) \times -10)$ $\Rightarrow 16200 = 800N - 10N^2 + 10N$ or e.g. $\Rightarrow 8100 = 400N - 5N^2 + 5N$ $\Rightarrow N^2 - 81N + 1620 = 0^*$	A1*	2.1
		(2)	
(c)	$N^2 - 81N + 1620 = 0 \Rightarrow (N - 45)(N - 36) = 0 \Rightarrow N = 45, 36$	M1	1.1b
	$(N =) 36$	A1	2.3
		(2)	
(5 marks)			

June 2024 Question 9 Paper 1

9. The first 3 terms of a geometric sequence are

$$3^{4k-5} \quad 9^{7-2k} \quad 3^{2(k-1)}$$

where k is a constant.

(a) Using algebra and making your reasoning clear, prove that $k = \frac{5}{2}$

(3)

(b) Hence find the sum to infinity of the geometric sequence.

(3)

ANSWER

Question	Scheme	Marks	AOs
9(a)	$\frac{9^{7-2k}}{3^{4k-5}} = \frac{3^{2(k-1)}}{9^{7-2k}} \Rightarrow \frac{3^{2(7-2k)}}{3^{4k-5}} = \frac{3^{2(k-1)}}{3^{2(7-2k)}}$	M1	3.1a
	$(3^{2(7-2k)})^2 = 3^{4k-5} \times 3^{2(k-1)}$ $\Rightarrow 28 - 8k = 6k - 7 \Rightarrow k = \dots$	dM1	1.1b
	$k = \frac{5}{2}^*$	A1*	2.1
		(3)	
(b)	$a = 3^{4(2.5)-5}$ and $r = \frac{9^{7-2(2.5)}}{3^{4(2.5)-5}} \Rightarrow$ one of $a = 243$ or $r = \frac{1}{3}$	M1	2.2a
	$S_{\infty} = \frac{a}{1-r} = \frac{"243"}{1-" \frac{1}{3} "}$	M1	1.1b
	$S_{\infty} = \frac{729}{2} (364.5)$ cao	A1	1.1b
		(3)	
(6 marks)			

June 2023 Question 2 Paper 2

2. A sequence $u_1, u_2, u_3, \dots$ is defined by

$$u_1 = 35$$

$$u_{n+1} = u_n + 7 \cos\left(\frac{n\pi}{2}\right) - 5(-1)^n$$

(a) (i) Show that $u_2 = 40$

(ii) Find the value of u_3 and the value of u_4

(3)

Given that the sequence is periodic with order 4

(b) (i) write down the value of u_5

(ii) find the value of $\sum_{r=1}^{25} u_r$

(3)

ANSWER

Question	Scheme	Marks	AOs
2(a)(i)	e.g. $(u_2 =) 35 + 7 \cos\left(\frac{\pi}{2}\right) - 5(-1)^1 = 40$ *	B1*	2.1
(ii)	$u_3 = 40 + 7 \cos\left(\frac{2\pi}{2}\right) - 5(-1)^2 (= 28)$ or $u_4 = "28" + 7 \cos\left(\frac{3\pi}{2}\right) - 5(-1)^3 (= 33)$	M1	1.1b
	$u_3 = 28$ and $u_4 = 33$	A1	1.1b
		(3)	
(b)(i)	$(u_5 =) 35$	B1	2.2a
(ii)	e.g. $\sum_{r=1}^{25} u_r = 6(35 + 40 + "28" + "33") + 35$	M1	3.1a
	$= 851$	A1	1.1b
		(3)	
(6 marks)			

June 2023 Question 9 Paper 1

9. The first three terms of a geometric sequence are

$$3k + 4 \quad 12 - 3k \quad k + 16$$

where k is a constant.

(a) Show that k satisfies the equation

$$3k^2 - 62k + 40 = 0 \quad (2)$$

Given that the sequence converges,

(b) (i) find the value of k , giving a reason for your answer,

(ii) find the value of S_{∞} (5)

ANSWER

Question	Scheme	Marks	AOs
9(a)	$\frac{12-3k}{3k+4} = \frac{k+16}{12-3k}$	M1	3.1a
	$3k^2 - 62k + 40 = 0 \quad *$	A1*	1.1b
		(2)	
(b)(i)	$3k^2 - 62k + 40 = 0 \Rightarrow k = \dots$	M1	1.1b
	States $k = 20$ and gives a reason e.g. that this gives a values of r such that $ r < 1$	A1	3.2a
	$a = 64$ and $r = -\frac{3}{4}$ (or allow $a = 6$ and $r = \frac{5}{3}$)	B1	1.1b
	$S_{\infty} = \frac{"64"}{1 - "(-\frac{3}{4})"} = \dots$	M1	3.1a
	$S_{\infty} = \frac{256}{7}$	A1	1.1b
		(5)	
(7 marks)			

June 2022 Question 15 Paper 2

15. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Given that the first three terms of a geometric series are

$$12 \cos \theta \quad 5 + 2 \sin \theta \quad \text{and} \quad 6 \tan \theta$$

- (a) show that

$$4 \sin^2 \theta - 52 \sin \theta + 25 = 0 \quad (3)$$

Given that θ is an obtuse angle measured in radians,

- (b) solve the equation in part (a) to find the exact value of θ (2)

- (c) show that the sum to infinity of the series can be expressed in the form

$$k(1 - \sqrt{3})$$

where k is a constant to be found. (5)

ANSWER

Question	Scheme	Marks	AOs
15(a)	Uses the common ratio $\frac{5+2\sin\theta}{12\cos\theta} = \frac{6\tan\theta}{5+2\sin\theta}$ o.e.	M1	3.1a
	Cross multiplies and uses $\tan\theta \times \cos\theta = \sin\theta$ $(5+2\sin\theta)^2 = 6 \times 12 \sin\theta$	dM1	1.1b
	Proceeds to given answer $25 + 20\sin\theta + 4\sin^2\theta = 72\sin\theta$ $\Rightarrow 4\sin^2\theta - 52\sin\theta + 25 = 0$ *	A1*	2.1
		(3)	
(b)	$4\sin^2\theta - 52\sin\theta + 25 = 0 \Rightarrow \sin\theta = \frac{1}{2}\left(\frac{25}{2}\right)$	M1	1.1b
	$\theta = \frac{5\pi}{6}$	A1	1.2
		(2)	
(c)	Attempts a value for either a or r e.g. $a = 12\cos\theta = 12 \times -\frac{\sqrt{3}}{2}$ or $r = \frac{5+2\sin\theta}{12\cos\theta} = \frac{5+2 \times \frac{1}{2}}{12 \times -\frac{\sqrt{3}}{2}}$	M1	3.1a
	" a " = $-6\sqrt{3}$ and " r " = $-\frac{1}{\sqrt{3}}$ o.e.	A1	1.1b
	Uses $S_{\infty} = \frac{a}{1-r} = \frac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}$	dM1	2.1
	Rationalises denominator $S_{\infty} = \frac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}} = \frac{-18}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$	ddM1	1.1b
	$(S_{\infty}) = 9(1-\sqrt{3})$	A1	2.1
		(5)	
(10 marks)			

June 2022 Question 13 Paper 1

13. (i) In an arithmetic series, the first term is a and the common difference is d .

Show that

$$S_n = \frac{n}{2} [2a + (n-1)d] \quad (3)$$

- (ii) James saves money over a number of weeks to buy a printer that costs £64

He saves £10 in week 1, £9.20 in week 2, £8.40 in week 3 and so on, so that the weekly amounts he saves form an arithmetic sequence.

Given that James takes n weeks to save exactly £64

- (a) show that

$$n^2 - 26n + 160 = 0 \quad (2)$$

- (b) Solve the equation

$$n^2 - 26n + 160 = 0 \quad (1)$$

- (c) Hence state the number of weeks James takes to save enough money to buy the printer, giving a brief reason for your answer.

(1)

ANSWER

Question	Scheme	Marks	AOs
13 (i)	States that $S = a + (a + d) + \dots + (a + (n - 1)d)$	B1	1.1a
	$S = a + (a + d) + (a + (n - 1)d)$ $S = (a + (n - 1)d) + (a + (n - 2)d) + \dots + a$	M1	3.1a
	Reaches $2S = n \times (2a + (n - 1)d)$ And so proves that $S = \frac{n}{2}[2a + (n - 1)d]$ *	A1*	2.1
		(3)	
(ii)	(a) $S = 10 + 9.20 + 8.40 + \dots$		
	$64 = \frac{n}{2}(20 - 0.8(n - 1))$ o.e	M1	3.1b
	$128 = 20n - 0.8n^2 + 0.8n$ $0.8n^2 - 20.8n + 128 = 0$ $n^2 - 26n + 160 = 0$ *	A1*	2.1
		(2)	
	(b) $n = 10, 16$	B1	1.1b
		(1)	
	(c) 10 weeks with a minimal correct reason. E.g. • He has saved up the amount by 10 weeks so he would not save for another 6 weeks • You would choose the smaller number • He starts saving negative amounts (in week 14) so 16 does not make sense	B1	2.3
		(1)	
(7 marks)			

Video solution:

<https://youtu.be/0c3gXTSNnBI>

November 2021 Question 3 Paper 1

3. The sequence $u_1, u_2, u_3, \dots$ is defined by

$$u_{n+1} = k - \frac{24}{u_n} \quad u_1 = 2$$

where k is an integer.

Given that $u_1 + 2u_2 + u_3 = 0$

(a) show that

$$3k^2 - 58k + 240 = 0 \quad (3)$$

(b) Find the value of k , giving a reason for your answer.

(2)

(c) Find the value of u_3

(1)

ANSWER

Question	Scheme	Marks	AOs
3(a)	$u_2 = k - 12, u_3 = k - \frac{24}{k-12}$	M1	1.1b
	$u_1 + 2u_2 + u_3 = 0 \Rightarrow 2 + 2(k-12) + k - \frac{24}{k-12} = 0$	dM1	1.1b
	$\Rightarrow 3k - 22 - \frac{24}{k-12} = 0 \Rightarrow (3k-22)(k-12) - 24 = 0$ $\Rightarrow 3k^2 - 36k - 22k + 264 - 24 = 0$ $\Rightarrow 3k^2 - 58k + 240 = 0^*$	A1*	2.1
		(3)	
(b)	$k = 6, \left(\frac{40}{3}\right)$	M1	1.1b
	$k = 6$ as k must be an integer	A1	2.3
		(2)	
(c)	$(u_3 =) 10$	B1	2.2a
		(1)	

(6 marks)

November 2021 Question 5 Paper 1

5. In this question you should show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A company made a profit of £20 000 in its first year of trading, Year 1

A model for future trading predicts that the yearly profit will increase by 8% each year, so that the yearly profits will form a geometric sequence.

According to the model,

- (a) show that the profit for Year 3 will be £23 328 (1)
- (b) find the first year when the yearly profit will exceed £65 000 (3)
- (c) find the total profit for the first 20 years of trading, giving your answer to the nearest £1000 (2)

ANSWER

Question	Scheme	Marks	AOs
5(a)	$u_3 = £20\,000 \times 1.08^2 = (£)23\,328^*$	B1*	1.1b
		(1)	
(b)	$20\,000 \times 1.08^{n-1} > 65\,000$	M1	1.1b
	$1.08^{n-1} > \frac{13}{4} \Rightarrow n-1 > \frac{\ln(3.25)}{\ln(1.08)}$ or e.g. $1.08^{n-1} > \frac{13}{4} \Rightarrow n-1 > \log_{1.08}\left(\frac{13}{4}\right)$	M1	3.1b
	Year 17	A1	3.2a
		(3)	
(c)	$S_{20} = \frac{20\,000(1-1.08^{20})}{1-1.08}$	M1	3.4
	Awrt (£) 915 000	A1	1.1b
		(2)	
(6 marks)			

November 2021 Question 1 Paper 2

1. In an arithmetic series

- the first term is 16
- the 21st term is 24

(a) Find the common difference of the series.

(2)

(b) Hence find the sum of the first 500 terms of the series.

(2)

ANSWER

Question	Scheme	Marks	AOs
1(a)	$16 + (21 - 1) \times d = 24 \Rightarrow d = \dots$	M1	1.1b
	$d = 0.4$	A1	1.1b
	Answer only scores both marks.		
		(2)	
(b)	$S_n = \frac{1}{2}n\{2a + (n-1)d\} \Rightarrow S_{500} = \frac{1}{2} \times 500\{2 \times 16 + 499 \times "0.4"\}$	M1	1.1b
	$= 57900$	A1	1.1b
	Answer only scores both marks		
		(2)	

November 2021 Question 9 Paper 2

9. Show that

$$\sum_{n=2}^{\infty} \left(\frac{3}{4}\right)^n \cos(180n)^{\circ} = \frac{9}{28}$$

(3)

ANSWER

Question	Scheme	Marks	AOs
9	$a = \left(\frac{3}{4}\right)^2 \text{ or } a = \frac{9}{16}$ or $r = -\frac{3}{4}$	B1	2.2a
	$\sum_{n=2}^{\infty} \left(\frac{3}{4}\right)^n \cos(180n)^{\circ} = \frac{\frac{9}{16}}{1 - \left(-\frac{3}{4}\right)} = \dots$	M1	3.1a
	$= \frac{9}{28}^*$	A1*	1.1b
		(3)	

November 2020 Question 5 Paper 1

5. A car has six forward gears.

The fastest speed of the car

- in 1st gear is 28 km h^{-1}
- in 6th gear is 115 km h^{-1}

Given that the fastest speed of the car in successive gears is modelled by an **arithmetic sequence**,

(a) find the fastest speed of the car in 3rd gear.

(3)

Given that the fastest speed of the car in successive gears is modelled by a **geometric sequence**,

(b) find the fastest speed of the car in 5th gear.

(3)

ANSWER

Question	Scheme	Marks	AOs
5 (a)	Uses $115 = 28 + 5d \Rightarrow d = (17.4)$	M1	3.1b
	Uses $28 + 2 \times "17.4" = \dots$	M1	3.4
	$= 62.8 \text{ (km h}^{-1} \text{)}$	A1	1.1b
		(3)	
(b)	Uses $115 = 28r^5 \Rightarrow r = (1.3265)$	M1	3.1b
	Uses $28 \times "1.3265^4" = \dots$ or $\frac{115}{"1.3265"}$	M1	3.4
	$= 86.7 \text{ (km h}^{-1} \text{)}$	A1	1.1b
		(3)	
(6 marks)			

Video solution:

<https://youtu.be/Tl827wheRr0>

Arancha Ruiz

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November 2020 Question 13 Paper 1

13. A sequence of numbers $a_1, a_2, a_3, \dots$ is defined by

$$a_{n+1} = \frac{k(a_n + 2)}{a_n} \quad n \in \mathbb{N}$$

where k is a constant.

Given that

- the sequence is a periodic sequence of order 3
- $a_1 = 2$

(a) show that

$$k^2 + k - 2 = 0 \quad (3)$$

(b) For this sequence explain why $k \neq 1$ (1)

(c) Find the value of

$$\sum_{r=1}^{80} a_r \quad (3)$$

ANSWER

Question	Scheme	Marks	AOs
13 (a)	Uses the sequence formula $a_{n+1} = \frac{k(a_n + 2)}{a_n}$ once with $a_1 = 2$	M1	1.1b
	$(a_1 = 2), a_2 = 2k, a_3 = k + 1, a_4 = \frac{k(k+3)}{k+1}$ Finds four consecutive terms and sets a_4 equal to a_1 (oe)	M1	3.1a
	$\frac{k(k+3)}{k+1} = 2 \Rightarrow k^2 + 3k = 2k + 2 \Rightarrow k^2 + k - 2 = 0$ *	A1*	2.1
		(3)	
(b)	States that when $k = 1$, all terms are the same and concludes that the sequence does not have a period of order 3	B1	2.3
		(1)	
(c)	Deduces the repeating terms are $a_{1/4} = 2, a_{2/5} = -4, a_{3/6} = -1,$	B1	2.2a
	$\sum_{n=1}^{80} a_k = 26 \times (2 + -4 + -1) + 2 + -4$	M1	3.1a
	$= -80$	A1	1.1b
		(3)	
(7 marks)			

November 2020 Question 15 Paper 2

15. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A geometric series has common ratio r and first term a .

Given $r \neq 1$ and $a \neq 0$

(a) prove that

$$S_n = \frac{a(1 - r^n)}{1 - r} \quad (4)$$

Given also that S_{10} is four times S_5

(b) find the exact value of r . (4)

ANSWER

Question	Scheme	Marks	AOs
15(a)	$S_n = a + ar + ar^2 + \dots + ar^{n-1}$	B1	1.2
	$rS_n = ar + ar^2 + ar^3 + \dots + ar^n \Rightarrow S_n - rS_n = \dots$	M1	2.1
	$S_n - rS_n = a - ar^n$	A1	1.1b
	$S_n(1-r) = a(1-r^n) \Rightarrow S_n = \frac{a(1-r^n)}{(1-r)}^*$	A1*	2.1
		(4)	
(b)	$\frac{a(1-r^{10})}{1-r} = 4 \times \frac{a(1-r^5)}{1-r}$ or $4 \times \frac{a(1-r^{10})}{1-r} = \frac{a(1-r^5)}{1-r}$ Equation in r^{10} and r^5 (and possibly $1-r$)	M1	3.1a
	$1-r^{10} = 4(1-r^5)$	A1	1.1b
	$r^{10} - 4r^5 + 3 = 0 \Rightarrow (r^5 - 1)(r^5 - 3) = 0 \Rightarrow r^5 = \dots$ or e.g. $1-r^{10} = 4(1-r^5) \Rightarrow (1-r^5)(1+r^5) = 4(1-r^5) \Rightarrow r^5 = \dots$	dM1	2.1
	$r = \sqrt[5]{3}$ oe only	A1	1.1b
		(4)	
(8 marks)			

June 2019 Question 11 Paper 1

11. A competitor is running a 20 kilometre race.

She runs each of the first 4 kilometres at a steady pace of 6 minutes per kilometre.
After the first 4 kilometres, she begins to slow down.

In order to estimate her finishing time, the time that she will take to complete each subsequent kilometre is modelled to be 5% greater than the time that she took to complete the previous kilometre.

Using the model,

(a) show that her time to run the first 6 kilometres is estimated to be 36 minutes 55 seconds, (2)

(b) show that her estimated time, in minutes, to run the r th kilometre, for $5 \leq r \leq 20$, is

$$6 \times 1.05^{r-4} \quad (1)$$

(c) estimate the total time, in minutes and seconds, that she will take to complete the race. (4)

ANSWER

Question	Scheme	Marks	AOs
11 (a)	Total time for 6 km = 24 minutes + $6 \times 1.05 + 6 \times 1.05^2$ minutes	M1	3.4
	= 36.915 minutes = 36 minutes 55 seconds *	A1*	1.1b
		(2)	
(b)	5th km is $6 \times 1.05 = 6 \times 1.05^1$ 6th km is $6 \times 1.05 \times 1.05 = 6 \times 1.05^2$ 7th km is $6 \times 1.05 \times 1.05 \times 1.05 = 6 \times 1.05^3$ Hence the time for the r^{th} km is $6 \times 1.05^{r-4}$	B1	3.4
		(1)	
(c)	Attempts the total time for the race = Eg. 24 minutes + $\sum_{r=5}^{r=20} 6 \times 1.05^{r-4}$ minutes	M1	3.1a
	Uses the series formula to find an allowable sum Eg. Time for 5 th to 20 th km = $\frac{6.3(1.05^{16} - 1)}{1.05 - 1} = (149.04)$	M1	3.4
	Correct calculation that leads to the total time Eg. Total time = $24 + \frac{6.3(1.05^{16} - 1)}{1.05 - 1}$	A1	1.1b
	Total time = awrt 173 minutes and 3 seconds	A1	1.1b
		(4)	
(7 marks)			

June 2019 Question 8 Paper 2

8. (i) Find the value of

$$\sum_{r=4}^{\infty} 20 \times \left(\frac{1}{2}\right)^r$$

(3)

(ii) Show that

$$\sum_{n=1}^{48} \log_5 \left(\frac{n+2}{n+1} \right) = 2$$

(3)

ANSWER

Question	Scheme	Marks	AOs
8 (i) Way 1	$\sum_{r=4}^{\infty} 20 \times \left(\frac{1}{2}\right)^r = 20\left(\frac{1}{2}\right)^4 + 20\left(\frac{1}{2}\right)^5 + 20\left(\frac{1}{2}\right)^6 + \dots$		
	$= \frac{20\left(\frac{1}{2}\right)^4}{1 - \frac{1}{2}}$	M1	1.1b
	$\{= (1.25)(2)\} = 2.5 \text{ o.e.}$	M1	3.1a
		A1	1.1b
		(3)	
(i) Way 2	$\sum_{r=4}^{\infty} 20 \times \left(\frac{1}{2}\right)^r = \sum_{r=1}^{\infty} 20 \times \left(\frac{1}{2}\right)^r - \sum_{r=1}^3 20 \times \left(\frac{1}{2}\right)^r$		
	$= \frac{10}{1 - \frac{1}{2}} - (10 + 5 + 2.5) \text{ or } = \frac{10}{1 - \frac{1}{2}} - \frac{10(1 - (\frac{1}{2})^3)}{1 - \frac{1}{2}}$	M1	1.1b
	$\{= 20 - 17.5\} = 2.5 \text{ o.e.}$	M1	3.1a
		A1	1.1b
		(3)	
(ii) Way 1	$\left\{ \sum_{n=1}^{48} \log_5 \left(\frac{n+2}{n+1} \right) \right\}$		
	$= \log_5 \left(\frac{3}{2} \right) + \log_5 \left(\frac{4}{3} \right) + \dots + \log_5 \left(\frac{50}{49} \right) = \log_5 \left(\frac{3}{2} \times \frac{4}{3} \times \dots \times \frac{50}{49} \right)$	M1	1.1b
	$= \log_5 \left(\frac{50}{2} \right) \text{ or } \log_5(25) = 2^*$	M1	3.1a
		A1*	2.1
		(3)	
(ii) Way 2	$\left\{ \sum_{n=1}^{48} \log_5 \left(\frac{n+2}{n+1} \right) \right\} = \sum_{n=1}^{48} (\log_5(n+2) - \log_5(n+1))$		
	$= (\log_5 3 + \log_5 4 + \dots + \log_5 50) - (\log_5 2 + \log_5 3 + \dots + \log_5 49)$	M1	1.1b
	$= \log_5 50 - \log_5 2 \text{ or } \log_5 \left(\frac{50}{2} \right) \text{ or } \log_5(25) = 2^*$	M1	3.1a
		A1*	2.1
		(3)	

(6 marks)

June 2018 Question 4 Paper 2

4. (i) Show that $\sum_{r=1}^{16} (3 + 5r + 2^r) = 131\,798$ (4)

(ii) A sequence $u_1, u_2, u_3, \dots$ is defined by

$$u_{n+1} = \frac{1}{u_n}, \quad u_1 = \frac{2}{3}$$

Find the exact value of $\sum_{r=1}^{100} u_r$ (3)

ANSWER

Question	Scheme	Marks	AOs
4	(i) $\sum_{r=1}^{16} (3+5r+2^r) = 131\,798$; (ii) $u_1, u_2, u_3, \dots, u_{n+1} = \frac{1}{u_n}, u_1 = \frac{2}{3}$		
(i) Way 1	$\left\{ \sum_{r=1}^{16} (3+5r+2^r) \right\} = \sum_{r=1}^{16} (3+5r) + \sum_{r=1}^{16} (2^r)$	M1	3.1a
	$= \frac{16}{2}(2(8)+15(5)) + \frac{2(2^{16}-1)}{2-1}$	M1	1.1b
	$= 728 + 131\,070 = 131\,798 *$	A1*	2.1
		(4)	
(i) Way 2	$\left\{ \sum_{r=1}^{16} (3+5r+2^r) \right\} = \sum_{r=1}^{16} 3 + \sum_{r=1}^{16} (5r) + \sum_{r=1}^{16} (2^r)$	M1	3.1a
	$= (3 \times 16) + \frac{16}{2}(2(5)+15(5)) + \frac{2(2^{16}-1)}{2-1}$	M1	1.1b
	$= 48 + 680 + 131\,070 = 131\,798 *$	A1*	2.1
		(4)	
(i) Way 3	Sum = $10 + 17 + 26 + 39 + 60 + 97 + 166 + 299 + 560 + 1077 + 2106$ $+ 4159 + 8260 + 16457 + 32846 + 65619 = 131\,798 *$	M1	3.1a
		M1	1.1b
		M1	1.1b
		A1*	2.1
		(4)	
(ii)	$\left\{ u_1 = \frac{2}{3} \right\}, u_2 = \frac{3}{2}, u_3 = \frac{2}{3}, \dots$ (can be implied by later working)	M1	1.1b
	$\left\{ \sum_{r=1}^{100} u_r = \right\} 50\left(\frac{2}{3}\right) + 50\left(\frac{3}{2}\right)$ or $50\left(\frac{2}{3} + \frac{3}{2}\right)$	M1	2.2a
	$= \frac{325}{3} \left(\text{or } 108\frac{1}{3} \text{ or } 108.\dot{3} \text{ or } \frac{1300}{12} \text{ or } \frac{650}{6} \right)$	A1	1.1b
		(3)	
(7 marks)			

June 2018 Question 4 Core 1

4. Each year, Andy pays into a savings scheme. In year one he pays in £600. His payments increase by £120 each year so that he pays £720 in year two, £840 in year three and so on, so that his payments form an arithmetic sequence.

(a) Find out how much Andy pays into the savings scheme in year ten.

(2)

Kim starts paying money into a different savings scheme at the same time as Andy. In year one she pays in £130. Her payments increase each year so that she pays £210 in year two, £290 in year three and so on, so that her payments form a different arithmetic sequence.

At the end of year N , Andy has paid, in total, twice as much money into his savings scheme as Kim has paid, in total, into her savings scheme.

(b) Find the value of N .

(5)

ANSWER

Question Number	Scheme		Marks
4 (a)	$a + (n-1)d = 600 + 9 \times 120$	This mark is for: $600 + 9 \times 120$ or $600 + 8 \times 120$	M1
	$= (£)1680$	1680 with or without the “£”	A1
	Answer only scores both marks		
	Listing M1: Lists ten terms starting £600, £720, £840, £960, ... A1: Identifies the 10 th term as (£)1680		
			(2)
(b)	Allow the use of n instead of N throughout in (b)		
	$d = 80$ for Kim	Identifies or uses $d = 80$ for Kim	B1
	$\frac{N}{2} \{2 \times 600 + (N-1) \times 120\}$ OR $\frac{N}{2} \{2 \times 130 + (N-1) \times 80\}$	Attempts a sum formula for Andy or Kim. A correct formula must be seen or implied with: $a = 600, d = 120$ for Andy or $a = 130, d = 80$ for Kim. If B0 was scored, allow M1 here if Kim’s incorrect “ d ” is used.	M1
	$\frac{N}{2} \{2 \times 600 + (N-1) \times 120\} = 2 \times \frac{N}{2} \{2 \times 130 + (N-1) \times 80\}$ A correct equation in any form		A1
	$20N = 360 \Rightarrow N = \dots$	Proceeds to find a value for N . (Allow if it leads to $N < 0$) Dependent on the first method mark and must be an equation that uses Andy’s and Kim’s sum.	dM1
	$(N=)18$	Ignore $N/n = 0$ and if a correct value of N is seen, isw any further reference to years etc.	A1
	See below for listing approach		
	If you see $N = 18$ with no working send to Review		
			(5)
			(7 marks)

June 2018 Question 6 Core 1

6. A sequence $a_1, a_2, a_3, \dots$ is defined by

$$a_1 = 4$$

$$a_{n+1} = \frac{a_n}{a_n + 1}, \quad n \geq 1, n \in \mathbb{N}$$

- (a) Find the values of a_2, a_3 and a_4

Write your answers as simplified fractions.

(3)

Given that

$$a_n = \frac{4}{pn + q}, \text{ where } p \text{ and } q \text{ are constants}$$

- (b) state the value of p and the value of q .

(2)

- (c) Hence calculate the value of N such that $a_N = \frac{4}{321}$

(2)

ANSWER

Question Number	Scheme		Marks
6 (a)	$a_1 = 4 \Rightarrow a_2 = \frac{4}{4+1}$	Attempts to use the given recurrence relation correctly at least once e.g. $a_2 = \frac{4}{4+1}$ or $a_3 = \frac{\text{their } a_2}{(\text{their } a_2)+1}$ or $a_4 = \frac{\text{their } a_3}{(\text{their } a_3)+1}$. May be implied by their term(s).	M1
	$\frac{4}{5}, \frac{4}{9}, \frac{4}{13}$	A1: Two of $\frac{4}{5}, \frac{4}{9}, \frac{4}{13}$ which may be un-simplified. Accept for example $0.8, \frac{0.8}{1.8}, \dots$ or $\frac{4}{5}, \frac{\frac{4}{5}}{1+\frac{4}{5}}, \dots$	A1A1
		A1: $\frac{4}{5}, \frac{4}{9}, \frac{4}{13}$ (Allow 0.8 for $\frac{4}{5}$)	
			(3)
(b)	$p = 4$ or e.g. $4 = \frac{4}{p+q}, \quad \frac{4}{5} = \frac{4}{2p+q}$ $\Rightarrow p = \dots \text{ or } q = \dots$	$a_n = \frac{4}{4n \pm \dots}$ or $p = 4$ OR Uses 2 terms to set up and solve two correct equations for their fractions in p and q to obtain a value for p or a value for q .	M1
	$a_n = \frac{4}{4n-3} \Rightarrow p = 4 \text{ and } q = -3$	Either $a_n = \frac{4}{4n-3}$ OR $p = 4 \text{ and } q = -3$	A1
	Correct answer only scores both marks.		
			(2)
(c)	$\frac{4}{4N-3} = \frac{4}{321} \Rightarrow N = \dots$	Solves their $\frac{4}{pN+q} = \frac{4}{321}$ to obtain a value for N or n .	M1
	$(N=)81$	Cao (ignore what they use for N)	A1
	Allow trial and improvement if 81 is clearly identified and then award both marks following a correct answer in (b) but just trying random values is M0		
			(2)
			(7 marks)

June 2018 Question 6 Core 2

6. A geometric series with common ratio $r = -0.9$ has sum to infinity 10 000

For this series,

(a) find the first term, (2)

(b) find the fifth term, (2)

(c) find the sum of the first twelve terms, giving this answer to the nearest integer. (3)

ANSWER

Question Number	Scheme	Marks
6.(a)	$10000 = \frac{a}{1 - (-0.9)}$ $a = 19000$	M1 A1 (2)
(b)	Use ar^4 $19000 \times (-0.9)^4 = 12465.9 \text{ (accept awrt 12466)}$	M1 A1 (2)
(c)	$S = \frac{a(1 - r^{12})}{1 - r} \text{ or lists and adds their first twelve terms with their } a$ $S = \frac{19000(1 - (-0.9)^{12})}{1 - (-0.9)} \text{ or } S = 10000(1 - (-0.9)^{12})$ $= 7176 \text{ only}$	M1 A1ft A1cso (3) [7]

Ch. 4: Binomial Expansion

June 2024 Question 2 Paper 1

2. (a) Find, in ascending powers of x , the first four terms of the binomial expansion of

$$(1 - 9x)^{\frac{1}{2}}$$

giving each term in simplest form.

(3)

- (b) Give a reason why $x = -\frac{2}{9}$ should **not** be used in the expansion to find an approximation to $\sqrt{3}$

(1)

ANSWER

Question	Scheme	Marks	AOs
2(a)	$\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(\pm 9x)^2$ or $\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(-9x)^3$	M1	1.1b
	$(1-9x)^{\frac{1}{2}} = 1 + \frac{1}{2}(-9x) + \frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(\pm 9x)^2 + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(-9x)^3$	A1	1.1b
	$(1-9x)^{\frac{1}{2}} = 1 - \frac{9}{2}x - \frac{81}{8}x^2 - \frac{729}{16}x^3$	A1	1.1b
		(3)	
(b)	Expansion is valid for $ x < \frac{1}{9}$ and $x = -\frac{2}{9}$ is outside this range.	B1	2.4
		(1)	
(4 marks)			

June 2022 Question 7 Paper 2

7. (a) Find the first four terms, in ascending powers of x , of the binomial expansion of

$$\sqrt{4-9x}$$

writing each term in simplest form.

(4)

A student uses this expansion with $x = \frac{1}{9}$ to find an approximation for $\sqrt{3}$

Using the answer to part (a) and without doing any calculations,

- (b) state whether this approximation will be an overestimate or an underestimate of $\sqrt{3}$ giving a brief reason for your answer.

(1)

ANSWER

Question	Scheme	Marks	AOs
7(a)	$\sqrt{4-9x} = 2(1 \pm \dots)^{\frac{1}{2}}$	B1	1.1b
	$\left(1 - \frac{9x}{4}\right)^{\frac{1}{2}} = \dots + \frac{\frac{1}{2} \times \left(-\frac{1}{2}\right) \left(\frac{9x}{4}\right)^2}{2!} \text{ or } \dots + \frac{\frac{1}{2} \times \left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right) \left(\frac{9x}{4}\right)^3}{3!}$	M1	1.1b
	$1 + \frac{1}{2} \times \left(-\frac{9x}{4}\right) + \frac{\frac{1}{2} \times \left(-\frac{1}{2}\right) \left(-\frac{9x}{4}\right)^2}{2!} + \frac{\frac{1}{2} \times \left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right) \left(-\frac{9x}{4}\right)^3}{3!}$	A1	1.1b
	$\sqrt{4-9x} = 2 - \frac{9x}{4} - \frac{81x^2}{64} - \frac{729x^3}{512}$	A1	1.1b
		(4)	
(b)	States that the approximation will be an overestimate since all terms (after the first one) in the expansion are negative (since $x > 0$)	B1	3.2b
		(1)	
(5 marks)			

November 2021 Question 9 Paper 1

9.

$$f(x) = \frac{50x^2 + 38x + 9}{(5x + 2)^2(1 - 2x)} \quad x \neq -\frac{2}{5} \quad x \neq \frac{1}{2}$$

Given that $f(x)$ can be expressed in the form

$$\frac{A}{5x + 2} + \frac{B}{(5x + 2)^2} + \frac{C}{1 - 2x}$$

where A , B and C are constants

(a) (i) find the value of B and the value of C

(ii) show that $A = 0$

(4)

(b) (i) Use binomial expansions to show that, in ascending powers of x

$$f(x) = p + qx + rx^2 + \dots$$

where p , q and r are simplified fractions to be found.

(ii) Find the range of values of x for which this expansion is valid.

