

Pure Year 1 exam questions Edexcel

NOTE:

Please be aware that in the Year 12 collections, you will find questions from the Year 13 papers. However, these questions are intentionally included because they align with Year 12 content and topics.

Happy studying!

Ch. 1: Algebraic Expressions	6
November 2021 Question 2	6
June 2019 Question 1 Paper 2 (A-Level)	7
June 2018 Question 1 Core 1	8
Ch. 2: Quadratics	9
June 2024 Question 9 Paper 2 (A-level)	9
June 2022 Question 5 Paper 1 (A Level)	12
November 2021 Question 6	13
November 2021 Question 12 Paper 1 (A-Level)	14
June 2019 Question 9	16
June 2018 Question 6	17
June 2018 Question 8 Paper 2 (A-level)	19
June 2018 Question 3 Core 1	22
Ch. 3: Equations and Inequalities	24
June 2023 Question 8	24
June 2023 Question 2	26
November 2021 Question 1	27
November 2020 Question 3	28
November 2020 Question 7 Paper 1 (A-Level)	29
June 2019 Question 2	31
June 2018 Question 7 Core 1	32
Ch. 4: Graphs and Transformations	34
June 2024 Question 3 Paper 2 (A-level)	34
June 2024 Question 5	35
June 2023 Question 15	37
June 2023 Question 4	39
June 2022 Question 11 Paper 1 (A Level)	40
June 2022 Question 7	42
June 2019 Question 7	44
June 2018 Question 9	46
June 2018 Question 5 Core 1	48
Ch. 5: Straight Line Graphs	50
June 2023 Question 10	50
June 2023 Question 7	52
November 2020 Question 4	54

June 2019 Question 1	55
June 2019 Question 4	56
June 2019 Question 7	57
June 2018 Question 4	59
June 2018 Question 8 Core 1	60
Ch. 6: Circles	64
June 2024 Question 14 Paper 2	64
June 2024 Question 10	65
June 2023 Question 6	67
June 2023 Question 10 Paper 1	68
June 2022 Question 11	69
June 2022 Question 3 Paper 1 (A Level)	71
November 2021 Question 7 Paper 1 (A-Level)	72
November 2021 Question 15	73
November 2020 Question 14 Paper 2 (A-Level)	75
June 2018 Question 14	77
June 2018 Question 6 Paper 1 (A-Level)	79
June 2018 Question 5 Core 2	81
Ch. 7: Algebraic Methods	82
June 2024 Question 1 (A-level)	82
June 2023 Question 14	83
June 2024 Question 2	84
June 2023 Question 17	86
June 2023 Question 7 Paper 1 (A-Level)	87
June 2023 Question 2 Paper 1 (A-Level)	88
June 2023 Question 17 Paper 1	90
June 2022 Question 11 Paper 2 (A Level)	91
June 2022 Question 2 Paper 1 (A Level)	92
June 2022 Question 14 Paper 1	93
June 2022 Question 2	94
November 2021 Question 10 Paper 1	95
November 2020 Question 16 Paper 2 (A-Level)	96
June 2019 Question 1 Paper 1 (A-Level)	97
June 2019 Question 10 Paper 1 (A-Level)	98
June 2018 Question 2	99
June 2018 Question 3 Core 2	100
Ch. 8: The Binomial Expansion	101
June 2024 Question 6	101
June 2023 Question 14	103
June 2022 Question 6	104
November 2021 Question 8	105
November 2020 Question 6	106
June 2019 Question 8	107

November 2020 Question 4 Paper 2 (A-Level)	108
June 2018 Question 11	109
June 2018 Question 1 Core 2	112
Ch. 9: Trigonometric Ratios	113
June 2024 Question 4	113
June 2023 Question 3	115
June 2022 Question 4	117
November 2021 Question 7	118
November 2020 Question 5	120
June 2019 Question 6	122
June 2018 Question 7	123
Ch. 10: Trigonometric Identities and Equations	124
June 2024 Question 13	124
June 2023 Question 12	125
June 2022 Question 13	128
November 2021 Question 12	130
November 2020 Question 9	132
June 2018 Question 12	134
June 2018 Question 8 Core 2	135
Ch. 11: Vectors	136
June 2024 Question 3	136
June 2023 Question 13	138
June 2023 Question 6 Paper 2	140
June 2022 Question 3	141
November 2021 Question 4	142
November 2020 Question 2	143
November 2020 Question 3 Paper 1	145
November 2020 Question 2 Paper 2 (A-Level)	146
June 2019 Question 10 Paper 2 (A-Level)	147
June 2018 Question 3	149
Ch. 12: Differentiation	150
June 2024 Question 15 Paper 2	150
June 2024 Question 1 Paper 2 (A-level)	152
June 2024 Question 4 Paper 1 (A-level)	153
June 2024 Question 12	154
June 2023 Question 1	156
June 2022 Question 4 Paper 2 (A Level)	157
June 2022 Question 15 Paper 1 (A Level)	158
June 2022 Question 6 Paper 1 (A Level)	160
June 2022 Question 12	162
November 2021 Question 5	164
November 2021 Question 5 Paper 2 (A-Level)	166
November 2021 Question 14 Paper 2 (A-Level)	168

November 2021 Question 16	172
November 2020 Question 1	174
June 2019 Question 5	175
June 2019 Question 13 Paper 2 (A-Level)	176
June 2018 Question 8	178
June 2018 Question 10	180
June 2018 Question 2 Paper 1	181
June 2018 Question 11 Paper 2 (A-Level)	182
June 2018 Question 10 Core 1	184
Ch. 13: Integration	187
June 2024 Question 10 Paper 1 (A-level)	187
June 2024 Question 8	189
June 2024 Question 1	191
June 2023 Question 16	192
June 2023 Question 5	193
June 2023 Question 5 Paper 2 (A-Level)	195
June 2023 Question 1 Paper 1 (A-Level)	196
June 2022 Question 8 Paper 2 (A Level)	197
June 2022 Question 10	199
June 2022 Question 1	201
November 2021 Question 3	202
November 2021 Question 7 Paper 2 (A-Level)	203
November 2021 Question 14	205
November 2020 Question 7	207
November 2020 Question 10	208
November 2020 Question 8 Paper 2 (A-Level)	209
June 2019 Question 3	210
June 2019 Question 8 Paper 1 (A-Level)	211
June 2018 Question 1	213
June 2018 Question 15	214
June 2018 Question 2 Core 1	217
June 2018 Question 9 Core 1	219
June 2018 Question 9 Core 2	222
Ch. 14: Exponentials and Logarithms	224
June 2024 Question 13 Paper 2 (A-level)	224
June 2024 Question 11	227
June 2024 Question 9	229
June 2024 Question 7	230
June 2023 Question 11	232
June 2023 Question 9	233
June 2023 Question 4 Paper 2 (A-Level)	234
June 2023 Question 3 Paper 2 (A-Level)	235
June 2023 Question 11 Paper 1 (A-Level)	236

June 2023 Question 6 Paper 1 (A-Level)	238
June 2022 Question 2 Paper 2 (A Level)	239
June 2022 Question 10 Paper 1 (A Level)	240
June 2022 Question 9	242
June 2022 Question 8	243
June 2022 Question 5	245
November 2021 Question 11	247
November 2021 Question 13	248
November 2021 Question 8 Paper 1 (A-Level)	250
November 2021 Question 3 Paper 2 (A-Level)	252
November 2021 Question 10 Paper 2 (A-Level)	253
November 2020 Question 8	255
November 2020 Question 2 Paper 1 (A-Level)	257
November 2020 Question 3 Paper 2 (A-Level)	258
November 2020 Question 5 Paper 2 (A-Level)	260
November 2020 Question 9 Paper 2 (A-Level)	261
June 2019 Question 7 Paper 1 (A-Level)	263
June 2019 Question 9 Paper 1 (A-Level)	265
June 2019 Question 9 Paper 2 (A-Level)	266
June 2018 Question 5	268
June 2018 Question 13	269
June 2018 Question 12 Paper 1	271
June 2018 Question 7 Core 2	273
June 2018 Question 3 Core 3	274

Ch. 1: Algebraic Expressions

November 2021 Question 2

2. In this question you should show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Given

$$\frac{9^{x-1}}{3^{y+2}} = 81$$

express y in terms of x , writing your answer in simplest form.

(3)

ANSWER

Question	Scheme	Marks	AOs
2	$\frac{9^{x-1}}{3^{y+2}} = 81 \Rightarrow \frac{3^{2x-2}}{3^{y+2}} = 3^4$ or $\frac{9^{x-1}}{3^{y+2}} = 81 \Rightarrow \frac{9^{x-1}}{9^{\frac{1}{2}(y+2)}} = 9^2$	M1	1.1b
	$\Rightarrow 2x - 2 - y - 2 = 4 \Rightarrow y =$ or $\Rightarrow x - 1 - \frac{1}{2}y - 1 = 2 \Rightarrow y =$	dM1	1.1b
	$\Rightarrow y = 2x - 8$	A1	1.1b
		(3)	
Alt	Eg. $\log_3 \left(\frac{9^{x-1}}{3^{y+2}} \right) = \log_3 81$	M1	1.1b
	$\Rightarrow (x-1)\log_3(9^{x-1}) - (y+2)\log_3(3^{y+2}) = 4$ $\Rightarrow 2(x-1) - y - 2 = 4 \Rightarrow y =$	dM1	1.1b
	$\Rightarrow y = 2x - 8$	A1	1.1b
(3 marks)			

June 2019 Question 1 Paper 2 (A-Level)

1. Given

$$2^x \times 4^y = \frac{1}{2\sqrt{2}}$$

express y as a function of x .

(3)

ANSWER

Way 1	$2^x \times 2^{2y} = 2^{-\frac{3}{2}}$	B1	1.1b
	$2^{x+2y} = 2^{-\frac{3}{2}} \Rightarrow x+2y = -\frac{3}{2} \Rightarrow y = \dots$	M1	2.1
	E.g. $y = -\frac{1}{2}x - \frac{3}{4}$ or $y = -\frac{1}{4}(2x+3)$	A1	1.1b
		(3)	
Way 2	$\log(2^x \times 4^y) = \log\left(\frac{1}{2\sqrt{2}}\right)$	B1	1.1b
	$\log 2^x + \log 4^y = \log\left(\frac{1}{2\sqrt{2}}\right)$ $\Rightarrow x \log 2 + y \log 4 = \log 1 - \log(2\sqrt{2}) \Rightarrow y = \dots$	M1	2.1
	$y = \frac{-\log(2\sqrt{2}) - x \log 2}{\log 4} \quad \left\{ \Rightarrow y = -\frac{1}{2}x - \frac{3}{4} \right\}$	A1	1.1b
		(3)	

June 2018 Question 1 Core 1

1. (i) Simplify

$$\sqrt{48} - \frac{6}{\sqrt{3}}$$

Write your answer in the form $a\sqrt{3}$, where a is an integer to be found.

(2)

(ii) Solve the equation

$$3^{6x-3} = 81$$

Write your answer as a rational number.

(3)

ANSWER

Question Number	Scheme		Marks
1.(i) Way 1	$\sqrt{48} = \sqrt{16}\sqrt{3}$ or $\frac{6}{\sqrt{3}} = 6\frac{\sqrt{3}}{3}$	Writes one of the terms of the given expression correctly in terms of $\sqrt{3}$	M1
	$\Rightarrow \sqrt{48} - \frac{6}{\sqrt{3}} = 2\sqrt{3}$	A correct answer of $2\sqrt{3}$. A correct answer with no working implies both marks.	A1
			(2)
(ii) Way 1	$81 = 3^4$ or $\log_3 81 = 6x - 3$	For $81 = 3^4$ or $\log_3 81 = 6x - 3$. This may be implied by subsequent work.	B1
	$3^{6x-3} = 3^4$ or $\log_3 81 = 6x - 3$ $\Rightarrow 4 = 6x - 3 \Rightarrow x = \dots$	Solves an equation of the form $6x - 3 = k$ where k is their power of 3.	M1
	$\Rightarrow x = \frac{4+3}{6} = \frac{7}{6}$	$\frac{7}{6}$ or $1\frac{1}{6}$ or 1.16 with a dot over the 6	A1
			(3)

Ch. 2: Quadratics

June 2024 Question 9 Paper 2 (A-level)

9.

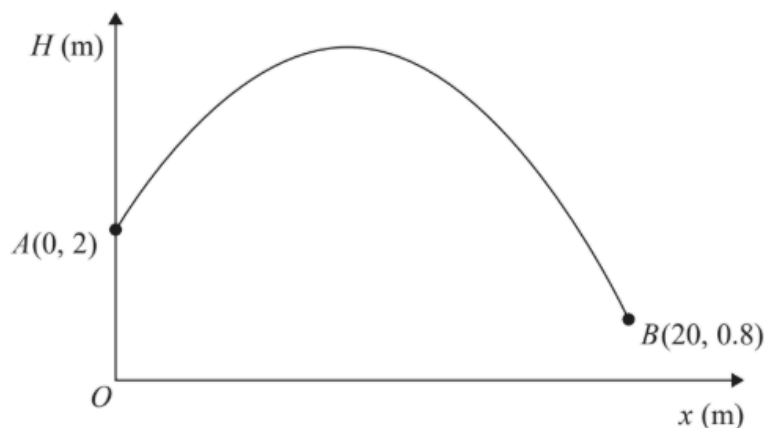


Figure 3

The graph in Figure 3 shows the path of a small ball.

The ball travels in a vertical plane above horizontal ground.

The ball is thrown from the point represented by A and caught at the point represented by B .

The height, H metres, of the ball above the ground has been plotted against the horizontal distance, x metres, measured from the point where the ball was thrown.

With respect to a fixed origin O , the point A has coordinates $(0, 2)$ and the point B has coordinates $(20, 0.8)$, as shown in Figure 3.

The ball reaches its maximum height when $x = 9$

A quadratic function, linking H with x , is used to model the path of the ball.

(a) Find H in terms of x . (4)

(b) Give one limitation of the model. (1)

Chandra is standing directly under the path of the ball at a point 16 m horizontally from O .

Chandra can catch the ball if the ball is less than 2.5 m above the ground.

(c) Use the model to determine if Chandra can catch the ball. (2)

ANSWER

Question	Scheme	Marks	AOs
9(a) Way 1	$H = \pm ax^2 \pm bx \pm c$ $x = 0, H = 2 \Rightarrow c = 2$ and either $x = 20, H = 0.8 \Rightarrow 0.8 = 400a + 20b + 2$ or $H = ax^2 + bx + c \Rightarrow \frac{dH}{dx} = 2ax + b$ $x = 9, \frac{dH}{dx} = 0 \Rightarrow 18a + b = 0$	M1	3.3
	$H = \pm ax^2 \pm bx \pm c$ $x = 0, H = 2 \Rightarrow c = 2$ and $x = 20, H = 0.8 \Rightarrow 0.8 = 20^2 a + 20b + 2$ and $H = ax^2 + bx + c \Rightarrow \frac{dH}{dx} = 2ax + b$ $x = 9, \frac{dH}{dx} = 0 \Rightarrow 18a + b = 0$	dM1	3.1b
	$0.8 = 400a + 20b + 2, 18a + b = 0 \Rightarrow a = \dots, b = \dots$	ddM1	1.1b
	$H = -0.03x^2 + 0.54x + 2$	A1	2.2a
		(4)	

(b)	Examples must focus on why the model may not be appropriate or give situations where the model would break down e.g.: • H is unlikely to be a quadratic function in x • The path is unlikely to be parabolic • Wind may affect the path of the ball • Wind may affect the distance the ball travels • Air resistance has not been considered • The ball is unlikely to travel in a vertical plane (as it may spin) • The ball is not a particle so has dimensions/size • The ground is unlikely to be horizontal • There may be trees (or other hazards) that would affect the path of the ball • The shape of the ball may affect the motion Condone statements (where the link to the model is not completely made) such as • The ball will spin • Ground is not flat • The ball is not a particle Do not accept statements that refer to the situation outside the range	B1	3.5b
	of the throw e.g. • The model is not valid for all values of x • H will become negative Do not accept statements that do not refer to the given model or single word vague answers e.g. • The distances may have been measured incorrectly • The ball is not modelled as a particle • “Friction”, “Spin”, “Force”, “air resistance” • It does not take into account the weight of the ball • It depends how good the thrower is • You cannot throw the ball the same way every time		
(c)	$x = 16 \Rightarrow H = -0.03(16)^2 + 0.54(16) + 2 = \dots$	(1)	
	$H = 2.96$ So Chandra would not be able to catch the ball	M1	3.4
		A1	3.2a
		(2)	
(7 marks)			

June 2022 Question 5 Paper 1 (A Level)

5. The height, h metres, of a tree, t years after being planted, is modelled by the equation

$$h^2 = at + b \quad 0 \leq t < 25$$

where a and b are constants.

Given that

- the height of the tree was 2.60 m, exactly 2 years after being planted
- the height of the tree was 5.10 m, exactly 10 years after being planted

- (a) find a complete equation for the model, giving the values of a and b to 3 significant figures.
(4)

Given that the height of the tree was 7 m, exactly 20 years after being planted

- (b) evaluate the model, giving reasons for your answer.
(2)

ANSWER

Question	Scheme	Marks	AOs
5 (a)	Attempts to use $h^2 = at + b$ with either $t = 2, h = 2.6$ or $t = 10, h = 5.1$	M1	3.1b
	Correct equations $2a + b = 6.76$ $10a + b = 26.01$	A1	1.1b
	Solves simultaneously to find values for a and b	dM1	1.1b
	$h^2 = 2.41t + 1.95$ cao	A1	3.3
		(4)	
(b)	Substitutes $t = 20$ into their $h^2 = 2.41t + 1.95$ and finds h or h^2 Or substitutes $h = 7$ into their $h^2 = 2.41t + 1.95$ and finds t	M1	3.4
	Compares the model with the true values and concludes "good model" with a minimal reason E.g. I Finds $h = 7.08$ (m) and states that it is a good model as 7.08 (m) is close to 7 (m) E.g. II Finds $t = 19.5$ years and states that the model is accurate as 19.5 (years) ≈ 20 (years)	A1	3.5a
		(2)	
(6 marks)			

November 2021 Question 6

6. In this question you should show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Using algebra, find all solutions of the equation

$$3x^3 - 17x^2 - 6x = 0 \quad (3)$$

(b) Hence find all real solutions of

$$3(y-2)^6 - 17(y-2)^4 - 6(y-2)^2 = 0 \quad (3)$$

ANSWER

Question	Scheme	Marks	AOs
6 (a)	$3x^3 - 17x^2 - 6x = 0 \Rightarrow x(3x^2 - 17x - 6) = 0$	M1	1.1a
	$\Rightarrow x(3x+1)(x-6) = 0$	dM1	1.1b
	$\Rightarrow x = 0, -\frac{1}{3}, 6$	A1	1.1b
		(3)	
(b)	Attempts to solve $(y-2)^2 = n$ where n is any solution ≥ 0 to (a)	M1	2.2a
	Two of $2, 2 \pm \sqrt{6}$	A1ft	1.1b
	All three of $2, 2 \pm \sqrt{6}$	A1	2.1
		(3)	
(6 marks)			

November 2021 Question 12 Paper 1 (A-Level)

12.

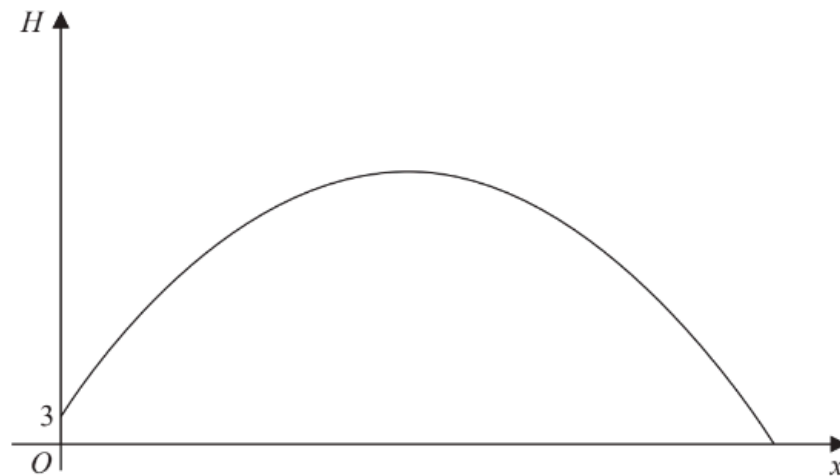


Figure 3

Figure 3 is a graph of the trajectory of a golf ball after the ball has been hit until it first hits the ground.

The vertical height, H metres, of the ball above the ground has been plotted against the horizontal distance travelled, x metres, measured from where the ball was hit.

The ball is modelled as a particle travelling in a vertical plane above horizontal ground.

Given that the ball

- is hit from a point on the top of a platform of vertical height 3 m above the ground
- reaches its maximum vertical height after travelling a horizontal distance of 90 m
- is at a vertical height of 27 m above the ground after travelling a horizontal distance of 120 m

Given also that H is modelled as a **quadratic** function in x

- (a) find H in terms of x (5)
- (b) Hence find, according to the model,
- the maximum vertical height of the ball above the ground,
 - the horizontal distance travelled by the ball, from when it was hit to when it first hits the ground, giving your answer to the nearest metre. (3)
- (c) The possible effects of wind or air resistance are two limitations of the model. Give one other limitation of this model. (1)

ANSWER

Question	Scheme	Marks	AOs
12(a)	$H = ax^2 + bx + c$ and $x=0, H=3 \Rightarrow H = ax^2 + bx + 3$	M1	3.3
	$H = ax^2 + bx + 3$ and $x=120, H=27 \Rightarrow 27 = 14400a + 120b + 3$	M1	3.1b
	or $\frac{dH}{dx} = 2ax + b = 0$ when $x = 90 \Rightarrow 180a + b = 0$	A1	1.1b
	$H = ax^2 + bx + 3$ and $x=120, H=27 \Rightarrow 27 = 14400a + 120b + 3$ and $\frac{dH}{dx} = 2ax + b = 0$ when $x = 90 \Rightarrow 180a + b = 0$ $\Rightarrow a = \dots, b = \dots$	dM1	3.1b
	$H = -\frac{1}{300}x^2 + \frac{3}{5}x + 3$ o.e.	A1	1.1b
		(5)	
(b)(i)	$x = 90 \Rightarrow H \left(= -\frac{1}{300}(90)^2 + \frac{3}{5}(90) + 3 \right) = 30\text{m}$	B1	3.4
(b)(ii)	$H = 0 \Rightarrow -\frac{1}{300}x^2 + \frac{3}{5}x + 3 = 0 \Rightarrow x = \dots$	M1	3.4
	$x = (-4.868\dots) 184.868\dots$ $\Rightarrow x = 185(\text{m})$	A1	3.2a
		(3)	
(c)	Examples must focus on why the model may not be appropriate or give values/situations where the model would break down: E.g. - The ground is unlikely to be horizontal - The ball is not a particle so has dimensions/size - The ball is unlikely to travel in a vertical plane (as it will spin) H is not likely to be a quadratic function in x	B1	3.5b
		(1)	
(9 marks)			

June 2019 Question 9

9. A company started mining tin in Riverdale on 1st January 2019.

A model to find the total mass of tin that will be mined by the company in Riverdale is given by the equation

$$T = 1200 - 3(n - 20)^2$$

where T tonnes is the total mass of tin mined in the n years after the start of mining.

Using this model,

- (a) calculate the mass of tin that will be mined up to 1st January 2020, (1)
- (b) deduce the maximum total mass of tin that could be mined, (1)
- (c) calculate the mass of tin that will be mined in 2023. (2)
- (d) State, giving reasons, the limitation on the values of n . (2)

ANSWER

Question	Scheme	Marks	AOs
9 (a)	117 tonnes	B1	3.4
		(1)	
(b)	1200 tonnes	B1	2.2a
		(1)	
(c)	Attempts $\{1200 - 3 \times (5 - 20)^2\} - \{1200 - 3 \times (4 - 20)^2\}$	M1	3.1a
	93 tonnes	A1	1.1b
		(2)	
(d)	States the model is only valid for values of n such that $n \leq 20$	B1	3.5b
	States that the total amount mined cannot decrease	B1	2.3
		(2)	
(6 marks)			

June 2018 Question 6

6.

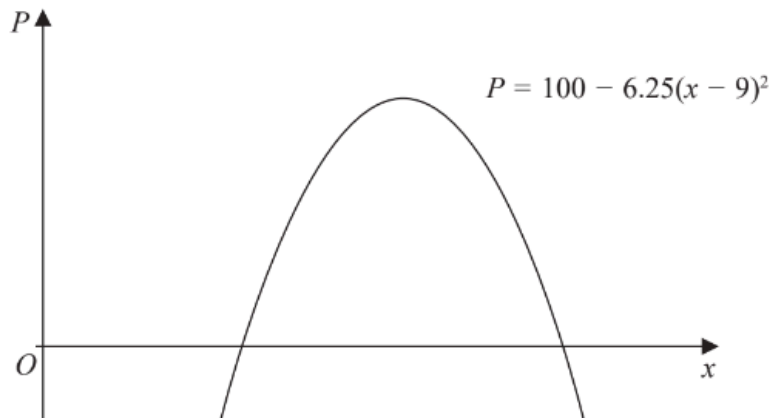


Figure 1

A company makes a particular type of children's toy.