(7)

ANSWER

Question	Scheme	Marks	AOs
9(a)(i)	$50x^2 + 38x + 9 \equiv A(5x+2)(1-2x) + B(1-2x) + C(5x+2)^2$ $\Rightarrow B = \dots$ or $C = \dots$	M1	1.1b
	$B = 1$ and $C = 2$	A1	1.1b
(a)(ii)	E.g. $x = 0$ $x = 0 \Rightarrow 9 = 2A + B + 4C$ $\Rightarrow 9 = 2A + 1 + 8 \Rightarrow A = \dots$	M1	2.1
	$A = 0^*$	A1*	1.1b
		(4)	
(b)(i)	$\frac{1}{(5x+2)^2} = (5x+2)^{-2} = 2^{-2} \left(1 + \frac{5}{2}x\right)^{-2}$ or $(5x+2)^{-2} = 2^{-2} + \dots$	M1	3.1a
	$\left(1 + \frac{5}{2}x\right)^{-2} = 1 - 2\left(\frac{5}{2}x\right) + \frac{-2(-2-1)}{2!}\left(\frac{5}{2}x\right)^2 + \dots$	M1	1.1b
	$2^{-2} \left(1 + \frac{5}{2}x\right)^{-2} = \frac{1}{4} - \frac{5}{4}x + \frac{75}{16}x^2 + \dots$	A1	1.1b
	$\frac{1}{(1-2x)} = (1-2x)^{-1} = 1 + 2x + \frac{-1(-1-1)}{2!}(2x)^2 + \dots$	M1	1.1b
	$\frac{1}{(5x+2)^2} + \frac{2}{1-2x} = \frac{1}{4} - \frac{5}{4}x + \frac{75}{16}x^2 + \dots + 2 + 4x + 8x^2 + \dots$	dM1	2.1
	$= \frac{9}{4} + \frac{11}{4}x + \frac{203}{16}x^2 + \dots$	A1	1.1b
(b)(ii)	$ x < \frac{2}{5}$	B1	2.2a
		(7)	
(11 marks)			

November 2020 Question 1 Paper 1

1. (a) Find the first four terms, in ascending powers of x , of the binomial expansion of

$$(1 + 8x)^{\frac{1}{2}}$$

giving each term in simplest form.

(3)

- (b) Explain how you could use $x = \frac{1}{32}$ in the expansion to find an approximation for $\sqrt{5}$

There is no need to carry out the calculation.

(2)

ANSWER

Question	Scheme	Marks	AOs
1 (a)	$(1 + 8x)^{\frac{1}{2}} = 1 + \frac{1}{2} \times 8x + \frac{\frac{1}{2} \times -\frac{1}{2}}{2!} \times (8x)^2 + \frac{\frac{1}{2} \times -\frac{1}{2} \times -\frac{3}{2}}{3!} \times (8x)^3$	M1 A1	1.1b 1.1b
	$= 1 + 4x - 8x^2 + 32x^3 + \dots$	A1	1.1b
		(3)	
(b)	Substitutes $x = \frac{1}{32}$ into $(1 + 8x)^{\frac{1}{2}}$ to give $\frac{\sqrt{5}}{2}$	M1	1.1b
	Explains that $x = \frac{1}{32}$ is substituted into $1 + 4x - 8x^2 + 32x^3$ and you multiply the result by 2	A1ft	2.4
		(2)	
(5 marks)			

June 2019 Question 4 Paper 1

4. (a) Find the first three terms, in ascending powers of x , of the binomial expansion of

$$\frac{1}{\sqrt{4-x}}$$

giving each coefficient in its simplest form.

(4)

The expansion can be used to find an approximation to $\sqrt{2}$

Possible values of x that could be substituted into this expansion are:

- $x = -14$ because $\frac{1}{\sqrt{4-x}} = \frac{1}{\sqrt{18}} = \frac{\sqrt{2}}{6}$
- $x = 2$ because $\frac{1}{\sqrt{4-x}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
- $x = -\frac{1}{2}$ because $\frac{1}{\sqrt{4-x}} = \frac{1}{\sqrt{\frac{9}{2}}} = \frac{\sqrt{2}}{3}$

- (b) Without evaluating your expansion,

- (i) state, giving a reason, which of the three values of x should not be used

(1)

- (ii) state, giving a reason, which of the three values of x would lead to the most accurate approximation to $\sqrt{2}$

(1)

ANSWER

4 (a)	$\frac{1}{\sqrt{4-x}} = (4-x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}} \times (1 \pm \dots)$	M1	2.1
	Uses a "correct" binomial expansion for their $(1+ax)^n = 1+nax + \frac{n(n-1)}{2}a^2x^2 +$	M1	1.1b
	$\left(1-\frac{x}{4}\right)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right) + \frac{\left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{2} \left(-\frac{x}{4}\right)^2$	A1	1.1b
	$\frac{1}{\sqrt{4-x}} = \frac{1}{2} + \frac{1}{16}x + \frac{3}{256}x^2$	A1	1.1b
		(4)	
(b) (i)	States $x = -14$ and gives a valid reason. Eg explains that the expansion is not valid for $ x > 4$	B1	2.4
		(1)	
(b)(ii)	States $x = -\frac{1}{2}$ and gives a valid reason. Eg. explains that it is closest to zero	B1	2.4
		(1)	
(6 marks)			

June 2018 Question 11 Paper 1

11. (a) Use binomial expansions to show that $\sqrt{\frac{1+4x}{1-x}} \approx 1 + \frac{5}{2}x - \frac{5}{8}x^2$ (6)

A student substitutes $x = \frac{1}{2}$ into both sides of the approximation shown in part (a) in an attempt to find an approximation to $\sqrt{6}$

- (b) Give a reason why the student **should not** use $x = \frac{1}{2}$ (1)

- (c) Substitute $x = \frac{1}{11}$ into

$$\sqrt{\frac{1+4x}{1-x}} = 1 + \frac{5}{2}x - \frac{5}{8}x^2$$

to obtain an approximation to $\sqrt{6}$. Give your answer as a fraction in its simplest form. (3)

ANSWER

Question	Scheme	Marks	AOs
11 (a)	$\sqrt{\frac{1+4x}{1-x}} = (1+4x)^{0.5} \times (1-x)^{-0.5}$	B1	3.1a
	$(1+4x)^{0.5} = 1 + 0.5 \times (4x) + \frac{0.5 \times -0.5}{2} \times (4x)^2$	M1	1.1b
	$(1-x)^{-0.5} = 1 + (-0.5)(-x) + \frac{(-0.5) \times (-1.5)}{2} (-x)^2$	M1	1.1b
	$(1+4x)^{0.5} = 1 + 2x - 2x^2$ and $(1-x)^{-0.5} = 1 + 0.5x + 0.375x^2$ oe	A1	1.1b
	$(1+4x)^{0.5} \times (1-x)^{-0.5} = (1+2x-2x^2 \dots) \times \left(1 + \frac{1}{2}x + \frac{3}{8}x^2 \dots\right)$ $= 1 + \frac{1}{2}x + \frac{3}{8}x^2 + 2x + x^2 - 2x^2 + \dots$ $= A + Bx + Cx^2$	dM1	2.1
(b)	$= 1 + \frac{5}{2}x - \frac{5}{8}x^2 \dots \dots *$	A1*	1.1b
		(6)	
	Expression is valid $ x < \frac{1}{4}$ Should not use $x = \frac{1}{2}$ as $\frac{1}{2} > \frac{1}{4}$	B1	2.3
(c)		(1)	
	Substitutes $x = \frac{1}{11}$ into $\sqrt{\frac{1+4x}{1-x}} \approx 1 + \frac{5}{2}x - \frac{5}{8}x^2$	M1	1.1b
	$\sqrt{\frac{3}{2}} = \frac{1183}{968}$	A1	1.1b
	(so $\sqrt{6}$ is) $\frac{1183}{484}$ or $\frac{2904}{1183}$	A1	2.1
		(3)	
	(10 marks)		

Ch. 5: Radians

June 2024 Question 5 Paper 5

5. Given that θ is small and in radians, use the small angle approximations to find an approximate numerical value of

$$\frac{\theta \tan 2\theta}{1 - \cos 3\theta}$$

(3)

ANSWER

Question	Scheme	Marks	AOs
5	One of $\theta \tan 2\theta = \theta \times 2\theta$ or $1 - \cos 3\theta = 1 - \left(1 - \frac{(3\theta)^2}{2}\right)$ or equivalents.	B1	1.1a
	$\frac{\theta \tan 2\theta}{1 - \cos 3\theta} = \frac{\theta \times 2\theta}{1 - \left(1 - \frac{(3\theta)^2}{2}\right)}$	M1	2.1
	$= \frac{4}{9}$ or exact equivalent.	A1	1.1b
		(3)	
(3 marks)			

June 2024 Question 11 Paper 1

11.

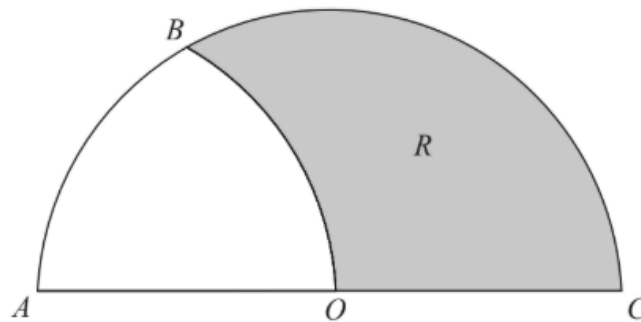


Figure 4

Figure 4 shows the design of a badge.

The shape $ABCOA$ is a semicircle with centre O and diameter 10 cm.

OB is the arc of a circle with centre A and radius 5 cm.

The region R , shown shaded in Figure 4, is bounded by the arc OB , the arc BC and the line OC .

Find the exact area of R .

Give your answer in the form $(a\sqrt{3} + b\pi)\text{ cm}^2$, where a and b are rational numbers.

(4)

ANSWER

Question	Scheme	Marks	AOs
11	Identifies angle $BAO = \frac{\pi}{3}$ or angle $BOC = \frac{2\pi}{3}$	B1	2.2a
	$\text{Area}(\text{segment}) = \frac{1}{2} \times 5^2 \times \frac{\pi}{3} - \frac{1}{2} \times 5^2 \sin \frac{\pi}{3} \left\{ = \frac{25\pi}{6} - \frac{25\sqrt{3}}{4} \right\}$ or $\text{Area}(AOB) = 2 \times \frac{1}{2} \times 5^2 \times \frac{\pi}{3} - \frac{1}{2} \times 5^2 \sin \frac{\pi}{3} \left\{ = \frac{25\pi}{3} - \frac{25\sqrt{3}}{4} \right\}$	M1	2.1
	$\text{Area of } R = \frac{1}{2} \times 5^2 \times \frac{2\pi}{3} - \left(\frac{1}{2} \times 5^2 \times \frac{\pi}{3} - \frac{1}{2} \times 5^2 \sin \frac{\pi}{3} \right)$ or $\text{Area of } R = \frac{1}{2} \times \pi \times 5^2 - \left(\frac{1}{2} \times 5^2 \times \frac{\pi}{3} - \frac{1}{2} \times 5^2 \sin \frac{\pi}{3} + \frac{1}{2} \times 5^2 \times \frac{\pi}{3} \right)$	dM1	3.1a
	$= \frac{25}{4} \sqrt{3} + \frac{25}{6} \pi \text{ (cm}^2\text{)}$	A1	1.1b
		(4)	
(4 marks)			

June 2023 Question 4 Paper 1

4. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation $y = f(x)$ where $x \in \mathbb{R}$

Given that

- $f'(x) = 2x + \frac{1}{2} \cos x$
 - the curve has a stationary point with x coordinate α
 - α is small
- (a) use the small angle approximation for $\cos x$ to estimate the value of α to 3 decimal places.

(3)

The point $P(0, 3)$ lies on C

- (b) Find the equation of the tangent to the curve at P , giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(2)

ANSWER

Question	Scheme	Marks	AOs
4a	$2\alpha + \frac{1}{2}\left(1 - \frac{\alpha^2}{2}\right)$	M1	1.2
	$2\alpha + \frac{1}{2}\left(1 - \frac{\alpha^2}{2}\right) = 0 \Rightarrow 2\alpha + \frac{1}{2} - \frac{\alpha^2}{4} = 0 \Rightarrow \alpha = \dots$	dM1	1.1b
	$\alpha = -0.243$ (3dp) only	A1	2.3
		(3)	
b	$f'(0) = \frac{1}{2}\cos 0 \Rightarrow \dots \Rightarrow y = \dots x + 3$	M1	1.1b
	$y = \frac{1}{2}x + 3$	A1	1.1b
		(2)	
(5 marks)			

June 2023 Question 8 Paper 1

8.

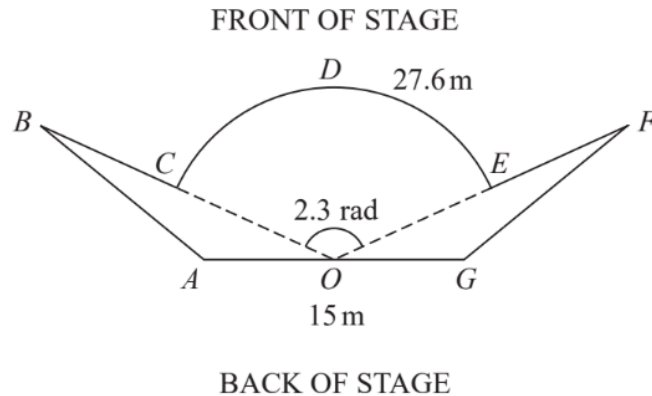


Diagram **NOT**
accurately drawn

Figure 1

Figure 1 shows the plan view of a stage.

The plan view shows two congruent triangles ABO and GFO joined to a sector $OCDEO$ of a circle, centre O , where

- angle $COE = 2.3$ radians
- arc length $CDE = 27.6$ m
- AOG is a straight line of length 15 m

(a) Show that $OC = 12$ m.

(2)

(b) Show that the size of angle AOB is 0.421 radians correct to 3 decimal places.

(2)

Given that the total length of the front of the stage, $BCDEF$, is 35 m,

(c) find the total area of the stage, giving your answer to the nearest square metre.

(6)

ANSWER

Question	Scheme	Marks	AOs
8(a)	$OC \times 2.3 = 27.6$	M1	1.1b
	e.g. $OC = \frac{27.6}{2.3} = 12 \text{ m} *$	A1*	2.1
		(2)	
(b)	e.g. $(2AOB) = \pi - 2.3$	M1	1.1b
	$\frac{\pi - 2.3}{2} \Rightarrow 0.421 \text{ rad} *$	A1*	2.1
		(2)	
(c)	$\text{Area } OCDE = \frac{1}{2} \times 12^2 \times 2.3$	M1	1.1b
	$= 165.6 \text{ (m}^2\text{)} \text{ (accept awrt 166)}$	A1	1.1b
	$(OB) = \frac{35 - 27.6}{2} + 12 = 15.7 \text{ m}$	B1	2.1
	$\text{Area of } OAB \text{ (or } OFG) = \frac{1}{2} \times 15.7 \times 7.5 \times \sin 0.421 \text{ (= 24.0...m}^2\text{)}$	M1	1.1b
	$\text{Total area} = 165.6 + 2 \times 24.1$	dM1	3.1a
	$= \text{awrt } 214 \text{ (m}^2\text{)}$	A1	1.1b
		(6)	
(10 marks)			

November 2021 question 4 Paper 2

4. Given that θ is small and measured in radians, use the small angle approximations to show that

$$4 \sin \frac{\theta}{2} + 3 \cos^2 \theta \approx a + b\theta + c\theta^2$$

where a , b and c are integers to be found.

(3)

ANSWER

Question	Scheme	Marks	AOs
4	Examples: $4 \sin \frac{\theta}{2} \approx 4 \left(\frac{\theta}{2} \right), \quad 3 \cos^2 \theta \approx 3 \left(1 - \frac{\theta^2}{2} \right)^2$ $3 \cos^2 \theta = 3(1 - \sin^2 \theta) \approx 3(1 - \theta^2)$ $3 \cos^2 \theta = 3 \frac{(\cos 2\theta + 1)}{2} \approx \frac{3}{2} \left(1 - \frac{4\theta^2}{2} + 1 \right)$	M1	1.1a
	Examples: $4 \sin \frac{\theta}{2} + 3 \cos^2 \theta \approx 4 \left(\frac{\theta}{2} \right) + 3 \left(1 - \frac{\theta^2}{2} \right)^2$ $4 \sin \frac{\theta}{2} + 3 \cos^2 \theta = 4 \left(\frac{\theta}{2} \right) + 3(1 - \sin^2 \theta) \approx 2\theta + 3(1 - \theta^2)$ $4 \sin \frac{\theta}{2} + 3 \cos^2 \theta = 4 \sin \frac{\theta}{2} + 3 \frac{(\cos 2\theta + 1)}{2} \approx 4 \left(\frac{\theta}{2} \right) + \frac{3}{2} \left(1 - \frac{4\theta^2}{2} + 1 \right)$	dM1	1.1b
	$= 2\theta + 3(1 - \theta^2 + \dots) = 3 + 2\theta - 3\theta^2$	A1	2.1
		(3)	
	(3 marks)		

November 2021 Question 6 Paper 2

6.

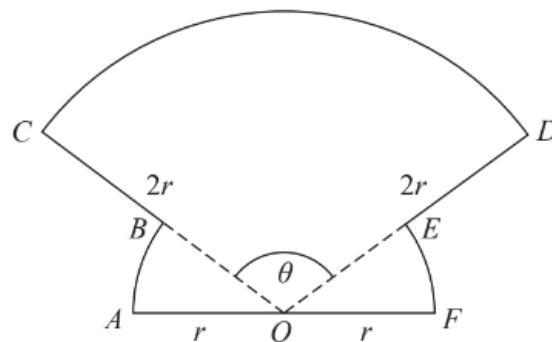


Figure 1

The shape $OABCDEF O$ shown in Figure 1 is a design for a logo.

In the design

- OAB is a sector of a circle centre O and radius r
- sector OFE is congruent to sector OAB
- ODC is a sector of a circle centre O and radius $2r$
- AOF is a straight line

Given that the size of angle COD is θ radians,

(a) write down, in terms of θ , the size of angle AOB

(1)

(b) Show that the area of the logo is

$$\frac{1}{2} r^2 (3\theta + \pi)$$

(2)

(c) Find the perimeter of the logo, giving your answer in simplest form in terms of r , θ and π .

(2)

ANSWER

Question	Scheme	Marks	AOs
6(a)	$\text{Angle } AOB = \frac{\pi - \theta}{2}$	B1	2.2a
		(1)	
(b)	$\text{Area} = 2 \times \frac{1}{2} r^2 \left(\frac{\pi - \theta}{2} \right) + \frac{1}{2} (2r)^2 \theta$	M1	2.1
	$= \frac{1}{2} r^2 \pi - \frac{1}{2} r^2 \theta + 2r^2 \theta = \frac{3}{2} r^2 \theta + \frac{1}{2} r^2 \pi = \frac{1}{2} r^2 (3\theta + \pi)^*$	A1*	1.1b
		(2)	
(c)	$\text{Perimeter} = 4r + 2r \left(\frac{\pi - \theta}{2} \right) + 2r\theta$	M1	3.1a
	$= 4r + r\pi + r\theta$ or e.g. $r(4 + \pi + \theta)$	A1	1.1b
		(2)	
(5 marks)			

November 2020 Question 11 Paper 1

11.

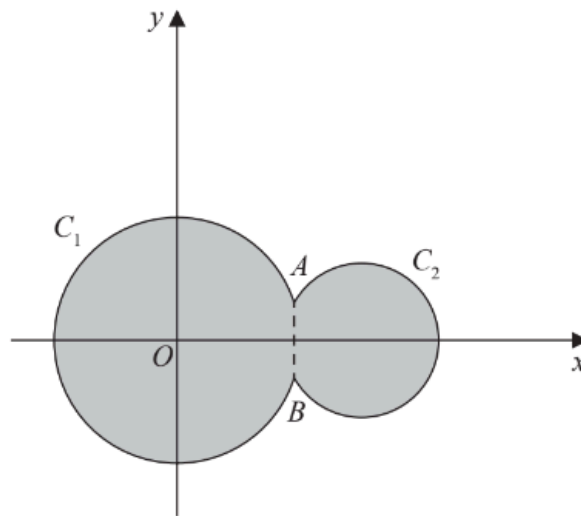


Figure 3

Circle C_1 has equation $x^2 + y^2 = 100$

Circle C_2 has equation $(x - 15)^2 + y^2 = 40$

The circles meet at points A and B as shown in Figure 3.

(a) Show that angle $AOB = 0.635$ radians to 3 significant figures, where O is the origin.

(4)

The region shown shaded in Figure 3 is bounded by C_1 and C_2

(b) Find the perimeter of the shaded region, giving your answer to one decimal place.

(4)

ANSWER

Question	Scheme	Marks	AOs
11 (a)	Solves $x^2 + y^2 = 100$ and $(x - 15)^2 + y^2 = 40$ simultaneously to find x or y E.g. $(x - 15)^2 + 100 - x^2 = 40 \Rightarrow x = \dots$	M1	3.1a
	Either $\Rightarrow -30x + 325 = 40 \Rightarrow x = 9.5$ Or $y = \frac{\sqrt{39}}{2} = \text{awrt } \pm 3.12$	A1	1.1b
	Attempts to find the angle AOB in circle C_1 Eg Attempts $\cos \alpha = \frac{9.5}{10}$ to find α then $\times 2$	M1	3.1a
	Angle $AOB = 2 \times \arccos\left(\frac{9.5}{10}\right) = 0.635 \text{ rads (3sf)}$ *	A1*	2.1
		(4)	
(b)	Attempts $10 \times (2\pi - 0.635) = 56.48$	M1	1.1b
	Attempts to find angle AXB or AXO in circle C_2 (see diagram) E.g. $\cos \beta = \frac{15 - 9.5}{\sqrt{40}} \Rightarrow \beta = \dots$ (Note $AXB = 1.03 \text{ rads}$)	M1	3.1a
	Attempts $10 \times (2\pi - 0.635) + \sqrt{40} \times (2\pi - 2\beta)$	dM1	2.1
	$= 89.7$	A1	1.1b
		(4)	
(8 marks)			

June 2019 Question 2 Paper 1

2.

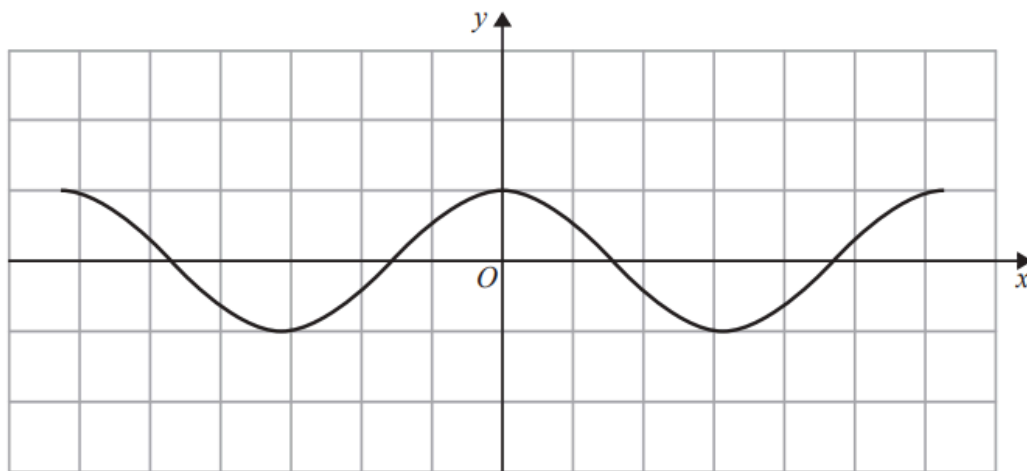


Figure 1

Figure 1 shows a plot of part of the curve with equation $y = \cos x$ where x is measured in radians. Diagram 1, on the opposite page, is a copy of Figure 1.

(a) Use Diagram 1 to show why the equation

$$\cos x - 2x - \frac{1}{2} = 0$$

has only one real root, giving a reason for your answer.

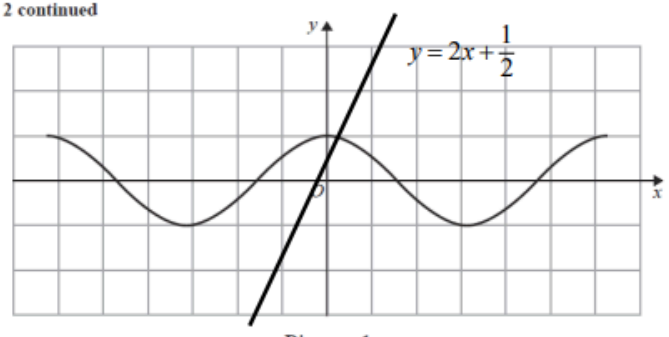
(2)

Given that the root of the equation is α , and that α is small,

(b) use the small angle approximation for $\cos x$ to estimate the value of α to 3 decimal places.

(3)

ANSWER

Question	Scheme	Marks	AOs
2(a)	2 continued  Diagram 1	B1	3.1a
	For an allowable linear graph and explaining that there is only one intersection	B1	2.4
		(2)	
(b)	$\cos x - 2x - \frac{1}{2} = 0 \Rightarrow 1 - \frac{x^2}{2} - 2x - \frac{1}{2} = 0$	M1	1.1b
	Solves their $x^2 + 4x - 1 = 0$	dM1	1.1b
	Allow awrt 0.236 but accept $-2 + \sqrt{5}$	A1	1.1b
		(3)	
(5 marks)			

June 2019 Question 3 Paper 2

3.

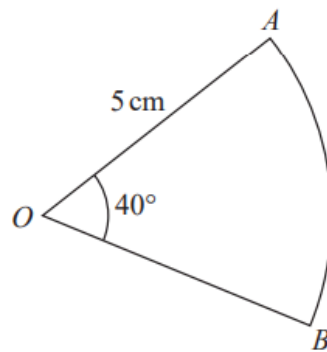
**Figure 1**

Figure 1 shows a sector AOB of a circle with centre O , radius 5 cm and angle $AOB = 40^\circ$

The attempt of a student to find the area of the sector is shown below.

$$\begin{aligned}\text{Area of sector} &= \frac{1}{2} r^2 \theta \\ &= \frac{1}{2} \times 5^2 \times 40 \\ &= 500 \text{ cm}^2\end{aligned}$$

(a) Explain the error made by this student.

(1)

(b) Write out a correct solution.

(2)

ANSWER

Question	Scheme	Marks	AOs
3 (a)	Allow explanations such as • student should have worked in radians • they did not convert degrees to radians • 40 should be in radians • θ should be in radians • angle (or θ) should be $\frac{40\pi}{180}$ or $\frac{2\pi}{9}$ • correct formula is $\pi r^2 \left(\frac{\theta}{360} \right)$ {where θ is in degrees} • correct formula is $\pi r^2 \left(\frac{40}{360} \right)$	B1	2.3
		(1)	
(b) Way 1	{Area of sector = } $\frac{1}{2} (5^2) \left(\frac{2\pi}{9} \right)$	M1	1.1b
	$= \frac{25}{9} \pi \text{ {cm}^2}$ or awrt 8.73 {cm ² }	A1	1.1b
		(2)	
(b) Way 2	{Area of sector = } $\pi (5^2) \left(\frac{40}{360} \right)$	M1	1.1b
	$= \frac{25}{9} \pi \text{ {cm}^2}$ or awrt 8.73 {cm ² }	A1	1.1b
		(2)	
(3 marks)			

June 2018 Question 1 Paper 1

1. Given that θ is small and is measured in radians, use the small angle approximations to find an approximate value of

$$\frac{1 - \cos 4\theta}{2\theta \sin 3\theta}$$

(3)

ANSWER

Question	Scheme	Marks	AOs
1	Attempts either $\sin 3\theta \approx 3\theta$ or $\cos 4\theta \approx 1 - \frac{(4\theta)^2}{2}$ in $\frac{1 - \cos 4\theta}{2\theta \sin 3\theta}$	M1	1.1b
	Attempts both $\sin 3\theta \approx 3\theta$ and $\cos 4\theta \approx 1 - \frac{(4\theta)^2}{2} \rightarrow \frac{1 - \left(1 - \frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta}$ and attempts to simplify	M1	2.1
	$= \frac{4}{3}$ oe	A1	1.1b
		(3)	
(3 marks)			

June 2018 Question 3 Paper 1

3.

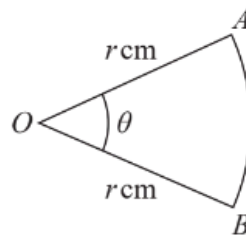


Figure 1

Figure 1 shows a sector AOB of a circle with centre O and radius r cm.

The angle AOB is θ radians.

The area of the sector AOB is 11 cm^2

Given that the perimeter of the sector is 4 times the length of the arc AB , find the exact value of r .

(4)

ANSWER

Question	Scheme	Marks	AOs
3	States or uses $\frac{1}{2}r^2\theta = 11$	B1	1.1b
	States or uses $2r + r\theta = 4r\theta$	B1	1.1b
	Attempts to solve, full method $r = \dots$	M1	3.1a
	$r = \sqrt{33}$	A1	1.1b
			[4]
(4 marks)			

June 2018 Question 4 Core 2

4.

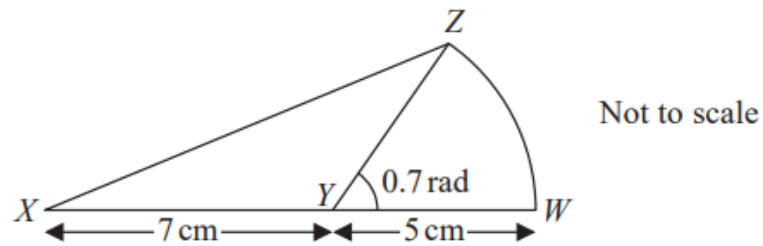


Figure 2

Figure 2 shows a flag $XYWZX$.

The flag consists of a triangle XYZ joined to a sector ZYW of a circle with radius 5 cm and centre Y .

The angle of the sector, angle ZYW , is 0.7 radians.

The points X , Y and W lie on a straight line with $XY = 7$ cm and $YW = 5$ cm.

Find

- (a) the area of the sector ZYW in cm^2 , (2)
- (b) the area of the flag, in cm^2 , to 2 decimal places, (3)
- (c) the length of the perimeter, $XYWZX$, of the flag, in cm to 2 decimal places. (4)

ANSWER

Question Number	Scheme	Marks
4. (a)	Usually answered in radians: Uses Area $ZYW = \frac{1}{2} \times 5^2 \times (\text{angle})$, $= 12.5 \times 0.7 = 8.75$ o.e. (cm ²)	M1 A1 (2)
(b)	Area of triangle $XYZ = \frac{1}{2} \times 7 \times 5 \times \sin Y = (11.273)$ (cm ²) Area of whole flag = "8.75" + "11.273", = 20.02 (cm ²)	M1 M1, A1 (3)
(c)	$(XZ^2) = 7^2 + 5^2 - 2 \times 7 \times 5 \cos(\pi - 0.7)$, Or $(XZ^2) = (7 + 5 \cos 0.7)^2 + (5 \sin 0.7)^2$ Use of arc length formula $s = 5\theta$ (= 3.5) Total perimeter = 12 + "3.5" + "11.293" = 26.79 cm	M1, M1 ddM1 A1 (4)
		(9 marks)

Ch. 6: Trigonometric Functions

June 2024 Question 8 Paper 2

8. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Prove that

$$\frac{1}{\operatorname{cosec} \theta - 1} + \frac{1}{\operatorname{cosec} \theta + 1} \equiv 2 \tan \theta \sec \theta \quad \theta \neq (90n)^\circ, n \in \mathbb{Z} \quad (3)$$

(b) Hence solve, for $0 < x < 90^\circ$, the equation

$$\frac{1}{\operatorname{cosec} 2x - 1} + \frac{1}{\operatorname{cosec} 2x + 1} = \cot 2x \sec 2x$$

Give each answer, in degrees, to one decimal place.

(4)

ANSWER

Question	Scheme	Marks	AOs
8(a) Way 1	$\left(\frac{1}{\operatorname{cosec} \theta - 1} + \frac{1}{\operatorname{cosec} \theta + 1} \right) \frac{\operatorname{cosec} \theta + 1 + \operatorname{cosec} \theta - 1}{(\operatorname{cosec} \theta - 1)(\operatorname{cosec} \theta + 1)}$	B1	1.1b
	$\equiv \frac{2 \operatorname{cosec} \theta}{\operatorname{cosec}^2 \theta - 1} \equiv \frac{2 \operatorname{cosec} \theta}{\cot^2 \theta}$ or e.g. $\equiv \frac{2 \sin \theta}{1 - \sin^2 \theta} = \frac{2 \sin \theta}{\cos^2 \theta}$	M1	1.1b
	$\frac{2 \operatorname{cosec} \theta}{\cot^2 \theta} \equiv \frac{2}{\sin \theta} \times \frac{\sin^2 \theta}{\cos^2 \theta} \equiv 2 \tan \theta \sec \theta^*$ or $\frac{2 \sin \theta}{\cos^2 \theta} \equiv 2 \tan \theta \sec \theta^*$	A1*	2.1
		(3)	

8(b)	$2 \tan 2x \sec 2x = \cot 2x \sec 2x$	B1	2.2a
	$2 \tan 2x \sec 2x - \cot 2x \sec 2x = 0$ $\Rightarrow \sec 2x (2 \tan 2x - \cot 2x) = 0$ $2 \tan 2x - \cot 2x = 0 \Rightarrow 2 \tan 2x = \cot 2x \Rightarrow \tan^2 2x = \frac{1}{2}$ or $2 \tan 2x - \cot 2x = 0 \Rightarrow 2 \frac{\sin 2x}{\cos 2x} = \frac{\cos 2x}{\sin 2x} \Rightarrow 2 \sin^2 2x = \cos^2 2x$ $\Rightarrow 2 \sin^2 2x = 1 - \sin^2 2x \Rightarrow \sin^2 2x = \frac{1}{3}$ or $2(1 - \cos^2 2x) = \cos^2 2x \Rightarrow \cos^2 2x = \frac{2}{3}$ or $2 \tan 2x = \cot 2x \Rightarrow \frac{4 \tan x}{1 - \tan^2 x} = \frac{1 - \tan^2 x}{2 \tan x}$ $\Rightarrow \tan^4 x - 10 \tan^2 x + 1 = 0$ or $2 \tan 2x - \cot 2x = 0 \Rightarrow 2 \frac{\sin 2x}{\cos 2x} = \frac{\cos 2x}{\sin 2x} \Rightarrow 2 \sin^2 2x = \cos^2 2x$ $2 \sin^2 2x = \cos^2 2x \Rightarrow 1 - \cos 4x = \frac{1}{2}(\cos 4x + 1) \Rightarrow \cos 4x = \frac{1}{3}$	M1	2.1
	$2x = \tan^{-1} \frac{1}{\sqrt{2}} = K \Rightarrow x = \frac{K}{2} \quad \text{or} \quad 2x = \sin^{-1} \frac{1}{\sqrt{3}} = K \Rightarrow x = \frac{K}{2}$ or $2x = \cos^{-1} \sqrt{\frac{2}{3}} = K \Rightarrow x = \frac{K}{2}$ or $\tan^2 x = 5 \pm 2\sqrt{6} \Rightarrow x = \tan^{-1}(\sqrt{5 \pm 2\sqrt{6}})$ or $\cos 4x = \frac{1}{3} \Rightarrow 4x = \cos^{-1}\left(\frac{1}{3}\right) = K \Rightarrow x = \frac{1}{4}K$	M1	1.1b
	$x = 17.6^\circ, 72.4^\circ$	A1	1.1b
		(4)	
(7 marks)			

November 2021 Question 10 Paper 1

10. In this question you should show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that $1 + \cos 2\theta + \sin 2\theta \neq 0$ prove that

$$\frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} \equiv \tan \theta$$

(4)

(b) Hence solve, for $0 < x < 180^\circ$

$$\frac{1 - \cos 4x + \sin 4x}{1 + \cos 4x + \sin 4x} = 3 \sin 2x$$

giving your answers to one decimal place where appropriate.