The annual profit made by the company is modelled by the equation

$$P = 100 - 6.25(x - 9)^2$$

where P is the profit measured in thousands of pounds and x is the selling price of the toy in pounds.

A sketch of P against x is shown in Figure 1.

Using the model,

- (a) explain why £15 is not a sensible selling price for the toy. (2)

Given that the company made an annual profit of more than £80 000

- (b) find, according to the model, the least possible selling price for the toy. (3)

The company wishes to maximise its annual profit.

State, according to the model,

- (c) (i) the maximum possible annual profit,
(ii) the selling price of the toy that maximises the annual profit. (2)

ANSWER

Question	Scheme	Marks	AOs
6 (a)	Attempts $P = 100 - 6.25(15 - 9)^2$	M1	3.4
	$= -125 \therefore$ not sensible as the company would make a loss	A1	2.4
		(2)	
(b)	Uses $P > 80 \Rightarrow (x - 9)^2 < 3.2$ or $P = 80 \Rightarrow (x - 9)^2 = 3.2$	M1	3.1b
	$\Rightarrow 9 - \sqrt{3.2} < x < 9 + \sqrt{3.2}$	dM1	1.1b
	Minimum Price = £7.22	A1	3.2a
		(3)	
(c)	States (i) maximum profit = £100 000 and (ii) selling price £9	B1 B1	3.2a 2.2a
		(2)	
(7 marks)			

Video solution:

<https://youtu.be/UFenaZppVpk>

June 2018 Question 8 Paper 2 (A-level)

8.

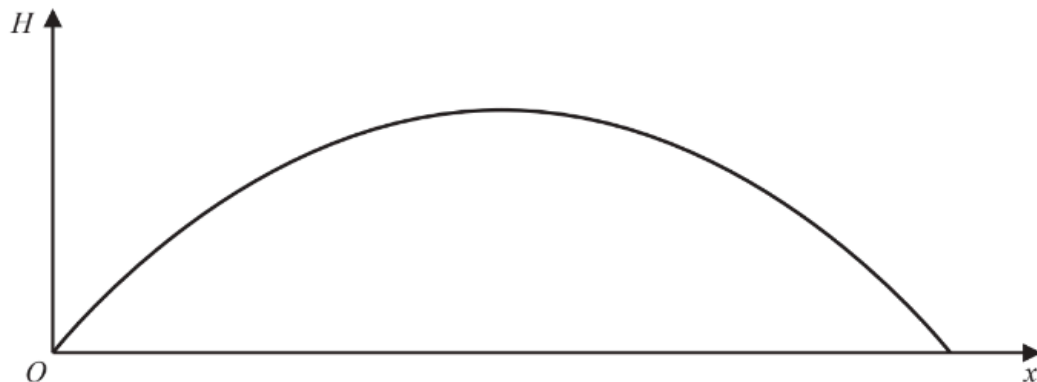


Figure 1

Figure 1 is a graph showing the trajectory of a rugby ball.

The height of the ball above the ground, H metres, has been plotted against the horizontal distance, x metres, measured from the point where the ball was kicked.

The ball travels in a vertical plane.

The ball reaches a maximum height of 12 metres and hits the ground at a point 40 metres from where it was kicked.

(a) Find a quadratic equation linking H with x that models this situation. (3)

The ball passes over the horizontal bar of a set of rugby posts that is perpendicular to the path of the ball. The bar is 3 metres above the ground.

(b) Use your equation to find the greatest horizontal distance of the bar from O . (3)

(c) Give one limitation of the model. (1)

ANSWER

Question	Scheme	Marks	AOs
8 (a) Way 1	$H = Ax(40 - x) \quad \{\text{or } H = Ax(x - 40)\}$	M1	3.3
	$x = 20, H = 12 \Rightarrow 12 = A(20)(40 - 20) \Rightarrow A = \frac{3}{100}$	dM1	3.1b
	$H = \frac{3}{100}x(40 - x) \text{ or } H = -\frac{3}{100}x(x - 40)$	A1	1.1b
		(3)	
(a) Way 2	$H = 12 - \lambda(x - 20)^2 \quad \{\text{or } H = 12 + \lambda(x - 20)^2\}$	M1	3.3
	$x = 40, H = 0 \Rightarrow 0 = 12 - \lambda(40 - 20)^2 \Rightarrow \lambda = \frac{3}{100}$	dM1	3.1b
	$H = 12 - \frac{3}{100}(x - 20)^2$	A1	1.1b
		(3)	
(a) Way 3	$H = ax^2 + bx + c$ (or deduces $H = ax^2 + bx$) Both $x = 0, H = 0 \Rightarrow 0 = 0 + 0 + c \Rightarrow c = 0$ and either $x = 40, H = 0 \Rightarrow 0 = 1600a + 40b$ or $x = 20, H = 12 \Rightarrow 12 = 400a + 20b$ or $\frac{-b}{2a} = 20 \Rightarrow b = -40a$	M1	3.3
	$b = -40a \Rightarrow 12 = 400a + 20(-40a) \Rightarrow a = -0.03$ so $b = -40(-0.03) = 1.2$	dM1	3.1b
	$H = -0.03x^2 + 1.2x$	A1	1.1b
		(3)	

(b)	$\{H = 3 \Rightarrow\} 3 = \frac{3}{100}x(40 - x) \Rightarrow x^2 - 40x + 100 = 0$ or $\{H = 3 \Rightarrow\} 3 = 12 - \frac{3}{100}(x - 20)^2 \Rightarrow (x - 20)^2 = 300$	M1	3.4
	e.g. $x = \frac{40 \pm \sqrt{1600 - 4(1)(100)}}{2(1)}$ or $x = 20 \pm \sqrt{300}$	dM1	1.1b
	$\{\text{chooses } 20 + \sqrt{300} \Rightarrow\}$ greatest distance = awrt 37.3 m	A1	3.2a
		(3)	
(c)	Gives a limitation of the model. Accept e.g. - the ground is horizontal - the ball needs to be kicked from the ground - the ball is modelled as a particle - the horizontal bar needs to be modelled as a line - there is no wind or air resistance on the ball - there is no spin on the ball - no obstacles in the trajectory (or path) of the ball - the trajectory of the ball is a perfect parabola	B1	3.5b
		(1)	
(7 marks)			

June 2018 Question 3 Core 1

3. $f(x) = x^2 - 10x + 23$

(a) Express $f(x)$ in the form $(x + a)^2 + b$, where a and b are constants to be found.

(2)

(b) Hence, or otherwise, find the exact solutions to the equation

$$x^2 - 10x + 23 = 0$$

(2)

(c) Use your answer to part (b) to find the larger solution to the equation

$$y - 10y^{0.5} + 23 = 0$$

Write your solution in the form $p + q\sqrt{r}$, where p , q and r are integers.

(2)

ANSWER

Question Number	Scheme		Marks
3.(a)	$x^2 - 10x + 23 = (x \pm 5)^2 \pm A$	For an attempt to complete the square. Note that if their $A = 23$ then this is M0 by the General Principles.	M1
	$(x-5)^2 - 2$	Correct expression. Ignore “= 0”.	A1
			(2)
(b)	$(x \pm 5)^2 - A \Rightarrow x = \dots$ or $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \dots$ $\left(x = \frac{10 \pm \sqrt{10^2 - 4(1)(23)}}{2} \right)$	Uses their completion of the square for positive A or uses the correct quadratic formula to obtain at least one value for x	M1
	$x = 5 \pm \sqrt{2}$	Correct exact values. If using the quadratic formula must reach as far as $\frac{10 \pm \sqrt{8}}{2}$	A1
			(2)
(c)	$(5 \pm \sqrt{2})^2 = 27 + 10\sqrt{2}$	Attempts to square any solution from part (b). Allow poor squaring e.g. $(5 + \sqrt{2})^2 = 25 + 2 = 27$. Do not allow for substituting e.g. $5 + \sqrt{2}$ into $x^2 - 10x + 23$.	M1
	$= 27 + 10\sqrt{2}$	Accept equivalent forms such as $27 + \sqrt{200}$. If any extra answers are given, this mark should be withheld.	A1
			(2)
	Allow candidates to start again:		
	$y - 10y^{0.5} + 23 = 0 \Rightarrow y^{0.5} = \frac{10 \pm \sqrt{10^2 - 4 \times 23}}{2} = 5 \pm \sqrt{2}$ $y = (5 \pm \sqrt{2})^2 = \dots$		M1
	$= 27 + 10\sqrt{2}$	Accept equivalent forms such as $27 + \sqrt{200}$. If any extra answers are given, this mark should be withheld.	A1
			(6 marks)

Ch. 3: Equations and Inequalities

June 2023 Question 8

8.

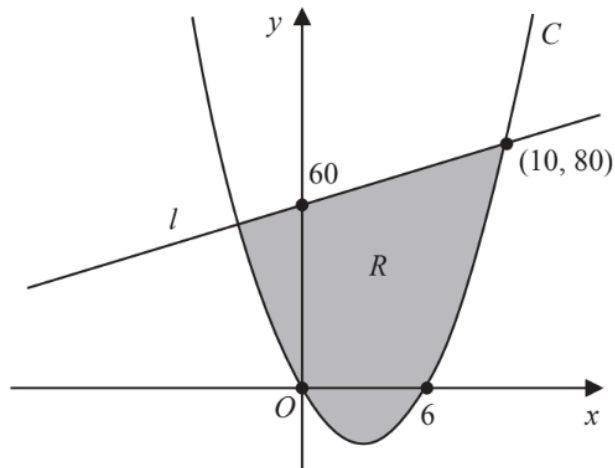


Figure 3

Figure 3 shows a sketch of a curve C and a straight line l .

Given that

- C has equation $y = f(x)$ where $f(x)$ is a quadratic expression in x
- C cuts the x -axis at 0 and 6
- l cuts the y -axis at 60 and intersects C at the point $(10, 80)$

use inequalities to define the region R shown shaded in Figure 3.

(5)

ANSWER

Question	Scheme	Marks	AOs
8	Complete method to find the RHS of an equation for l e.g., Attempts gradient $= \frac{80-60}{10} \{=2\}$ and uses intercept $= 60$	M1	1.1b
	$\{y=\} 2x+60$	A1	1.1b
	Deduces the RHS of the equation for C is $\{y=\} ax(x-6)$ and attempts to use $(10,80)$ to find the value of a	M1	3.1a
	Equation of C is $\{y=\} 2x(x-6)$	A1	1.1b
	$2x(x-6) \leq y \leq 2x+60$	B1ft	2.5
		(5)	
(5 marks)			

June 2023 Question 2

2. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Using the substitution $u = \sqrt{x}$ or otherwise, solve

$$6x + 7\sqrt{x} - 20 = 0$$

(4)

ANSWER

Question	Scheme	Marks	AOs
2	Let $u = \sqrt{x}$ $6x + 7\sqrt{x} - 20 = 0 \Rightarrow 6u^2 + 7u - 20 = 0$ $\Rightarrow (3u - 4)(2u + 5) = 0$	M1A1	1.1b 1.1b
	Attempts $\sqrt{x} = \frac{4}{3}, -\frac{5}{2} \Rightarrow x = \dots$	M1	1.1b
	$x = \frac{16}{9}$ only	A1 cso	2.3
		(4)	
(4 marks)			

November 2021 Question 1

1. In this question you should show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Using algebra, solve the inequality

$$x^2 - x > 20$$

writing your answer in set notation.

(3)

ANSWER

Question	Scheme	Marks	AOs
1	Finds critical values $x^2 - x > 20 \Rightarrow x^2 - x - 20 > 0 \Rightarrow x = (5, -4)$	M1	1.1b
	Chooses outside region for their values Eg. $x > 5, x < -4$	M1	1.1b
	Presents solution in set notation $\{x : x < -4\} \cup \{x : x > 5\}$ oe	A1	2.5
		(3)	
(3 marks)			

November 2020 Question 3

3. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(i) Solve the equation

$$x\sqrt{2} - \sqrt{18} = x$$

writing the answer as a surd in simplest form.

(3)

(ii) Solve the equation

$$4^{3x-2} = \frac{1}{2\sqrt{2}}$$

(3)

ANSWER

Question	Scheme	Marks	AOs
3 (i)	$x\sqrt{2} - \sqrt{18} = x \Rightarrow x(\sqrt{2} - 1) = \sqrt{18} \Rightarrow x = \frac{\sqrt{18}}{\sqrt{2} - 1}$	M1	1.1b
	$\Rightarrow x = \frac{\sqrt{18}}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1}$	dM1	3.1a
	$x = \frac{\sqrt{18}(\sqrt{2} + 1)}{1} = 6 + 3\sqrt{2}$	A1	1.1b
		(3)	
(ii)	$4^{3x-2} = \frac{1}{2\sqrt{2}} \Rightarrow 2^{6x-4} = 2^{-\frac{3}{2}}$	M1	2.5
	$6x - 4 = -\frac{3}{2} \Rightarrow x = \dots$	dM1	1.1b
	$x = \frac{5}{12}$	A1	1.1b
		(3)	
(6 marks)			

November 2020 Question 7 Paper 1 (A-Level)

7.