(4)

ANSWER

Question	Scheme	Marks	AOs
10(a)	$\frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} = \frac{1 - (1 - 2\sin^2 \theta) + 2\sin \theta \cos \theta}{1 + \cos 2\theta + \sin 2\theta}$ or $\frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} = \frac{1 - \cos 2\theta + \sin 2\theta}{1 + (2\cos^2 \theta - 1) + 2\sin \theta \cos \theta}$	M1	2.1
	$\frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} = \frac{1 - (1 - 2\sin^2 \theta) + 2\sin \theta \cos \theta}{1 + (2\cos^2 \theta - 1) + 2\sin \theta \cos \theta}$	A1	1.1b
	$= \frac{2\sin^2 \theta + 2\sin \theta \cos \theta}{2\cos^2 \theta + 2\sin \theta \cos \theta} = \frac{2\sin \theta (\sin \theta + \cos \theta)}{2\cos \theta (\cos \theta + \sin \theta)}$	dM1	2.1
	$= \frac{\sin \theta}{\cos \theta} = \tan \theta^*$	A1*	1.1b
		(4)	
(b)	$\frac{1 - \cos 4x + \sin 4x}{1 + \cos 4x + \sin 4x} = 3 \sin 2x \Rightarrow \tan 2x = 3 \sin 2x \quad \text{o.e}$	M1	3.1a
	$\Rightarrow \sin 2x - 3 \sin 2x \cos 2x = 0$ $\Rightarrow \sin 2x (1 - 3 \cos 2x) = 0$ $\Rightarrow (\sin 2x = 0,) \cos 2x = \frac{1}{3}$	A1	1.1b
	$x = 90^\circ, \text{ awrt } 35.3^\circ, \text{ awrt } 144.7^\circ$	A1 A1	1.1b 2.1
		(4)	
(8 marks)			

November 2020 Question 12 Paper 1

12. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\operatorname{cosec} \theta - \sin \theta \equiv \cos \theta \cot \theta \quad \theta \neq (180n)^\circ \quad n \in \mathbb{Z} \quad (3)$$

(b) Hence, or otherwise, solve for $0 < x < 180^\circ$

$$\operatorname{cosec} x - \sin x = \cos x \cot(3x - 50^\circ) \quad (5)$$

ANSWER

Question	Scheme	Marks	AOs
12 (a)	States or uses $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$	B1	1.2
	$\operatorname{cosec} \theta - \sin \theta = \frac{1}{\sin \theta} - \sin \theta = \frac{1 - \sin^2 \theta}{\sin \theta}$	M1	2.1
	$= \frac{\cos^2 \theta}{\sin \theta} = \cos \theta \times \frac{\cos \theta}{\sin \theta} = \cos \theta \cot \theta \quad *$	A1*	2.1
		(3)	
(b)	$\operatorname{cosec} x - \sin x = \cos x \cot(3x - 50^\circ)$ $\Rightarrow \cos x \cot x = \cos x \cot(3x - 50^\circ)$		
	$\cot x = \cot(3x - 50^\circ) \Rightarrow x = 3x - 50^\circ$	M1	3.1a
	$x = 25^\circ$	A1	1.1b
	Also $\cot x = \cot(3x - 50^\circ) \Rightarrow x + 180^\circ = 3x - 50^\circ$	M1	2.1
	$x = 115^\circ$	A1	1.1b
	Deduces $x = 90^\circ$	B1	2.2a
		(5)	
(8 marks)			

November 2020 Question 10 Paper 2

10. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\cos 3A \equiv 4 \cos^3 A - 3 \cos A \quad (4)$$

(b) Hence solve, for $-90^\circ \leq x \leq 180^\circ$, the equation

$$1 - \cos 3x = \sin^2 x \quad (4)$$

ANSWER

Question	Scheme	Marks	AOs
10 (a)	$\cos 3A = \cos (2A + A) = \cos 2A \cos A - \sin 2A \sin A$	M1	3.1a
	$= (2 \cos^2 A - 1) \cos A - (2 \sin A \cos A) \sin A$	dM1	1.1b
	$= (2 \cos^2 A - 1) \cos A - 2 \cos A (1 - \cos^2 A)$	ddM1	2.1
	$= 4 \cos^3 A - 3 \cos A^*$	A1*	1.1b
		(4)	
(b)	$1 - \cos 3x = \sin^2 x \Rightarrow \cos^2 x + 3 \cos x - 4 \cos^3 x = 0$	M1	1.1b
	$\Rightarrow \cos x (4 \cos^2 x - \cos x - 3) = 0$	dM1	3.1a
	$\Rightarrow \cos x (4 \cos x + 3)(\cos x - 1) = 0$		
	$\Rightarrow \cos x = \dots$		
	Two of $-90^\circ, 0, 90^\circ$, awrt 139°	A1	1.1b
	All four of $-90^\circ, 0, 90^\circ$, awrt 139°	A1	2.1
		(4)	
(8 marks)			

June 2019 Question 12 Paper 2

12. (a) Prove

$$\frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} \equiv 2 \cot 2\theta \quad \theta \neq (90n)^\circ, n \in \mathbb{Z} \quad (4)$$

(b) Hence solve, for $90^\circ < \theta < 180^\circ$, the equation

$$\frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} = 4$$

giving any solutions to one decimal place.

(3)

ANSWER

Question	Scheme	Marks	AOs
12	$\frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} = 2 \cot 2\theta$		
(a) Way 1	{LHS=} $\frac{\cos 3\theta \cos \theta + \sin 3\theta \sin \theta}{\sin \theta \cos \theta}$	M1	3.1a
	$= \frac{\cos(3\theta - \theta)}{\sin \theta \cos \theta} \left\{ = \frac{\cos 2\theta}{\sin \theta \cos \theta} \right\}$	A1	2.1
	$= \frac{\cos 2\theta}{\frac{1}{2} \sin 2\theta} = 2 \cot 2\theta *$	dM1	1.1b
		A1 *	2.1
		(4)	
(a) Way 2	{LHS=} $\frac{\cos 2\theta \cos \theta - \sin 2\theta \sin \theta}{\sin \theta} + \frac{\sin 2\theta \cos \theta + \cos 2\theta \sin \theta}{\cos \theta}$		
	$= \frac{\cos 2\theta \cos^2 \theta - \sin 2\theta \sin \theta \cos \theta + \sin 2\theta \cos \theta \sin \theta + \cos 2\theta \sin^2 \theta}{\sin \theta \cos \theta}$	M1	3.1a
	$= \frac{\cos 2\theta(\cos^2 \theta + \sin^2 \theta)}{\sin \theta \cos \theta} \left\{ = \frac{\cos 2\theta}{\sin \theta \cos \theta} \right\}$	A1	2.1
	$= \frac{\cos 2\theta}{\frac{1}{2} \sin 2\theta} = 2 \cot 2\theta *$	dM1	1.1b
		A1 *	2.1
		(4)	
(b) Way 1	$\left\{ \frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} = 4 \Rightarrow \right\} 2 \cot 2\theta = 4 \Rightarrow 2 \left(\frac{1}{\tan 2\theta} \right) = 4$	M1	1.1b
	Rearranges to give $\tan 2\theta = k; k \neq 0$ and applies $\arctan k$	dM1	1.1b
	$\left\{ 90^\circ < \theta < 180^\circ, \tan 2\theta = \frac{1}{2} \Rightarrow \right\}$		
	Only one solution of $\theta = 103.3^\circ$ (1 dp) or awrt 103.3°	A1	2.2a
		(3)	
(b) Way 2	$\left\{ \frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} = 4 \Rightarrow \right\} 2 \cot 2\theta = 4 \Rightarrow \frac{2}{\tan 2\theta} = 4$	M1	1.1b
	$\frac{2}{\left(\frac{2 \tan \theta}{1 - \tan^2 \theta} \right)} = 4 \Rightarrow 2(1 - \tan^2 \theta) = 8 \tan \theta$		
	$\Rightarrow \tan^2 \theta + 4 \tan \theta - 1 = 0 \Rightarrow \tan \theta = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-1)}}{2(1)}$	dM1	1.1b
	$\{\Rightarrow \tan \theta = -2 \pm \sqrt{5}\} \Rightarrow \tan \theta = k; k \neq 0 \Rightarrow$ applies $\arctan k$		
	$\{90^\circ < \theta < 180^\circ, \tan \theta = -2 - \sqrt{5} \Rightarrow\}$		
	Only one solution of $\theta = 103.3^\circ$ (1 dp) or awrt 103.3°	A1	2.2a
		(3)	
(7 marks)			

June 2018 Question 7 Paper 2

7. (i) Solve, for $0 \leq x < \frac{\pi}{2}$, the equation

$$4 \sin x = \sec x$$

(4)

- (ii) Solve, for $0 \leq \theta < 360^\circ$, the equation

$$5 \sin \theta - 5 \cos \theta = 2$$

giving your answers to one decimal place.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(5)

ANSWER

Question	Scheme	Marks	AOs
7	(i) $4\sin x = \sec x, 0 \leq x < \frac{\pi}{2}$; (ii) $5\sin \theta - 5\cos \theta = 2, 0 \leq \theta < 360^\circ$		
(i) Way 1	For $\sec x = \frac{1}{\cos x}$	B1	1.2
	$\{4\sin x = \sec x \Rightarrow\} 4\sin x \cos x = 1 \Rightarrow 2\sin 2x = 1 \Rightarrow \sin 2x = \frac{1}{2}$	M1	3.1a
	$x = \frac{1}{2}\arcsin\left(\frac{1}{2}\right)$ or $\frac{1}{2}\left(\pi - \arcsin\left(\frac{1}{2}\right)\right) \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$	dM1 A1	1.1b 1.1b
		(4)	
(i) Way 2	For $\sec x = \frac{1}{\cos x}$	B1	1.2
	$\{4\sin x = \sec x \Rightarrow\} 4\sin x \cos x = 1 \Rightarrow 16\sin^2 x \cos^2 x = 1$ $16\sin^2 x(1 - \sin^2 x) = 1$ $16(1 - \cos^2 x)\cos^2 x = 1$ $16\sin^4 x - 16\sin^2 x + 1 = 0$ $16\cos^4 x - 16\cos^2 x + 1 = 0$ $\sin^2 x$ or $\cos^2 x = \frac{16 \pm \sqrt{192}}{32} \left\{ = \frac{2 \pm \sqrt{3}}{4} \text{ or } 0.933..., 0.066... \right\}$	M1	3.1a
	$x = \arcsin\left(\sqrt{\frac{2 \pm \sqrt{3}}{4}}\right)$ or $x = \arccos\left(\sqrt{\frac{2 \pm \sqrt{3}}{4}}\right) \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$	dM1 A1	1.1b 1.1b
		(4)	

(ii)	Complete strategy, i.e.		M1	3.1a
	- Expresses $5\sin\theta - 5\cos\theta = 2$ in the form $R\sin(\theta - \alpha) = 2$, finds both R and α, and proceeds to $\sin(\theta - \alpha) = k, k < 1, k \neq 0$ - Applies $(5\sin\theta - 5\cos\theta)^2 = 2^2$, followed by applying both $\cos^2\theta + \sin^2\theta = 1$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to proceed to $\sin 2\theta = k, k < 1, k \neq 0$			
	$R = \sqrt{50}$ $\tan\alpha = 1 \Rightarrow \alpha = 45^\circ$	$(5\sin\theta - 5\cos\theta)^2 = 2^2 \Rightarrow$ $25\sin^2\theta + 25\cos^2\theta - 50\sin\theta\cos\theta = 4$ $\Rightarrow 25 - 25\sin 2\theta = 4$	M1	1.1b
	$\sin(\theta - 45^\circ) = \frac{2}{\sqrt{50}}$	$\sin 2\theta = \frac{21}{25}$	A1	1.1b
	dependent on the first M mark e.g. $\theta = \arcsin\left(\frac{2}{\sqrt{50}}\right) + 45^\circ$ e.g. $\theta = \frac{1}{2}\left(\arcsin\left(\frac{21}{25}\right)\right)$		dM1	1.1b
	$\theta = \text{awrt } 61.4^\circ, \text{ awrt } 208.6^\circ$		A1	2.1
	Note: Working in radians does not affect any of the first 4 marks			
			(5)	
(9 marks)				

June 2018 Question 12 Paper 2

12. (a) Prove that

$$1 - \cos 2\theta \equiv \tan \theta \sin 2\theta, \quad \theta \neq \frac{(2n+1)\pi}{2}, \quad n \in \mathbb{Z} \quad (3)$$

(b) Hence solve, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$, the equation

$$(\sec^2 x - 5)(1 - \cos 2x) = 3 \tan^2 x \sin 2x$$

Give any non-exact answer to 3 decimal places where appropriate.

(6)

ANSWER

Question	Scheme	Marks	AOs
12	$1 - \cos 2\theta \equiv \tan \theta \sin 2\theta, \theta \neq \frac{(2n+1)\pi}{2}, n \in \mathbb{Z}$		
(a) Way 1	$\tan \theta \sin 2\theta = \left(\frac{\sin \theta}{\cos \theta}\right)(2 \sin \theta \cos \theta)$	M1	1.1b
	$= \left(\frac{\sin \theta}{\cancel{\cos \theta}}\right)(2 \sin \theta \cancel{\cos \theta}) = 2 \sin^2 \theta = 1 - \cos 2\theta *$	M1	1.1b
		A1*	2.1
		(3)	
(a) Way 2	$1 - \cos 2\theta = 1 - (1 - 2 \sin^2 \theta) = 2 \sin^2 \theta$	M1	1.1b
	$= \left(\frac{\sin \theta}{\cos \theta}\right)(2 \sin \theta \cos \theta) = \tan \theta \sin 2\theta *$	M1	1.1b
		A1*	2.1
		(3)	
	$(\sec^2 x - 5)(1 - \cos 2x) = 3 \tan^2 x \sin 2x, -\frac{\pi}{2} < x < \frac{\pi}{2}$		
(b) Way 1	$(\sec^2 x - 5) \tan x \sin 2x = 3 \tan^2 x \sin 2x$ or $(\sec^2 x - 5)(1 - \cos 2x) = 3 \tan x(1 - \cos 2x)$		
	Deduces $x = 0$	B1	2.2a
	Uses $\sec^2 x = 1 + \tan^2 x$ and cancels/factorises out $\tan x$ or $(1 - \cos 2x)$ e.g. $(1 + \tan^2 x - 3 \tan x - 5) \tan x = 0$ or $(1 + \tan^2 x - 3 \tan x - 5)(1 - \cos 2x) = 0$ or $1 + \tan^2 x - 5 = 3 \tan x$	M1	2.1
	$\tan^2 x - 3 \tan x - 4 = 0$	A1	1.1b
	$(\tan x - 4)(\tan x + 1) = 0 \Rightarrow \tan x = \dots$	M1	1.1b
	$x = -\frac{\pi}{4}, 1.326$	A1	1.1b
		A1	1.1b
		(6)	
(9 marks)			

Ch. 7: Trigonometry and Modelling

June 2024 Question 12 Paper 1

12. (a) Express $140 \cos \theta - 480 \sin \theta$ in the form $K \cos(\theta + \alpha)$

where $K > 0$ and $0 < \alpha < 90^\circ$

State the value of K and give the value of α , in degrees, to 2 decimal places.

(3)

A scientist studies the number of rabbits and the number of foxes in a wood for one year.

The number of rabbits, R , is modelled by the equation

$$R = A + 140 \cos(30t)^\circ - 480 \sin(30t)^\circ$$

where t months is the time after the start of the year and A is a constant.

Given that, during the year, the maximum number of rabbits in the wood is 1500

(b) (i) find a complete equation for this model.

(ii) Hence write down the minimum number of rabbits in the wood during the year according to the model.

(2)

The actual number of rabbits in the wood is at its minimum value in the middle of April.

(c) Use this information to comment on the model for the number of rabbits.

(2)

The number of foxes, F , in the wood during the same year is modelled by the equation

$$F = 100 + 70 \sin(30t + 70)^\circ$$

The number of foxes is at its minimum value after T months.

(d) Find, according to the models, the number of **rabbits** in the wood at time T months.

(4)

ANSWER

Question	Scheme	Marks	AOs
12(a)	$K = 500$	B1	1.1b
	$\tan \alpha = \frac{480}{140} \Rightarrow \alpha = \dots$	M1	1.1b
	$\alpha = \text{awrt } 73.74^\circ \text{ or } 500 \cos(\theta + 73.74)^\circ$	A1	1.1b
		(3)	
(b)(i)	$R = 1000 + 500 \cos(30t + 73.74)^\circ$ or $R = 1000 + 140 \cos(30t)^\circ - 480 \sin(30t)^\circ$	B1ft	3.3
(b)(ii)	$\{R_{\min} = \} 500$	B1ft	3.4
		(2)	
(c)	$t = 3.5 \Rightarrow R = "1000" + "500" \cos(30(3.5) + "73.74")^\circ = \dots$	M1	3.4
	$R = \text{awrt } 500.1 \dots$ so the model is reliable	A1	3.5a
		(2)	
(d)	$\sin(30t + 70)^\circ = -1 \Rightarrow 30t + 70 = 270 \Rightarrow 30t = \dots \text{ (or } t = \dots)$	M1	3.4
	$30t = 200 \left(\text{or } t = \frac{20}{3} \right)$	A1	1.1b
	$R = "1000" + "500" \cos\left(30\left(\frac{20}{3}\right) + "73.74" \right)^\circ$ or $R = "1000" + 140 \cos("200")^\circ - 480 \sin("200")^\circ$	dM1	3.4
	$R = 1032 \text{ (or } 1033)$	A1	1.1b
		(4)	
(11 marks)			

June 2023 Question 15 Paper 2

15. A student attempts to answer the following question:

Given that x is an obtuse angle, use algebra to prove by contradiction that

$$\sin x - \cos x \geq 1$$

The student starts the proof with:

Assume that $\sin x - \cos x < 1$ when x is an obtuse angle

$$\Rightarrow (\sin x - \cos x)^2 < 1$$

$$\Rightarrow \dots$$

The start of the student's proof is reprinted below.

Complete the proof.

(3)

Assume that $\sin x - \cos x < 1$ when x is an obtuse angle

$$\Rightarrow (\sin x - \cos x)^2 < 1$$

ANSWER

Question	Scheme	Marks	AOs
15	$(\sin x - \cos x)^2 < 1 \Rightarrow \sin^2 x - 2 \sin x \cos x + \cos^2 x < 1$ o.e.	M1	1.1b
	Examples: $1 - 2 \sin x \cos x < 1$, $1 - \sin 2x < 1$, $-2 \sin x \cos x < 0$, $-\sin 2x < 0$	A1	2.2a
	As x is obtuse then $-2 \sin x \cos x$ is positive because $\sin x > 0$ and $\cos x < 0$ so we have a contradiction. Therefore $\sin x - \cos x \geq 1$ *	A1*	2.4
(3 marks)			

June 2023 Question 14 Paper 2

14. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2 \tan \theta (8 \cos \theta + 23 \sin^2 \theta) = 8 \sin 2\theta (1 + \tan^2 \theta)$$

may be written as

$$\sin 2\theta (A \cos^2 \theta + B \cos \theta + C) = 0$$

where A , B and C are constants to be found.

(3)

(b) Hence, solve for $360^\circ \leq x \leq 540^\circ$

$$2 \tan x (8 \cos x + 23 \sin^2 x) = 8 \sin 2x (1 + \tan^2 x) \quad x \in \mathbb{R} \quad x \neq 450^\circ$$

(4)

ANSWER

Question	Scheme	Marks	AOs
14(a)	e.g. $2 \frac{\sin \theta}{\cos \theta} (8 \cos \theta + 23 (1 - \cos^2 \theta)) = 8 \times 2 \sin \theta \cos \theta \sec^2 \theta$	B1	1.2
	$2 \tan \theta (8 \cos \theta + 23 \sin^2 \theta) = 8 \sin 2\theta \sec^2 \theta$ $\Rightarrow 2 \sin \theta \cos \theta (8 \cos \theta + 23 (1 - \cos^2 \theta)) = 8 \sin 2\theta$ $\sin 2\theta (8 \cos \theta + 23 (1 - \cos^2 \theta)) = 8 \sin 2\theta$ $\sin 2\theta (23 \cos^2 \theta - 8 \cos \theta - 15) = 0$	M1A1	2.1 2.2a
		(3)	
(b)	$\sin 2x (23 \cos^2 x - 8 \cos x - 15) = 0$		
	$\sin 2x = 0 \Rightarrow x = 360^\circ \text{ or } 540^\circ$	B1	2.2a
	$23 \cos^2 x - 8 \cos x - 15 \Rightarrow \cos x = -\frac{15}{23}$	M1	1.1b
	$\cos x = -\frac{15}{23} \Rightarrow x = \dots$	dM1	1.1b
	$x = 360^\circ, 540^\circ \text{ and awrt } 491^\circ \text{ only}$	A1	2.3
		(4)	
(7 marks)			

June 2023 Question 8 Paper 2

8. (a) Express $2\cos\theta + 8\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants,

$$R > 0 \text{ and } 0 < \alpha < \frac{\pi}{2}$$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

The first three terms of an arithmetic sequence are

$$\cos x \quad \cos x + \sin x \quad \cos x + 2\sin x \quad x \neq n\pi$$

Given that S_9 represents the sum of the first 9 terms of this sequence as x varies,

- (b) (i) find the exact maximum value of S_9

- (ii) deduce the smallest positive value of x at which this maximum value of S_9 occurs.

(3)

ANSWER

Question	Scheme	Marks	AOs
8(a)	$R = \sqrt{2^2 + 8^2} = \sqrt{68} = 2\sqrt{17}$	B1	1.1b
	$2\cos\theta + 8\sin\theta = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$ $2 = R\cos\alpha \quad 8 = R\sin\alpha$ $\tan\alpha = \frac{8}{2} \Rightarrow \alpha = \dots$	M1	1.1b
	$\alpha = \text{awrt } 1.326$	A1	2.2a
		(3)	
(b)(i)	$4.5 \times 2\sqrt{17}$	M1	1.1b
	$9\sqrt{17}$	A1	2.2a
(ii)	awrt 1.33	B1ft	2.2a
		(3)	
(6 marks)			

June 2023 Question 13 Paper 1

13. On a roller coaster ride, passengers travel in carriages around a track.

On the ride, carriages complete multiple circuits of the track such that

- the maximum vertical height of a carriage above the ground is 60 m
- a carriage starts a circuit at a vertical height of 2 m above the ground
- the ground is horizontal

The vertical height, H m, of a carriage above the ground, t seconds after the carriage starts the first circuit, is modelled by the equation

$$H = a - b(t - 20)^2$$

where a and b are positive constants.

(a) Find a complete equation for the model.

(3)

(b) Use the model to determine the height of the carriage above the ground when $t = 40$

(1)

In an alternative model, the vertical height, H m, of a carriage above the ground, t seconds after the carriage starts the first circuit, is given by

$$H = 29 \cos(9t + \alpha)^\circ + \beta \quad 0 \leq \alpha < 360^\circ$$

where α and β are constants.

(c) Find a complete equation for the alternative model.

(2)

Given that the carriage moves continuously for 2 minutes,

(d) give a reason why the alternative model would be more appropriate.

(1)

ANSWER

Question	Scheme	Marks	AOs
13(a)	$a = 60$	B1	3.1b
	$2 = "60" - b(-20)^2 \Rightarrow b = \dots$	M1	3.4
	$H = 60 - 0.145(t - 20)^2$	A1	3.3
		(3)	
(b)	Height = 2 m	B1	3.4
		(1)	
(c)	$\alpha = 180$ or $\beta = 31$	M1	3.4
	$H = 29 \cos(9t + 180)^\circ + 31$	A1	3.3
		(2)	
(d)	e.g. "The model allows for more than one circuit"	B1	3.5a
		(1)	
(7 marks)			

June 2022 Question 9 Paper 2

9.

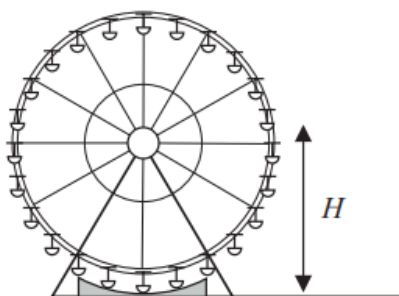


Figure 4

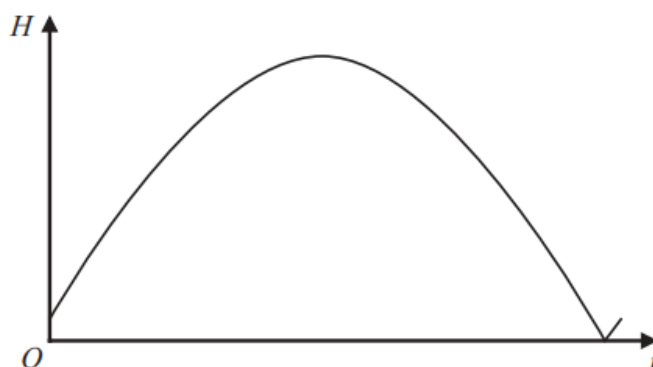


Figure 5

Figure 4 shows a sketch of a Ferris wheel.

The height above the ground, H m, of a passenger on the Ferris wheel, t seconds after the wheel starts turning, is modelled by the equation

$$H = |A \sin(bt + \alpha)|$$

where A , b and α are constants.

Figure 5 shows a sketch of the graph of H against t , for one revolution of the wheel.

Given that

- the maximum height of the passenger above the ground is 50 m
- the passenger is 1 m above the ground when the wheel starts turning
- the wheel takes 720 seconds to complete one revolution

(a) find a complete equation for the model, giving the exact value of A , the exact value of b and the value of α to 3 significant figures.

(4)

(b) Explain why an equation of the form

$$H = |A \sin(bt + \alpha)| + d$$

where d is a positive constant, would be a more appropriate model.

(1)

ANSWER

Question	Scheme	Marks	AOs
9(a)	Deduces that $A = \pm 50$ or $b = \frac{1}{4}$	B1	3.4
	Deduces that $A = \pm 50$ and $b = \frac{1}{4}$	B1	3.4
	Uses $t = 0, H = 1 \Rightarrow \alpha = \dots$ E.g. $1 = 50 \sin(\alpha)^\circ \Rightarrow \alpha = \dots$	M1	3.4
	$H = \left \pm 50 \sin\left(\frac{1}{4}t + 1.15\right)^\circ \right $	A1	3.3
		(4)	
(b)	E.g. the minimum height above the ground of the passenger on the original model was 0 m or Adding “d” means the passenger does not touch the ground.	B1	3.5b
		(1)	
(5 marks)			

Video solution:

<https://youtu.be/MAvbZ55bRtE>

June 2022 Question 14 Paper 1

14. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that

$$2 \sin(x - 60^\circ) = \cos(x - 30^\circ)$$

show that

$$\tan x = 3\sqrt{3} \quad (4)$$

(b) Hence or otherwise solve, for $0 \leq \theta < 180^\circ$

$$2 \sin 2\theta = \cos(2\theta + 30^\circ)$$

giving your answers to one decimal place.

(4)

ANSWER

Question	Scheme	Marks	AOs
14(a)	Attempts to use both $\sin(x - 60^\circ) = \pm \sin x \cos 60^\circ \pm \cos x \sin 60^\circ$ $\cos(x - 30^\circ) = \pm \cos x \cos 30^\circ \pm \sin x \sin 30^\circ$	M1	2.1
	Correct equation $2 \sin x \cos 60^\circ - 2 \cos x \sin 60^\circ = \cos x \cos 30^\circ + \sin x \sin 30^\circ$	A1	1.1b
	Either uses $\frac{\sin x}{\cos x} = \tan x$ and attempts to make $\tan x$ the subject E.g. $(2 \cos 60^\circ - \sin 30^\circ) \tan x = \cos 30^\circ + 2 \sin 60^\circ$ Or attempts $\sin 30^\circ$ etc with at least two correct and collects terms in $\sin x$ and $\cos x$ E.g. $\left(2 \times \frac{1}{2} - \frac{1}{2}\right) \sin x = \left(2 \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right) \cos x$	M1	2.1
	Proceeds to given answer showing all key steps E.g. $\frac{1}{2} \tan x = \frac{3\sqrt{3}}{2} \Rightarrow \tan x = 3\sqrt{3}$ *	A1*	1.1b
		(4)	
(b)	Deduces that $x = 2\theta + 60^\circ$	B1	2.2a
	$\tan(2\theta + 60^\circ) = 3\sqrt{3} \Rightarrow 2\theta + 60^\circ = 79.1^\circ, 259.1^\circ, \dots$	M1	1.1b
	Correct method to find one value of θ E.g. $\theta = \frac{79.1^\circ - 60^\circ}{2}$	dM1	1.1b
	$\theta = \text{awrt } 9.6^\circ, 99.6^\circ$ (See note)	A1	2.1
		(4)	
(8 marks)			

November 2021 Question 15 Paper 2

15. (a) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and the value of α in radians to 3 decimal places.

(3)

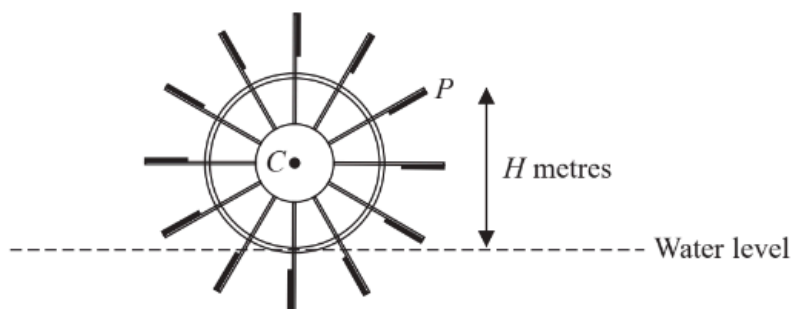


Figure 6

Figure 6 shows the cross-section of a water wheel.

The wheel is free to rotate about a fixed axis through the point C .

The point P is at the end of one of the paddles of the wheel, as shown in Figure 6.

The water level is assumed to be horizontal and of constant height.

The vertical height, H metres, of P above the water level is modelled by the equation

$$H = 3 + 4\cos(0.5t) - 2\sin(0.5t)$$

where t is the time in seconds after the wheel starts rotating.

Using the model, find

- (b) (i) the maximum height of P above the water level,
(ii) the value of t when this maximum height first occurs, giving your answer to one decimal place.

(3)

In a single revolution of the wheel, P is below the water level for a total of T seconds.

According to the model,

- (c) find the value of T giving your answer to 3 significant figures.

(Solutions based entirely on calculator technology are not acceptable.)

(4)

In reality, the water level may not be of constant height.

- (d) Explain how the equation of the model should be refined to take this into account.

(1)

ANSWER

Question	Scheme	Marks	AOs
15(a)	$R = \sqrt{5}$	B1	1.1b
	$\tan \alpha = \frac{1}{2}$ or $\sin \alpha = \frac{1}{\sqrt{5}}$ or $\cos \alpha = \frac{2}{\sqrt{5}} \Rightarrow \alpha = \dots$	M1	1.1b
	$\alpha = 0.464$	A1	1.1b
		(3)	
(b)(i)	$3 + 2\sqrt{5}$	B1ft	3.4
(ii)	$\cos(0.5t + 0.464) = 1 \Rightarrow 0.5t + 0.464 = 2\pi$ $\Rightarrow t = \dots$	M1	3.4
	$t = 11.6$	A1	1.1b
		(3)	
(c)	$3 + 2\sqrt{5} \cos(0.5t + 0.464) = 0$ $\cos(0.5t + 0.464) = -\frac{3}{2\sqrt{5}}$	M1	3.4
	$\cos(0.5t + 0.464) = -\frac{3}{2\sqrt{5}} \Rightarrow 0.5t + 0.464 = \cos^{-1}\left(-\frac{3}{2\sqrt{5}}\right)$ $\Rightarrow t = 2\left(\cos^{-1}\left(-\frac{3}{2\sqrt{5}}\right) - 0.464\right)$	dM1	1.1b
	So the time required is e.g.: $2(3.977\dots - 0.464) - 2(2.306\dots - 0.464)$	dM1	3.1b
	$= 3.34$	A1	1.1b
		(4)	
(d)	e.g. the “3” would need to vary	B1	3.5c
		(1)	

Video solution:

<https://youtu.be/cKe2VWz8jMM>

November 2020 Question 6 Paper 1

6. (a) Express $\sin x + 2 \cos x$ in the form $R \sin(x + \alpha)$ where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

The temperature, $\theta^\circ\text{C}$, inside a room on a given day is modelled by the equation

$$\theta = 5 + \sin\left(\frac{\pi t}{12} - 3\right) + 2 \cos\left(\frac{\pi t}{12} - 3\right) \quad 0 \leq t < 24$$

where t is the number of hours after midnight.

Using the equation of the model and your answer to part (a),

- (b) deduce the maximum temperature of the room during this day,

(1)

- (c) find the time of day when the maximum temperature occurs, giving your answer to the nearest minute.

(3)

ANSWER

Question	Scheme	Marks	AOs
6 (a)	$R = \sqrt{5}$	B1	1.1b
	$\tan \alpha = 2 \Rightarrow \alpha = \dots$	M1	1.1b
	$\alpha = 1.107$	A1	1.1b
		(3)	
	$\theta = 5 + \sqrt{5} \sin\left(\frac{\pi t}{12} + 1.107 - 3\right)$		
(b)	$(5 + \sqrt{5})^\circ\text{C}$ or awrt 7.24°C	B1ft	2.2a
		(1)	
(c)	$\frac{\pi t}{12} + 1.107 - 3 = \frac{\pi}{2} \Rightarrow t =$	M1	3.1b
	$t = \text{awrt } 13.2$	A1	1.1b
	Either 13:14 or 1:14 pm or 13 hours 14 minutes after midnight.	A1	3.2a
		(3)	
(7 marks)			

June 2019 Question 6 Paper 1

6. (a) Solve, for $-180^\circ \leq \theta \leq 180^\circ$, the equation

$$5 \sin 2\theta = 9 \tan \theta$$

giving your answers, where necessary, to one decimal place.

[Solutions based entirely on graphical or numerical methods are not acceptable.]

(6)

- (b) Deduce the smallest positive solution to the equation

$$5 \sin(2x - 50^\circ) = 9 \tan(x - 25^\circ)$$

(2)

ANSWER

Question	Scheme	Marks	AOs
6 (a)	$5 \sin 2\theta = 9 \tan \theta \Rightarrow 10 \sin \theta \cos \theta = 9 \times \frac{\sin \theta}{\cos \theta}$ $A \cos^2 \theta = B \quad \text{or} \quad C \sin^2 \theta = D \quad \text{or} \quad P \cos^2 \theta \sin \theta = Q \sin \theta$	M1	3.1a
	For a correct simplified equation in one trigonometric function Eg $10 \cos^2 \theta = 9 \quad 10 \sin^2 \theta = 1 \quad \text{oe}$	A1	1.1b
	Correct order of operations For example $10 \cos^2 \theta = 9 \Rightarrow \theta = \arccos(\pm) \sqrt{\frac{9}{10}}$	dM1	2.1
	Any one of the four values awrt $\theta = \pm 18.4^\circ, \pm 161.6^\circ$	A1	1.1b
	All four values $\theta = \text{awrt } \pm 18.4^\circ, \pm 161.6^\circ$	A1	1.1b
	$\theta = 0^\circ, \pm 180^\circ$	B1	1.1b
		(6)	
(b)	Attempts to solve $x - 25^\circ = -18.4^\circ$	M1	1.1b
	$x = 6.6^\circ$	A1ft	2.2a
		(2)	
(8 marks)			

June 2019 Question 12 Paper 1

12. $f(x) = 10e^{-0.25x} \sin x, \quad x \geq 0$

- (a) Show that the x coordinates of the turning points of the curve with equation $y = f(x)$ satisfy the equation $\tan x = 4$ (4)

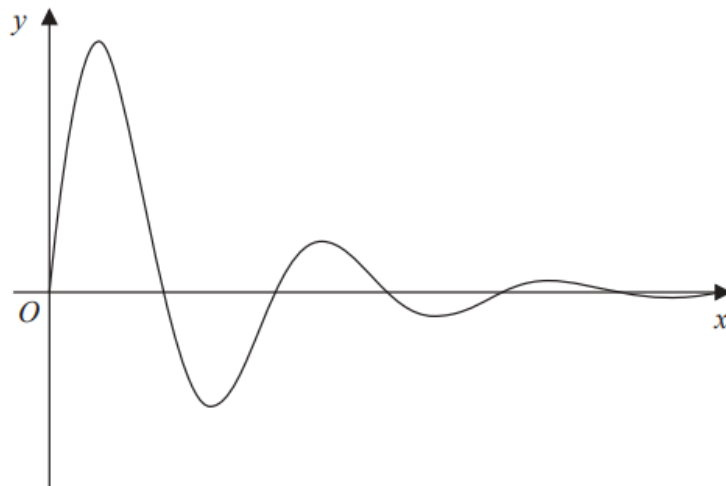


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$.

- (b) Sketch the graph of H against t where

$$H(t) = |10e^{-0.25t} \sin t| \quad t \geq 0$$

showing the long-term behaviour of this curve.

(2)

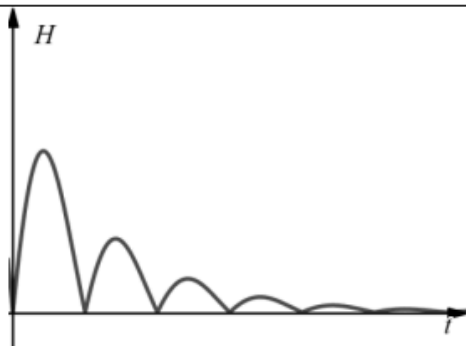
The function $H(t)$ is used to model the height, in metres, of a ball above the ground t seconds after it has been kicked.

Using this model, find

- (c) the maximum height of the ball above the ground between the first and second bounce. (3)

- (d) Explain why this model should not be used to predict the time of each bounce. (1)

ANSWER

Question	Scheme	Marks	AOs
12 (a)	$f(x) = 10e^{-0.25x} \sin x$		
	$\Rightarrow f'(x) = -2.5e^{-0.25x} \sin x + 10e^{-0.25x} \cos x$ oe	M1 A1	1.1b 1.1b
	$f'(x) = 0 \Rightarrow -2.5e^{-0.25x} \sin x + 10e^{-0.25x} \cos x = 0$	M1	2.1
	$\frac{\sin x}{\cos x} = \frac{10}{2.5} \Rightarrow \tan x = 4^*$	A1*	1.1b
		(4)	
(b)	 "Correct" shape for 2 loops Fully correct with decreasing heights	M1 A1	1.1b 1.1b
		(2)	
(c)	Solves $\tan x = 4$ and substitutes answer into $H(t)$	M1	3.1a
	$H(4.47) = 10e^{-0.25 \times 4.47} \sin 4.47 $	M1	1.1b
	awrt 3.18 (metres)	A1	3.2a
		(3)	
(d)	The times between each bounce should not stay the same when the heights of each bounce is getting smaller	B1	3.5b
		(1)	
(10 marks)			

June 2018 Question 8 Paper 1

8. The depth of water, D metres, in a harbour on a particular day is modelled by the formula

$$D = 5 + 2 \sin(30t)^\circ \quad 0 \leq t < 24$$

where t is the number of hours after midnight.

A boat enters the harbour at 6:30 am and it takes 2 hours to load its cargo.

The boat requires the depth of water to be at least 3.8 metres before it can leave the harbour.

- (a) Find the depth of the water in the harbour when the boat enters the harbour.

(1)

- (b) Find, to the nearest minute, the earliest time the boat can leave the harbour.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(4)

ANSWER

Question	Scheme	Marks	AOs
8 (a)	$D = 5 + 2 \sin(30 \times 6.5)^\circ = \text{awrt } 4.48 \text{ m}$ with units	B1	3.4
		(1)	
(b)	$3.8 = 5 + 2 \sin(30t)^\circ \Rightarrow \sin(30t)^\circ = -0.6$	M1	1.1b
		A1	1.1b
	$t = 10.77$	dM1	3.1a
	10:46 a.m. or 10:47 a.m.	A1	3.2a
		(4)	
(5 marks)			

Ch. 8: Parametric Equations

June 2024 Question 10 Paper 2

10.

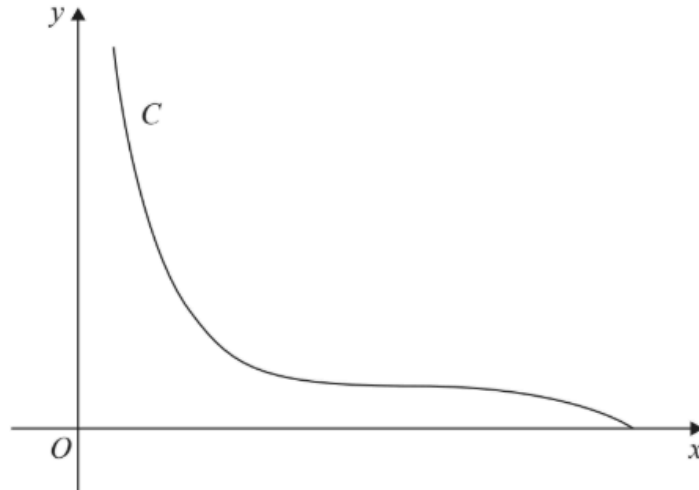


Figure 4

Figure 4 shows a sketch of the curve C with parametric equations

$$x = (t+3)^2 \quad y = 1-t^3 \quad -2 \leq t \leq 1$$

The point P with coordinates $(4, 2)$ lies on C .

(a) Using parametric differentiation, show that the tangent to C at P has equation

$$3x + 4y = 20 \quad (5)$$

The curve C is used to model the profile of a slide at a water park.

Units are in metres, with y being the height of the slide above water level.

(b) Find, according to the model, the greatest height of the slide above water level.

(1)

ANSWER

Question	Scheme	Marks	AOs
10(a)	$x = 4, y = 2 \Rightarrow t = -1$	B1	2.2a
	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = -3t^2 \times \frac{1}{2(t+3)}$	M1	1.1b
	$\frac{dy}{dx} = -3(-1)^2 \times \frac{1}{2(-1+3)} = -\frac{3}{4}$	M1	1.1b
	$\Rightarrow y - 2 = -\frac{3}{4}(x - 4)$ or $\Rightarrow y = -\frac{3}{4}x + c \rightarrow 2 = -\frac{3}{4} \times 4 + c \Rightarrow c \dots$	ddM1	2.1
	$y - 2 = -\frac{3}{4}(x - 4) \Rightarrow 4y - 8 = -3x + 12$ or $c = 5 \Rightarrow y = -\frac{3}{4}x + 5$ $\Rightarrow 3x + 4y = 20^*$	A1*	1.1b
		(5)	
(b)	Maximum height is 9m	B1	3.4
		(1)	
(6 marks)			

June 2023 Question 9 Paper 2

9. The curve C has parametric equations

$$x = t^2 + 6t - 16 \quad y = 6 \ln(t + 3) \quad t > -3$$

(a) Show that a Cartesian equation for C is

$$y = A \ln(x + B) \quad x > -B$$

where A and B are integers to be found.

(3)

The curve C cuts the y -axis at the point P

(b) Show that the equation of the tangent to C at P can be written in the form

$$ax + by = c \ln 5$$

where a , b and c are integers to be found.

(4)

ANSWER

Question	Scheme	Marks	AOs
9(a) Way 1	$x = (t + 3)^2 - 25$	M1	1.1b
	$\Rightarrow x + 25 = (t + 3)^2 \Rightarrow (x + 25)^{\frac{1}{2}} = (t + 3) \Rightarrow y = \dots$	M1	2.1
	$y = 6 \ln(x + 25)^{\frac{1}{2}} \Rightarrow y = 3 \ln(x + 25)$	A1cso	1.1b
		(3)	
(b)	$x = 0, y = 3 \ln 25$ oe e.g. $6 \ln 5$	B1ft	2.2a
	$\frac{dy}{dx} = \frac{3}{x + "25"} \Rightarrow \frac{dy}{dx} = \frac{3}{0 + "25"} \quad \left(= \frac{3}{25} \right)$ or $\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\left(\frac{6}{t+3} \right)}{2t+6} \Rightarrow \frac{6}{2 \times 2 + 6} \quad \left(= \frac{6}{50} = \frac{3}{25} \right)$	M1	2.1
	$y - "3 \ln 25" = " \frac{3}{25} "(x \{-0\})$	dM1	3.1a
	$25y - 3x = 150 \ln 5$	A1	2.2a
		(4)	
(7 marks)			
Notes			
Choose the mark scheme that best matches their chosen method.			

November 2021 Question 13 Paper 1

13. A curve C has parametric equations

$$x = \frac{t^2 + 5}{t^2 + 1} \quad y = \frac{4t}{t^2 + 1} \quad t \in \mathbb{R}$$

Show that all points on C satisfy

$$(x - 3)^2 + y^2 = 4 \quad (3)$$

ANSWER

Question	Scheme	Marks	AOs
13	$(x - 3)^2 + y^2 = \left(\frac{t^2 + 5}{t^2 + 1} - 3 \right)^2 + \left(\frac{4t}{t^2 + 1} \right)^2$	M1	3.1a
	$= \frac{(2 - 2t^2)^2 + 16t^2}{(t^2 + 1)^2} = \frac{4 + 8t^2 + 4t^4}{(t^2 + 1)^2}$	dM1	1.1b
	$\frac{4(t^4 + 2t^2 + 1)}{(t^2 + 1)^2} = \frac{4(t^2 + 1)^2}{(t^2 + 1)^2} = 4^*$	A1*	2.1
		(3)	

June 2019 Question 4 Paper 2

4.

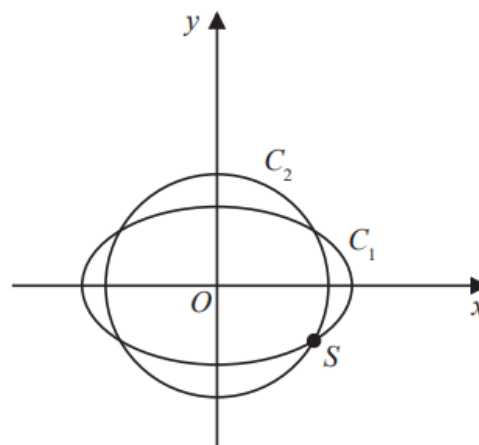


Figure 2

The curve C_1 with parametric equations

$$x = 10 \cos t, \quad y = 4\sqrt{2} \sin t, \quad 0 \leq t < 2\pi$$

meets the circle C_2 with equation

$$x^2 + y^2 = 66$$

at four distinct points as shown in Figure 2.

Given that one of these points, S , lies in the 4th quadrant, find the Cartesian coordinates of S .
(6)

ANSWER

Question	Scheme		Marks	AOs
4	$C_1: x=10\cos t, y=4\sqrt{2}\sin t, 0 \leq t < 2\pi; C_2: x^2 + y^2 = 66$			
Way 1	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$		M1	3.1a
	$100(1 - \sin^2 t) + 32\sin^2 t = 66$	$100\cos^2 t + 32(1 - \cos^2 t) = 66$	M1	2.1
			A1	1.1b
	$100 - 68\sin^2 t = 66 \Rightarrow \sin^2 t = \frac{1}{2}$ $\Rightarrow \sin t = \dots$	$68\cos^2 t + 32 = 66 \Rightarrow \cos^2 t = \frac{1}{2}$ $\Rightarrow \cos t = \dots$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the x-coordinate and value of the corresponding y-coordinate. Note: These may not be in the correct quadrant		M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2}, y = -4$ or $S = (\text{awrt } 7.07, -4)$		A1	3.2a
			(6)	
Way 2	$\{\cos^2 t + \sin^2 t = 1 \Rightarrow \left(\frac{x}{10}\right)^2 + \left(\frac{y}{4\sqrt{2}}\right)^2 = 1 \Rightarrow 32x^2 + 100y^2 = 3200\}$		M1	3.1a
	$\frac{x^2}{100} + \frac{66 - x^2}{32} = 1$	$\frac{66 - y^2}{100} + \frac{y^2}{32} = 1$	M1	2.1
			A1	1.1b
	$32x^2 + 6600 - 100x^2 = 3200$ $x^2 = 50 \Rightarrow x = \dots$	$2112 - 32y^2 + 100y^2 = 3200$ $y^2 = 16 \Rightarrow y = \dots$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the corresponding x-coordinate or y-coordinate. Note: These may not be in the correct quadrant		M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2}, y = -4$ or $S = (\text{awrt } 7.07, -4)$		A1	3.2a
			(6)	

June 2018 Question 14 Paper 1

14. A curve C has parametric equations

$$x = 3 + 2 \sin t, \quad y = 4 + 2 \cos 2t, \quad 0 \leq t < 2\pi$$

(a) Show that all points on C satisfy $y = 6 - (x - 3)^2$ (2)

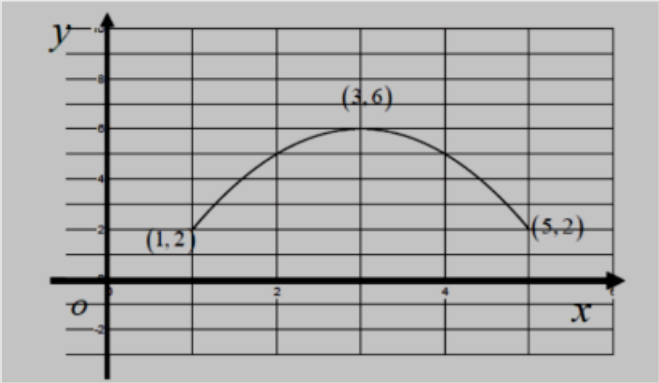
(b) (i) Sketch the curve C .

(ii) Explain briefly why C does not include all points of $y = 6 - (x - 3)^2$, $x \in \mathbb{R}$ (3)

The line with equation $x + y = k$, where k is a constant, intersects C at two distinct points.

(c) State the range of values of k , writing your answer in set notation. (5)

ANSWER

Question	Scheme	Marks	AOs
14(a)	Attempts to use $\cos 2t = 1 - 2\sin^2 t \Rightarrow \frac{y-4}{2} = 1 - 2\left(\frac{x-3}{2}\right)^2$	M1	2.1
	$\Rightarrow y - 4 = 2 - 4 \times \frac{(x-3)^2}{4} \Rightarrow y = 6 - (x-3)^2 *$	A1*	1.1b
		(2)	
(b)	 shaped parabola Fully correct with 'ends' at (1,2) & (5,2) Suitable reason : Eg states as $x = 3 + 2\sin t, 1 \leq x \leq 5$	M1	1.1b
		A1	1.1b
		B1	2.4
		(3)	
(c)	Either finds the lower value for $k = 7$ or deduces that $k < \frac{37}{4}$	B1	2.2a

	Finds where $x + y = k$ meets $y = 6 - (x - 3)^2$ $\Rightarrow k - x = 6 - (x - 3)^2$ and proceeds to 3TQ in x or y	M1	3.1a
	Correct 3TQ in x $x^2 - 7x + (k + 3) = 0$ Or y $y^2 + (7 - 2k)y + (k^2 - 6k + 3) = 0$	A1	1.1b
	Uses $b^2 - 4ac = 0 \Rightarrow 49 - 4 \times 1 \times (k + 3) = 0 \Rightarrow k = \left(\frac{37}{4}\right)$ or $(7 - 2k)^2 - 4 \times 1 \times (k^2 - 6k + 3) = 0 \Rightarrow k = \left(\frac{37}{4}\right)$	M1	2.1
	Range of values for $k = \left\{k : 7 \leq k < \frac{37}{4}\right\}$	A1	2.5
		(5)	
(10 marks)			

Ch. 9: Differentiation

June 2024 Question 5 Paper 1

5. The function f is defined by

$$f(x) = \frac{2x-3}{x^2+4} \quad x \in \mathbb{R}$$

(a) Show that

$$f'(x) = \frac{ax^2 + bx + c}{(x^2 + 4)^2}$$

where a , b and c are constants to be found.

(3)

(b) Hence, using algebra, find the values of x for which f is decreasing.
You must show each step in your working.

(3)

ANSWER

Question	Scheme	Marks	AOs
5(a)	$f(x) = \frac{2x-3}{x^2+4} \Rightarrow f'(x) = \frac{2(x^2+4) - 2x(2x-3)}{(x^2+4)^2}$	M1	1.1b
	or	A1	1.1b
	$f(x) = (2x-3)(x^2+4)^{-1} \Rightarrow f'(x) = 2(x^2+4)^{-1} - 2x(2x-3)(x^2+4)^{-2}$		
	$f'(x) = \frac{-2x^2 + 6x + 8}{(x^2+4)^2}$	A1	1.1b
		(3)	
(b)	$-2x^2 + 6x + 8 = 0 \Rightarrow -2(x+1)(x-4) = 0 \Rightarrow x = -1, 4$	B1ft (M1 on EPEN)	1.1b
	Chooses correct region for their numerator and their critical values $x < -1$ or $x > 4$	M1 A1	1.1b 2.2a
		(3)	
	(6 marks)		

June 2023 Question 7 Paper 2

7. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

A curve has equation

$$x^3 + 2xy + 3y^2 = 47$$

- (a) Find $\frac{dy}{dx}$ in terms of x and y

(4)

The point $P(-2, 5)$ lies on the curve.

- (b) Find the equation of the normal to the curve at P , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers to be found.

(3)

ANSWER

Question	Scheme	Marks	AOs
7(a)	$x^3 \rightarrow \dots x^2$ and $3y^2 \rightarrow \dots y \frac{dy}{dx}$	M1	1.1b
	$2xy \rightarrow 2y + 2x \frac{dy}{dx}$	B1	1.1b
	$3x^2 + 2x \frac{dy}{dx} + 2y + 6y \frac{dy}{dx} = \dots \Rightarrow \frac{dy}{dx} = \dots$	M1	2.1
	$\frac{dy}{dx} = -\frac{2y+3x^2}{2x+6y}$	A1	1.1b
		(4)	
(b)	$\frac{dy}{dx} = -\frac{2(5)+3(-2)^2}{2(-2)+6(5)}$ or e.g. $3(-2)^2 + 2(-2)\frac{dy}{dx} + 2 \times 5 + 6 \times 5 \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \dots \left(-\frac{11}{13}\right)$	M1	1.1b
	$y - 5 = -\frac{13}{11}(x + 2)$	dM1	1.1b
	$13x - 11y + 81 = 0$	A1	2.2a
		(3)	
(7 marks)			

June 2023 Question 1 Paper 2

1.

$$f(x) = x^3 + 2x^2 - 8x + 5$$

(a) Find $f''(x)$

(2)

(b) (i) Solve $f''(x) = 0$

(ii) Hence find the range of values of x for which $f(x)$ is concave.

(2)

ANSWER

Question	Scheme	Marks	AOs
1(a)	$\{f'(x) = \dots x^2 + \dots x + \dots \Rightarrow \{f''(x) = \dots x + \dots$	M1	1.1b
	$\{f'(x) = \dots 3x^2 + 4x - 8 \Rightarrow \{f''(x) = \dots 6x + 4$	A1cso	1.1b
		(2)	
(b)(i)	$"6x + 4" = 0 \Rightarrow x = "-\frac{2}{3}"$	B1ft	1.1b
(ii)	$x \leqslant "-\frac{2}{3}" \text{ or } x < "-\frac{2}{3}"$	B1ft	2.2a
		(2)	
(4 marks)			

June 2023 Question 12 Paper 1

12. $y = \sin x$

where x is measured in radians.

Use differentiation from first principles to show that

$$\frac{dy}{dx} = \cos x$$

You may

- use without proof the formula for $\sin(A \pm B)$
- assume that as $h \rightarrow 0$, $\frac{\sin h}{h} \rightarrow 1$ and $\frac{\cos h - 1}{h} \rightarrow 0$

(5)

ANSWER

Question	Scheme	Marks	AOs
12	$\frac{\sin(x+h) - \sin x}{h}$	B1	2.1
	$\frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$	M1	1.1b
		A1	1.1b
	$(\text{As } h \rightarrow 0), \sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \left(\frac{\sin h}{h} \right) \rightarrow 0 \times \sin x + 1 \times \cos x$	dM1	2.1
	so $\frac{dy}{dx} = \cos x$ *	A1*	2.5
(5 marks)			

June 2022 Question 16 Paper 2

16.

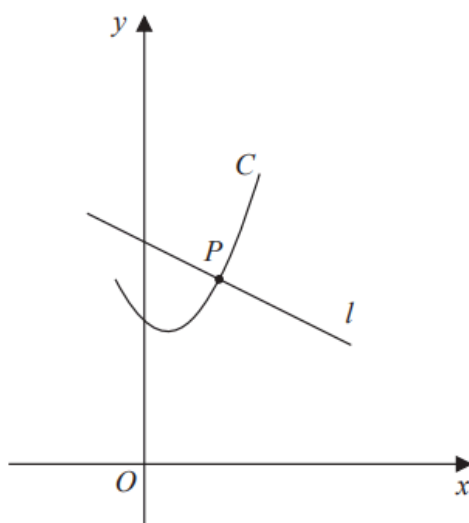


Figure 6

Figure 6 shows a sketch of the curve C with parametric equations

$$x = 2 \tan t + 1 \quad y = 2 \sec^2 t + 3 \quad -\frac{\pi}{4} \leq t \leq \frac{\pi}{3}$$

The line l is the normal to C at the point P where $t = \frac{\pi}{4}$

(a) Using parametric differentiation, show that an equation for l is

$$y = -\frac{1}{2}x + \frac{17}{2} \quad (5)$$

(b) Show that all points on C satisfy the equation

$$y = \frac{1}{2}(x-1)^2 + 5 \quad (2)$$

The straight line with equation

$$y = -\frac{1}{2}x + k \quad \text{where } k \text{ is a constant}$$

intersects C at two distinct points.

(c) Find the range of possible values for k .

(5)

ANSWER

Question	Scheme	Marks	AOs
16(a)	Attempts $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4\sec^2 t \tan t}{2\sec^2 t} (= 2 \tan t)$	M1 A1	1.1b 1.1b
	At $t = \frac{\pi}{4}$, $\frac{dy}{dx} = 2, x = 3, y = 7$	M1	2.1
	Attempts equation of normal $y - 7 = -\frac{1}{2}(x - 3)$	M1	1.1b
	$y = -\frac{1}{2}x + \frac{17}{2}$ *	A1*	2.1
		(5)	
(b)	Attempts to use $\sec^2 t = 1 + \tan^2 t \Rightarrow \frac{y-3}{2} = 1 + \left(\frac{x-1}{2}\right)^2$	M1	3.1a
	$\Rightarrow y - 3 = 2 + \frac{(x-1)^2}{2} \Rightarrow y = \frac{1}{2}(x-1)^2 + 5$ *	A1*	2.1
		(2)	

(c)	Attempts the lower limit for k: $\frac{1}{2}(x-1)^2 + 5 = -\frac{1}{2}x + k \Rightarrow x^2 - x + (11 - 2k) = 0$ $b^2 - 4ac = 1 - 4(11 - 2k) = 0 \Rightarrow k = \dots$	M1	2.1
	$(k =) \frac{43}{8}$	A1	1.1b
	Attempts the upper limit for k: $(x, y)_{t=-\frac{\pi}{4}} : t = -\frac{\pi}{4} \Rightarrow x = 2 \tan\left(-\frac{\pi}{4}\right) + 1 = -1, y = 2 \sec^2\left(-\frac{\pi}{4}\right) + 3 = 7$ $(-1, 7), y = -\frac{1}{2}x + k \Rightarrow 7 = \frac{1}{2} + k \Rightarrow k = \dots$	M1	2.1
	$(k =) \frac{13}{2}$	A1	1.1b
	$\frac{43}{8} < k \leq \frac{13}{2}$	A1	2.2a
		(5)	
(12 marks)			

June 2022 Question 12 Paper 2

12. The function f is defined by

$$f(x) = \frac{e^{3x}}{4x^2 + k}$$

where k is a positive constant.

(a) Show that

$$f'(x) = (12x^2 - 8x + 3k)g(x)$$

where $g(x)$ is a function to be found.

(3)

Given that the curve with equation $y = f(x)$ has at least one stationary point,

(b) find the range of possible values of k .

(3)

ANSWER

Question	Scheme	Marks	AOs
12(a)	$f(x) = \frac{e^{3x}}{4x^2 + k} \Rightarrow f'(x) = \frac{(4x^2 + k)3e^{3x} - 8xe^{3x}}{(4x^2 + k)^2}$	M1	1.1b
	or	A1	1.1b
	$f(x) = e^{3x} (4x^2 + k)^{-1} \Rightarrow f'(x) = 3e^{3x} (4x^2 + k)^{-1} - 8xe^{3x} (4x^2 + k)^{-2}$		
	$f'(x) = \frac{(12x^2 - 8x + 3k)e^{3x}}{(4x^2 + k)^2}$	A1	2.1
		(3)	
(b)	If $y = f(x)$ has at least one stationary point then		
	$12x^2 - 8x + 3k = 0$ has at least one root	B1	2.2a
	Applies $b^2 - 4ac (\geq) 0$ with $a = 12, b = -8, c = 3k$	M1	2.1
	$0 < k \leq \frac{4}{9}$	A1	1.1b
		(3)	
(6 marks)			

Video solution:

<https://youtu.be/lKpbO3FbB1I>

Arancha Ruiz

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November 2021 Question 14 Paper 1

14. Given that

$$y = \frac{x-4}{2+\sqrt{x}} \quad x > 0$$

show that

$$\frac{dy}{dx} = \frac{1}{A\sqrt{x}} \quad x > 0$$

where A is a constant to be found.

(4)

ANSWER

Question	Scheme	Marks	AOs
14	$y = \frac{x-4}{2+\sqrt{x}} \Rightarrow \frac{dy}{dx} = \frac{2+\sqrt{x} - (x-4)\frac{1}{2}x^{-\frac{1}{2}}}{(2+\sqrt{x})^2}$	M1 A1	2.1 1.1b
	$= \frac{2+\sqrt{x} - (x-4)\frac{1}{2}x^{-\frac{1}{2}}}{(2+\sqrt{x})^2} = \frac{2+\sqrt{x} - \frac{1}{2}\sqrt{x} + 2x^{-\frac{1}{2}}}{(2+\sqrt{x})^2} = \frac{2\sqrt{x} + \frac{1}{2}x + 2}{\sqrt{x}(2+\sqrt{x})^2}$	M1	1.1b
	$= \frac{x+4\sqrt{x}+4}{2\sqrt{x}(2+\sqrt{x})^2} = \frac{(2+\sqrt{x})^2}{2\sqrt{x}(2+\sqrt{x})^2} = \frac{1}{2\sqrt{x}}$	A1	2.1
		(4)	
(4 marks)			

November 2021 Question 8 Paper 2

8. The curve C has equation

$$px^3 + qxy + 3y^2 = 26$$

where p and q are constants.

(a) Show that

$$\frac{dy}{dx} = \frac{apx^2 + bqy}{qx + cy}$$

where a , b and c are integers to be found.

(4)

Given that

- the point $P(-1, -4)$ lies on C
- the normal to C at P has equation $19x + 26y + 123 = 0$

(b) find the value of p and the value of q .

(5)

ANSWER

Question	Scheme	Marks	AOs
8(a)	$\frac{d}{dx}(3y^2) = 6y \frac{dy}{dx}$ or $\frac{d}{dx}(qxy) = qx \frac{dy}{dx} + qy$	M1	2.1
	$3px^2 + qx \frac{dy}{dx} + qy + 6y \frac{dy}{dx} = 0$	A1	1.1b
	$(qx + 6y) \frac{dy}{dx} = -3px^2 - qy \Rightarrow \frac{dy}{dx} = \dots$	dM1	2.1
	$\frac{dy}{dx} = \frac{-3px^2 - qy}{qx + 6y}$	A1	1.1b
		(4)	
(b)	$p(-1)^3 + q(-1)(-4) + 3(-4)^2 = 26$	M1	1.1b
	$19x + 26y + 123 = 0 \Rightarrow m = -\frac{19}{26}$	B1	2.2a
	$\frac{-3p(-1)^2 - q(-4)}{q(-1) + 6(-4)} = \frac{26}{19} \quad \text{or} \quad \frac{q(-1) + 6(-4)}{3p(-1)^2 + q(-4)} = -\frac{19}{26}$	M1	3.1a
	$p - 4q = 22, \quad 57p - 102q = 624 \Rightarrow p = \dots, q = \dots$	dM1	1.1b
	$p = 2, \quad q = -5$	A1	1.1b
		(5)	
(9 marks)			

November 2021 Question 13 Paper 2

13. The curve C has parametric equations

$$x = \sin 2\theta \quad y = \operatorname{cosec}^3 \theta \quad 0 < \theta < \frac{\pi}{2}$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of θ (3)

(b) Hence find the exact value of the gradient of the tangent to C at the point where $y = 8$ (3)

ANSWER

Question	Scheme	Marks	AOs
13(a)	$y = \operatorname{cosec}^3 \theta \Rightarrow \frac{dy}{d\theta} = -3\operatorname{cosec}^2 \theta \operatorname{cosec} \theta \cot \theta$	B1	1.1b
	$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	M1	1.1b
	$\frac{dy}{dx} = \frac{-3\operatorname{cosec}^3 \theta \cot \theta}{2 \cos 2\theta}$	A1	1.1b
		(3)	
(b)	$y = 8 \Rightarrow \operatorname{cosec}^3 \theta = 8 \Rightarrow \sin^3 \theta = \frac{1}{8} \Rightarrow \sin \theta = \frac{1}{2}$	M1	3.1a
	$\theta = \frac{\pi}{6} \Rightarrow \frac{dy}{dx} = \frac{-3\operatorname{cosec}^3 \left(\frac{\pi}{6}\right) \cot \left(\frac{\pi}{6}\right)}{2 \cos \left(\frac{2\pi}{6}\right)} = \dots$ or $\sin \theta = \frac{1}{2} \Rightarrow \frac{dy}{dx} = \frac{-3}{2(1-2\sin^2 \theta)} \times \frac{\cos \theta}{\sin \theta} = \frac{-3 \times 8 \times \frac{\sqrt{3}}{2}}{2 \left(1 - 2 \times \frac{1}{4}\right)}$	M1	2.1
	$= -24\sqrt{3}$	A1	2.2a
		(3)	
(6 marks)			

November 2020 Question 9 Paper 1

9.