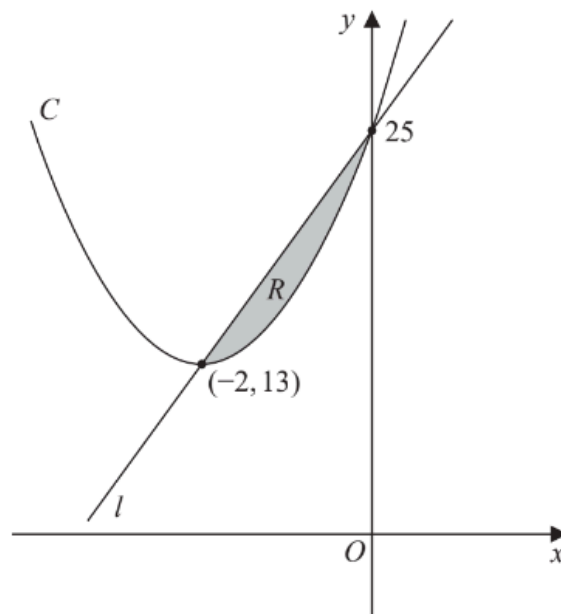


Figure 1

Figure 1 shows a sketch of a curve C with equation $y = f(x)$ and a straight line l .

The curve C meets l at the points $(-2, 13)$ and $(0, 25)$ as shown.

The shaded region R is bounded by C and l as shown in Figure 1.

Given that

- $f(x)$ is a quadratic function in x
- $(-2, 13)$ is the minimum turning point of $y = f(x)$

use inequalities to define R .

(5)

ANSWER

Question	Scheme	Marks	AOs
7	Attempts equation of line Eg Substitutes $(-2,13)$ into $y = mx + 25$ and finds m	M1	1.1b
	Equation of l is $y = 6x + 25$	A1	1.1b
	Attempts equation of C Eg Attempts to use the intercept $(0,25)$ within the equation $y = a(x \pm 2)^2 + 13$, in order to find a	M1	3.1a
	Equation of C is $y = 3(x+2)^2 + 13$ or $y = 3x^2 + 12x + 25$	A1	1.1b
	Region R is defined by $3(x+2)^2 + 13 < y < 6x + 25$ o.e.	B1ft	2.5
		(5)	
(5 marks)			

June 2019 Question 2

2. Find, using algebra, all real solutions to the equation

(i) $16a^2 = 2\sqrt{a}$ (4)

(ii) $b^4 + 7b^2 - 18 = 0$ (4)

ANSWER

Question	Scheme		Marks	AOs
2(i)	$16a^2 = 2\sqrt{a} \Rightarrow a^{\frac{3}{2}} = \frac{1}{8}$	$16a^2 - 2\sqrt{a} = 0$ $\Rightarrow 2a^{\frac{1}{2}}\left(8a^{\frac{3}{2}} - 1\right) = 0$ $\Rightarrow a^{\frac{3}{2}} = \frac{1}{8}$	M1	1.1b
	$\Rightarrow a = \left(\frac{1}{8}\right)^{\frac{2}{3}}$	$\Rightarrow a = \left(\frac{1}{8}\right)^{\frac{2}{3}}$	M1	1.1b
	$\Rightarrow a = \frac{1}{4}$	$\Rightarrow a = \frac{1}{4}$	A1	1.1b
	Deduces that $a = 0$ is a solution		B1	2.2a
			(4)	
(ii)	$b^4 + 7b^2 - 18 = 0 \Rightarrow (b^2 + 9)(b^2 - 2) = 0$		M1	1.1b
	$b^2 = -9, 2$		A1	1.1b
	$b^2 = k \Rightarrow b = \sqrt{k}, k > 0$		dM1	2.3
	$b = \sqrt{2}, -\sqrt{2}$ only		A1	1.1b
			(4)	
(8 marks)				

June 2018 Question 7 Core 1

7. The equation $20x^2 = 4kx - 13kx^2 + 2$, where k is a constant, has no real roots.

(a) Show that k satisfies the inequality

$$2k^2 + 13k + 20 < 0 \quad (4)$$

(b) Find the set of possible values for k .

(4)

ANSWER

Question Number	Scheme		Marks
7(a)	$b^2 - 4ac = (4k)^2 - 4(-2)(20 + 13k)$	Attempts to use $b^2 - 4ac$ with $a = \pm(20 \pm 13k)$, $b = \pm 4k$, $c = \pm 2$. This could be as part of the quadratic formula or as $b^2 < 4ac$ or as $b^2 > 4ac$ or as $\sqrt{b^2 - 4ac}$ etc. If it is part of the quadratic formula only look for use of $b^2 - 4ac$. There must be no x 's. If they gather to the lhs, condone the omission of the “-” on the “ $4k$ ”.	M1
	$(4k)^2 - 4(-2)(20 + 13k)$	For a correct un-simplified expression.	A1
	$b^2 - 4ac < 0$ $\Rightarrow (4k)^2 - 4(-2)(20 + 13k) < 0$	Uses $b^2 - 4ac < 0$ or e.g. $b^2 < 4ac$ with their values of a , b and c in terms of k . The “ < 0 ” must appear before the final printed answer but can appear as $b^2 - 4ac < 0$ at the start.	M1
	$16k^2 + 160 + 104k < 0$ $\Rightarrow 2k^2 + 13k + 20 < 0^*$	Reaches the printed answer with no errors, including bracketing errors, or contradictory statements and sufficient working shown. Note that the statement $(20 + 13k)x^2 - 4kx - 2 < 0$ or starting with e.g. $20x^2 < 4kx - 13kx^2 + 2$ would be an error.	A1*
			(4)

(b)	$2k^2 + 13k + 20 = 0 \Rightarrow k = \dots$ e.g. $(2k + 5)(k + 4) = 0 \Rightarrow k = \dots$	Attempt to solve the given quadratic to find 2 values for k . See general guidance.	M1
	$\Rightarrow k = -\frac{5}{2}, -4$	Both correct. May be implied by e.g. $k < -\frac{5}{2}$, $k < -4$ or seen on a sketch. If they use the quadratic formula allow $\frac{-13 \pm 3}{4}$ for this mark but not $\sqrt{9}$ for 3 and allow e.g. $-\frac{13}{4} \pm \frac{3}{4}$ if they complete the square.	A1
	$-4 < k < -\frac{5}{2}$ Allow equivalent values e.g. $-\frac{10}{4}$ i.e. the critical values must be in the form $\frac{a}{b}$ where a and b are integers	M1: Chooses 'inside' region for their critical values i.e. Lower Limit $< k <$ Upper Limit or e.g. Lower Limit $\leq k \leq$ Upper Limit A1: Allow $k \in (-4, -\frac{5}{2})$ or just $(-4, -\frac{5}{2})$ and allow $k > -4$ and $k < -2.5$ and $-\frac{5}{2} > k > -4$ but $k > -4$, $k < -\frac{5}{2}$ scores M1A0. $-\frac{5}{2} < k < -4$ is M0A0	M1A1
	Allow working in terms of x in (b) but the answer must be in terms of k for the final mark.		
			(4)
			(8 marks)

Ch. 4: Graphs and Transformations

June 2024 Question 3 Paper 2 (A-level)

3. The point $P(3, -2)$ lies on the curve with equation $y = f(x)$, $x \in \mathbb{R}$

Find the coordinates of the point to which P is mapped when the curve with equation $y = f(x)$ is transformed to the curve with equation

(i) $y = f(x - 2)$

(ii) $y = f(2x)$

(iii) $y = 3f(-x) + 5$

(4)

ANSWER

Question	Scheme	Marks	AOs
3(i)	$(5, -2)$ or e.g. $x = 5, y = -2$ o.e.	B1	1.1b
		(1)	
(ii)	$(1.5, -2)$ or e.g. $x = 1.5, y = -2$ o.e.	B1	1.1b
		(1)	
(iii)	$(-3, \dots)$ or $(\dots, -1)$ or $x = -3$ or $y = -1$ o.e.	B1	1.1b
	$(-3, -1)$ or $x = -3$ and $y = -1$ o.e.	B1	1.1b
		(2)	
(4 marks)			

June 2024 Question 5

5. The curve C_1 has equation

$$y = \frac{6}{x} + 3$$

(a) (i) Sketch C_1 stating the coordinates of any points where the curve cuts the coordinate axes.

(ii) State the equations of any asymptotes to the curve C_1

(3)

The curve C_2 has equation

$$y = 3x^2 - 4x - 10$$

(b) Show that C_1 and C_2 intersect when

$$3x^3 - 4x^2 - 13x - 6 = 0$$

(2)

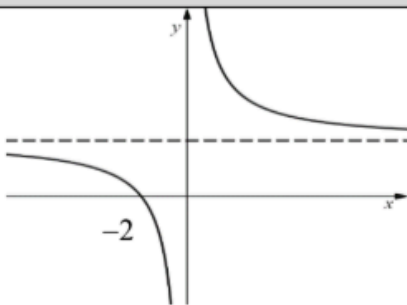
Given that the x coordinate of one of the points of intersection is $-\frac{2}{3}$

(c) use algebra to find the x coordinates of the other points of intersection between C_1 and C_2

(Solutions relying on calculator technology are not acceptable.)

(4)

ANSWER

Question	Scheme	Marks	AOs
5(a)(i)		M1 A1	1.1b 1.1b
(ii)	Asymptotes: $y = 3, x = 0$	B1	1.1b
		(3)	
(b)	e.g. $\frac{6}{x} + 3 = 3x^2 - 4x - 10 \Rightarrow 6 + 3x = 3x^3 - 4x^2 - 10x$	M1	1.1b
	$3x^3 - 4x^2 - 13x - 6 = 0$ *	A1*	1.1b
		(2)	
(c)	$\begin{array}{r} x^2 - 2x \pm \dots \\ \text{e.g. } 3x + 2 \overline{) 3x^3 - 4x^2 - 13x - 6} \end{array}$	M1	3.1a
	$x^2 - 2x - 3 (= 0)$ or $3x^2 - 6x - 9 (= 0)$	A1	1.1b
	e.g. $(3x + 2)(x - 3)(x + 1) = 0 \Rightarrow x = \dots$	dM1	1.1b
	$x = -1, x = 3$	A1cso	1.1b
		(4)	
(9 marks)			

June 2023 Question 15

15.

In this question you must show detailed reasoning.

Solutions relying on calculator technology are not acceptable.

The curve C_1 has equation $y = 8 - 10x + 6x^2 - x^3$

The curve C_2 has equation $y = x^2 - 12x + 14$

(a) Verify that when $x = 1$ the curves C_1 and C_2 intersect.

(2)

The curves also intersect when $x = k$.

Given that $k < 0$

(b) use algebra to find the exact value of k .

(5)

ANSWER

Question	Scheme	Marks	AOs
15 (a)	Attempts both $y = 8 - 10 \times 1 + 6 \times 1^2 - 1^3$ and $y = 1^2 - 12 \times 1 + 14$	M1	1.1b
	Achieves $y = 3$ for both equations and gives a minimal conclusion / statement, e.g., $(1, 3)$ lies on both curves so they intersect at $x = 1$	A1	1.1b
		(2)	
(b)	(Curves intersect when) $x^2 - 12x + 14 = 8 - 10x + 6x^2 - x^3$ $\Rightarrow x^3 - 5x^2 - 2x + 6 = 0$	M1	1.1b
	For the key step in dividing by $(x - 1)$ $x^3 - 5x^2 - 2x + 6 = (x - 1)(x^2 + px \pm 6)$	dM1	3.1a
	$x^3 - 5x^2 - 2x + 6 = (x - 1)(x^2 - 4x - 6)$	A1	1.1b
	Solves $x^2 - 4x - 6 = 0$ $(x - 2)^2 = 10 \Rightarrow x = \dots$	ddM1	1.1b
	$x = 2 - \sqrt{10}$ only	A1	1.1b
		(5)	
(7 marks)			

June 2023 Question 4

4. (a) Sketch the curve with equation

$$y = \frac{k}{x} \quad x \neq 0$$

where k is a positive constant.

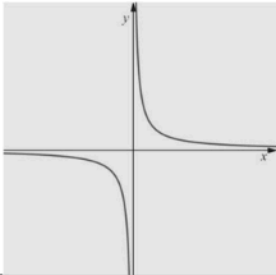
(2)

- (b) Hence or otherwise, solve

$$\frac{16}{x} \leq 2$$

(3)

ANSWER

Question	Scheme	Marks	AOs	
4(a)		Shape in quadrant 1 or 3	M1	1.1b
		Shape and Position	A1	1.1b
			(2)	
(b)	Deduces that $x < 0$	B1	2.2a	
	Attempts $\frac{16}{x} \dots 2 \Rightarrow x \dots \pm \frac{16}{2}$	M1	1.1b	
	$x < 0$ or $x \geq 8$	A1 cso	2.2a	
		(3)		
(5 marks)				

June 2022 Question 11 Paper 1 (A Level)

11.

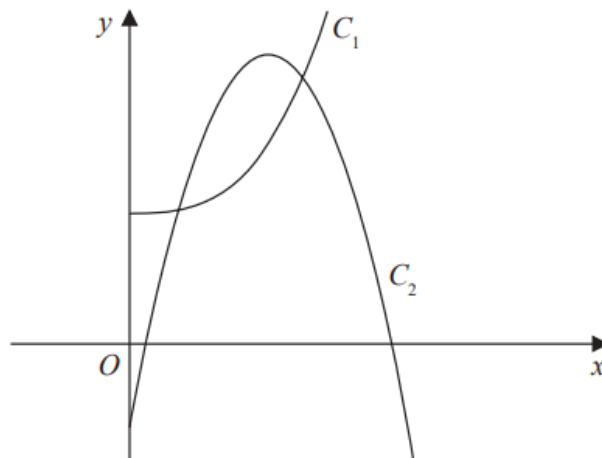


Figure 4

Figure 4 shows a sketch of part of the curve C_1 with equation

$$y = 2x^3 + 10 \quad x > 0$$

and part of the curve C_2 with equation

$$y = 42x - 15x^2 - 7 \quad x > 0$$

- (a) Verify that the curves intersect at $x = \frac{1}{2}$

(2)

The curves intersect again at the point P

- (b) Using algebra and showing all stages of working, find the exact x coordinate of P

(5)

ANSWER

Question	Scheme	Marks	AOs
11 (a)	Substitutes $x = \frac{1}{2}$ into $y = 2x^3 + 10$ and $y = 42x - 15x^2 - 7$ and finds the y values for both	M1	1.1b
	Achieves $\frac{41}{4}$ o.e. for both and makes a valid conclusion. *	A1*	2.4
		(2)	
(b)	Sets $42x - 15x^2 - 7 = 2x^3 + 10 \Rightarrow 2x^3 + 15x^2 - 42x + 17 = 0$	M1	1.1b
	Deduces that $(2x - 1)$ is a factor and attempts to divide	dM1	2.1
	$2x^3 + 15x^2 - 42x + 17 = (2x - 1)(x^2 + 8x - 17)$	A1	1.1b
	Solves their $x^2 + 8x - 17 = 0$ using suitable method	M1	1.1b
	Deduces $x = -4 + \sqrt{33}$ (see note)	A1	2.2a
		(5)	
(7 marks)			

Video solution:

https://youtu.be/aUIOj6-F_x8

June 2022 Question 7

7. (a) Factorise completely $9x - x^3$

(2)

The curve C has equation

$$y = 9x - x^3$$

- (b) Sketch C showing the coordinates of the points at which the curve cuts the x -axis.

(2)

The line l has equation $y = k$ where k is a constant.

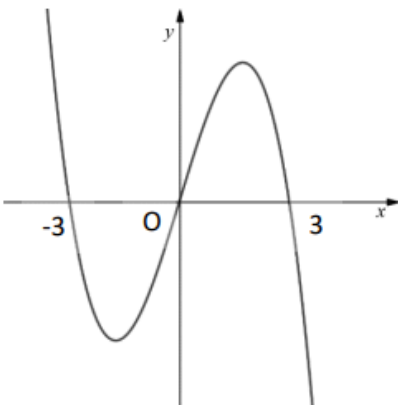
Given that C and l intersect at 3 distinct points,

- (c) find the range of values for k , writing your answer in set notation.

Solutions relying on calculator technology are not acceptable.

(3)

ANSWER

Question	Scheme	Marks	AOs	
7(a)	$9x - x^3 = x(9 - x^2)$	M1	1.1b	
	$9x - x^3 = x(3 - x)(3 + x)$ oe	A1	1.1b	
		(2)		
(b)		A cubic with correct orientation	B1	1.1b
		Passes through origin, (3, 0) and (-3, 0)	B1	1.1b
		(2)		
(c)	$y = 9x - x^3 \Rightarrow \frac{dy}{dx} = 9 - 3x^2 = 0 \Rightarrow x = (\pm)\sqrt{3} \Rightarrow y = \dots$	M1	3.1a	
	$y = (\pm)6\sqrt{3}$	A1	1.1b	
	$\{k \in \mathbb{R} : -6\sqrt{3} < k < 6\sqrt{3}\}$ oe	A1ft	2.5	
		(3)		
(7 marks)				

Video solution:

<https://youtu.be/SEvw1T8kZbl>

June 2019 Question 7

7. The curve C has equation

$$y = \frac{k^2}{x} + 1 \quad x \in \mathbb{R}, x \neq 0$$

where k is a constant.

(a) Sketch C stating the equation of the horizontal asymptote.

(3)

The line l has equation $y = -2x + 5$

(b) Show that the x coordinate of any point of intersection of l with C is given by a solution of the equation

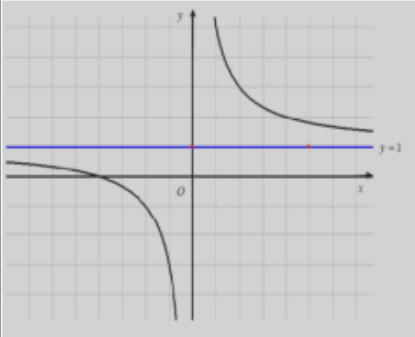
$$2x^2 - 4x + k^2 = 0$$

(2)

(c) Hence find the exact values of k for which l is a tangent to C .

(3)

ANSWER

Question	Scheme	Marks	AOs
7 (a)	 $\frac{1}{x}$ shape in 1st quadrant Correct Asymptote $y = 1$	M1	1.1b
		A1	1.1b
		B1	1.2
		(3)	
(b)	Combines equations $\Rightarrow \frac{k^2}{x} + 1 = -2x + 5$	M1	1.1b
	$(\times x) \Rightarrow k^2 + 1x = -2x^2 + 5x \Rightarrow 2x^2 - 4x + k^2 = 0^*$	A1*	2.1
		(2)	
(c)	Attempts to set $b^2 - 4ac = 0$	M1	3.1a
	$8k^2 = 16$	A1	1.1b
	$k = \pm\sqrt{2}$	A1	1.1b
		(3)	
(8 marks)			

June 2018 Question 9

9.

$$g(x) = 4x^3 - 12x^2 - 15x + 50$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $g(x)$.

(2)

(b) Hence show that $g(x)$ can be written in the form $g(x) = (x + 2)(ax + b)^2$, where a and b are integers to be found.

(4)

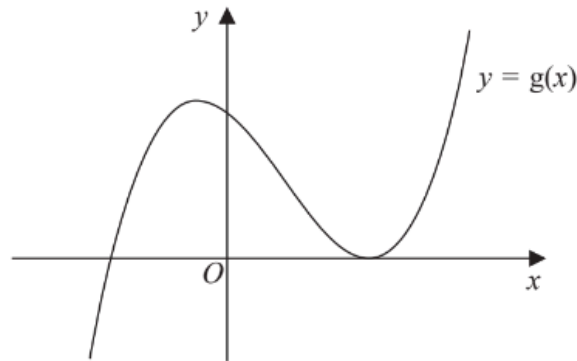


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = g(x)$

(c) Use your answer to part (b), and the sketch, to deduce the values of x for which

(i) $g(x) \leq 0$

(ii) $g(2x) = 0$

(3)

ANSWER

Question	Scheme	Marks	AOs
9(a)	$(g(-2)) = 4 \times -8 - 12 \times 4 - 15 \times -2 + 50$	M1	1.1b
	$g(-2) = 0 \Rightarrow (x+2)$ is a factor	A1	2.4
		(2)	
(b)	$4x^3 - 12x^2 - 15x + 50 = (x+2)(4x^2 - 20x + 25)$	M1 A1	1.1b 1.1b
	$= (x+2)(2x-5)^2$	M1 A1	1.1b 1.1b
		(4)	
(c)	(i) $x \leq -2, x = 2.5$	M1 A1ft	1.1b 1.1b
	(ii) $x = -1, x = 1.25$	B1ft	2.2a
		(3)	
(9 marks)			

June 2018 Question 5 Core 1

5.

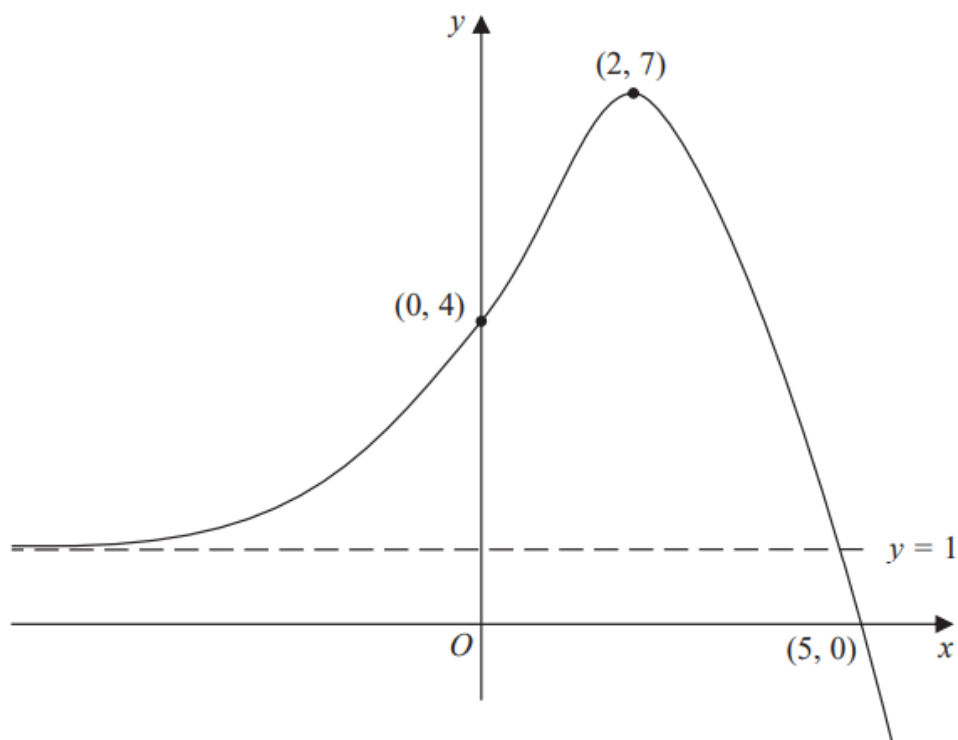


Figure 1

Figure 1 shows the sketch of a curve with equation $y = f(x)$, $x \in \mathbb{R}$.

The curve crosses the y -axis at $(0, 4)$ and crosses the x -axis at $(5, 0)$.

The curve has a single turning point, a maximum, at $(2, 7)$.

The line with equation $y = 1$ is the only asymptote to the curve.

(a) State the coordinates of the turning point on the curve with equation $y = f(x - 2)$. (1)

(b) State the solution of the equation $f(2x) = 0$ (1)

(c) State the equation of the asymptote to the curve with equation $y = f(-x)$. (1)

Given that the line with equation $y = k$, where k is a constant, meets the curve $y = f(x)$ at only one point,

(d) state the set of possible values for k . (2)

ANSWER

Arancha Ruiz

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Question Number	Scheme		Marks
5(a)	$(4, 7)$	Accept $(4, 7)$ or $x = 4, y = 7$ or a sketch of $y = f(x - 2)$ with a maximum point marked at $(4, 7)$. (Condone missing brackets) There should be no other coordinates.	B1
			(1)
(b)	$(x =) 2.5$	Allow $(2.5, 0)$ (condone missing brackets) but no other values or points. Allow a sketch of $f(2x)$ with the only x -intercept marked at $x = 2.5$ (Allow $(0, 2.5)$ marked in the correct place.	B1
			(1)
(c)	$y = 1$ (oe e.g. $y - 1 = 0$)	Must be an equation and not just '1' and no other asymptotes stated.	B1
			(1)
(d)	$k \leq 1$ or $k = 7$	Either of $k \leq 1$ or $k = 7$ Accept either of $y \leq 1$ or $y = 7$ Note that $k = 7$ may sometimes be seen embedded in e.g. $k = 0, 1, 7$ and can score B1 here.	B1
	$k \leq 1 \quad k = 7$	Both correct and in terms of k with no other solutions.	B1
			(2)
			(5 marks)

Ch. 5: Straight Line Graphs

June 2023 Question 10

10.