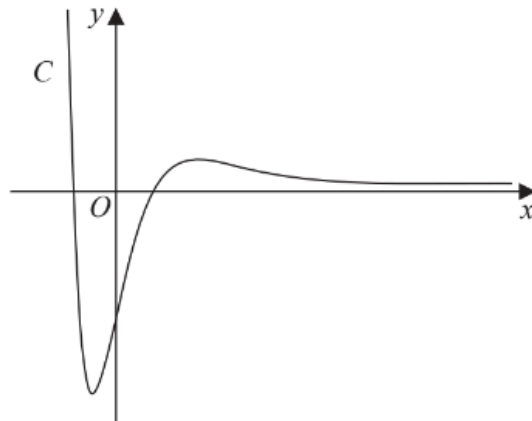


Figure 2

Figure 2 shows a sketch of the curve C with equation $y = f(x)$ where

$$f(x) = 4(x^2 - 2)e^{-2x} \quad x \in \mathbb{R}$$

(a) Show that $f'(x) = 8(2 + x - x^2)e^{-2x}$ (3)

(b) Hence find, in simplest form, the exact coordinates of the stationary points of C . (3)

The function g and the function h are defined by

$$g(x) = 2f(x) \quad x \in \mathbb{R}$$

$$h(x) = 2f(x) - 3 \quad x \geq 0$$

(c) Find
 (i) the range of g
 (ii) the range of h (3)

ANSWER

Question	Scheme	Marks	AOs
9(a)	$f(x) = 4(x^2 - 2)e^{-2x}$		
	Differentiates to $e^{-2x} \times 8x + 4(x^2 - 2) \times -2e^{-2x}$	M1 A1	1.1b 1.1b
	$f'(x) = 8e^{-2x} \{x - (x^2 - 2)\} = 8(2 + x - x^2)e^{-2x}$ *	A1*	2.1
		(3)	
(b)	States roots of $f'(x) = 0$ $x = -1, 2$	B1	1.1b
	Substitutes one x value to find a y value	M1	1.1b
	Stationary points are $(-1, -4e^2)$ and $(2, 8e^{-4})$	A1	1.1b
		(3)	
(c)	(i) Range $[-8e^2, \infty)$ o.e. such as $g(x) \geq -8e^2$	B1ft	2.5
	(ii) For - Either attempting to find $2f(0) - 3 = 2 \times -8 - 3 = (-19)$ and identifying this as the lower bound - Or attempting to find $2 \times "8e^{-4}" - 3$ and identifying this as the upper bound	M1	3.1a
	Range $[-19, 16e^{-4} - 3]$	A1	1.1b
		(3)	

November 2020 Question 15 Paper 1

15. The curve C has equation

$$x^2 \tan y = 9 \quad 0 < y < \frac{\pi}{2}$$

(a) Show that

$$\frac{dy}{dx} = \frac{-18x}{x^4 + 81}$$

(4)

(b) Prove that C has a point of inflection at $x = \sqrt[4]{27}$

(3)

ANSWER

Question	Scheme	Marks	AOs
15 (a)	$x^2 \tan y = 9 \Rightarrow 2x \tan y + x^2 \sec^2 y \frac{dy}{dx} = 0$	M1 A1	3.1a 1.1b
	Full method to get $\frac{dy}{dx}$ in terms of x using $\sec^2 y = 1 + \tan^2 y = 1 + f(x)$	M1	1.1b
	$\frac{dy}{dx} = \frac{-2x \times \frac{9}{x^2}}{x^2 \left(1 + \frac{81}{x^4}\right)} = \frac{-18x}{x^4 + 81} *$	A1*	2.1
		(4)	
(b)	$\frac{dy}{dx} = \frac{-18x}{x^4 + 81}$ $\Rightarrow \frac{d^2y}{dx^2} = \frac{-18 \times (x^4 + 81) - (-18x)(4x^3)}{(x^4 + 81)^2} = \frac{54(x^4 - 27)}{(x^4 + 81)^2} \text{ o.e.}$	M1 A1	1.1b 1.1b
	States that when $x < \sqrt[4]{27} \Rightarrow \frac{d^2y}{dx^2} < 0$ when $x = \sqrt[4]{27} \Rightarrow \frac{d^2y}{dx^2} = 0$ AND when $x > \sqrt[4]{27} \Rightarrow \frac{d^2y}{dx^2} > 0$ giving a point of inflection when $x = \sqrt[4]{27}$	A1	2.4
		(3)	
(7 marks)			

November 2020 Question 13 Paper 2

13. The function g is defined by

$$g(x) = \frac{3\ln(x) - 7}{\ln(x) - 2} \quad x > 0 \quad x \neq k$$

where k is a constant.

(a) Deduce the value of k .

(1)

(b) Prove that

$$g'(x) > 0$$

for all values of x in the domain of g .

(3)

(c) Find the range of values of a for which

$$g(a) > 0$$

(2)

ANSWER

Question	Scheme	Marks	AOs
13(a)	$k = e^2$ or $x \neq e^2$	B1	2.2a
		(1)	
(b)	$g'(x) = \frac{(\ln x - 2) \times \frac{3}{x} - (3 \ln x - 7) \times \frac{1}{x}}{(\ln x - 2)^2} = \frac{1}{x(\ln x - 2)^2}$ or $g'(x) = \frac{d}{dx} \left(3 - (\ln(x) - 2)^{-1} \right) = (\ln x - 2)^{-2} \times \frac{1}{x} = \frac{1}{x(\ln x - 2)^2}$ or $g'(x) = (\ln x - 2)^{-1} \times \frac{3}{x} - (3 \ln x - 7)(\ln x - 2)^{-2} \times \frac{1}{x} = \frac{1}{x(\ln x - 2)^2}$	M1 A1	1.1b 2.1
	As $x > 0$ (or $1/x > 0$) AND $\ln x - 2$ is squared so $g'(x) > 0$	A1cso	2.4
		(3)	
(c)	Attempts to solve either $3 \ln x - 7 \dots 0$ or $\ln x - 2 \dots 0$ or $3 \ln a - 7 \dots 0$ or $\ln a - 2 \dots 0$ where $\dots$ is “=” or “>” to reach a value for x or a but may be seen as an inequality e.g. $x > \dots$ or $a > \dots$	M1	3.1a
	$0 < a < e^2, \quad a > e^{\frac{7}{3}}$	A1	2.2a
		(2)	
(6 marks)			

June 2019 Question 3 Paper 1

3.
$$y = \frac{5x^2 + 10x}{(x+1)^2} \quad x \neq -1$$

(a) Show that $\frac{dy}{dx} = \frac{A}{(x+1)^n}$ where A and n are constants to be found. (4)

(b) Hence deduce the range of values for x for which $\frac{dy}{dx} < 0$ (1)

ANSWER

Question	Scheme	Marks	AOs
3 (a)	Correct method used in attempting to differentiate $y = \frac{5x^2 + 10x}{(x+1)^2}$	M1	3.1a
	$\frac{dy}{dx} = \frac{(x+1)^2 \times (10x+10) - (5x^2 + 10x) \times 2(x+1)}{(x+1)^4}$ oe	A1	1.1b
	Factorises/Cancel term in $(x+1)$ and attempts to simplify $\frac{dy}{dx} = \frac{(x+1) \times (10x+10) - (5x^2 + 10x) \times 2}{(x+1)^3} = \frac{A}{(x+1)^3}$	M1	2.1
	$\frac{dy}{dx} = \frac{10}{(x+1)^3}$	A1	1.1b
		(4)	
(b)	For $x < -1$ Follow through on their $\frac{dy}{dx} = \frac{A}{(x+1)^n}, n = 1, 3$	B1ft	2.2a
		(1)	
(5 marks)			

June 2019 Question 14 Paper 1

14. The curve C , in the standard Cartesian plane, is defined by the equation

$$x = 4 \sin 2y \quad \frac{-\pi}{4} < y < \frac{\pi}{4}$$

The curve C passes through the origin O

(a) Find the value of $\frac{dy}{dx}$ at the origin. (2)

(b) (i) Use the small angle approximation for $\sin 2y$ to find an equation linking x and y for points close to the origin.

(ii) Explain the relationship between the answers to (a) and (b)(i). (2)

(c) Show that, for all points (x, y) lying on C ,

$$\frac{dy}{dx} = \frac{1}{a\sqrt{b-x^2}}$$

where a and b are constants to be found. (3)

ANSWER

Question	Scheme	Marks	AOs
14 (a)	Attempts to differentiate $x = 4 \sin 2y$ and inverts $\frac{dx}{dy} = 8 \cos 2y \Rightarrow \frac{dy}{dx} = \frac{1}{8 \cos 2y}$	M1	1.1b
	At (0,0) $\frac{dy}{dx} = \frac{1}{8}$	A1	1.1b
		(2)	
(b)	(i) Uses $\sin 2y \approx 2y$ when y is small to obtain $x \approx 8y$	B1	1.1b
	(ii) The value found in (a) is the gradient of the line found in (b)(i)	B1	2.4
		(2)	
(c)	Uses their $\frac{dy}{dx}$ as a function of y and, using both $\sin^2 2y + \cos^2 2y = 1$ and $x = 4 \sin 2y$ in an attempt to write $\frac{dy}{dx}$ or $\frac{dx}{dy}$ as a function of x Allow for $\frac{dy}{dx} = k \frac{1}{\cos 2y} = \dots \frac{1}{\sqrt{1 - (\dots)^2}}$	M1	2.1
	A correct answer $\frac{dy}{dx} = \frac{1}{8\sqrt{1 - \left(\frac{x}{4}\right)^2}}$ or $\frac{dx}{dy} = 8\sqrt{1 - \left(\frac{x}{4}\right)^2}$	A1	1.1b
	and in the correct form $\frac{dy}{dx} = \frac{1}{2\sqrt{16 - x^2}}$	A1	1.1b
		(3)	
(7 marks)			

June 2018 Question 5 Paper 1

5. Given that

$$y = \frac{3 \sin \theta}{2 \sin \theta + 2 \cos \theta} \quad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

show that

$$\frac{dy}{d\theta} = \frac{A}{1 + \sin 2\theta} \quad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

where A is a rational constant to be found.

(5)

ANSWER

Question	Scheme	Marks	AOs
5	$\frac{dy}{d\theta} = \frac{(2 \sin \theta + 2 \cos \theta)3 \cos \theta - 3 \sin \theta(2 \cos \theta - 2 \sin \theta)}{(2 \sin \theta + 2 \cos \theta)^2}$	M1 A1	1.1b 1.1b
	Expands and uses $\sin^2 \theta + \cos^2 \theta = 1$ at least once in the numerator or the denominator or uses $2 \sin \theta \cos \theta = \sin 2\theta$ in $\Rightarrow \frac{dy}{d\theta} = \frac{\dots}{\dots C \sin \theta \cos \theta}$	M1	3.1a
	Expands and uses $\sin^2 \theta + \cos^2 \theta = 1$ the numerator and the denominator AND uses $2 \sin \theta \cos \theta = \sin 2\theta$ in $\Rightarrow \frac{dy}{d\theta} = \frac{P}{Q + R \sin 2\theta}$	M1	2.1
	$\Rightarrow \frac{dy}{d\theta} = \frac{3}{2 + 2 \sin 2\theta} = \frac{\frac{3}{2}}{1 + \sin 2\theta}$	A1	1.1b
(5 marks)			

June 2018 Question 9 Paper 1

9.

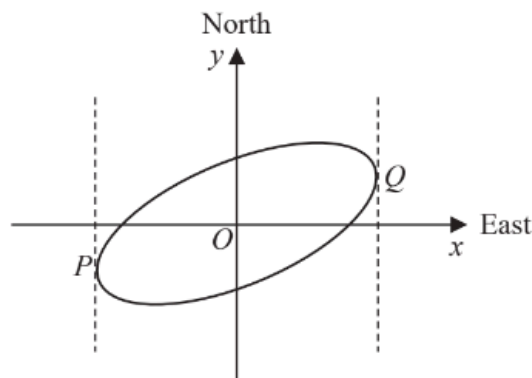


Figure 4

Figure 4 shows a sketch of the curve with equation $x^2 - 2xy + 3y^2 = 50$

- (a) Show that $\frac{dy}{dx} = \frac{y - x}{3y - x}$ (4)

The curve is used to model the shape of a cycle track with both x and y measured in km.

The points P and Q represent points that are furthest west and furthest east of the origin O , as shown in Figure 4.

Using part (a),

- (b) find the exact coordinates of the point P . (5)

- (c) Explain briefly how to find the coordinates of the point that is furthest north of the origin O . (You **do not** need to carry out this calculation). (1)

ANSWER

Question	Scheme	Marks	AOs
9(a)	Either $3y^2 \rightarrow Ay \frac{dy}{dx}$ or $2xy \rightarrow 2x \frac{dy}{dx} + 2y$	M1	2.1
	$2x - 2x \frac{dy}{dx} - 2y + 6y \frac{dy}{dx} = 0$	A1	1.1b
	$(6y - 2x) \frac{dy}{dx} = 2y - 2x$	M1	2.1
	$\frac{dy}{dx} = \frac{2y - 2x}{6y - 2x} = \frac{y - x}{3y - x} *$	A1*	1.1b
		(4)	
(b)	$\left(\text{At } P \text{ and } Q \frac{dy}{dx} \rightarrow \infty \Rightarrow \right)$ Deduces that $3y - x = 0$	M1	2.2a
	Solves $y = \frac{1}{3}x$ and $x^2 - 2xy + 3y^2 = 50$ simultaneously	M1	3.1a
	$\Rightarrow x = (\pm)5\sqrt{3}$ OR $\Rightarrow y = (\pm)\frac{5}{3}\sqrt{3}$	A1	1.1b
	Using $y = \frac{1}{3}x \Rightarrow x = ..$ AND $y = ..$	dM1	1.1b
	$P = \left(-5\sqrt{3}, -\frac{5}{3}\sqrt{3} \right)$	A1	2.2a
		(5)	
(c)	Explains that you need to solve $y = x$ and $x^2 - 2xy + 3y^2 = 50$ simultaneously and choose the positive solution	B1ft	2.4
		(1)	
(10 marks)			

June 2018 Question 9 Paper 2

9. Given that θ is measured in radians, prove, from first principles, that

$$\frac{d}{d\theta}(\cos \theta) = -\sin \theta$$

You may assume the formula for $\cos(A \pm B)$ and that as $h \rightarrow 0$, $\frac{\sin h}{h} \rightarrow 1$ and $\frac{\cos h - 1}{h} \rightarrow 0$
(5)

ANSWER

Question	Scheme	Marks	AOs
9	$\frac{d}{d\theta}(\cos \theta) = -\sin \theta$; as $h \rightarrow 0$, $\frac{\sin h}{h} \rightarrow 1$ and $\frac{\cos h - 1}{h} \rightarrow 0$		
	$\frac{\cos(\theta + h) - \cos \theta}{h}$	B1	2.1
	$= \frac{\cos \theta \cos h - \sin \theta \sin h - \cos \theta}{h}$	M1	1.1b
		A1	1.1b
	$= -\frac{\sin h}{h} \sin \theta + \left(\frac{\cos h - 1}{h}\right) \cos \theta$		
	As $h \rightarrow 0$, $-\frac{\sin h}{h} \sin \theta + \left(\frac{\cos h - 1}{h}\right) \cos \theta \rightarrow -1 \sin \theta + 0 \cos \theta$	dM1	2.1
	so $\frac{d}{d\theta}(\cos \theta) = -\sin \theta$ *	A1*	2.5
		(5)	
(5 marks)			

June 2018 Question 14 Paper 2

14. A scientist is studying a population of mice on an island.

The number of mice, N , in the population, t months after the start of the study, is modelled by the equation

$$N = \frac{900}{3 + 7e^{-0.25t}}, \quad t \in \mathbb{R}, \quad t \geq 0$$

(a) Find the number of mice in the population at the start of the study.

(1)

(b) Show that the rate of growth $\frac{dN}{dt}$ is given by $\frac{dN}{dt} = \frac{N(300 - N)}{1200}$

(4)

The rate of growth is a maximum after T months.

(c) Find, according to the model, the value of T .

(4)

According to the model, the maximum number of mice on the island is P .

(d) State the value of P .

(1)

ANSWER

Question	Scheme	Marks	AOs
14	$N = \frac{900}{3 + 7e^{-0.25t}} = 900(3 + 7e^{-0.25t})^{-1}, t \in \mathbb{R}, t \geq 0; \quad \frac{dN}{dt} = \frac{N(300 - N)}{1200}$		
(a)	90	B1	3.4
		(1)	
(b) Way 1	$\frac{dN}{dt} = -900(3 + 7e^{-0.25t})^{-2} (7(-0.25)e^{-0.25t}) \quad \left\{ = \frac{900(0.25)(7)e^{-0.25t}}{(3 + 7e^{-0.25t})^2} \right\}$	M1	2.1
		A1	1.1b
	$\Rightarrow \frac{dN}{dt} = \frac{900(0.25) \left(\left(\frac{900}{N} - 3 \right) \right)}{\left(\frac{900}{N} \right)^2}$	dM1	2.1
	correct algebra leading to $\frac{dN}{dt} = \frac{N(300 - N)}{1200} \quad *$	A1*	1.1b
		(4)	
(b) Way 2	$\frac{dN}{dt} = -900(3 + 7e^{-0.25t})^{-2} (7(-0.25)e^{-0.25t}) \quad \left\{ = \frac{900(0.25)(7)e^{-0.25t}}{(3 + 7e^{-0.25t})^2} \right\}$	M1	2.1
		A1	1.1b
	$\frac{N(300 - N)}{1200} = \frac{\left(\frac{900}{3 + 7e^{-0.25t}} \right) \left(300 - \frac{900}{3 + 7e^{-0.25t}} \right)}{1200}$	dM1	2.1
	LHS = $\frac{1575e^{-0.25t}}{(3 + 7e^{-0.25t})^2}$ o.e., RHS = $\frac{900(300(3 + 7e^{-0.25t}) - 900)}{1200(3 + 7e^{-0.25t})^2} = \frac{1575e^{-0.25t}}{(3 + 7e^{-0.25t})^2}$ o.e. and states hence $\frac{dN}{dt} = \frac{N(300 - N)}{1200}$ (or LHS = RHS) *	A1*	1.1b
		(4)	
(c)	Deduces $N = 150$ (can be implied)	B1	2.2a
	so $150 = \frac{900}{3 + 7e^{-0.25T}} \Rightarrow e^{-0.25T} = \frac{3}{7}$	M1	3.4
	$T = -4 \ln\left(\frac{3}{7}\right)$ or $T = \text{awrt } 3.4$ (months)	dM1	1.1b
		A1	1.1b
		(4)	
(d)	either one of 299 or 300	B1	3.4
		(1)	
(10 marks)			

June 2018 Question 1 Core 3

1. Given $y = 2x(3x - 1)^5$,

(a) find $\frac{dy}{dx}$, giving your answer as a single fully factorised expression. (4)

(b) Hence find the set of values of x for which $\frac{dy}{dx} \leq 0$ (2)

ANSWER

Question Number	Scheme	Marks
1.(a)	$y = 2x(3x - 1)^5 \Rightarrow \frac{dy}{dx} = 2(3x - 1)^5 + 30x(3x - 1)^4$ $\Rightarrow \left(\frac{dy}{dx}\right) = 2(3x - 1)^4 \{(3x - 1) + 15x\} = 2(3x - 1)^4 (18x - 1)$	M1A1 M1A1 (4)
(b)	$\frac{dy}{dx} \leq 0 \Rightarrow 2(3x - 1)^4 (18x - 1) \leq 0 \Rightarrow x \leq \frac{1}{18} \quad x = \frac{1}{3}$	B1ft, B1 (2) (6 marks)

Ch. 10: Numerical Methods

June 2024 Question 6 Paper 2

6.

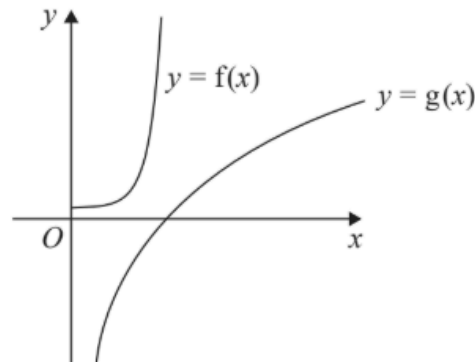


Figure 1

Figure 1 shows a sketch of the curves with equations $y = f(x)$ and $y = g(x)$ where

$$\begin{aligned} f(x) &= e^{4x^2 - 1} & x > 0 \\ g(x) &= 8 \ln x & x > 0 \end{aligned}$$

(a) Find

(i) $f'(x)$

(ii) $g'(x)$

(2)

Given that $f'(x) = g'(x)$ at $x = \alpha$

(b) show that α satisfies the equation

$$4x^2 + 2 \ln x - 1 = 0$$

(2)

The iterative formula

$$x_{n+1} = \sqrt{\frac{1 - 2 \ln x_n}{4}}$$

is used with $x_1 = 0.6$ to find an approximate value for α

(c) Calculate, giving each answer to 4 decimal places,

(i) the value of x_2

(ii) the value of α

(3)

ANSWER

Question	Scheme	Marks	AOs
6(a)(i)	$(f'(x) =) 8xe^{4x^2-1}$ or e.g. $\frac{8xe^{4x^2}}{e}$ oe	B1	1.1b
	$(g'(x) =) \frac{8}{x}$ or e.g. $8x^{-1}$ oe	B1	1.2
		(2)	
(b)	$8xe^{4x^2-1} = \frac{8}{x} \Rightarrow e^{4x^2-1} = \frac{1}{x^2} \Rightarrow 4x^2 - 1 = \ln \frac{1}{x^2}$	M1	1.1b
	$4x^2 - 1 = \ln \frac{1}{x^2} \Rightarrow 4x^2 - 1 = -2 \ln x$ $\Rightarrow 4x^2 + 2 \ln x - 1 = 0^*$	A1*	2.1
		(2)	
(c)(i)	$x_1 = 0.6 \Rightarrow x_2 = \sqrt{\frac{1-2 \ln 0.6}{4}}$	M1	1.1b
	$(x_2 =) 0.7109$	A1	1.1b
	$(\alpha =) 0.6706$	B1 (A1 In ePEN)	1.1b
		(3)	
(7 marks)			

June 2024 Question 3 Paper 1

3.
$$f(x) = x + \tan\left(\frac{1}{2}x\right) \quad \pi < x < \frac{3\pi}{2}$$

Given that the equation $f(x) = 0$ has a single root α

(a) show that α lies in the interval $[3.6, 3.7]$

(2)

(b) Find $f'(x)$

(2)

(c) Using 3.7 as a first approximation for α , apply the Newton–Raphson method once to obtain a second approximation for α . Give your answer to 3 decimal places.

(2)

ANSWER

Question	Scheme	Marks	AOs
3(a)	$\{f(3.6) = \} 3.6 + \tan\left(\frac{1}{2}(3.6)\right) = -0.686... < 0$ and $\{f(3.7) = \} 3.7 + \tan\left(\frac{1}{2}(3.7)\right) = 0.211... > 0$	M1	1.1b
	<u>Change of sign</u> and function is <u>continuous</u> in the interval $\Rightarrow$ <u>conclusion</u> e.g. “there is a root in $[3.6, 3.7]$ ” *	A1*	2.4
		(2)	
(b)	Use of $\tan\left(\frac{1}{2}x\right) \rightarrow \dots \sec^2\left(\frac{1}{2}x\right)$	M1	1.1b
	$\{f'(x) = \} 1 + \frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$	A1	1.1b
		(2)	
(c)	Attempts $3.7 - \frac{3.7 + \tan\left(\frac{1}{2}(3.7)\right)}{1 + \frac{1}{2}\sec^2\left(\frac{1}{2}(3.7)\right)} = \dots$ (N.B. $f(3.7) = 0.211...$ and $f'(3.7) = 7.58...$)	M1	1.1b
	$\alpha = \text{awrt } 3.672$	A1	1.1b
		(2)	
(6 marks)			

June 2023 Question 15 Paper 1

15. A curve has equation $y = f(x)$, where

$$f(x) = \frac{7xe^x}{\sqrt{e^{3x} - 2}} \quad x > \ln \sqrt[3]{2}$$

(a) Show that

$$f'(x) = \frac{7e^x(e^{3x}(2-x) + Ax + B)}{2(e^{3x} - 2)^{\frac{3}{2}}}$$

where A and B are constants to be found.

(5)

(b) Hence show that the x coordinates of the turning points of the curve are solutions of the equation

$$x = \frac{2e^{3x} - 4}{e^{3x} + 4}$$

(2)

The equation $x = \frac{2e^{3x} - 4}{e^{3x} + 4}$ has two positive roots α and β where $\beta > \alpha$

A student uses the iteration formula

$$x_{n+1} = \frac{2e^{3x_n} - 4}{e^{3x_n} + 4}$$

in an attempt to find approximations for α and β

Diagram 1 shows a plot of part of the curve with equation $y = \frac{2e^{3x} - 4}{e^{3x} + 4}$ and part of the line with equation $y = x$

Using Diagram 1 on page 42

(c) draw a staircase diagram to show that the iteration formula starting with $x_1 = 1$ can be used to find an approximation for β

(1)

Use the iteration formula with $x_1 = 1$, to find, to 3 decimal places,

- (d) (i) the value of x_2
(ii) the value of β

(3)

Using a suitable interval and a suitable function that should be stated

- (e) show that $\alpha = 0.432$ to 3 decimal places.

(2)

Only use the copy of Diagram 1 if you need to redraw your answer to part (c).

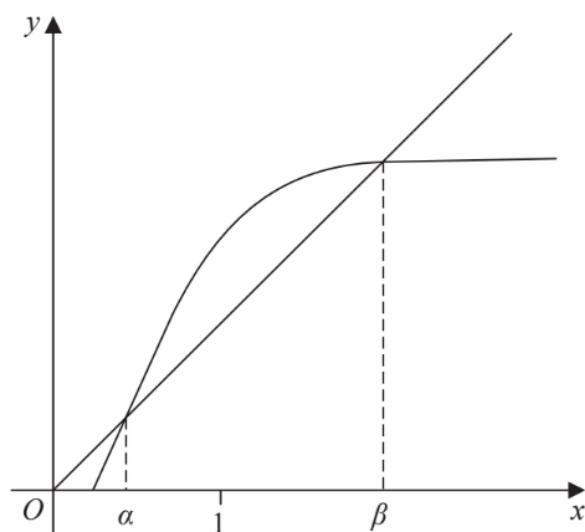
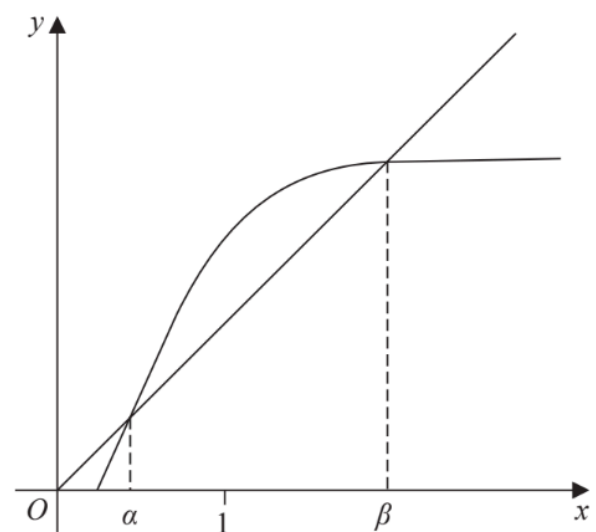


Diagram 1



copy of Diagram 1

ANSWER

Question	Scheme	Marks	AOs
15(a)	$\dots xe^x + \dots e^x$	M1	1.1b
	$k(xe^x + e^x)$	A1	1.1b
	$\frac{d}{dx}(\sqrt{e^{3x}-2}) = \frac{1}{2} \times 3e^{3x}(e^{3x}-2)^{-\frac{1}{2}}$	B1	1.1b
	$(f'(x) =) \frac{(e^{3x}-2)^{\frac{1}{2}}(7xe^x + 7e^x) - \frac{3}{2}e^{3x}(e^{3x}-2)^{-\frac{1}{2}} \times 7xe^x}{e^{3x}-2}$	dM1	2.1
	$f'(x) = \frac{7e^x(e^{3x}(2-x) - 4x - 4)}{2(e^{3x}-2)^{\frac{3}{2}}}$	A1	1.1b
		(5)	
(b)	$e^{3x}(2-x) - 4x - 4 = 0 \Rightarrow x(\dots e^{3x} \pm \dots) = \dots e^{3x} \pm \dots$	M1	1.1b
	$\Rightarrow x = \frac{2e^{3x}-4}{e^{3x}+4} *$	A1*	2.1
		(2)	
(c)	Draws a vertical line $x=1$ up to the curve then across to the line $y=x$ then up to the curve finishing at the root (need to see a minimum of 2 vertical and horizontal lines tending to the root)	B1	2.1
		(1)	
(d)(i)	$x_2 = \frac{2e^3-4}{e^3+4} = 1.5017756\dots$	M1	1.1b
	$x_2 = \text{awrt } 1.502$	A1	1.1b
	$\beta = 1.968$	dB1	2.2b
		(3)	
(e)	$h(x) = \frac{2e^{3x}-4}{e^{3x}+4} - x$ $h(0.4315) = -0.000297\dots \quad h(0.4325) = 0.000947\dots$	M1	3.1a
	Both calculations correct and e.g. states: - There is a change of sign - e.g $f'(x)$ is continuous $\alpha = 0.432$ (to 3dp)	A1cao	2.4
		(2)	
(13 marks)			

June 2023 Question 5 Paper 1

5. A continuous curve has equation $y = f(x)$.

The table shows corresponding values of x and y for this curve, where a and b are constants.

x	3	3.2	3.4	3.6	3.8	4
y	a	16.8	b	20.2	18.7	13.5

The trapezium rule is used, with all the y values in the table, to find an approximate area under the curve between $x = 3$ and $x = 4$

Given that this area is 17.59

- (a) show that $a + 2b = 51$

(3)

Given also that the sum of all the y values in the table is 97.2

- (b) find the value of a and the value of b

(3)

ANSWER

Question	Scheme	Marks	AOs
5(a)	$h = 0.2$	B1	1.1b
	$\frac{1}{2} \times 0.2 \times \{a + 13.5 + 2(16.8 + b + 20.2 + 18.7)\} = 17.59$	M1	1.1b
	e.g. $\Rightarrow a + 13.5 + 2b + 111.4 = 175.9 \Rightarrow a + 2b = 51^*$	A1*	2.1
		(3)	
(b)	$a + 16.8 + b + 20.2 + 18.7 + 13.5 = 97.2 \Rightarrow a + b = 28 \Rightarrow a = \dots$ (or $b = \dots$)	M1	3.1a
	$a = 5$ or $b = 23$	A1	1.1b
	$a = 5$ and $b = 23$	A1	1.1b
		(3)	
(6 marks)			

June 2022 Question 6 Paper 2

6.

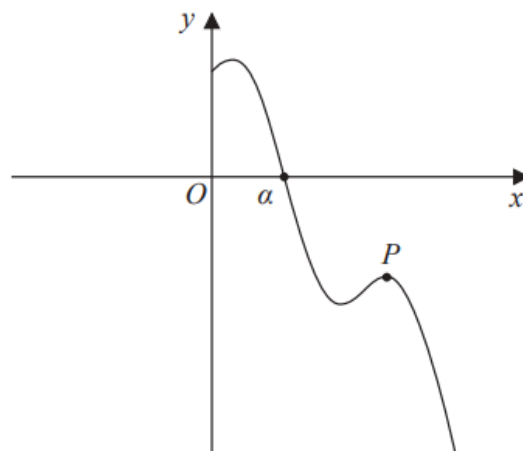


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$ where

$$f(x) = 8 \sin\left(\frac{1}{2}x\right) - 3x + 9 \quad x > 0$$

and x is measured in radians.

The point P , shown in Figure 2, is a local maximum point on the curve.

Using calculus and the sketch in Figure 2,

(a) find the x coordinate of P , giving your answer to 3 significant figures.

(4)

The curve crosses the x -axis at $x = \alpha$, as shown in Figure 2.

Given that, to 3 decimal places, $f(4) = 4.274$ and $f(5) = -1.212$

(b) explain why α must lie in the interval $[4, 5]$

(1)

(c) Taking $x_0 = 5$ as a first approximation to α , apply the Newton-Raphson method once to $f(x)$ to obtain a second approximation to α .

Show your method and give your answer to 3 significant figures.

(2)

ANSWER

Question	Scheme	Marks	AOs
6(a)	$(f'(x) =) 4 \cos\left(\frac{1}{2}x\right) - 3$	M1 A1	1.1b 1.1b
	Sets $f'(x) = 4 \cos\left(\frac{1}{2}x\right) - 3 = 0 \Rightarrow x =$	dM1	3.1a
	$x = 14.0$ Cao	A1	3.2a
		(4)	
(b)	Explains that $f(4) > 0$, $f(5) < 0$ and the function is continuous	B1	2.4
		(1)	
(c)	Attempts $x_1 = 5 - \frac{8 \sin 2.5 - 15 + 9}{"4 \cos 2.5 - 3"}$ (NB $f(5) = -1.212...$ and $f'(5) = -6.204...$)	M1	1.1b
	$x_1 = \text{awrt } 4.80$	A1	1.1b
		(2)	
(7 marks)			

November 2021 Question 4 Paper 1

4. The curve with equation $y = f(x)$ where

$$f(x) = x^2 + \ln(2x^2 - 4x + 5)$$

has a single turning point at $x = \alpha$

- (a) Show that α is a solution of the equation

$$2x^3 - 4x^2 + 7x - 2 = 0$$

(4)

The iterative formula

$$x_{n+1} = \frac{1}{7}(2 + 4x_n^2 - 2x_n^3)$$

is used to find an approximate value for α .

Starting with $x_1 = 0.3$

- (b) calculate, giving each answer to 4 decimal places,

(i) the value of x_2

(ii) the value of x_4

(3)

Using a suitable interval and a suitable function that should be stated,

- (c) show that α is 0.341 to 3 decimal places.

(2)

ANSWER

Question	Scheme	Marks	AOs
4(a)	$f'(x) = 2x + \frac{4x-4}{2x^2-4x+5}$	M1 A1	1.1b 1.1b
	$2x + \frac{4x-4}{2x^2-4x+5} = 0 \Rightarrow 2x(2x^2-4x+5) + 4x-4 = 0$	dM1	1.1b
	$2x^3 - 4x^2 + 7x - 2 = 0^*$	A1*	2.1
		(4)	
(b)	(i) $x_2 = \frac{1}{7}(2 + 4(0.3)^2 - 2(0.3)^3)$	M1	1.1b
	$x_2 = 0.3294$	A1	1.1b
	(ii) $x_4 = 0.3398$	A1	1.1b
		(3)	
(c)	$h(x) = 2x^3 - 4x^2 + 7x - 2$ $h(0.3415) = 0.00366... \quad h(0.3405) = -0.00130...$	M1	3.1a
	States: - there is a change of sign $f'(x)$ is continuous $\alpha = 0.341$ to 3dp	A1	2.4
		(2)	
(9 marks)			

November 2020 Question 7 Paper 2

7.

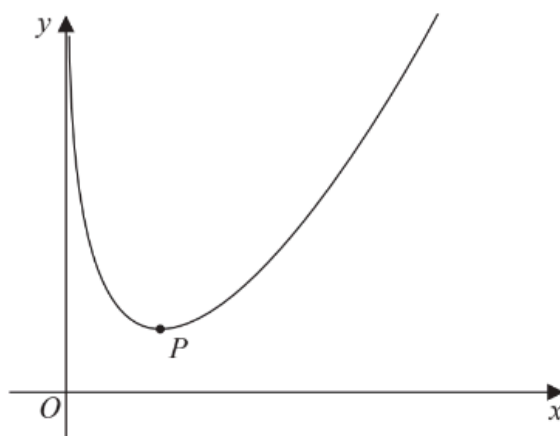


Figure 1

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{4x^2 + x}{2\sqrt{x}} - 4\ln x \quad x > 0$$

(a) Show that

$$\frac{dy}{dx} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}} \quad (4)$$

The point P , shown in Figure 1, is the minimum turning point on C .

(b) Show that the x coordinate of P is a solution of

$$x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12} \right)^{\frac{2}{3}} \quad (3)$$

(c) Use the iteration formula

$$x_{n+1} = \left(\frac{4}{3} - \frac{\sqrt{x_n}}{12} \right)^{\frac{2}{3}} \quad \text{with } x_1 = 2$$

to find (i) the value of x_2 to 5 decimal places,

(ii) the x coordinate of P to 5 decimal places.

(3)

ANSWER

7(a)	$\ln x \rightarrow \frac{1}{x}$	B1	1.1a
	Method to differentiate $\frac{4x^2 + x}{2\sqrt{x}}$ – see notes	M1	1.1b
	E.g. $2 \times \frac{3}{2}x^{\frac{1}{2}} + \frac{1}{2} \times \frac{1}{2}x^{-\frac{1}{2}}$	A1	1.1b
	$\frac{dy}{dx} = 3\sqrt{x} + \frac{1}{4\sqrt{x}} - \frac{4}{x} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}}$ *	A1*	2.1
		(4)	
(b)	$12x^2 + x - 16\sqrt{x} = 0 \Rightarrow 12x^{\frac{3}{2}} + x^{\frac{1}{2}} - 16 = 0$	M1	1.1b
	E.g. $12x^{\frac{3}{2}} = 16 - \sqrt{x}$	dM1	1.1b
	$x^{\frac{3}{2}} = \frac{4}{3} - \frac{\sqrt{x}}{12} \Rightarrow x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}$ *	A1*	2.1
		(3)	
(c)	$x_2 = \sqrt[3]{\left(\frac{4}{3} - \frac{\sqrt{2}}{12}\right)^2}$	M1	1.1b
	$x_2 = \text{awrt } 1.13894$	A1	1.1b
	$x = 1.15650$	A1	2.2a
		(3)	
(10 marks)			

June 2019 Question 11 Paper 2

11.

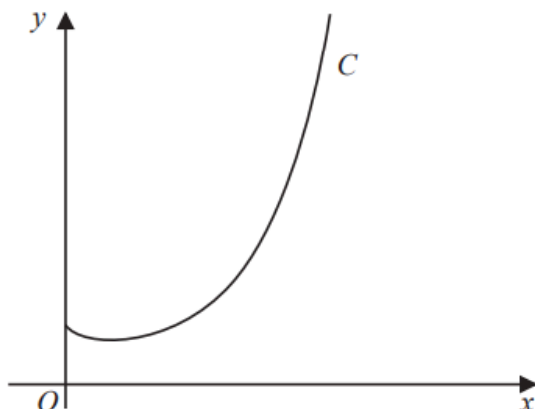


Figure 8

Figure 8 shows a sketch of the curve C with equation $y = x^x$, $x > 0$

- (a) Find, by firstly taking logarithms, the x coordinate of the turning point of C .
(Solutions based entirely on graphical or numerical methods are not acceptable.)

(5)

The point $P(\alpha, 2)$ lies on C .

- (b) Show that $1.5 < \alpha < 1.6$

(2)

A possible iteration formula that could be used in an attempt to find α is

$$x_{n+1} = 2x_n^{1-x_n}$$

Using this formula with $x_1 = 1.5$

- (c) find x_4 to 3 decimal places,

(2)

- (d) describe the long-term behaviour of x_n

(2)

ANSWER

Question	Scheme	Marks	AOs	
11 (a) Way 1	$\{y = x^x \Rightarrow\} \ln y = x \ln x$	B1	1.1a	
	$\frac{1}{y} \frac{dy}{dx} = 1 + \ln x$	M1	1.1b	
		A1	2.1	
	$\left\{ \frac{dy}{dx} = 0 \Rightarrow \right\} \frac{x}{x} + \ln x = 0 \text{ or } 1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = \dots$	M1	1.1b	
	$x = e^{-1} \text{ or awrt } 0.368$	A1	1.1b	
	Note: $k \neq 0$	(5)		
(a) Way 2	$\{y = x^x \Rightarrow\} y = e^{x \ln x}$	B1	1.1a	
	$\frac{dy}{dx} = \left(\frac{x}{x} + \ln x \right) e^{x \ln x}$	M1	1.1b	
		A1	2.1	
	$\left\{ \frac{dy}{dx} = 0 \Rightarrow \right\} \frac{x}{x} + \ln x = 0 \text{ or } 1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = \dots$	M1	1.1b	
	$x = e^{-1} \text{ or awrt } 0.368$	A1	1.1b	
	Note: $k \neq 0$	(5)		
(b) Way 1	Attempts both $1.5^{1.5} = 1.8\dots$ and $1.6^{1.6} = 2.1\dots$ and at least one result is correct to awrt 1 dp	M1	1.1b	
	$1.8\dots < 2$ and $2.1\dots > 2$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1	
		(2)		
(c)	Attempts $x_{n+1} = 2x_n^{1-x_n}$ at least once with $x_1 = 1.5$	M1	1.1b	
	Can be implied by $2(1.5)^{1-1.5}$ or awrt 1.63			
	$\{x_4 = 1.67313\dots \Rightarrow\} x_4 = 1.673 \text{ (3 dp) cao}$	A1	1.1b	
		(2)		
(d)	Give 1 st B1 for any of - oscillates - periodic - non-convergent - divergent - fluctuates - goes up and down 1, 2, 1, 2, 1, 2 - alternates (condone)	Give B1 B1 for any of - periodic {sequence} with period 2 - oscillates between 1 and 2	B1	2.5
		Condone B1 B1 for any of - fluctuates between 1 and 2 - keep getting 1, 2 - alternates between 1 and 2 - goes up and down between 1 and 2 1, 2, 1, 2, 1, 2, ...	B1	2.5
			(2)	
(11 marks)				

June 2018 Question 4 Paper 1

4. The curve with equation $y = 2 \ln(8 - x)$ meets the line $y = x$ at a single point, $x = \alpha$.

(a) Show that $3 < \alpha < 4$

(2)

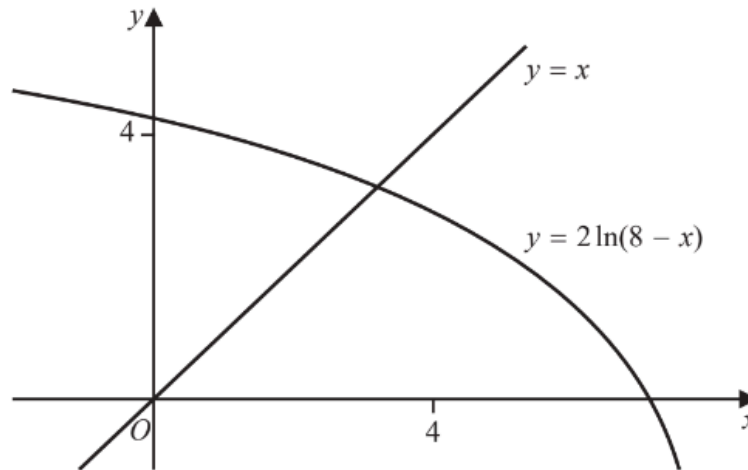


Figure 2

Figure 2 shows the graph of $y = 2 \ln(8 - x)$ and the graph of $y = x$.

A student uses the iteration formula

$$x_{n+1} = 2 \ln(8 - x_n), \quad n \in \mathbb{N}$$

in an attempt to find an approximation for α .

Using the graph and starting with $x_1 = 4$

(b) determine whether or not this iteration formula can be used to find an approximation for α , justifying your answer.

(2)

ANSWER

Question	Scheme	Marks	AOs
4 (a)	Attempts $f(3) =$ and $f(4) =$ where $f(x) = \pm(2\ln(8-x)-x)$	M1	2.1
	$f(3) = (2\ln(5)-x) = (+)0.22$ and $f(4) = (2\ln(4)-4) = -1.23$ <u>Change of sign</u> and function <u>continuous</u> in interval $[3,4] \Rightarrow$ <u>Root</u> *	A1*	2.4
		(2)	
(b)	For annotating the graph by drawing a cobweb diagram starting at $x_1 = 4$ It should have at least two spirals	M1	2.4
	Deduces that the iteration formula can be used to find an approximation for α because the cobweb spirals inwards for the cobweb diagram	A1	2.2a
		(2)	
(4 marks)			

June 2018 Question 5 Paper 2

5. The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root.

(a) Show that, for this equation, the Newton-Raphson formula can be written

$$x_{n+1} = \frac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n} \quad (3)$$

Using the formula given in part (a) with $x_1 = 1$

(b) find the values of x_2 and x_3 (2)

(c) Explain why, for this question, the Newton-Raphson method cannot be used with $x_1 = 0$ (1)

ANSWER

Question	Scheme	Marks	AOs
5	The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root		
(a)	$\{f(x) = 2x^3 + x^2 - 1 \Rightarrow f'(x) = 6x^2 + 2x$	B1	1.1b
	$\left\{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \Rightarrow \left\{x_{n+1}\right\} = x_n - \frac{2x_n^3 + x_n^2 - 1}{6x_n^2 + 2x_n}\right.$	M1	1.1b
	$= \frac{x_n(6x_n^2 + 2x_n) - (2x_n^3 + x_n^2 - 1)}{6x_n^2 + 2x_n} \Rightarrow x_{n+1} = \frac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n} *$	A1*	2.1
		(3)	
(b)	$\{x_1 = 1 \Rightarrow x_2 = \frac{4(1)^3 + (1)^2 + 1}{6(1)^2 + 2(1)} \text{ or } x_2 = 1 - \frac{2(1)^3 + (1)^2 - 1}{6(1)^2 + 2(1)}$	M1	1.1b
	$\Rightarrow x_2 = \frac{3}{4}, x_3 = \frac{2}{3}$	A1	1.1b
		(2)	
(c)	Accept any reasons why the Newton-Raphson method cannot be used with $x_1 = 0$ which refer or allude to either the stationary point or the tangent. E.g. • There is a stationary point at $x = 0$ • Tangent to the curve (or $y = 2x^3 + x^2 - 1$) would not meet the x-axis • Tangent to the curve (or $y = 2x^3 + x^2 - 1$) is horizontal	B1	2.3
		(1)	
(6 marks)			

June 2018 Question 4 Core 3

4.

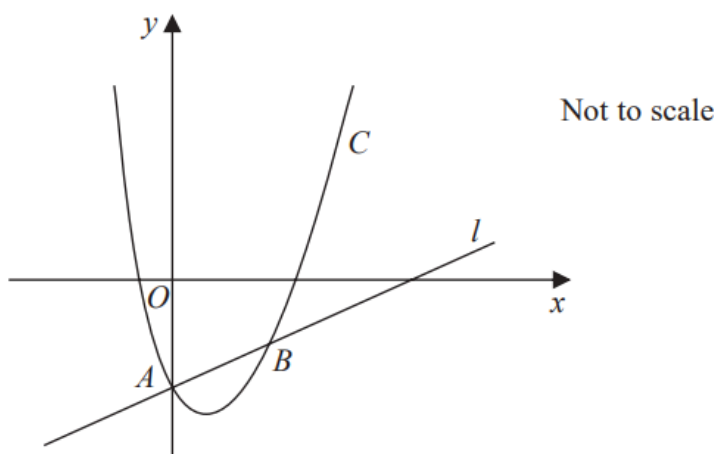


Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = e^{-2x} + x^2 - 3$$

The curve C crosses the y -axis at the point A .

The line l is the normal to C at the point A .

- (a) Find the equation of l , writing your answer in the form $y = mx + c$, where m and c are constants.

(5)

The line l meets C again at the point B , as shown in Figure 1.

- (b) Show that the x coordinate of B is a solution of

$$x = \sqrt{1 + \frac{1}{2}x - e^{-2x}}$$

(2)

Using the iterative formula

$$x_{n+1} = \sqrt{1 + \frac{1}{2}x_n - e^{-2x_n}}$$

with $x_1 = 1$

- (c) find x_2 and x_3 to 3 decimal places.

(2)

ANSWER

Question Number	Scheme	Marks
4.(a)	$\frac{dy}{dx} = -2e^{-2x} + 2x$ $\text{At } x=0 \quad \frac{dy}{dx} = -2 \Rightarrow \frac{dx}{dy} = \frac{1}{2}$ $\text{Equation of normal is } y - (-2) = \frac{1}{2}(x - 0) \Rightarrow y = \frac{1}{2}x - 2$	M1A1 M1 M1 A1 (5)
(b)	$y = e^{-2x} + x^2 - 3 \text{ meets } y = \frac{1}{2}x - 2 \text{ when } e^{-2x} + x^2 - 3 = \frac{1}{2}x - 2$ $x^2 = 1 + \frac{1}{2}x - e^{-2x}$ $x = \sqrt{1 + \frac{1}{2}x - e^{-2x}} \quad *$	M1 A1* (2)
(c)	$x_2 = \sqrt{1 + 0.5 - e^{-2}}$ $x_2 = 1.168, x_3 = 1.220$	M1 A1 (2) (9 marks)

Ch. 11: Integration

June 2024 Question 12 Paper 2

12. (a) Express $\frac{1}{V(25 - V)}$ in partial fractions. (2)

The volume, V microlitres, of a plant cell t hours after the plant is watered is modelled by the differential equation

$$\frac{dV}{dt} = \frac{1}{10}V(25 - V)$$

The plant cell has an initial volume of 20 microlitres.

- (b) Find, according to the model, the time taken, in minutes, for the volume of the plant cell to reach 24 microlitres. (5)

- (c) Show that

$$V = \frac{A}{e^{-kt} + B}$$

where A , B and k are constants to be found. (3)

The model predicts that there is an upper limit, L microlitres, on the volume of the plant cell.

- (d) Find the value of L , giving a reason for your answer. (2)

ANSWER

Question	Scheme	Marks	AOs
12(a)	$\frac{1}{V(25-V)} = \frac{P}{V} + \frac{Q}{25-V}$ e.g. $1 = P(25-V) + QV$ $V = 0$ or $V = 25$ leading to $P = \dots$ or $Q = \dots$	M1	1.1b
	$\frac{1}{V(25-V)} = \frac{1}{25V} + \frac{1}{25(25-V)}$	A1	1.1b
		(2)	
(b) Way 1	$\int \frac{1}{V(25-V)} dV = \int \frac{1}{25V} + \frac{1}{25(25-V)} dV = \frac{1}{25} \ln V - \frac{1}{25} \ln(25-V)$	M1	3.1a
	$\frac{1}{25} \ln V - \frac{1}{25} \ln(25-V) = \frac{1}{10} t (+c)$	A1ft	1.1b
	$t = 0, V = 20 \Rightarrow \frac{1}{25} \ln 20 - \frac{1}{25} \ln(25-20) = c \left(\Rightarrow c = \frac{1}{25} \ln 4 \right)$	M1	3.4
	$V = 24 \Rightarrow t = \frac{2}{5} \ln 24 - \frac{2}{5} \ln(25-24) - \frac{2}{5} \ln 4$	dM1	3.1b
	$= 43 \text{ (or exact } 24 \ln 6)$	A1	3.2a
		(5)	
(c)	$\frac{1}{25} \ln V - \frac{1}{25} \ln(25-V) = \frac{1}{10} t + \frac{1}{25} \ln 4$ $\ln V - \ln(25-V) = 2.5t + \ln 4$ $\ln \frac{V}{4(25-V)} = 2.5t \Rightarrow \frac{V}{4(25-V)} = e^{2.5t}$	M1	2.1
	$\Rightarrow V = 4e^{2.5t} (25-V) \Rightarrow V + 4Ve^{2.5t} = 100e^{2.5t} \Rightarrow V = \dots$	M1	2.1
	$\Rightarrow V = \frac{100e^{2.5t}}{1 + 4e^{2.5t}} = \frac{100}{e^{-2.5t} + 4}$	A1	1.1b
		(3)	
(d)	25 (microlitres)	B1	2.2a
	Since e.g. As $t \rightarrow \infty$, $e^{-2.5t} \rightarrow 0$	B1	2.4
		(2)	
(12 marks)			

June 2024 Question 11 Paper 2

11.

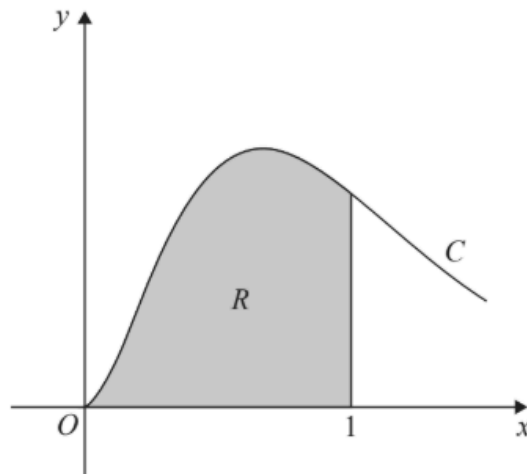


Figure 5

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 5 shows a sketch of part of the curve C with equation

$$y = 8x^2 e^{-3x} \quad x \geq 0$$

The finite region R , shown shaded in Figure 5, is bounded by

- the curve C
- the line with equation $x = 1$
- the x -axis

Find the exact area of R , giving your answer in the form

$$A + Be^{-3}$$

where A and B are rational numbers to be found.

(5)

ANSWER

Question	Scheme	Marks	AOs
11	$\int 8x^2 e^{-3x} dx = -\frac{8x^2}{3} e^{-3x} + \int \frac{16x}{3} e^{-3x} dx$	M1 A1	2.1 1.1b
	$= -\frac{8x^2}{3} e^{-3x} - \frac{16x}{9} e^{-3x} + \int \frac{16}{9} e^{-3x} dx$	dM1	1.1b
	$\left[-\frac{8x^2}{3} e^{-3x} - \frac{16x}{9} e^{-3x} - \frac{16}{27} e^{-3x} \right]_0^1$ $= -\frac{8}{3} e^{-3} - \frac{16}{9} e^{-3} - \frac{16}{27} e^{-3} - \left(-0 - 0 - \frac{16}{27} \right)$	M1	2.1
	$= \frac{16}{27} - \frac{136}{27} e^{-3}$	A1	1.1b
		(5)	
(5 marks)			

June 2024 Question 14 Paper 1

14. A balloon is being inflated.

In a simple model,

- the balloon is modelled as a sphere
- the rate of increase of the radius of the balloon is inversely proportional to the square root of the radius of the balloon

At time t seconds, the radius of the balloon is r cm.

(a) Write down a differential equation to model this situation.

(1)

At the instant when $t = 10$

- the radius is 16 cm
- the radius is increasing at a rate of 0.9 cm s^{-1}

(b) Solve the differential equation to show that

$$r^{\frac{3}{2}} = 5.4t + 10$$

(5)

(c) Hence find the radius of the balloon when $t = 20$

Give your answer to the nearest millimetre.

(2)

(d) Suggest a limitation of the model.

(1)

ANSWER

Question	Scheme	Marks	AOs
14(a)	$\frac{dr}{dt} = \frac{k}{\sqrt{r}}$	B1	3.3
		(1)	
(b)	$0.9 = \frac{k}{\sqrt{16}} \Rightarrow k = 3.6$	B1	3.4
	$\int \sqrt{r} \, dr = \int "3.6" \, dt \Rightarrow \dots$	M1	2.1
	$\frac{2}{3} r^{\frac{3}{2}} = "3.6" t \quad \{+c\}$	A1	1.1b
	$t = 10, r = 16 \Rightarrow \frac{2}{3} \times 16^{\frac{3}{2}} = 3.6 \times 10 + c \Rightarrow c = \dots$	dM1	3.4
	$r^{\frac{3}{2}} = 5.4t + 10 \quad *$	A1*	1.1b
		(5)	
(c)	$t = 20 \Rightarrow r = (5.4(20) + 10)^{\frac{2}{3}} = \dots$	M1	3.4
	$r = 24.1 \text{ cm}$	A1	1.1b
		(2)	
(d)	(The model will not hold indefinitely as) the balloon may burst	B1	3.5b
		(1)	
(9 marks)			

June 2024 Question 13 Paper 1

13. (a) Given that a is a positive constant, use the substitution $x = a \sin^2 \theta$ to show that

$$\int_0^a x^{\frac{1}{2}} \sqrt{a-x} \, dx = \frac{1}{2} a^2 \int_0^{\frac{\pi}{2}} \sin^2 2\theta \, d\theta \quad (4)$$

(b) Hence use algebraic integration to show that

$$\int_0^a x^{\frac{1}{2}} \sqrt{a-x} \, dx = k\pi a^2$$

where k is a constant to be found.

(4)

ANSWER

Question	Scheme	Marks	AOs
13(a)	$x = a \sin^2 \theta \Rightarrow \frac{dx}{d\theta} = 2a \sin \theta \cos \theta$	B1	1.1b
	$\int x^{\frac{1}{2}} \sqrt{a-x} dx = \int \sqrt{a} \sin \theta \sqrt{a-a \sin^2 \theta} \times 2a \sin \theta \cos \theta \{d\theta\}$	M1	2.1
	$= \int \sqrt{a} \sin \theta \sqrt{a} \cos \theta \times 2a \sin \theta \cos \theta d\theta = 2a^2 \int \sin^2 \theta \cos^2 \theta d\theta$ $= 2a^2 \int \left(\frac{1}{2} \sin 2\theta\right)^2 d\theta$	dM1	3.1a
	Replaces or considers limits $\{x=0 \Rightarrow\} \theta=0$, $\{x=a \Rightarrow\} \theta=\frac{\pi}{2}$ $= \frac{1}{2} a^2 \int_0^{\frac{\pi}{2}} \sin^2 2\theta d\theta$ *	A1*	1.1b
		(4)	
(b)	$\dots \int \sin^2 2\theta d\theta \rightarrow \dots \int \frac{1-\cos 4\theta}{2} d\theta$	M1	1.1b
	$\rightarrow \dots \left(\frac{\theta}{2} - \frac{1}{8} \sin 4\theta\right)$	dM1	2.1
	$\mu \int \sin^2 2\theta d\theta \rightarrow \frac{\mu}{2} \left(\theta - \frac{1}{4} \sin 4\theta\right)$	A1	1.1b
	$\left\{ \int_0^a x^{\frac{1}{2}} \sqrt{a-x} dx \right\} = \frac{a^2}{4} \left(\frac{\pi}{2} - 0 - 0\right) = \frac{1}{8} \pi a^2$	A1	1.1b
		(4)	
(8 marks)			

June 2024 Question 7 Paper 1

7.

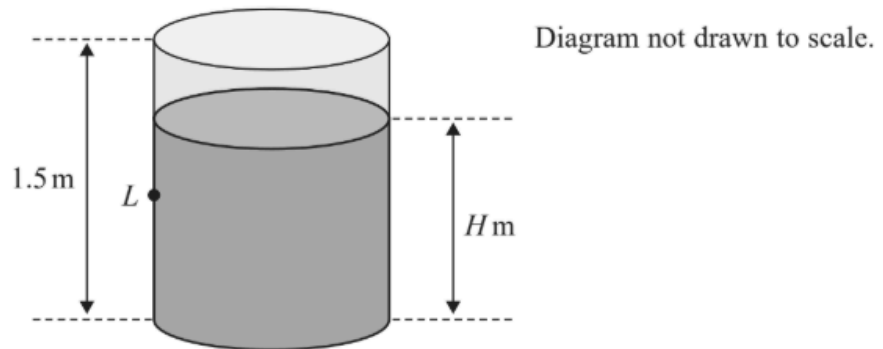


Figure 2

Figure 2 shows a cylindrical tank of height 1.5 m.

Initially the tank is full of water.

The water starts to leak from a small hole, at a point L , in the side of the tank.

While the tank is leaking, the depth, H metres, of the water in the tank is modelled by the differential equation

$$\frac{dH}{dt} = -0.12e^{-0.2t}$$

where t hours is the time after the leak starts.

Using the model,

(a) show that

$$H = Ae^{-0.2t} + B$$

where A and B are constants to be found,

(3)

(b) find the time taken for the depth of the water to decrease to 1.2 m. Give your answer in hours and minutes, to the nearest minute.

(3)

In the long term, the water level in the tank falls to the same height as the hole.

(c) Find, according to the model, the height of the hole from the bottom of the tank.

(2)

ANSWER

Question	Scheme	Marks	AOs
7(a)	$\{H =\} 0.6e^{-0.2t} \{+c\}$	M1	1.1b
	$t = 0, H = 1.5 \Rightarrow 1.5 = "0.6" + c$ $\Rightarrow c = 0.9$	dM1	3.4
	$\Rightarrow H = 0.6e^{-0.2t} + 0.9$	A1	2.1
		(3)	
(b)	$1.2 = 0.6e^{-0.2t} + 0.9 \Rightarrow 0.6e^{-0.2t} = 0.3$	M1	3.4
	$e^{-0.2t} = \frac{1}{2}$ $\Rightarrow t = -5 \ln\left(\frac{1}{2}\right)$	dM1	1.1b
	$\{t =\} 3 \text{ hours } 28 \text{ minutes}$	A1	3.2a
		(3)	
(c)	$\{\text{As } t \text{ gets large } H \rightarrow\} 0.9$	M1	3.1b
	0.9 m or 90 cm	A1ft	2.2b
		(2)	
(8 marks)			

June 2023 Question 13 Paper 2

13. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

- (a) Find the first three terms, in ascending powers of x , of the binomial expansion of

$$(3 + x)^{-2}$$

writing each term in simplest form.

(4)

- (b) Using the answer to part (a) and using algebraic integration, estimate the value of

$$\int_{0.2}^{0.4} \frac{6x}{(3 + x)^2} dx$$

giving your answer to 4 significant figures.

(4)

- (c) Find, using algebraic integration, the exact value of

$$\int_{0.2}^{0.4} \frac{6x}{(3 + x)^2} dx$$

giving your answer in the form $a \ln b + c$, where a , b and c are constants to be found.

(5)

ANSWER

Question	Scheme	Marks	AOs
13(a)	$3^{-2}\left(1+\frac{x}{3}\right)^{-2} = 3^{-2}(1+...x+...x^2)$	M1	1.1b
	$(-2)\left(\frac{x}{3}\right)$ or $\frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^2$	M1	1.1b
	$\left(1+\frac{x}{3}\right)^{-2} = 1+(-2)\left(\frac{x}{3}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^2$	A1	1.1b
	$3^{-2}\left(1+\frac{x}{3}\right)^{-2} = \frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$	A1	2.1
		(4)	
(b)	$\int 6x \times \left(\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}\right) dx = \int \left(\frac{2x}{3}-\frac{4x^2}{9}+\frac{2x^3}{9}\right) dx = ...$	M1	1.1b
	$\int \left(\frac{2x}{3}-\frac{4x^2}{9}+\frac{2x^3}{9}\right) dx = \frac{x^2}{3}-\frac{4x^3}{27}+\frac{x^4}{18}$ oe	A1	1.1b
	$\left[\frac{x^2}{3}-\frac{4x^3}{27}+\frac{x^4}{18}\right]_{0.2}^{0.4} = \left(\frac{(0.4)^2}{3}-\frac{4(0.4)^3}{27}+\frac{(0.4)^4}{18}\right) - \left(\frac{(0.2)^2}{3}-\frac{4(0.2)^3}{27}+\frac{(0.2)^4}{18}\right)$	dM1	3.1a
	= awrt 0.03304 or $\frac{223}{6750}$	A1	1.1b
		(4)	
(c)	Overall problem-solving mark (see notes)	M1	3.1a
	$u = 3 + x \Rightarrow \int_{3.2}^{3.4} f(u) du \Rightarrow \int_{3.2}^{3.4} \frac{6(u-3)}{u^2} du = \int_{3.2}^{3.4} \frac{6}{u} - \frac{18}{u^2} du \Rightarrow ... \ln u + ... u^{-1}$	M1	1.1b
	$\int_{3.2}^{3.4} \frac{6(u-3)}{u^2} du = \int_{3.2}^{3.4} \frac{6}{u} - \frac{18}{u^2} du \Rightarrow 6 \ln u + 18u^{-1}$	A1	1.1b
	$\left[6 \ln u + 18u^{-1}\right]_{3.2}^{3.4} = \left(6 \ln 3.4 + \frac{18}{3.4}\right) - \left(6 \ln 3.2 + \frac{18}{3.2}\right) = ...$	ddM1	1.1b
	$6 \ln\left(\frac{17}{16}\right) - \frac{45}{136}$ oe	A1	2.1
		(5)	

June 2023 Question 11 Paper 2

11.

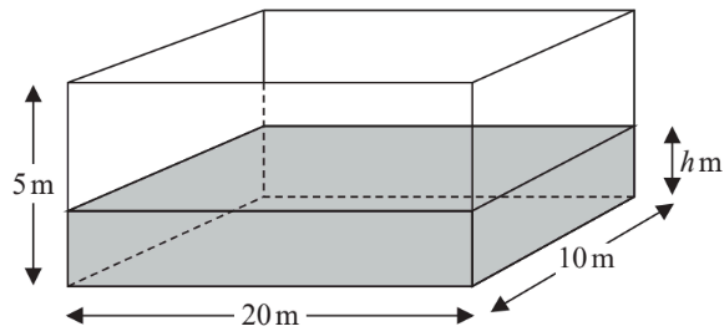


Figure 1

A tank in the shape of a cuboid is being filled with water.

The base of the tank measures 20 m by 10 m and the height of the tank is 5 m, as shown in Figure 1.

At time t minutes after water started flowing into the tank the height of the water was h m and the volume of water in the tank was V m³

In a model of this situation

- the sides of the tank have negligible thickness
- the rate of change of V is inversely proportional to the square root of h

(a) Show that

$$\frac{dh}{dt} = \frac{\lambda}{\sqrt{h}}$$

where λ is a constant.

(3)

Given that

- initially the height of the water in the tank was 1.44 m
- exactly 8 minutes after water started flowing into the tank the height of the water was 3.24 m

(b) use the model to find an equation linking h with t , giving your answer in the form

$$h^{\frac{3}{2}} = At + B$$

where A and B are constants to be found.

(5)

(c) Hence find the time taken, from when water started flowing into the tank, for the tank to be completely full.

(2)

ANSWER

Question	Scheme	Marks	AOs
11(a)	$\frac{dV}{dh} = 200$ oe e.g. $\frac{dh}{dV} = \frac{1}{200}$	B1	1.1b
	$\left(\frac{dh}{dt}\right) \frac{dh}{dV} \times \frac{dV}{dt} = \frac{1}{200} \times \frac{k}{\sqrt{h}}$	M1	3.1a
	$\frac{dh}{dt} = \frac{\lambda}{\sqrt{h}}$ *	A1*	2.1
		(3)	
(b)	$\frac{dh}{dt} = \frac{\lambda}{\sqrt{h}} \Rightarrow \int h^{\frac{1}{2}} dh = \int \lambda dt \Rightarrow \dots h^{\frac{3}{2}} = \lambda t \{+c\}$	M1	1.1b
	$\frac{2}{3} h^{\frac{3}{2}} = \lambda t \{+c\}$ oe e.g. $\frac{h^{\frac{3}{2}}}{\frac{3}{2}} = \lambda t \{+c\}$	A1	1.1b
	$\frac{2}{3} (1.44)^{\frac{3}{2}} = \lambda \times 0 + c \Rightarrow c = 1.152 \left(= \frac{144}{125} \right)$	dM1	3.4
	$\frac{2}{3} (3.24)^{\frac{3}{2}} = \lambda \times 8 + "1.152" \Rightarrow \lambda = 0.342 \left(= \frac{171}{500} \right)$	ddM1	3.1b
	$h^{\frac{3}{2}} = 0.513t + 1.728$ oe e.g. $h^{\frac{3}{2}} = \frac{513}{1000}t + \frac{216}{125}$	A1	3.3
		(5)	

June 2023 Question 10 Paper 2

10. $f(x) = \frac{3kx - 18}{(x + 4)(x - 2)}$ where k is a positive constant

(a) Express $f(x)$ in partial fractions in terms of k .

(3)

(b) Hence find the exact value of k for which

$$\int_{-3}^1 f(x) \, dx = 21$$

(4)

ANSWER

Question	Scheme	Marks	AOs
10(a)	e.g. $\frac{3kx-18}{(x+4)(x-2)} \equiv \frac{A}{x+4} + \frac{B}{x-2} \Rightarrow 3kx-18 \equiv A(x-2) + B(x+4)$ or $\frac{3kx-18}{(x+4)(x-2)} \equiv \frac{A}{x-2} + \frac{B}{x+4} \Rightarrow 3kx-18 \equiv A(x+4) + B(x-2)$	B1	1.1b
	$6k-18=6B \Rightarrow B=\dots$ or $-12k-18=-6A \Rightarrow A=\dots$ or $3kx-18 \equiv (A+B)x + 4B - 2A \Rightarrow A+B=3k, -18=4B-2A$ $\Rightarrow A=\dots$ or $B=\dots$	M1	1.1b
	$\frac{2k+3}{x+4} + \frac{k-3}{x-2}$	A1	1.1b
		(3)	
(b)	$\int \left(\frac{2k+3}{x+4} + \frac{k-3}{x-2} \right) dx = \dots \ln(x+4) + \dots \ln(x-2)$	M1	1.2
	$(2k+3) \ln(x+4) + (k-3) \ln(x-2)$	A1ft	1.1b
	$(2k+3) \ln(5) - (k-3) \ln(5) \Rightarrow (k+6) \ln 5 = 21 \Rightarrow k = \dots$	dM1	3.1a
	$(k =) \frac{21}{\ln 5} - 6$	A1	2.2a
		(4)	
(7 marks)			

June 2023 Question 5 Paper 2

5. The curve C has equation $y = f(x)$

The curve

- passes through the point $P(3, -10)$
- has a turning point at P

Given that

$$\frac{dy}{dx} = 2x^3 - 9x^2 + 5x + k$$

where k is a constant,

(a) show that $k = 12$

(2)

(b) Hence find the coordinates of the point where C crosses the y -axis.

(3)

ANSWER

Question	Scheme	Marks	AOs
5(a)	$2(3)^3 - 9(3)^2 + 5(3) + k = 0 \Rightarrow k = \dots$	M1	1.1b
	$54 - 81 + 15 + k = 0 \Rightarrow k = 12^* \text{ or } -12 + k = 0 \Rightarrow k = 12^*$	A1*	1.1b
		(2)	
	(a) Alternative by verification:		
	$2(3)^3 - 9(3)^2 + 5(3) + 12 = 0$	M1	1.1b
	$54 - 81 + 15 + 12 = 0 \text{ Hence } k = 12^*$	A1*	1.1b
		(2)	
(b)	$\int (2x^3 - 9x^2 + 5x + 12) dx \dots x^4 \pm \dots x^3 \pm \dots x^2 \pm \dots x \pm \dots$	M1	3.1a
	$\frac{1}{2}(3)^4 - 3(3)^3 + \frac{5}{2}(3)^2 + 12(3) + c = -10 \Rightarrow c = \dots$	dM1	1.1b
	$(0, -28)$	A1	2.2a
		(3)	
(5 marks)			

June 2023 Question 1 Paper 1

1. Find

$$\int \frac{x^{\frac{1}{2}}(2x-5)}{3} dx$$

writing each term in simplest form.