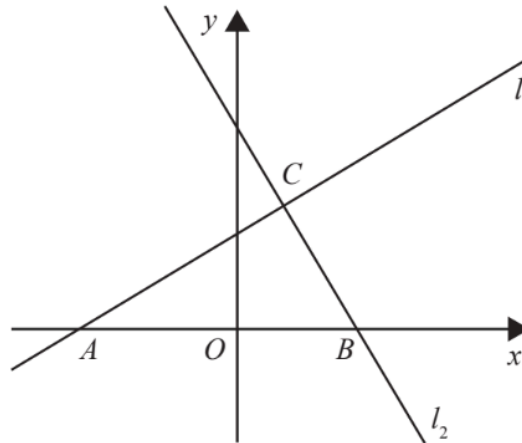


Figure 4

The line l_1 has equation $y = \frac{3}{5}x + 6$

The line l_2 is perpendicular to l_1 and passes through the point $B(8,0)$, as shown in the sketch in Figure 4.

(a) Show that an equation for line l_2 is

$$5x + 3y = 40$$

(3)

Given that

- lines l_1 and l_2 intersect at the point C
- line l_1 crosses the x -axis at the point A

(b) find the exact area of triangle ABC , giving your answer as a fully simplified fraction in the form $\frac{p}{q}$

(5)

ANSWER

Question	Scheme	Marks	AOs
10 (a)	Deduces that the gradient of line l_2 is $-\frac{5}{3}$	B1	1.1b
	Complete attempt to find the equation of line l_2 e.g., $y - 0 = -\frac{1}{m_1}(x - 8)$	M1	1.1b
	$5x + 3y = 40$ *	A1*	2.1
		(3)	
(b)	Deduces $A(-10, 0)$	B1	2.2a
	Attempts to solve $y = \frac{3}{5}x + 6$ and $5x + 3y = 40$ simultaneously to find the y coordinate of their point of intersection	M1	1.1b
	y coordinate of C is $\frac{135}{17}$ o.e.	A1	1.1b
	Complete attempt at area $ABC = \frac{1}{2} \times (8 + "10") \times "\frac{135}{17}"$	dM1	2.1
	$= \frac{1215}{17}$	A1	1.1b
		(5)	
(8 marks)			

June 2023 Question 7

7. The distance a particular car can travel in a journey starting with a full tank of fuel was investigated.
- From a full tank of fuel, 40 litres remained in the car's fuel tank after the car had travelled 80 km
 - From a full tank of fuel, 25 litres remained in the car's fuel tank after the car had travelled 200 km

Using a **linear model**, with V litres being the volume of fuel remaining in the car's fuel tank and d km being the distance the car had travelled,

- (a) find an equation linking V with d .

(4)

Given that, on a particular journey

- the fuel tank of the car was initially full
- the car continued until it ran out of fuel

find, according to the model,

- (b) (i) the initial volume of fuel that was in the fuel tank of the car,
(ii) the distance that the car travelled on this journey.

(3)

In fact the car travelled 320 km on this journey.

- (c) Evaluate the model in light of this information.

(1)

ANSWER

Question	Scheme	Marks	AOs
7 (a)	Uses or implies that $V = ad + b$	B1	3.3
	Uses both $40 = 80a + b$ and $25 = 200a + b$ to get either a or b	M1	3.1b
	Uses both $40 = 80a + b$ and $25 = 200a + b$ to get both a and b	dM1	1.1b
	$\Rightarrow V = -\frac{1}{8}d + 50$ o.e.	A1	1.1b
		(4)	
(b)(i)(ii)	States either that the initial volume was 50 {litres} or that the distance travelled was 400 {km}	B1 ft	3.4
	Attempts to find both answers by solving $0 = -\frac{1}{8}d + 50$ and $0 = 400 - 8V$	M1	3.4
	States both that the initial volume was 50 litres and that the distance travelled was 400 km	A1	3.2b
		(3)	
(c)	States, e.g., “Poor model” as 320km is significantly less than 400 km.	B1 ft	3.5a
		(1)	
(8 marks)			

November 2020 Question 4

4. In 1997 the average CO₂ emissions of new cars in the UK was 190 g/km.

In 2005 the average CO₂ emissions of new cars in the UK had fallen to 169 g/km.

Given A g/km is the average CO₂ emissions of new cars in the UK n years after 1997 and using a linear model,

- (a) form an equation linking A with n .

(3)

In 2016 the average CO₂ emissions of new cars in the UK was 120 g/km.

- (b) Comment on the suitability of your model in light of this information.

(3)

ANSWER

Question	Scheme	Marks	AOs
4 (a)	Attempts $A = mn + c$ with either (0,190) or (8,169) Or attempts gradient eg $m = \pm \frac{190-169}{8} (= -2.625)$	M1	3.3
	Full method to find a linear equation linking A with n E.g. Solves $190 = 0n + c$ and $169 = 8n + c$ simultaneously	dM1	3.1b
	$A = -2.625n + 190$	A1	1.1b
		(3)	
(b)	Attempts $A = -2.625 \times 19 + 190 = \dots$	M1	3.4
	$A = 140.125 \text{ g km}^{-1}$	A1	1.1b
	It is predicting a much higher value and so is not suitable	B1ft	3.5a
		(3)	
(6 marks)			

June 2019 Question 1

1. The line l_1 has equation $2x + 4y - 3 = 0$

The line l_2 has equation $y = mx + 7$, where m is a constant.

Given that l_1 and l_2 are perpendicular,

- (a) find the value of m .

(2)

The lines l_1 and l_2 meet at the point P .

- (b) Find the x coordinate of P .

(2)

ANSWER

Question	Scheme	Marks	AOs
1(a)	$2x + 4y - 3 = 0 \Rightarrow y = \mp \frac{2}{4}x + \dots$ Gradient of perpendicular = $\pm \frac{4}{2}$	M1	1.1b
	Either $m = 2$ or $y = 2x + 7$	A1	1.1b
		(2)	
(b)	Combines 'their' $y = 2x + 7$ with $2x + 4y - 3 = 0 \Rightarrow 2x + 4(2x + 7) - 3 = 0 \Rightarrow x = \dots$	M1	1.1b
	$x = -2.5$ oe	A1	1.1b
		(2)	
(4 marks)			

June 2019 Question 4

4. A tree was planted in the ground.
Its height, H metres, was measured t years after planting.

Exactly 3 years after planting, the height of the tree was 2.35 metres.

Exactly 6 years after planting, the height of the tree was 3.28 metres.

Using a linear model,

- (a) find an equation linking H with t .

(3)

The height of the tree was approximately 140 cm when it was planted.

- (b) Explain whether or not this fact supports the use of the linear model in part (a).

(2)

ANSWER

Question	Scheme	Marks	AOs
4 (a)	Attempts $H = mt + c$ with both (3,2.35) and (6,3.28)	M1	3.3
	Method to find both m and c	dM1	3.1a
	$H = 0.31t + 1.42$ oe	A1	1.1b
		(3)	
(b)	Uses the model and states that the initial height is their 'b'	B1ft	3.4
	Compares 140 cm with their 1.42 (m) and makes a valid comment. In the case where $H = 0.31t + 1.42$ it should be this fact supports the use of the linear model as the values are close.	B1ft	3.5a
		(2)	
(5 marks)			

June 2019 Question 7

7. A small factory makes bars of soap.

On any day, the total cost to the factory, £ y , of making x bars of soap is modelled to be the sum of two separate elements:

- a fixed cost
- a cost that is proportional to the number of bars of soap that are made that day

(a) Write down a general equation linking y with x , for this model.

(1)

The bars of soap are sold for £2 each.

On a day when 800 bars of soap are made and sold, the factory makes a profit of £500

On a day when 300 bars of soap are made and sold, the factory makes a loss of £80

Using the above information,

(b) show that $y = 0.84x + 428$

(3)

(c) With reference to the model, interpret the significance of the value 0.84 in the equation.

(1)

Assuming that each bar of soap is sold on the day it is made,

(d) find the least number of bars of soap that must be made on any given day for the factory to make a profit that day.

(2)

ANSWER

Question	Scheme	Marks	AOs
7	$\pounds y$ is the total cost of making x bars of soap Bars of soap are sold for $\pounds 2$ each		
(a)	$y = kx + c$ {where k and c are constants}	B1	3.3
	Note: Work for (a) cannot be recovered in (b) or (c)	(1)	
(b) Way 1	Either $x = 800 \Rightarrow y = 2(800) - 500 \Rightarrow \{1100 \Rightarrow (x, y) = (800, 1100)\}$ $x = 300 \Rightarrow y = 2(300) + 80 \Rightarrow \{680 \Rightarrow (x, y) = (300, 680)\}$	M1	3.1b
	Applies (800, their 1100) and (300, their 680) to give two equations $1100 = 800k + c$ and $680 = 300k + c \Rightarrow k, c = \dots$	dM1	1.1b
	Solves correctly to find $k = 0.84, c = 428$ and states $y = 0.84x + 428$ *	A1*	2.1
	Note: the answer $y = 0.84x + 428$ must be stated in (b)	(3)	
(b) Way 2	Either $x = 800 \Rightarrow y = 2(800) - 500 \Rightarrow \{1100 \Rightarrow (x, y) = (800, 1100)\}$ $x = 300 \Rightarrow y = 2(300) + 80 \Rightarrow \{680 \Rightarrow (x, y) = (300, 680)\}$	M1	3.1b
	Complete method for finding both $k = \dots$ and $c = \dots$ e.g. $k = \frac{1100 - 680}{800 - 300} \Rightarrow \{0.84\}$ $(800, 1100) \Rightarrow 1100 = 800(0.84) + c \Rightarrow c = \dots$	dM1	1.1b
	Solves to find $k = 0.84, c = 428$ and states $y = 0.84x + 428$ *	A1*	2.1
	Note: the answer $y = 0.84x + 428$ must be stated in (b)	(3)	
(c)	Allow any of {0.84, in $\pounds$ s} represents - the cost of {making} each extra bar {of soap} - the direct cost of {making} a bar {of soap} - the marginal cost of {making} a bar {of soap} - the cost of {making} a bar {of soap} (Condone this answer) Note: Do not allow {0.84, in $\pounds$s} is the profit per bar {of soap} {0.84, in $\pounds$s} is the (selling) price per bar {of soap}	B1	3.4
		(1)	
(d) Way 1	{Let n be the least number of bars required to make a profit}		
	$2n = 0.84n + 428 \Rightarrow n = \dots$ (Condone $2x = 0.84x + 428 \Rightarrow x = \dots$)	M1	3.4
	Answer of 369 {bars}	A1	3.2a
		(2)	
(d) Way 2	- Trial 1: $n = 368 \Rightarrow y = (0.84)(368) + 428 \Rightarrow y = 737.12$ {revenue = $2(368) = 736$ or loss = 1.12} - Trial 2: $n = 369 \Rightarrow y = (0.84)(369) + 428 \Rightarrow y = 737.96$ {revenue = $2(369) = 738$ or profit = 0.04}	M1	3.4
	leading to an answer of 369 {bars}	A1	3.2a
		(2)	
(7 marks)			

June 2018 Question 4

4. The line l_1 has equation $4y - 3x = 10$

The line l_2 passes through the points $(5, -1)$ and $(-1, 8)$.

Determine, giving full reasons for your answer, whether lines l_1 and l_2 are parallel, perpendicular or neither.

(4)

ANSWER

Question	Scheme	Marks	AOs
4	States gradient of $4y - 3x = 10$ is $\frac{3}{4}$ oe or rewrites as $y = \frac{3}{4}x + \dots$	B1	1.1b
	Attempts to find gradient of line joining $(5, -1)$ and $(-1, 8)$	M1	1.1b
	$= \frac{-1 - 8}{5 - (-1)} = -\frac{3}{2}$	A1	1.1b
	States neither with suitable reasons	A1	2.4
		(4)	
(4 marks)			

June 2018 Question 8 Core 1

8.

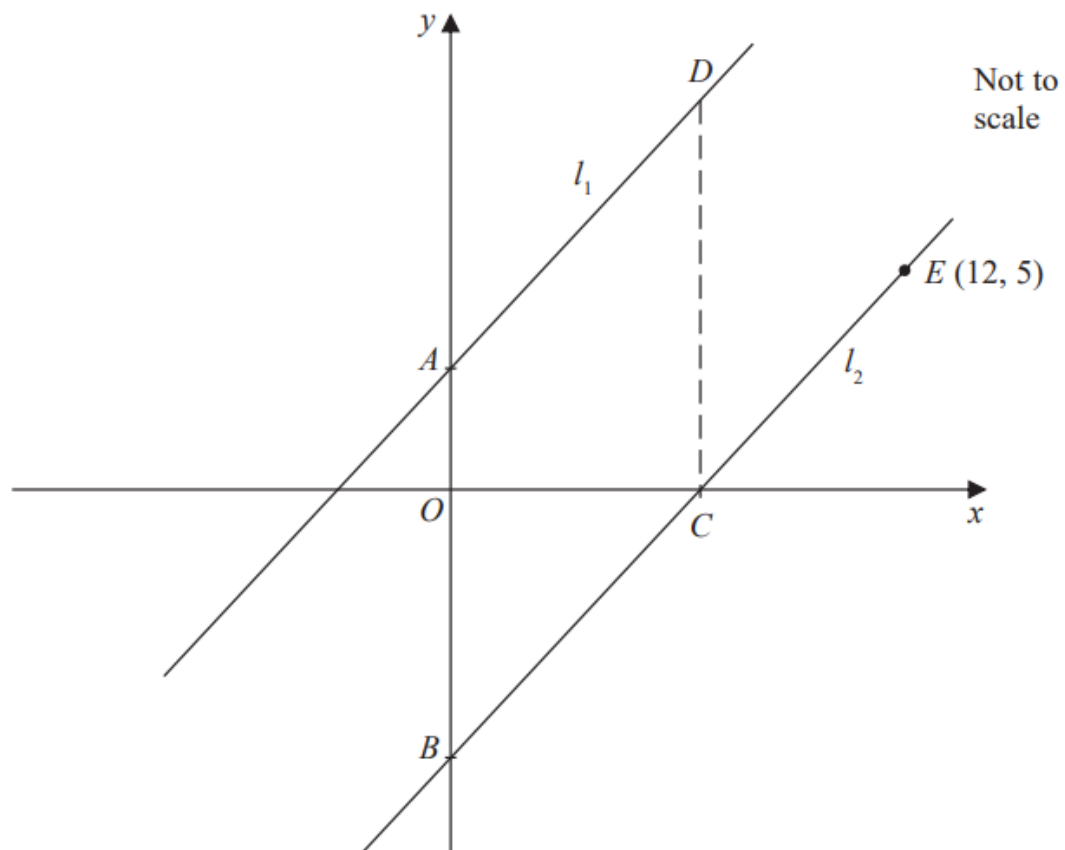


Figure 2

Figure 2 shows the straight line l_1 with equation $4y = 5x + 12$

(a) State the gradient of l_1

(1)

The line l_2 is parallel to l_1 and passes through the point $E (12, 5)$, as shown in Figure 2.

- (b) Find the equation of l_2 . Write your answer in the form $y = mx + c$, where m and c are constants to be determined.

(3)

The line l_2 cuts the x -axis at the point C and the y -axis at the point B .

- (c) Find the coordinates of

(i) the point B ,

(ii) the point C .

(2)

The line l_1 cuts the y -axis at the point A .

The point D lies on l_1 such that $ABCD$ is a parallelogram, as shown in Figure 2.

- (d) Find the area of $ABCD$.

(2)

ANSWER

Question Number	Scheme		Marks
8(a)	$\frac{5}{4}$ oe	$\frac{5}{4}$ or exact equivalents such as 1.25 but not $\frac{5}{4}x$.	B1
			(1)
(b)	$y = \frac{5}{4}x + c$	Uses a line with a parallel gradient $\frac{5}{4}$ oe or their gradient from part (a). Evidence is $y = \frac{5}{4}x + c$ or similar.	M1
	$12,5 \Rightarrow 5 = \frac{5}{4} \times 12 + c \Rightarrow c = ..$	Method of finding an equation of a line with numerical gradient and passing through 12,5 . Score even for the perpendicular line. Must be seen in part (a).	M1
	$y = \frac{5}{4}x - 10$	Correct equation. Allow $-\frac{40}{4}$ for -10	A1
			(3)
(c)	$B = 0, -10$	$B = 0, -10$ Follow through on their 'c'. Allow also if -10 is marked in the correct place on the diagram. Allow $x = 0, y = -10$ (the $x = 0$ may be seen "embedded" but not just $y = -10$ with no evidence that $x = 0$)	B1ft
	$C = 8, 0$	$C = 8, 0$ Correct coordinates. Allow also if 8 is marked in the correct place on the diagram. Allow $y = 0, x = 8$ (the $y = 0$ may be seen "embedded" but not just $x = 8$ with no evidence that $y = 0$)	B1
	Do not penalise lack of "0" twice so penalise it at the first occurrence but check the diagram if necessary.		
			(2)

(d) Way 1	Area of Parallelogram = $3 + '10' \times '8'$	Uses area of parallelogram = $bh = 3 + '10' \times '8'$ Follow through on their 10 and their 8	M1
	= 104	cao	A1
	Correct answer only scores both marks		(2)
(d) Way 2	Trapezium $AOCD$ + Triangle OCB $= \frac{1}{2} 3 + 3 + '10' \times '8' + \frac{1}{2} \times '8' \times '10'$	A correct method using their values for $AOCD + OCB$.	M1
	= 104	cao	A1
			(2)
(d) Way 3	2 Triangles + Rectangle $= 2 \times \frac{1}{2} '8' \times '10' + '8' \times 3$	A correct method using their values for $2 \times OBC$ + rectangle.	M1
	= 104	cao	A1
			(2)
(d) Way 4	Triangle ACD + Triangle ACB $= 2 \times \frac{1}{2} '10' + 3 \times '8'$	A correct method using their values for $ACD + ABC$.	M1
	= 104	cao	A1
			(2)
			(8 marks)

Ch. 6: Circles

June 2024 Question 14 Paper 2

14. The circle C_1 has equation

$$x^2 + y^2 - 6x + 14y + 33 = 0$$

(a) Find

- (i) the coordinates of the centre of C_1
- (ii) the radius of C_1

(3)

A different circle C_2

- has centre with coordinates $(-6, -8)$
- has radius k , where k is a constant

Given that C_1 and C_2 intersect at 2 distinct points,

(b) find the range of values of k , writing your answer in set notation.

(5)

ANSWER

Question	Scheme	Marks	AOs
14(a)(i) (ii)	Centre is $(3, -7)$	B1	1.1b
	$(x-3)^2 + (y+7)^2 = 49 + 9 - 33 \Rightarrow r^2 = \dots(25)$	M1	1.1b
	$r = 5$	A1	1.1b
		(3)	
(b)	Distance between centres = $\sqrt{(3+6)^2 + (-7+8)^2} = \sqrt{82}$	M1 A1ft	3.1a 1.1b
	" $\sqrt{82}$ " - "5" or " $\sqrt{82}$ " + "5"	dM1	3.1a
	$\sqrt{82} - 5$ and $\sqrt{82} + 5$	A1	2.2a
	$\{k : \sqrt{82} - 5 < k\} \cap \{k : k < \sqrt{82} + 5\}$ or e.g. $\{k : \sqrt{82} - 5 < k < \sqrt{82} + 5\}$	A1	2.5
		(5)	
(8 marks)			

June 2024 Question 10

10.

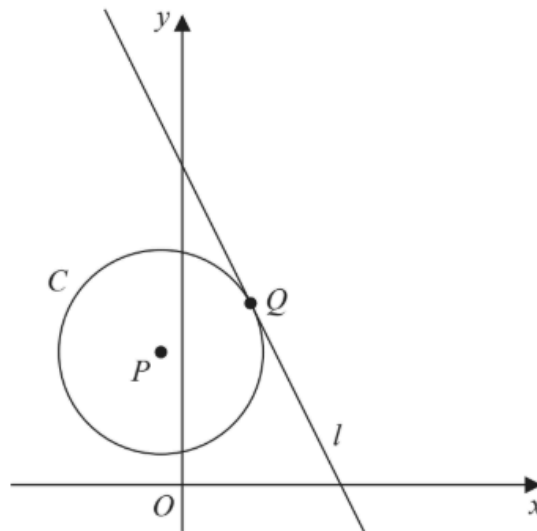


Figure 4

Figure 4 shows a sketch of the circle C

- the point $P(-1, k + 8)$ is the centre of C
- the point $Q(3, k^2 - 2k)$ lies on C
- k is a positive constant
- the line l is the tangent to C at Q

Given that the gradient of l is -2

(a) show that

$$k^2 - 3k - 10 = 0$$

(4)

(b) Hence find an equation for C

(4)

ANSWER

Question	Scheme	Marks	AOs
10(a)	tangent = $-2 \rightarrow$ normal = $\frac{1}{2}$	B1	2.2a
	$\frac{k^2 - 2k - k - 8}{3 - -1}$	M1	1.1b
	$\frac{1}{2} = \frac{k^2 - 2k - k - 8}{3 - -1} \Rightarrow \frac{1}{2} = \frac{k^2}{4} - \frac{2k}{4} - \frac{k}{4} - 2$	dM1	1.1b
	$\Rightarrow k^2 - 3k - 10 = 0$ *	A1*	2.1
	(4)		
(b)	$k = 5$	B1	2.3
	y coordinate of P is 13	B1ft	1.1b
	Attempts $PQ \left(= \sqrt{(3 - -1)^2 + ("25 - 15 - 8")^2} \right)$ (or PQ^2)	M1	3.1a
	$(x + 1)^2 + (y - 13)^2 = 20$	A1	1.1b
	(4)		
(8 marks)			

June 2023 Question 6

6. The circle C has equation

$$x^2 + y^2 - 6x + 10y + k = 0$$

where k is a constant.

(a) Find the coordinates of the centre of C .

(2)

Given that C does not cut or touch the x -axis,

(b) find the range of possible values for k .

(3)

ANSWER

Question	Scheme	Marks	AOs
6 (a)	$x^2 + y^2 - 6x + 10y + k = 0$		
	$(x-3)^2 + (y+5)^2 \pm \dots = \dots$	M1	1.1b
	Centre $(3, -5)$	A1	1.1b
		(2)	
(b)	Deduces that $k = 9$ is a critical point	B1ft	2.2a
	Recognises that radius > 0 " 9 " + " 25 " - $k > 0$	M1	3.1a
	$9 < k < 34$	A1	1.1b
		(3)	
(5 marks)			

June 2023 Question 10 Paper 1

10. A circle C has equation

$$x^2 + y^2 + 6kx - 2ky + 7 = 0$$

where k is a constant.

(a) Find in terms of k ,

(i) the coordinates of the centre of C

(ii) the radius of C

(3)

The line with equation $y = 2x - 1$ intersects C at 2 distinct points.

(b) Find the range of possible values of k .

(6)

ANSWER

Question	Scheme	Marks	AOs
10(a)(i) (ii)	Centre $(-3k, k)$	B1	2.2a
	$(x+3k)^2 - 9k^2 + (y-k)^2 - k^2 + 7 = 0 \Rightarrow (x+3k)^2 + (y-k)^2 = \dots$	M1	1.1b
	Radius $\sqrt{10k^2 - 7}$	A1ft	2.2a
		(3)	
(b)	$x^2 + (2x-1)^2 + 6kx - 2k(2x-1) + 7 = 0 \Rightarrow \dots x^2 + (pk+q)x + rk + s (=0)$	M1	1.1a
	$5x^2 + (2k-4)x + 2k+8 (=0)$	A1	1.1b
	$(2k-4)^2 - 4 \times 5 \times (2k+8) = 0 \Rightarrow k = \dots$	dM1	2.1
	Critical values $= 7 \pm \sqrt{85}$	A1	1.1b
	$k < "7 - \sqrt{85}"$ or $k > "7 + \sqrt{85}"$ o.e.	ddM1	3.1a
	$k < 7 - \sqrt{85}$ or $k > 7 + \sqrt{85}$ o.e.	A1	2.5
		(6)	
(9 marks)			

June 2022 Question 11

11.

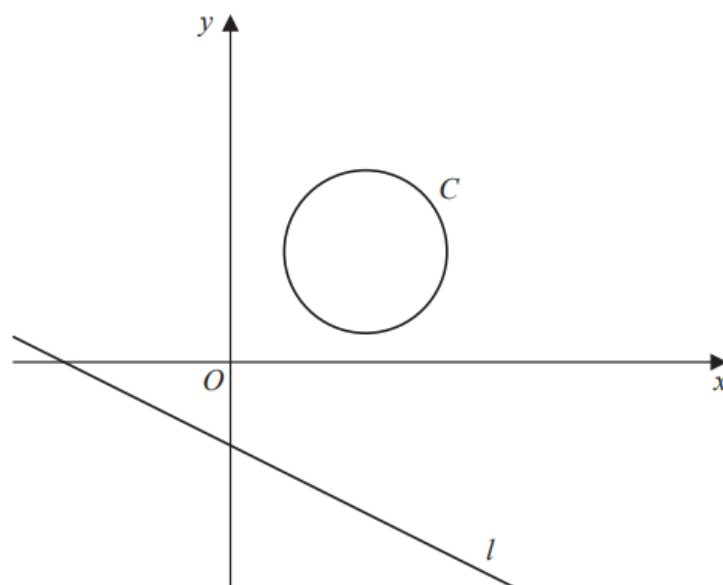


Figure 3

Figure 3 shows the circle C with equation

$$x^2 + y^2 - 10x - 8y + 32 = 0$$

and the line l with equation

$$2y + x + 6 = 0$$

(a) Find

- (i) the coordinates of the centre of C ,
- (ii) the radius of C .