(4)

ANSWER

Question	Scheme	Marks	AOs
1	$x^{\frac{1}{2}}(2x-5) = \dots x^{\frac{3}{2}} + \dots x^{\frac{1}{2}} \text{ or } \frac{x^{\frac{1}{2}}(2x-5)}{3} = \frac{\dots x^{\frac{3}{2}} + \dots x^{\frac{1}{2}}}{3}$	M1	1.1b
	$\frac{2x^{\frac{3}{2}}}{3} - \frac{5x^{\frac{1}{2}}}{3}$	A1	1.1b
	$\int \frac{2x^{\frac{3}{2}}}{3} - \frac{5x^{\frac{1}{2}}}{3} dx = \dots x^{\frac{5}{2}} \pm \dots x^{\frac{3}{2}} (+c)$	dM1	1.1b
	$\frac{4}{15}x^{\frac{5}{2}} - \frac{10}{9}x^{\frac{3}{2}} + c$	A1	1.1b
		(4)	
(4 marks)			

June 2022 Question 14 Paper 2

14. (a) Express $\frac{3}{(2x-1)(x+1)}$ in partial fractions. (3)

When chemical A and chemical B are mixed, oxygen is produced.

A scientist mixed these two chemicals and measured the total volume of oxygen produced over a period of time.

The total volume of oxygen produced, $V\text{ m}^3$, t hours after the chemicals were mixed, is modelled by the differential equation

$$\frac{dV}{dt} = \frac{3V}{(2t-1)(t+1)} \quad V \geq 0 \quad t \geq k$$

where k is a constant.

Given that exactly 2 hours after the chemicals were mixed, a total volume of 3 m^3 of oxygen had been produced,

- (b) solve the differential equation to show that

$$V = \frac{3(2t-1)}{(t+1)} \quad (5)$$

The scientist noticed that

- there was a **time delay** between the chemicals being mixed and oxygen being produced
- there was a **limit** to the total volume of oxygen produced

Deduce from the model

- (c) (i) the **time delay** giving your answer in minutes,
 (ii) the **limit** giving your answer in m^3 (2)

ANSWER

Question	Scheme	Marks	AOs
14(a)	$\frac{3}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow A = \dots, B = \dots$	M1	1.1b
	Either $A = 2$ or $B = -1$	A1	1.1b
	$\frac{3}{(2x-1)(x+1)} = \frac{2}{2x-1} - \frac{1}{x+1}$	A1	1.1b
		(3)	
(b)	$\int \frac{1}{V} dV = \int \frac{3}{(2t-1)(t+1)} dt$	B1	1.1a
	$\int \frac{2}{2t-1} - \frac{1}{t+1} dt = \dots \ln(2t-1) - \dots \ln(t+1) (+c)$	M1	3.1a
	$\ln V = \ln(2t-1) - \ln(t+1) (+c)$	A1ft	1.1b
	Substitutes $t = 2, V = 3 \Rightarrow c = (\ln 3)$	M1	3.4
	$\ln V = \ln(2t-1) - \ln(t+1) + \ln 3$ $V = \frac{3(2t-1)}{(t+1)} *$	A1*	2.1
		(5)	

June 2022 Question 5 Paper 2

5. The table below shows corresponding values of x and y for $y = \log_3 2x$

The values of y are given to 2 decimal places as appropriate.

x	3	4.5	6	7.5	9
y	1.63	2	2.26	2.46	2.63

- (a) Using the trapezium rule with all the values of y in the table, find an estimate for

$$\int_3^9 \log_3 2x \, dx$$

(3)

Using your answer to part (a) and making your method clear, estimate

(b) (i) $\int_3^9 \log_3 (2x)^{10} \, dx$

(ii) $\int_3^9 \log_3 18x \, dx$

(3)

ANSWER

Question	Scheme	Marks	AOs
5(a)	States or uses $h = 1.5$	B1	1.1a
	Full attempt at the trapezium rule $= \frac{\dots}{2} \{1.63 + 2.63 + 2 \times (2 + 2.26 + 2.46)\}$	M1	1.1b
	= awrt 13.3 or $\frac{531}{40}$	A1	1.1b
		(3)	
(b)(i)	$\int_3^9 \log_3 (2x)^{10} dx = 10 \times "13.3" = \text{awrt } 133 \text{ or e.g. } \frac{531}{4}$	B1ft	2.2a
(ii)	$\int_3^9 \log_3 18x dx = \int_3^9 \log_3 (9 \times 2x) dx = \int_3^9 2 + \log_3 2x dx$ $= [2x]_3^9 + \int_3^9 \log_3 2x dx = 18 - 6 + \int_3^9 \log_3 2x dx = \dots$	M1	3.1a
	Awrt 25.3 or $\frac{1011}{40}$	A1ft	1.1b
		(3)	
(6 marks)			

June 2022 Question 16 Paper 1

16.

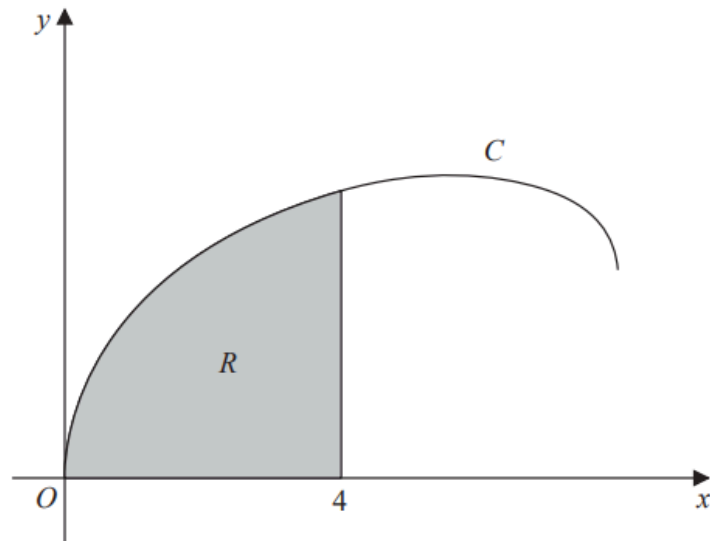


Figure 6

Figure 6 shows a sketch of the curve C with parametric equations

$$x = 8 \sin^2 t \quad y = 2 \sin 2t + 3 \sin t \quad 0 \leq t \leq \frac{\pi}{2}$$

The region R , shown shaded in Figure 6, is bounded by C , the x -axis and the line with equation $x = 4$

(a) Show that the area of R is given by

$$\int_0^a (8 - 8 \cos 4t + 48 \sin^2 t \cos t) dt$$

where a is a constant to be found.

(5)

(b) Hence, using algebraic integration, find the exact area of R .

(4)

ANSWER

Question	Scheme	Marks	AOs
16 (a)	Attempts $y \cdot \frac{dx}{dt} = (2 \sin 2t + 3 \sin t) \times 16 \sin t \cos t$ and uses $\sin 2t = 2 \sin t \cos t$	M1	2.1
	Correct expanded integrand. Usually for one of $(R) = \int \underline{48 \sin^2 t \cos t + 16 \sin^2 2t} dt$ $(R) = \int \underline{48 \sin^2 t \cos t + 64 \sin^2 t \cos^2 t} dt$ $(R) = \int \underline{24 \sin 2t \sin t + 16 \sin^2 2t} dt$	A1	1.1b
	Attempts to use $\cos 4t = 1 - 2 \sin^2 2t = (1 - 8 \sin^2 t \cos^2 t)$	M1	1.1b
	$R = \int_0^a 8 - 8 \cos 4t + 48 \sin^2 t \cos t dt$ *	A1*	2.1
	Deduces $a = \frac{\pi}{4}$	B1	2.2a
		(5)	
(b)	$\int 8 - 8 \cos 4t + 48 \sin^2 t \cos t dt = 8t - 2 \sin 4t + 16 \sin^3 t$	M1 A1	2.1 1.1b
	$\left[8t - 2 \sin 4t + 16 \sin^3 t \right]_0^{\frac{\pi}{4}} = 2\pi + 4\sqrt{2}$	M1 A1	2.1 1.1b
		(4)	
(9 marks)			

June 2022 Question 12 Paper 1

12. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Show that

$$\int_1^{e^2} x^3 \ln x \, dx = ae^8 + b$$

where a and b are rational constants to be found.

(5)

ANSWER

Question	Scheme	Marks	AOs
12	$\int x^3 \ln x \, dx = \frac{x^4}{4} \ln x - \int \frac{x^4}{4} \times \frac{1}{x} \, dx$	M1	1.1b
	$= \frac{x^4}{4} \ln x - \frac{x^4}{16} (+c)$	M1 A1	1.1b 1.1b
	$\int_1^{e^2} x^3 \ln x \, dx = \left[\frac{x^4}{4} \ln x - \frac{x^4}{16} \right]_1^{e^2} = \left(\frac{e^8}{4} \ln e^2 - \frac{e^8}{16} \right) - \left(-\frac{1^4}{16} \right)$	M1	2.1
	$= \frac{7}{16} e^8 + \frac{1}{16}$	A1	1.1b
		(5)	
(5 marks)			

June 2022 Question 4 Paper 1

4. (a) Express $\lim_{\delta x \rightarrow 0} \sum_{x=2.1}^{6.3} \frac{2}{x} \delta x$ as an integral.

(1)

(b) Hence show that

$$\lim_{\delta x \rightarrow 0} \sum_{x=2.1}^{6.3} \frac{2}{x} \delta x = \ln k$$

where k is a constant to be found.

(2)

ANSWER

Question	Scheme	Marks	AOs
4 (a)	$\lim_{\delta x \rightarrow 0} \sum_{x=2.1}^{6.3} \frac{2}{x} \delta x = \int_{2.1}^{6.3} \frac{2}{x} dx$	B1	1.2
		(1)	
(b)	$= [2 \ln x]_{2.1}^{6.3} = 2 \ln 6.3 - 2 \ln 2.1$	M1	1.1b
	$= \ln 9$ CSO	A1	1.1b
		(2)	
(3 marks)			

November 2021 Question 11 Paper 1

11.

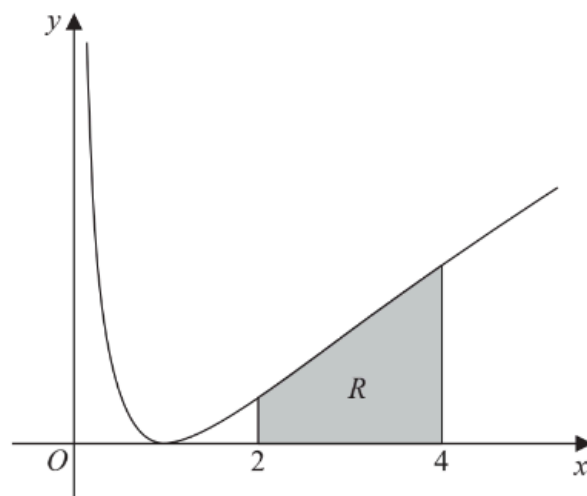


Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = (\ln x)^2 \quad x > 0$$

The finite region R , shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 2$, the x -axis and the line with equation $x = 4$

The table below shows corresponding values of x and y , with the values of y given to 4 decimal places.

x	2	2.5	3	3.5	4
y	0.4805	0.8396	1.2069	1.5694	1.9218

- (a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of R , giving your answer to 3 significant figures.

(3)

- (b) Use algebraic integration to find the exact area of R , giving your answer in the form

$$y = a(\ln 2)^2 + b \ln 2 + c$$

where a , b and c are integers to be found.

(5)

ANSWER

Question	Scheme	Marks	AOs
11(a)	$h = 0.5$	B1	1.1b
	$A \approx \frac{1}{2} \times \frac{1}{2} \{0.4805 + 1.9218 + 2(0.8396 + 1.2069 + 1.5694)\}$	M1	1.1b
	$= 2.41$	A1	1.1b
		(3)	
(b)	$\int (\ln x)^2 dx = x(\ln x)^2 - \int x \times \frac{2 \ln x}{x} dx$	M1 A1	3.1a 1.1b
	$= x(\ln x)^2 - 2 \int \ln x dx = x(\ln x)^2 - 2(x \ln x - \int dx)$ $= x(\ln x)^2 - 2 \int \ln x dx = x(\ln x)^2 - 2x \ln x + 2x$	dM1	2.1
	$\int_2^4 (\ln x)^2 dx = \left[x(\ln x)^2 - 2x \ln x + 2x \right]_2^4$ $= 4(\ln 4)^2 - 2 \times 4 \ln 4 + 2 \times 4 - (2(\ln 2)^2 - 2 \times 2 \ln 2 + 2 \times 2)$ $= 4(2 \ln 2)^2 - 16 \ln 2 + 8 - 2(\ln 2)^2 + 4 \ln 2 - 4$	ddM1	2.1
	$= 14(\ln 2)^2 - 12 \ln 2 + 4$	A1	1.1b
		(5)	
	(8 marks)		

November 2021 Question 12 Paper 2

12. (a) Use the substitution $u = 1 + \sqrt{x}$ to show that

$$\int_0^{16} \frac{x}{1 + \sqrt{x}} \, dx = \int_p^q \frac{2(u-1)^3}{u} \, du$$

where p and q are constants to be found.

(3)

(b) Hence show that

$$\int_0^{16} \frac{x}{1 + \sqrt{x}} \, dx = A - B \ln 5$$

where A and B are constants to be found.

(4)

ANSWER

Question	Scheme	Marks	AOs
12(a)	$u = 1 + \sqrt{x} \Rightarrow x = (u-1)^2 \Rightarrow \frac{dx}{du} = 2(u-1)$ or $u = 1 + \sqrt{x} \Rightarrow \frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$	B1	1.1b
	$\int \frac{x}{1+\sqrt{x}} dx = \int \frac{(u-1)^2}{u} 2(u-1) du$ or $\int \frac{x}{1+\sqrt{x}} dx = \int \frac{x}{u} \times 2x^{\frac{1}{2}} du = \int \frac{2x^{\frac{3}{2}}}{u} du = \int \frac{2(u-1)^3}{u} du$	M1	2.1
	$\int_0^{16} \frac{x}{1+\sqrt{x}} dx = \int_1^5 \frac{2(u-1)^3}{u} du$	A1	1.1b
		(3)	
(b)	$2 \int \frac{u^3 - 3u^2 + 3u - 1}{u} du = 2 \int \left(u^2 - 3u + 3 - \frac{1}{u} \right) du = \dots$	M1	3.1a
	$= (2) \left[\frac{u^3}{3} - \frac{3u^2}{2} + 3u - \ln u \right]$	A1	1.1b
	$= 2 \left[\frac{5^3}{3} - \frac{3(5)^2}{2} + 3(5) - \ln 5 - \left(\frac{1}{3} - \frac{3}{2} + 3 - \ln 1 \right) \right]$	dM1	2.1
	$= \frac{104}{3} - 2 \ln 5$	A1	1.1b
		(4)	
(7 marks)			

November 2021 Question 14 Paper 2

14.

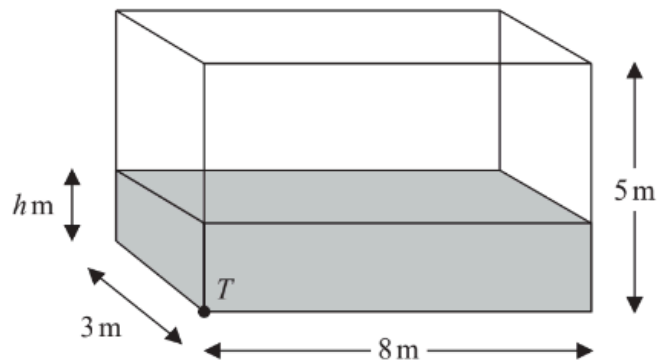


Figure 5

Water flows at a constant rate into a large tank.

The tank is a cuboid, with all sides of negligible thickness.

The base of the tank measures 8 m by 3 m and the height of the tank is 5 m.

There is a tap at a point T at the bottom of the tank, as shown in Figure 5.

At time t minutes after the tap has been opened

- the depth of water in the tank is h metres
- water is flowing into the tank at a constant rate of 0.48 m^3 per minute
- water is modelled as leaving the tank through the tap at a rate of $0.1h \text{ m}^3$ per minute

(a) Show that, according to the model,

$$1200 \frac{dh}{dt} = 24 - 5h \quad (4)$$

Given that when the tap was opened, the depth of water in the tank was 2 m,

(b) show that, according to the model,

$$h = A + Be^{-kt}$$

where A , B and k are constants to be found.

(6)

Given that the tap remains open,

(c) determine, according to the model, whether the tank will ever become full, giving a reason for your answer.

(2)

ANSWER

Question	Scheme	Marks	AOs
14(a)	$\frac{dV}{dt} = 0.48 - 0.1h$	B1	3.1b
	$V = 24h \Rightarrow \frac{dV}{dh} = 24 \text{ or } \frac{dh}{dV} = \frac{1}{24}$	B1	3.1b
	$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV} = \frac{0.48 - 0.1h}{24}$ or e.g. $\frac{dV}{dt} = \frac{dV}{dh} \frac{dh}{dt} \Rightarrow 0.48 - 0.1h = 24 \frac{dh}{dt}$	M1	2.1
	$1200 \frac{dh}{dt} = 24 - 5h^*$	A1*	1.1b
		(4)	
(b)	$1200 \frac{dh}{dt} = 24 - 5h \Rightarrow \int \frac{1200}{24 - 5h} dh = \int dt$ $\Rightarrow \text{e.g. } \alpha \ln(24 - 5h) = t(+c) \text{ oe}$ or $1200 \frac{dh}{dt} = 24 - 5h \Rightarrow \frac{dt}{dh} = \frac{1200}{24 - 5h}$ $\Rightarrow \text{e.g. } t(+c) = \alpha \ln(24 - 5h) \text{ oe}$	M1	3.1a
	$t = -240 \ln(24 - 5h)(+c) \text{ oe}$	A1	1.1b
	$t = 0, h = 2 \Rightarrow 0 = -240 \ln(24 - 10) + c \Rightarrow c = \dots(240 \ln 14)$	M1	3.4

	$t = 240 \ln(14) - 240 \ln(24 - 5h)$	A1	1.1b
	$t = 240 \ln \frac{14}{24 - 5h} \Rightarrow \frac{t}{240} = \ln \frac{14}{24 - 5h} \Rightarrow e^{\frac{t}{240}} = \frac{14}{24 - 5h}$ $\Rightarrow 14e^{-\frac{t}{240}} = 24 - 5h \Rightarrow h = \dots$	ddM1	2.1
	$h = 4.8 - 2.8e^{-\frac{t}{240}}$ oe e.g. $h = \frac{24}{5} - \frac{14}{5}e^{-\frac{t}{240}}$	A1	3.3
		(6)	
(c)	Examples: - As $t \rightarrow \infty, e^{-\frac{t}{240}} \rightarrow 0$ - When $h > 4.8, \frac{dV}{dt} < 0$ - Flow in = flow out at max h so $0.1h = 4.8 \rightarrow h = 4.8$ - As $e^{-\frac{t}{240}} > 0, h < 4.8$ $h = 5 \Rightarrow \frac{dV}{dt} = -0.02$ or $\frac{dh}{dt} = -\frac{1}{1200}$ $\frac{dh}{dt} = 0 \Rightarrow h = 4.8$ $h = 5 \Rightarrow 4.8 - 2.8e^{-\frac{t}{240}} = 5 \Rightarrow e^{-\frac{t}{240}} < 0$	M1	3.1b
	- The limit for h (according to the model) is 4.8m and the tank is 5m high so the tank will never become full - If $h = 5$ the tank would be emptying so can never be full - The equation can't be solved when $h = 5$	A1	3.2a
		(2)	

Video solution:

<https://youtu.be/TNLQCDmVqpM>

November 2020 Question 8 Paper 1

8. A new smartphone was released by a company.

The company monitored the total number of phones sold, n , at time t days after the phone was released.

The company observed that, during this time,

the rate of increase of n was proportional to n

Use this information to write down a suitable equation for n in terms of t .

(You do not need to evaluate any unknown constants in your equation.)

(2)

ANSWER

Question	Scheme	Marks	AOs
8	Any equation involving an exponential of the correct form. See notes	M1	3.1b
	$n = Ae^{kt}$ (where A and k are positive constants)	A1	1.1b
		(2)	
(2 marks)			

November 2020 Question 10 Paper 1

10. (a) Use the substitution $x = u^2 + 1$ to show that

$$\int_5^{10} \frac{3 \, dx}{(x-1)(3+2\sqrt{x-1})} = \int_p^q \frac{6 \, du}{u(3+2u)}$$

where p and q are positive constants to be found.

(4)

(b) Hence, using algebraic integration, show that

$$\int_5^{10} \frac{3 \, dx}{(x-1)(3+2\sqrt{x-1})} = \ln a$$

where a is a rational constant to be found.

(6)

ANSWER

Question	Scheme	Marks	AOs
10 (a)	$x = u^2 + 1 \Rightarrow dx = 2u du$ oe	B1	1.1b
	Full substitution $\int \frac{3dx}{(x-1)(3+2\sqrt{x-1})} = \int \frac{3 \times 2u du}{(u^2+1-1)(3+2u)}$	M1	1.1b
	Finds correct limits e.g. $p = 2, q = 3$	B1	1.1b
	$= \int \frac{3 \times 2 \cancel{u} du}{u^2(3+2u)} = \int \frac{6 du}{u(3+2u)} *$	A1*	2.1
		(4)	
(b)	$\frac{6}{u(3+2u)} = \frac{A}{u} + \frac{B}{3+2u} \Rightarrow A = \dots, B = \dots$	M1	1.1b
	Correct PF. $\frac{6}{u(3+2u)} = \frac{2}{u} - \frac{4}{3+2u}$	A1	1.1b
	$\int \frac{6 du}{u(3+2u)} = 2 \ln u - 2 \ln(3+2u) \quad (+c)$	dM1 A1ft	3.1a 1.1b
	Uses limits $u = "3", u = "2"$ with some correct $\ln$ work leading to $k \ln b$ E.g. $(2 \ln 3 - 2 \ln 9) - (2 \ln 2 - 2 \ln 7) = 2 \ln \frac{7}{6}$	M1	1.1b
	$\ln \frac{49}{36}$	A1	2.1
		(6)	
(10 marks)			

Video solution:

<https://youtu.be/tsl94ty27S8>

November 2020 Question 14 Paper 1

14. A large spherical balloon is deflating.

At time t seconds the balloon has radius r cm and volume V cm³

The volume of the balloon is modelled as decreasing at a constant rate.

(a) Using this model, show that

$$\frac{dr}{dt} = -\frac{k}{r^2}$$

where k is a positive constant.

(3)

Given that

- the initial radius of the balloon is 40 cm
- after 5 seconds the radius of the balloon is 20 cm
- the volume of the balloon continues to decrease at a constant rate until the balloon is empty

(b) solve the differential equation to find a complete equation linking r and t .

(5)

(c) Find the limitation on the values of t for which the equation in part (b) is valid.

(2)

ANSWER

Question	Scheme	Marks	AOs
14 (a)	Uses the model to state $\frac{dV}{dt} = -c$ (for positive constant c)	B1	3.1b
	Uses $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ with their $\frac{dV}{dt} = -c$ and $\frac{dV}{dr} = 4\pi r^2$	M1	2.1
	$-c = 4\pi r^2 \times \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = -\frac{c}{4\pi r^2} = -\frac{k}{r^2}$ *	A1*	2.2a
		(3)	
(b)	$\frac{dr}{dt} = -\frac{k}{r^2} \Rightarrow \int r^2 dr = \int -k dt$ and integrates with one side "correct"	M1	2.1
	$\frac{r^3}{3} = -kt (+\alpha)$	A1	1.1b
	Uses $t = 0, r = 40 \Rightarrow \alpha = \dots$ $\alpha = \frac{64000}{3}$	M1	1.1b
	Uses $t = 5, r = 20$ & $\alpha = \dots \Rightarrow k = \dots$	M1	3.4
	$r^3 = 64000 - 11200t$ or exact equivalent	A1	3.3
		(5)	
(c)	Uses the equation of their model and proceeds to a limiting value for t E.g. " $64000 - 11200t$ " ... $0 \Rightarrow t \dots$	M1	3.4
	For times up to and including $\frac{40}{7}$ seconds	A1ft	3.5b
		(2)	
(10 marks)			

Video solution:

<https://youtu.be/lvzf21s7C4E>

November 2020 Question 1 Paper 2

- 1 The table below shows corresponding values of x and y for $y = \sqrt{\frac{x}{1+x}}$

The values of y are given to 4 significant figures.

x	0.5	1	1.5	2	2.5
y	0.5774	0.7071	0.7746	0.8165	0.8452

- (a) Use the trapezium rule, with all the values of y in the table, to find an estimate for

$$\int_{0.5}^{2.5} \sqrt{\frac{x}{1+x}} dx$$

giving your answer to 3 significant figures.

(3)

- (b) Using your answer to part (a), deduce an estimate for $\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}} dx$

(1)

Given that

$$\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}} dx = 4.535 \text{ to 4 significant figures}$$

- (c) comment on the accuracy of your answer to part (b).

(1)

ANSWER

Question	Scheme	Marks	AOs
1(a)	$h = 0.5$	B1	1.1a
	$A \approx \frac{0.5}{2} \{0.5774 + 0.8452 + 2(0.7071 + 0.7746 + 0.8165)\}$	M1	1.1b
	= awrt 1.50	A1	1.1b
	For reference: The integration on a calculator gives 1.511549071 The full accuracy for y values gives 1.504726147 The accuracy from the table gives 1.50475		
		(3)	
(b)	$3 \times \text{their (a)}$ If (a) is correct, allow awrt 4.50 or awrt 4.51 even with no working. Only allow 4.5 if (a) is correct and working is shown e.g. 3×1.5 If (a) is incorrect allow $3 \times \text{their (a)}$ given to at least 3sf but do not be too concerned about the accuracy (as they may use rounded or rounded value from (a))	B1 ft	2.2a
	For reference the integration on a calculator gives 4.534647213		
		(1)	

(c)	<u>This mark depends on the B1 having been awarded in part (b) with awrt 4.5</u> Look for a sensible comment. Some examples: • The answer is accurate to 2 sf or one decimal place • Answer to (b) is accurate as $4.535 \approx 4.50$ • Very accurate as 4.535 to 2 sf is 4.5 • $4.51425 < 4.535$ so my answer is underestimate but not too far off • It is an underestimate but quite close • It is a very good estimate • High accuracy • (Quite) accurate • It is less than 1% out • $4.535 - 4.5 = 0.035$ so not far out But not just “it is an underestimate” or Calculates percentage error correctly using awrt 4.50 or awrt 4.51 or 4.5 (No comment is necessary in these cases although one may be given) Examples: $\left \frac{4.535 - 4.50}{4.535} \right \times 100 = 0.77\% \quad \text{or} \quad \left \frac{4.535 - 4.51}{4.535} \right \times 100 = 0.55\% \quad \text{or}$ $\left \frac{4.535 - 4.51425}{4.535} \right \times 100 = 0.46\% \quad \text{or} \quad \left \frac{4.50}{4.535} \right \times 100 = 99\%$ In these cases don't be too concerned about accuracy e.g. allow 1sf. This mark should be withheld if there are any contradictory statements	B1	3.2b
		(1)	
(5 marks)			

November 2020 Question

6. (a) Given that

$$\frac{x^2 + 8x - 3}{x + 2} \equiv Ax + B + \frac{C}{x + 2} \quad x \in \mathbb{R} \quad x \neq -2$$

find the values of the constants A , B and C

(3)

(b) Hence, using algebraic integration, find the exact value of

$$\int_0^6 \frac{x^2 + 8x - 3}{x + 2} dx$$

giving your answer in the form $a + b \ln 2$ where a and b are integers to be found.

(4)

ANSWER

6(a)	$x^2 + 8x - 3 = (Ax + B)(x + 2) + C \text{ or } Ax(x + 2) + B(x + 2) + C$ $\Rightarrow A = \dots, B = \dots, C = \dots$ or $\begin{array}{r} x+6 \\ x+2 \overline{) x^2 + 8x - 3} \\ \underline{x^2 + 2x} \\ 6x - 3 \\ \underline{6x + 12} \\ -15 \end{array}$	M1	1.1b
	Two of $A = 1, B = 6, C = -15$	A1	1.1b
	All three of $A = 1, B = 6, C = -15$	A1	1.1b
		(3)	
6(b)	$\int \frac{x^2 + 8x - 3}{x + 2} dx = \int x + 6 - \frac{15}{x + 2} dx = \dots - 15 \ln(x + 2)$	M1	1.1b
	$= \frac{1}{2}x^2 + 6x - 15 \ln(x + 2) \quad (+c)$	A1ft	1.1b
	$\int_0^6 \frac{x^2 + 8x - 3}{x + 2} dx = \left[\frac{1}{2}x^2 + 6x - 15 \ln(x + 2) \right]_0^6$ $= (18 + 36 - 15 \ln 8) - (0 + 0 - 15 \ln 2)$ $= 18 + 36 - (15 - 45) \ln 2 \text{ or e.g. } 18 + 36 + 15 \ln \left(\frac{2}{8} \right)$	M1	2.1
	$= 54 - 30 \ln 2$	A1	1.1b
		(4)	
(7 marks)			

November 2020 Question 12 Paper 2

12.

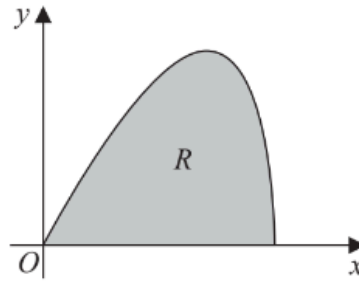


Figure 3

The curve shown in Figure 3 has parametric equations

$$x = 6 \sin t \quad y = 5 \sin 2t \quad 0 \leq t \leq \frac{\pi}{2}$$

The region R , shown shaded in Figure 3, is bounded by the curve and the x -axis.

(a) (i) Show that the area of R is given by $\int_0^{\frac{\pi}{2}} 60 \sin t \cos^2 t \, dt$

(3)

(ii) Hence show, by algebraic integration, that the area of R is exactly 20

(3)

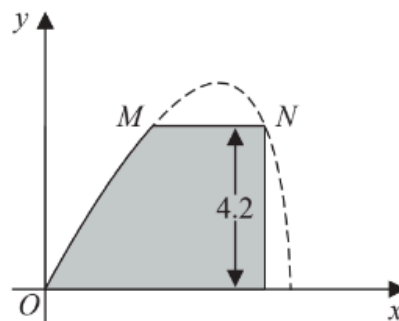


Figure 4

Part of the curve is used to model the profile of a small dam, shown shaded in Figure 4. Using the model and given that

- x and y are in metres
- the vertical wall of the dam is 4.2 metres high
- there is a horizontal walkway of width MN along the top of the dam

(b) calculate the width of the walkway.

(5)

ANSWER

Question	Scheme	Marks	AOs
12(a)(i)	$y \times \frac{dx}{dt} = 5 \sin 2t \times 6 \cos t$ or $5 \times 2 \sin t \cos t \times 6 \cos t$	M1	1.2
	$(\text{Area} =) \int 5 \sin 2t \times 6 \cos t \, dt = \int 5 \times 2 \sin t \cos t \times 6 \cos t \, dt$ or $\int 5 \sin 2t \times 6 \cos t \, dt = \int 60 \sin t \cos^2 t \, dt$	dM1	1.1b
	$(\text{Area} =) \int_0^{\frac{\pi}{2}} 60 \sin t \cos^2 t \, dt *$	A1*	2.1*
		(3)	
(a)(ii)	$\int 60 \sin t \cos^2 t \, dt = -20 \cos^3 t$	M1 A1	1.1b 1.1b
	$\text{Area} = \left[-20 \cos^3 t \right]_0^{\frac{\pi}{2}} = 0 - (-20) = 20 *$	A1*	2.1
		(3)	
(b)	$5 \sin 2t = 4.2 \Rightarrow \sin 2t = \frac{4.2}{5}$	M1	3.4
	$t = 0.4986..., 1.072...$	A1	1.1b
	Attempts to find the x values at both t values	dM1	3.4
	$t = 0.4986... \Rightarrow x = 2.869...$ $t = 1.072 \Rightarrow x = 5.269...$	A1	1.1b
	Width of path = 2.40 metres	A1	3.2a
		(5)	

(11 marks)

June 2019 Question 13 Paper 1

13. The curve C with equation

$$y = \frac{p - 3x}{(2x - q)(x + 3)} \quad x \in \mathbb{R}, x \neq -3, x \neq 2$$

where p and q are constants, passes through the point $\left(3, \frac{1}{2}\right)$ and has two vertical asymptotes with equations $x = 2$ and $x = -3$

(a) (i) Explain why you can deduce that $q = 4$

(ii) Show that $p = 15$

(3)

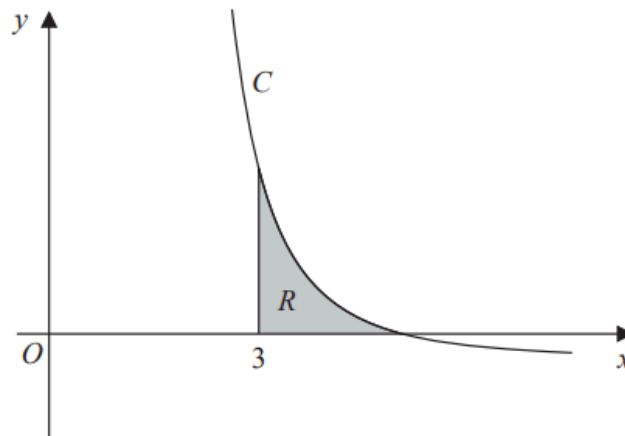


Figure 4

Figure 4 shows a sketch of part of the curve C . The region R , shown shaded in Figure 4, is bounded by the curve C , the x -axis and the line with equation $x = 3$

(b) Show that the exact value of the area of R is $a \ln 2 + b \ln 3$, where a and b are rational constants to be found.

(8)

ANSWER

Question	Scheme	Marks	AOs
13 (a)	(i) Explains $2x - q = 0$ when $x = 2$ oe Hence $q = 4$ *	B1*	2.4
	(ii) Substitutes $\left(3, \frac{1}{2}\right)$ into $y = \frac{p-3x}{(2x-4)(x+3)}$ and solves	M1	1.1b
	$\frac{1}{2} = \frac{p-9}{(2) \times (6)} \Rightarrow p-9=6 \Rightarrow p=15$ *	A1*	2.1
		(3)	
(b)	Attempts to write $\frac{15-3x}{(2x-4)(x+3)}$ in PF's and integrates using lns between 3 and another value of x .	M1	3.1a
	$\frac{15-3x}{(2x-4)(x+3)} = \frac{A}{(2x-4)} + \frac{B}{(x+3)}$ leading to A and B	M1	1.1b
	$\frac{15-3x}{(2x-4)(x+3)} = \frac{1.8}{(2x-4)} - \frac{2.4}{(x+3)}$ or $\frac{0.9}{(x-2)} - \frac{2.4}{(x+3)}$ oe	A1	1.1b
	$I = \int \frac{15-3x}{(2x-4)(x+3)} dx = m \ln(2x-4) + n \ln(x+3) + (c)$	M1	1.1b
	$I = \int \frac{15-3x}{(2x-4)(x+3)} dx = 0.9 \ln(2x-4) - 2.4 \ln(x+3)$ oe	A1ft	1.1b
	Deduces that Area Either $\int_3^5 \frac{15-3x}{(2x-4)(x+3)} dx$ Or $[.....]_3^5$	B1	2.2a
	Uses correct ln work seen at least once for $\ln 6 = \ln 2 + \ln 3$ or $\ln 8 = 3 \ln 2$ $[0.9 \ln(6) - 2.4 \ln(8)] - [0.9 \ln(2) - 2.4 \ln(6)]$ $= 3.3 \ln 6 - 7.2 \ln 2 - 0.9 \ln 2$ $= 3.3 \ln 3 - 4.8 \ln 2$	dM1	2.1
		A1	1.1b
		(8)	
(11marks)			

June 2019 Question 5 Paper 2

5.

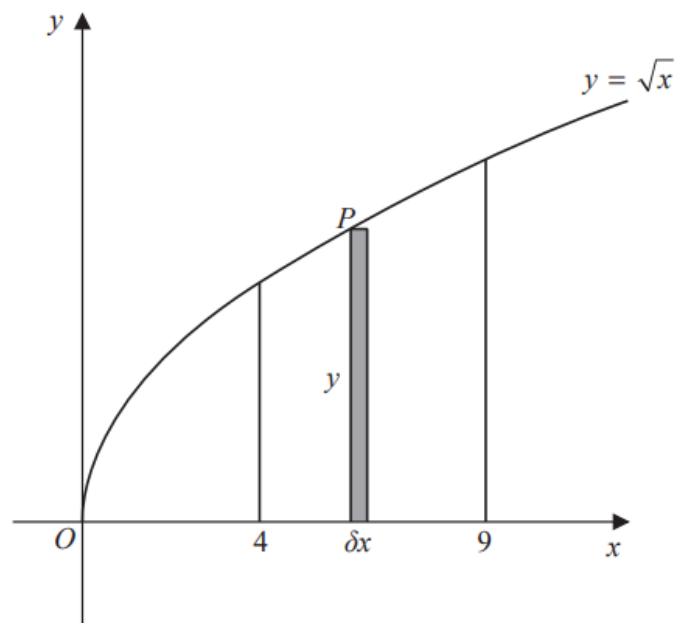


Figure 3

Figure 3 shows a sketch of the curve with equation $y = \sqrt{x}$

The point $P(x, y)$ lies on the curve.

The rectangle, shown shaded on Figure 3, has height y and width δx .

Calculate

$$\lim_{\delta x \rightarrow 0} \sum_{x=4}^9 \sqrt{x} \delta x$$

(3)

ANSWER

Question	Scheme	Marks	AOs
5	States $\left\{ \lim_{\delta x \rightarrow 0} \sum_{x=4}^9 \sqrt{x} \delta x \text{ is } \right\} \int_4^9 \sqrt{x} dx$	B1	1.2
	$= \left[\frac{2}{3} x^{\frac{3}{2}} \right]_4^9$	M1	1.1b
	$= \frac{2}{3} \times 9^{\frac{3}{2}} - \frac{2}{3} \times 4^{\frac{3}{2}} = \frac{54}{3} - \frac{16}{3}$		
	$= \frac{38}{3} \text{ or } 12\frac{2}{3} \text{ or awrt } 12.7$	A1	1.1b
		(3)	
(3 marks)			

June 2019 Question 14 Paper 2

14. (a) Use the substitution $u = 4 - \sqrt{h}$ to show that

$$\int \frac{dh}{4 - \sqrt{h}} = -8 \ln|4 - \sqrt{h}| - 2\sqrt{h} + k$$

where k is a constant

(6)

A team of scientists is studying a species of slow growing tree.

The rate of change in height of a tree in this species is modelled by the differential equation

$$\frac{dh}{dt} = \frac{t^{0.25}(4 - \sqrt{h})}{20}$$

where h is the height in metres and t is the time, measured in years, after the tree is planted.

- (b) Find, according to the model, the range in heights of trees in this species.

(2)

One of these trees is one metre high when it is first planted.

According to the model,

- (c) calculate the time this tree would take to reach a height of 12 metres, giving your answer to 3 significant figures.

(7)

ANSWER

Question	Scheme	Marks	AOs
14 (a)	$\{u = 4 - \sqrt{h} \Rightarrow \} \frac{du}{dh} = -\frac{1}{2}h^{-\frac{1}{2}} \text{ or } \frac{dh}{du} = -2(4-u) \text{ or } \frac{dh}{du} = -2\sqrt{h}$	B1	1.1b
	$\left\{ \int \frac{dh}{4-\sqrt{h}} = \right\} \int \frac{-2(4-u)}{u} du$	M1	2.1
	$= \int \left(-\frac{8}{u} + 2 \right) du$	M1	1.1b
	$= -8\ln u + 2u \{+c\}$	M1	1.1b
		A1	1.1b
	$= -8\ln 4-\sqrt{h} + 2(4-\sqrt{h}) + c = -8\ln 4-\sqrt{h} - 2\sqrt{h} + k *$	A1*	2.1
		(6)	
(b)	$\left\{ \frac{dh}{dt} = \frac{t^{0.25}(4-\sqrt{h})}{20} = 0 \Rightarrow \right\} 4-\sqrt{h} = 0$	M1	3.4
	Deduces any of $0 < h < 16$, $0 \leq h < 16$, $0 < h \leq 16$, $0 \leq h \leq 16$, $h < 16$, $h \leq 16$ or all values up to 16	A1	2.2a
		(2)	
(c) Way 1	$\int \frac{1}{(4-\sqrt{h})} dh = \int \frac{1}{20} t^{0.25} dt$	B1	1.1b
	$-8\ln 4-\sqrt{h} - 2\sqrt{h} = \frac{1}{25} t^{1.25} \{+c\}$	M1	1.1b
		A1	1.1b
	$\{t=0, h=1 \Rightarrow \} -8\ln(4-1) - 2\sqrt{1} = \frac{1}{25}(0)^{1.25} + c$	M1	3.4
	$\Rightarrow c = -8\ln(3) - 2 \Rightarrow -8\ln 4-\sqrt{h} - 2\sqrt{h} = \frac{1}{25} t^{1.25} - 8\ln(3) - 2$	dM1	3.1a
	$\{h=12 \Rightarrow \} -8\ln 4-\sqrt{12} - 2\sqrt{12} = \frac{1}{25} t^{1.25} - 8\ln(3) - 2$		
	$t^{1.25} = 221.2795202... \Rightarrow t = \sqrt[1.25]{221.2795202...} \text{ or } t = (221.2795202...)^{0.8}$	M1	1.1b
	$t = 75.154... \Rightarrow t = 75.2 \text{ (years) (3 sf) or awrt 75.2 (years)}$	A1	1.1b
	Note: You can recover work for part (c) in part (b)	(7)	
(c) Way 2	$\int_1^{12} \frac{20}{(4-\sqrt{h})} dh = \int_0^T t^{0.25} dt$	B1	1.1b
	$\left[20(-8\ln 4-\sqrt{h} - 2\sqrt{h}) \right]_1^{12} = \left[\frac{4}{5} t^{1.25} \right]_0^T$	M1	1.1b
		A1	1.1b
	$20(-8\ln(4-\sqrt{12}) - 2\sqrt{12}) - 20(-8\ln(4-1) - 2\sqrt{1}) = \frac{4}{5} T^{1.25} - 0$	M1	3.4
		dM1	3.1a
	$T^{1.25} = 221.2795202... \Rightarrow T = \sqrt[1.25]{221.2795202...} \text{ or } T = (221.2795202...)^{0.8}$	M1	1.1b
	$T = 75.154... \Rightarrow T = 75.2 \text{ (years) (3 sf) or awrt 75.2 (years)}$	A1	1.1b
	Note: You can recover work for part (c) in part (b)	(7)	

(15 marks)

June 2018 Question 7 Paper 1

7. Given that $k \in \mathbb{Z}^+$

(a) show that $\int_k^{3k} \frac{2}{(3x-k)} dx$ is independent of k , (4)

(b) show that $\int_k^{2k} \frac{2}{(2x-k)^2} dx$ is inversely proportional to k . (3)

ANSWER

Question	Scheme	Marks	AOs
7 (a)	$\int \frac{2}{(3x-k)} dx = \frac{2}{3} \ln(3x-k)$	M1 A1	1.1a 1.1b
	$\int_k^{3k} \frac{2}{(3x-k)} dx = \frac{2}{3} \ln(9k-k) - \frac{2}{3} \ln(3k-k)$	dM1	1.1b
	$= \frac{2}{3} \ln\left(\frac{8k}{2k}\right) = \frac{2}{3} \ln 4$ oe	A1	2.1
		(4)	
(b)	$\int \frac{2}{(2x-k)^2} dx = -\frac{1}{(2x-k)}$	M1	1.1b
	$\int_k^{2k} \frac{2}{(2x-k)^2} dx = -\frac{1}{(4k-k)} + \frac{1}{(2k-k)}$	dM1	1.1b
	$= \frac{2}{3k} \left(\propto \frac{1}{k} \right)$	A1	2.1
		(3)	
(7 marks)			

June 2018 Question 10 Paper 1

10. The height above ground, H metres, of a passenger on a roller coaster can be modelled by the differential equation

$$\frac{dH}{dt} = \frac{H \cos(0.25t)}{40}$$

where t is the time, in seconds, from the start of the ride.

Given that the passenger is 5 m above the ground at the start of the ride,

- (a) show that $H = 5e^{0.1 \sin(0.25t)}$ (5)

- (b) State the maximum height of the passenger above the ground. (1)

The passenger reaches the maximum height, for the second time, T seconds after the start of the ride.

- (c) Find the value of T . (2)

ANSWER

Question	Scheme	Marks	AOs
10(a)	$\frac{dH}{dt} = \frac{H \cos 0.25t}{40} \Rightarrow \int \frac{1}{H} dH = \int \frac{\cos 0.25t}{40} dt$	M1	3.1a
	$\ln H = \frac{1}{10} \sin 0.25t (+c)$	M1	1.1b
	Substitutes $t = 0, H = 5 \Rightarrow c = \ln(5)$	A1	1.1b
	$\ln\left(\frac{H}{5}\right) = \frac{1}{10} \sin 0.25t \Rightarrow H = 5e^{0.1 \sin 0.25t} *$	dM1	3.4
		A1*	2.1
(b)		(5)	
	Max height = $5e^{0.1} = 5.53 \text{ m}$ (Condone lack of units)	B1	3.4
(c)		(1)	
	Sets $0.25t = \frac{5\pi}{2}$	M1	3.1b
	31.4	A1	1.1b
		(2)	
(8 marks)			

June 2018 Question 13 Paper 1

13. Show that

$$\int_0^2 2x\sqrt{x+2} \, dx = \frac{32}{15}(2 + \sqrt{2})$$

(7)

ANSWER

Question	Scheme for Substitution	Marks	AOs
13	Chooses a suitable method for $\int_0^2 2x\sqrt{x+2} \, dx$ Award for - Using a valid substitution $u = \dots$, changing the terms to u's - integrating and using appropriate limits .	M1	3.1a
	Substitution $u = \sqrt{x+2} \Rightarrow \frac{dx}{du} = 2u$ oe	B1	1.1b
	Substitution $u = x+2 \Rightarrow \frac{dx}{du} = 1$ oe	B1	1.1b
	$\int 2x\sqrt{x+2} \, dx$ $= \int A(u^2 \pm 2)u^2 \, du$	M1	1.1b
	$= \int 2x\sqrt{x+2} \, dx$ $= \int A(u \pm 2)\sqrt{u} \, du$	M1	1.1b
	$= Pu^5 \pm Qu^3$	dM1	2.1
	$= \frac{4}{5}u^5 - \frac{8}{3}u^3$	A1	1.1b
	Uses limits 2 and $\sqrt{2}$ the correct way around	ddM1	1.1b
	Uses limits 4 and 2 the correct way around	ddM1	1.1b
	$= \frac{32}{15}(2 + \sqrt{2}) *$	A1*	2.1
		(7)	
(7 marks)			

June 2018 Question 10 Paper 2

10. A spherical mint of radius 5 mm is placed in the mouth and sucked.
Four minutes later, the radius of the mint is 3 mm.

In a simple model, the rate of decrease of the radius of the mint is inversely proportional to the square of the radius.

Using this model and all the information given,

- (a) find an equation linking the radius of the mint and the time.
(You should define the variables that you use.) (5)
- (b) Hence find the total time taken for the mint to completely dissolve. Give your answer in minutes and seconds to the nearest second. (2)
- (c) Suggest a limitation of the model. (1)

ANSWER

Question	Scheme	Marks	AOs
10 (a)	$\frac{dr}{dt} \propto \pm \frac{1}{r^2}$ or $\frac{dr}{dt} = \pm \frac{k}{r^2}$ (for k or a numerical k)	M1	3.3
	$\int r^2 dr = \int \pm k dt \Rightarrow \dots$ (for k or a numerical k)	M1	2.1
	$\frac{1}{3}r^3 = \pm kt \{+ c\}$	A1	1.1b
	$t=0, r=5$ and $t=4, r=3$ gives $\frac{1}{3}r^3 = -\frac{49}{6}t + \frac{125}{3}$, where r, in mm, is the radius {of the mint} and t, in minutes, is the time from when it {the mint} was placed in the mouth $t=0, r=5$ and $t=240, r=3$ gives $\frac{1}{3}r^3 = -\frac{49}{360}t + \frac{125}{3}$, where r, in mm, is the radius {of the mint} and t, in seconds, is the time from when it {the mint} was placed in the mouth	M1	3.1a
		A1	1.1b
		(5)	
(b)	$r=0 \Rightarrow 0 = -\frac{49}{6}t + \frac{125}{3} \Rightarrow 0 = -49t + 250 \Rightarrow t = \dots$	M1	3.4
	time = 5 minutes 6 seconds	A1	1.1b
		(2)	
(c)	Suggests a suitable limitation of the model. E.g. - Model does not consider how the mint is sucked - Model does not consider whether the mint is bitten - Model is limited for times up to 5 minutes 6 seconds, o.e. - Not valid for times greater than 5 minutes 6 seconds, o.e. - Mint may not retain the shape of a sphere (or have uniform radius) as it is being sucked - The model indicates that the radius of the mint is negative after it dissolves - Model does not consider the temperature in the mouth - Model does not consider rate of saliva production - Mint could be swallowed before it dissolves in the mouth	B1	3.5b
		(1)	
(8 marks)			

June 2018 Question 13 Paper 2

13.

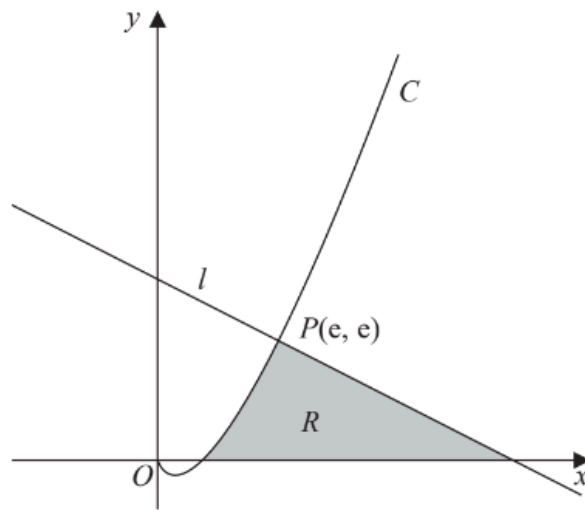


Figure 2

Figure 2 shows a sketch of part of the curve C with equation $y = x \ln x$, $x > 0$

The line l is the normal to C at the point $P(e, e)$

The region R , shown shaded in Figure 2, is bounded by the curve C , the line l and the x -axis.

Show that the exact area of R is $Ae^2 + B$ where A and B are rational numbers to be found.

(10)

ANSWER

Question	Scheme	Marks	AOs
13	$C: y = x \ln x$; l is a normal to C at $P(e, e)$ Let x_A be the x -coordinate of where l cuts the x -axis		
	$\frac{dy}{dx} = \ln x + x \left(\frac{1}{x} \right) \quad \{= 1 + \ln x\}$	M1	2.1
		A1	1.1b
	$x = e, m_T = 2 \Rightarrow m_N = -\frac{1}{2} \Rightarrow y - e = -\frac{1}{2}(x - e)$ $y = 0 \Rightarrow -e = -\frac{1}{2}(x - e) \Rightarrow x = \dots$	M1	3.1a
	l meets x -axis at $x = 3e$ (allow $x = 2e + e \ln e$)	A1	1.1b
	{Areas:} either $\int_1^e x \ln x \, dx = [\dots]_1^e = \dots$ or $\frac{1}{2}((\text{their } x_A) - e)e$	M1	2.1
	$\left\{ \int x \ln x \, dx = \right\} \frac{1}{2} x^2 \ln x - \int \frac{1}{x} \cdot \left(\frac{x^2}{2} \right) \{dx\}$	M1	2.1
	$\left\{ = \frac{1}{2} x^2 \ln x - \int \frac{1}{2} x \{dx\} \right\} = \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2$	dM1	1.1b
		A1	1.1b
	$\text{Area}(R_1) = \int_1^e x \ln x \, dx = [\dots]_1^e = \dots$; $\text{Area}(R_2) = \frac{1}{2}((\text{their } x_A) - e)e$ and so, $\text{Area}(R) = \text{Area}(R_1) + \text{Area}(R_2) \quad \{= \frac{1}{4}e^2 + \frac{1}{4} + e^2\}$	M1	3.1a
	$\text{Area}(R) = \frac{5}{4}e^2 + \frac{1}{4}$	A1	1.1b
		(10)	

June 2018 Question 1 Core 2

1.

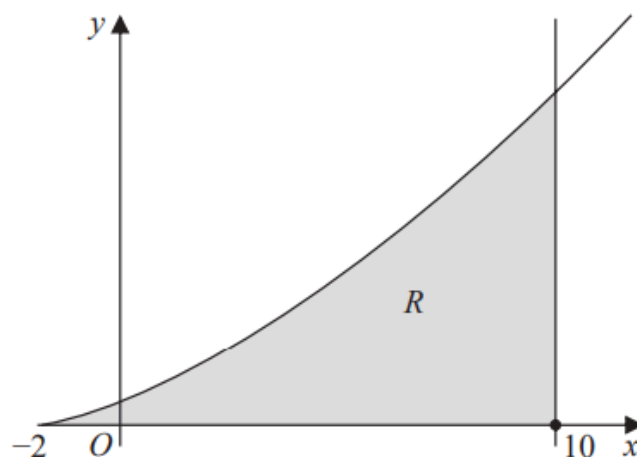


Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = \frac{(x+2)^{\frac{3}{2}}}{4}, \quad x \geq -2$$

The finite region R , shown shaded in Figure 1, is bounded by the curve, the x -axis and the line with equation $x = 10$

The table below shows corresponding values of x and y for $y = \frac{(x+2)^{\frac{3}{2}}}{4}$

(a) Complete the table, giving values of y corresponding to $x = 2$ and $x = 6$

x	-2	2	6	10
y	0			$6\sqrt{3}$

(1)

(b) Use the trapezium rule, with all the values of y from the completed table, to find an approximate value for the area of R , giving your answer to 3 decimal places.

(4)

ANSWER

Question Number	Scheme					Marks
1.	x	-2	2	6	10	
	y	0	2	$4\sqrt{2}$	$6\sqrt{3}$	
(a)	{At $x=2$,} $y=2$ and {At $x=6$,} $y=4\sqrt{2}$ or $2\sqrt{8}$ or awrt 5.7					B1 cao (1)
(b)	$\frac{1}{2} \times 4$; or $h=4$ $\left\{ 0 + 6\sqrt{3} + 2(\text{their } 2 + \text{their } 4\sqrt{2}) \right\}$ For structure of {.....} $\frac{1}{2} \times 4 \left\{ 0 + 6\sqrt{3} + 2(2 + 4\sqrt{2}) \right\} = 2(25.706) = 51.412.. = \text{awrt } 51.412$					B1 oe M1 A1ft A1 (4) (5 marks)

Ch. 12: Vectors

June 2024 Question 7 Paper 2

7.

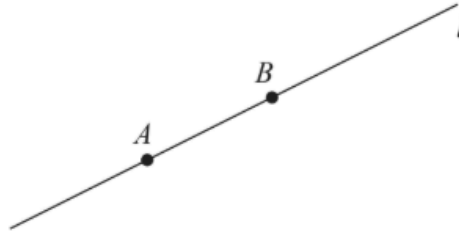


Figure 2

Figure 2 shows a sketch of the straight line l .

Line l passes through the points A and B .

Relative to a fixed origin O

- the point A has position vector $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$
- the point B has position vector $5\mathbf{i} + 6\mathbf{j} + 8\mathbf{k}$

(a) Find $\vec{AB}$

(1)

Given that a point P lies on l such that

$$|\vec{AP}| = 2|\vec{BP}|$$

(b) find the possible position vectors of P .

(4)

ANSWER

Question	Scheme	Marks	AOs
7(a)	$(\overrightarrow{AB}) = 3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}$	B1	1.1b
		(1)	
(b)	Examples: $\overrightarrow{OP} = \overrightarrow{OA} + 2\overrightarrow{AB} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + 2(3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = \dots$ or $\overrightarrow{OP} = \overrightarrow{OB} + \overrightarrow{AB} = 5\mathbf{i} + 6\mathbf{j} + 8\mathbf{k} + (3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = \dots$ or $\overrightarrow{OP} = \overrightarrow{OA} + \frac{2}{3}\overrightarrow{AB} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + \frac{2}{3}(3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = \dots$ or $\overrightarrow{OP} = \overrightarrow{OB} + \frac{1}{3}\overrightarrow{BA} = 5\mathbf{i} + 6\mathbf{j} + 8\mathbf{k} - \frac{1}{3}(3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = \dots$	M1	3.1a
	$8\mathbf{i} + 15\mathbf{j} + 11\mathbf{k}$ or $4\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$	A1	1.1b
	Examples: $\overrightarrow{OP} = \overrightarrow{OA} + 2\overrightarrow{AB} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + 2(3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = \dots$ or $\overrightarrow{OP} = \overrightarrow{OB} + \overrightarrow{AB} = 5\mathbf{i} + 6\mathbf{j} + 8\mathbf{k} + (3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = \dots$ and $\overrightarrow{OP} = \overrightarrow{OA} + \frac{2}{3}\overrightarrow{AB} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + \frac{2}{3}(3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = \dots$ or $\overrightarrow{OP} = \overrightarrow{OB} + \frac{1}{3}\overrightarrow{BA} = 5\mathbf{i} + 6\mathbf{j} + 8\mathbf{k} - \frac{1}{3}(3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = \dots$	dM1	3.1a
	$8\mathbf{i} + 15\mathbf{j} + 11\mathbf{k}$ and $4\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$	A1	2.2a
		(4)	
(5 marks)			

June 2023 Question 6 Paper 2

6. Relative to a fixed origin O ,

- A is the point with position vector $12\mathbf{i}$
- B is the point with position vector $16\mathbf{j}$
- C is the point with position vector $(50\mathbf{i} + 136\mathbf{j})$
- D is the point with position vector $(22\mathbf{i} + 24\mathbf{j})$

(a) Show that AD is parallel to BC .

(2)

Points A , B , C and D are used to model the vertices of a running track in the shape of a quadrilateral.

Runners complete one lap by running along all four sides of the track.

The lengths of the sides are measured in metres.

Given that a particular runner takes exactly 5 minutes to complete 2 laps,

(b) calculate the average speed of this runner, giving the answer in kilometres per hour.

(4)

ANSWER

Question	Scheme	Marks	AOs
6(a)	$\overrightarrow{AD} = 10\mathbf{i} + 24\mathbf{j}$ and $\overrightarrow{BC} = 50\mathbf{i} + 120\mathbf{j}$	M1	1.1b
	$\overrightarrow{AD} = \frac{1}{5}\overrightarrow{BC}$ therefore AD is parallel to BC *	A1*cs0	2.2a
		(2)	
(b)	Attempt to find at least two lengths between AB , BC , CD and AD $ \overrightarrow{BC} = \sqrt{50^2 + 120^2} = 130$, $ \overrightarrow{DA} = \sqrt{10^2 + 24^2} = 26$ $ \overrightarrow{AB} = \sqrt{12^2 + 16^2} = 20$, $ \overrightarrow{CD} = \sqrt{28^2 + 112^2} = 28\sqrt{17}$ (awrt 115 m)	M1	1.1b
		A1	1.1b
	Average speed = $\frac{2(26+130+20+28\sqrt{17})}{\frac{5}{60} \times 1000}$	dM1	3.1b
	awrt = 6.99 (km/h)	A1	3.2a
		(4)	
(6 marks)			

June 2023 Question 3 Paper 1

3. Relative to a fixed origin O

- the point A has position vector $5\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$
- the point B has position vector $2\mathbf{i} + 4\mathbf{j} + a\mathbf{k}$

where a is a positive integer.

(a) Show that $|\vec{OA}| = \sqrt{38}$

(1)

(b) Find the smallest value of a for which

$$|\vec{OB}| > |\vec{OA}|$$

(2)

ANSWER

Question	Scheme	Marks	AOs
3(a)	$(\vec{OA} = \sqrt{5^2 + 3^2 + 2^2} = \sqrt{38} \quad *$	B1*	1.1b
(b)	$ \vec{OB} = \sqrt{2^2 + 4^2 + a^2} = \sqrt{20 + a^2}$ so when $a = 5$ $ \vec{OB} = \sqrt{20 + 25} = \sqrt{45}$	M1	1.1b
	$= 5$	A1cso	2.3
		(2)	
(3 marks)			

June 2022 Question 13 Paper 2

13. Relative to a fixed origin O

- the point A has position vector $4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$
- the point B has position vector $4\mathbf{j} + 6\mathbf{k}$
- the point C has position vector $-16\mathbf{i} + p\mathbf{j} + 10\mathbf{k}$

where p is a constant.

Given that A , B and C lie on a straight line,

(a) find the value of p .

(3)

The line segment OB is extended to a point D so that $\vec{CD}$ is parallel to $\vec{OA}$

(b) Find $|\vec{OD}|$, writing your answer as a fully simplified surd.

(3)

ANSWER

Question	Scheme	Marks	AOs
13(a)	Attempts two of the relevant vectors $\pm \overrightarrow{AB} = \pm(-4\mathbf{i} + 7\mathbf{j} + \mathbf{k})$ $\pm \overrightarrow{AC} = \pm(-20\mathbf{i} + (p+3)\mathbf{j} + 5\mathbf{k})$ $\pm \overrightarrow{BC} = \pm(-16\mathbf{i} + (p-4)\mathbf{j} + 4\mathbf{k})$	M1	3.1a
	Uses two of the three vectors in such a way as to find the value of p . E.g. $p+3=5 \times 7$	dM1	2.1
	$p = 32$	A1	1.1b
		(3)	
(b)	Deduces that $\overrightarrow{OD} = \lambda \overrightarrow{OB} = 4\lambda \mathbf{j} + 6\lambda \mathbf{k}$ and attempts $\overrightarrow{CD} = 16\mathbf{i} + (4\lambda - 32)\mathbf{j} + (6\lambda - 10)\mathbf{k}$	M1	3.1a
	Correct attempt at λ using the fact that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$ $\overrightarrow{CD} = 16\mathbf{i} + (4\lambda - 32)\mathbf{j} + (6\lambda - 10)\mathbf{k}$ $\overrightarrow{OA} = 4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ $4\lambda - 32 = -12 \Rightarrow \lambda = \dots$ OR $6\lambda - 10 = 20 \Rightarrow \lambda = \dots$	dM1	1.1b
	$ \overrightarrow{OD} = 5 \times \sqrt{4^2 + 6^2} = 10\sqrt{13}$	A1	1.1b
		(3)	

Video solution:

<https://youtu.be/8uADn2CLsVs>

June 2022 Question 9 Paper 1

9.

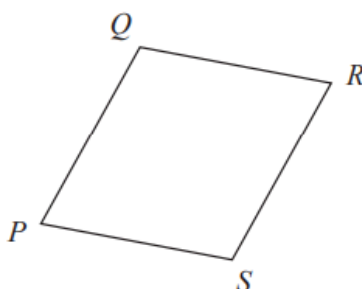


Figure 3

Figure 3 shows a sketch of a parallelogram $PQRS$.

Given that

- $\vec{PQ} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$
- $\vec{QR} = 5\mathbf{i} - 2\mathbf{k}$

(a) show that parallelogram $PQRS$ is a rhombus.

(2)

(b) Find the exact area of the rhombus $PQRS$.

(4)

ANSWER

Question	Scheme	Marks	AOs
9(a)	Attempts both $ \vec{PQ} = \sqrt{2^2 + 3^2 + (-4)^2}$ and $ \vec{QR} = \sqrt{5^2 + (-2)^2}$	M1	3.1a
	States that $ \vec{PQ} = \vec{QR} = \sqrt{29}$ so $PQRS$ is a rhombus	A1	2.4
		(2)	
(b)	Attempts BOTH $\vec{PR} = \vec{PQ} + \vec{QR} = 7\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$ AND $\vec{QS} = -\vec{PQ} + \vec{PS} = 3\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$	M1	3.1a
	Correct $\vec{PR} = 7\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$ and $\vec{QS} = 3\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$	A1	1.1b
	Correct method for area $PQRS$. E.g. $\frac{1}{2} \times \vec{PR} \times \vec{QS} $	dM1	2.1
	$= \sqrt{517}$	A1	1.1b
		(4)	
(6 marks)			

November 2021 Question 6 Paper 1

6.

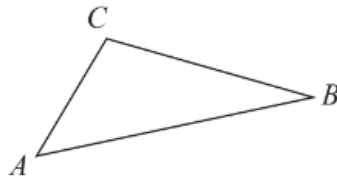


Figure 1

Figure 1 shows a sketch of triangle ABC .

Given that

- $\vec{AB} = -3\mathbf{i} - 4\mathbf{j} - 5\mathbf{k}$
- $\vec{BC} = \mathbf{i} + \mathbf{j} + 4\mathbf{k}$

(a) find $\vec{AC}$

(2)

(b) show that $\cos ABC = \frac{9}{10}$

(3)

ANSWER

Question	Scheme	Marks	AOs
6(a)	$\vec{AC} = \vec{AB} + \vec{BC} = -3\mathbf{i} - 4\mathbf{j} - 5\mathbf{k} + \mathbf{i} + \mathbf{j} + 4\mathbf{k} = \dots$	M1	1.1b
	$= -2\mathbf{i} - 3\mathbf{j} - \mathbf{k}$	A1	1.1b
		(2)	
(b)	At least 2 of $(AC^2) = 2^2 + 3^2 + 1^2, (AB^2) = 3^2 + 4^2 + 5^2, (BC^2) = 1^2 + 1^2 + 4^2$	M1	1.1b
	$2^2 + 3^2 + 1^2 = 3^2 + 4^2 + 5^2 + 1^2 + 1^2 + 4^2 - 2\sqrt{3^2 + 4^2 + 5^2}\sqrt{1^2 + 1^2 + 4^2} \cos ABC$	M1	3.1a
	$14 = 50 + 18 - 2\sqrt{50}\sqrt{18} \cos ABC$ $\Rightarrow \cos ABC = \frac{50 + 18 - 14}{2\sqrt{50}\sqrt{18}} = \frac{9}{10}^*$	A1*	2.1
		(3)	
	(b) Alternative		
	$AB^2 = 3^2 + 4^2 + 5^2, BC^2 = 1^2 + 1^2 + 4^2$	M1	1.1b
	$\vec{BA} \cdot \vec{BC} = (3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}) \cdot (\mathbf{i} + \mathbf{j} + 4\mathbf{k}) = 27 = \sqrt{3^2 + 4^2 + 5^2}\sqrt{1^2 + 1^2 + 4^2} \cos ABC$	M1	3.1a
	$27 = \sqrt{50}\sqrt{18} \cos ABC \Rightarrow \cos ABC = \frac{27}{\sqrt{50}\sqrt{18}} = \frac{9}{10}^*$	A1*	2.1
(5 marks)			

June 2018 Question 2 Paper 2

2. Relative to a fixed origin O ,

the point A has position vector $(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$,

the point B has position vector $(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$,

and the point C has position vector $(a\mathbf{i} + 5\mathbf{j} - 2\mathbf{k})$, where a is a constant and $a < 0$

D is the point such that $\overrightarrow{AB} = \overrightarrow{BD}$.

(a) Find the position vector of D .

(2)

Given $|\overrightarrow{AC}| = 4$

(b) find the value of a .

(3)

ANSWER

Question	Scheme	Marks	AOs
2	$\vec{OA} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$, $\vec{OB} = 4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, $\vec{OC} = a\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$, $a < 0$ $\vec{AB} = \vec{BD}$, $ \vec{AB} = 4$		
(a)	E.g. $\vec{OD} = \vec{OB} + \vec{BD} = \vec{OB} + \vec{AB}$ or $\vec{OD} = \vec{OB} + \vec{BD} = \vec{OB} + \vec{AB} = \vec{OB} + \vec{OB} - \vec{OA} = 2\vec{OB} - \vec{OA}$ or $\vec{OD} = \vec{OB} + \vec{BD} = \vec{OB} + \vec{AB} = \vec{OA} + \vec{AB} + \vec{AB} = \vec{OA} + 2\vec{AB}$		
	$= \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} \left\{ = \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ -5 \\ 7 \end{pmatrix} \right\}$ or $= \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + 2 \left(\begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} \right) \left\{ = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + 2 \begin{pmatrix} 2 \\ -5 \\ 7 \end{pmatrix} \right\}$	M1	3.1a
	$= \begin{pmatrix} 6 \\ -7 \\ 10 \end{pmatrix} \text{ or } 6\mathbf{i} - 7\mathbf{j} + 10\mathbf{k}$	A1	1.1b
		(2)	
(b)	$(a-2)^2 + (5-3)^2 + (-2-4)^2$	M1	1.1b
	$\{ \vec{AC} = 4 \Rightarrow (a-2)^2 + (5-3)^2 + (-2-4)^2 = (4)^2$ $\Rightarrow (a-2)^2 = 8 \Rightarrow a = \dots$ or $\Rightarrow a^2 - 4a - 4 = 0 \Rightarrow a = \dots$	dM1	2.1
	$(\text{as } a < 0 \Rightarrow) a = 2 - 2\sqrt{2} \text{ (or } a = 2 - \sqrt{8})$	A1	1.1b
		(3)	
(5 marks)			