(3)

(b) Find the shortest distance between C and l .

(5)

ANSWER

Question	Scheme	Marks	AOs
11(a)	$(x \pm 5)^2 + (y \pm 4)^2$	M1	1.1b
	(i) Centre is (5, 4)	A1	1.1b
	(ii) Radius is 3	A1	1.1b
		(3)	
(b)	$2y + x + 6 = 0 \Rightarrow y = -\frac{1}{2}x + \dots \Rightarrow -\frac{1}{2} \rightarrow 2$	B1	2.2a
	$m_N = 2 \Rightarrow y - 4 = 2(x - 5)$ $y - 4 = 2(x - 5), 2y + x + 6 = 0 \Rightarrow x = \dots, y = \dots$	M1	3.1a
	Intersection is at $\left(\frac{6}{5}, -\frac{18}{5}\right)$ oe	A1	1.1b
	Distance from centre to intersection is $\sqrt{\left(5 - \frac{6}{5}\right)^2 + \left(4 + \frac{18}{5}\right)^2}$ So distance required is $\sqrt{\left(5 - \frac{6}{5}\right)^2 + \left(4 + \frac{18}{5}\right)^2} - 3$	dM1	3.1a
	$= \frac{19\sqrt{5}}{5} - 3$ (or awrt 5.50)	A1	1.1b
		(5)	
(8 marks)			

Video solution:

<https://youtu.be/a5vqlR4UYmA>

June 2022 Question 3 Paper 1 (A Level)

3. A circle has equation

$$x^2 + y^2 - 10x + 16y = 80$$

(a) Find

- (i) the coordinates of the centre of the circle,
- (ii) the radius of the circle.

(3)

Given that P is the point on the circle that is furthest away from the origin O ,

(b) find the exact length OP

(2)

ANSWER

Question	Scheme	Marks	AOs
3 (a)	(i) $x^2 + y^2 - 10x + 16y = 80 \Rightarrow (x-5)^2 + (y+8)^2 = \dots$	M1	1.1b
	Centre $(5, -8)$	A1	1.1b
	(ii) Radius 13	A1	1.1b
		(3)	
(b)	Attempts $\sqrt{5^2 + 8^2} + 13$	M1	3.1a
	$13 + \sqrt{89}$ but ft on their centre and radius	A1ft	1.1b
		(2)	
(5 marks)			

November 2021 Question 7 Paper 1 (A-Level)

7. The circle C has equation

$$x^2 + y^2 - 10x + 4y + 11 = 0$$

(a) Find

- (i) the coordinates of the centre of C ,
- (ii) the exact radius of C , giving your answer as a simplified surd.

(4)

The line l has equation $y = 3x + k$ where k is a constant.

Given that l is a tangent to C ,

(b) find the possible values of k , giving your answers as simplified surds.

(5)

ANSWER

Question	Scheme	Marks	AOs
7(a)(i)	$(x-5)^2 + (y+2)^2 = \dots$	M1	1.1b
	$(5, -2)$	A1	1.1b
(ii)	$r = \sqrt{5^2 + 2^2 - 11}$	M1	1.1b
	$r = 3\sqrt{2}$	A1	1.1b
		(4)	
(b)	$y = 3x + k \Rightarrow x^2 + (3x + k)^2 - 10x + 4(3x + k) + 11 = 0$ $\Rightarrow x^2 + 9x^2 + 6kx + k^2 - 10x + 12x + 4k + 11 = 0$ $\Rightarrow 10x^2 + (6k + 2)x + k^2 + 4k + 11 = 0$	M1	2.1
	$b^2 - 4ac = 0 \Rightarrow (6k + 2)^2 - 4 \times 10 \times (k^2 + 4k + 11) = 0$	A1	1.1b
	$\Rightarrow 4k^2 + 136k + 436 = 0 \Rightarrow k = \dots$	M1	3.1a
	$k = -17 \pm 6\sqrt{5}$	M1	1.1b
		A1	2.2a
		(5)	
(9 marks)			

November 2021 Question 15

15.

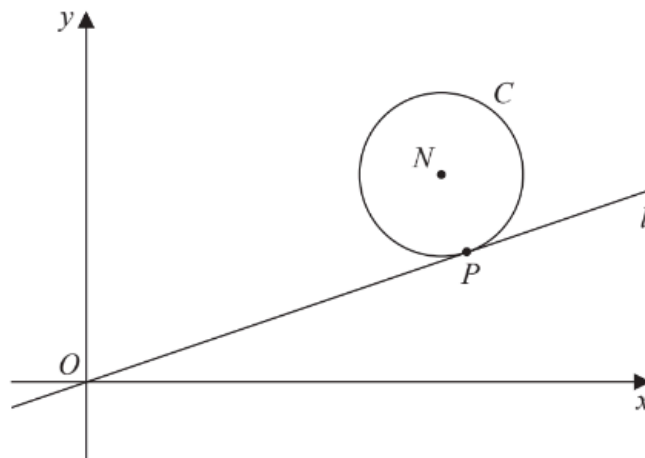


Figure 4

Figure 4 shows a sketch of a circle C with centre $N(7, 4)$

The line l with equation $y = \frac{1}{3}x$ is a tangent to C at the point P .

Find

(a) the equation of line PN in the form $y = mx + c$, where m and c are constants, (2)

(b) an equation for C . (4)

The line with equation $y = \frac{1}{3}x + k$, where k is a non-zero constant, is also a tangent to C .

(c) Find the value of k . (3)

ANSWER

Question	Scheme	Marks	AOs
15 (a)	Deduces the line has gradient "-3" and point (7,4) Eg $y - 4 = -3(x - 7)$	M1	2.2a
	$y = -3x + 25$	A1	1.1b
		(2)	
(b)	Solves $y = -3x + 25$ and $y = \frac{1}{3}x$ simultaneously	M1	3.1a
	$P = \left(\frac{15}{2}, \frac{5}{2}\right)$ oe	A1	1.1b
	Length $PN = \sqrt{\left(\frac{15}{2} - 7\right)^2 + \left(4 - \frac{5}{2}\right)^2} = \left(\sqrt{\frac{5}{2}}\right)$	M1	1.1b
	Equation of C is $(x - 7)^2 + (y - 4)^2 = \frac{5}{2}$ o.e.	A1	1.1b
		(4)	
(c)	Attempts to find where $y = \frac{1}{3}x + k$ meets C using vectors Eg: $\begin{pmatrix} 7.5 \\ 2.5 \end{pmatrix} + 2 \times \begin{pmatrix} -0.5 \\ 1.5 \end{pmatrix}$	M1	3.1a
	Substitutes their $\left(\frac{13}{2}, \frac{11}{2}\right)$ in $y = \frac{1}{3}x + k$ to find k	M1	2.1
	$k = \frac{10}{3}$	A1	1.1b
		(3)	
(9 marks)			

November 2020 Question 14 Paper 2 (A-Level)

14. A circle C with radius r

- lies only in the 1st quadrant
- touches the x -axis and touches the y -axis

The line l has equation $2x + y = 12$

(a) Show that the x coordinates of the points of intersection of l with C satisfy

$$5x^2 + (2r - 48)x + (r^2 - 24r + 144) = 0 \quad (3)$$

Given also that l is a tangent to C ,

(b) find the two possible values of r , giving your answers as fully simplified surds. (4)

ANSWER

Question	Scheme	Marks	AOs
14 (a)	C is $(x-r)^2 + (y-r)^2 = r^2$ or $x^2 + y^2 - 2rx - 2ry + r^2 = 0$	B1	2.2a
	$y = 12 - 2x$, $x^2 + y^2 - 2rx - 2ry + r^2 = 0$ $\Rightarrow x^2 + (12 - 2x)^2 - 2rx - 2r(12 - 2x) + r^2 = 0$ or	M1	1.1b
	$y = 12 - 2x$, $(x-r)^2 + (y-r)^2 = r^2$ $\Rightarrow (x-r)^2 + (12 - 2x - r)^2 = r^2$		
	$x^2 + 144 - 48x + 4x^2 - 2rx - 24r + 4rx + r^2 = 0$ $\Rightarrow 5x^2 + (2r - 48)x + (r^2 - 24r + 144) = 0$ *	A1*	2.1
		(3)	
(b)	$b^2 - 4ac = 0 \Rightarrow (2r - 48)^2 - 4 \times 5 \times (r^2 - 24r + 144) = 0$	M1	3.1a
	$r^2 - 18r + 36 = 0$ or any multiple of this equation	A1	1.1b
	$\Rightarrow (r - 9)^2 - 81 + 36 = 0 \Rightarrow r = \dots$	dM1	1.1b
	$r = 9 \pm 3\sqrt{5}$	A1	1.1b
		(4)	
(7 marks)			

Video solution:

<https://youtu.be/DZBSLH6mEgw>

June 2018 Question 14

14. The circle C has equation

$$x^2 + y^2 - 6x + 10y + 9 = 0$$

(a) Find

- (i) the coordinates of the centre of C
- (ii) the radius of C

(3)

The line with equation $y = kx$, where k is a constant, cuts C at two distinct points.

(b) Find the range of values for k .

(6)

ANSWER

Question	Scheme	Marks	AOs
14 (a)	Attempts to complete the square $(x \pm 3)^2 + (y \pm 5)^2 = \dots$	M1	1.1b
(b)	(i) Centre $(3, -5)$	A1	1.1b
	(ii) Radius 5	A1	1.1b
		(3)	
	Uses a sketch or otherwise to deduce $k = 0$ is a critical value	B1	2.2a
	Substitute $y = kx$ in $x^2 + y^2 - 6x + 10y + 9 = 0$	M1	3.1a
	Collects terms to form correct 3TQ $(1 + k^2)x^2 + (10k - 6)x + 9 = 0$	A1	1.1b
	Attempts $b^2 - 4ac \dots 0$ for their a, b and c leading to values for k $"(10k - 6)^2 - 36(1 + k^2) \dots 0" \rightarrow k = \dots, \dots$ $\left(0 \text{ and } \frac{15}{8}\right)$	M1	1.1b
	Uses $b^2 - 4ac > 0$ and chooses the outside region (see note) for their critical values (Both a and b must have been expressions in k)	dM1	3.1a
	Deduces $k < 0, k > \frac{15}{8}$ oe	A1	2.2a
		(6)	
(9 marks)			

Video solution:

<https://youtu.be/mdhsyR5xWdo>

June 2018 Question 6 Paper 1 (A-Level)

6.

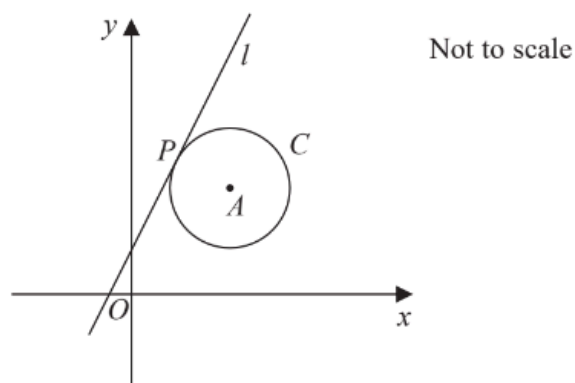


Figure 3

The circle C has centre A with coordinates $(7, 5)$.

The line l , with equation $y = 2x + 1$, is the tangent to C at the point P , as shown in Figure 3.

(a) Show that an equation of the line PA is $2y + x = 17$ (3)

(b) Find an equation for C . (4)

The line with equation $y = 2x + k$, $k \neq 1$ is also a tangent to C .

(c) Find the value of the constant k . (3)

ANSWER

Question	Scheme	Marks	AOs
6 (a)	Deduces that gradient of PA is $-\frac{1}{2}$	M1	2.2a
	Finding the equation of a line with gradient " $-\frac{1}{2}$ " and point $(7, 5)$ $y - 5 = -\frac{1}{2}(x - 7)$	M1	1.1b
	Completes proof $2y + x = 17$ *	A1*	1.1b
		(3)	
(b)	Solves $2y + x = 17$ and $y = 2x + 1$ simultaneously	M1	2.1
	$P = (3, 7)$	A1	1.1b
	Length $PA = \sqrt{(3-7)^2 + (7-5)^2} = (\sqrt{20})$	M1	1.1b
	Equation of C is $(x-7)^2 + (y-5)^2 = 20$	A1	1.1b
		(4)	
(c)	Attempts to find where $y = 2x + k$ meets C using $\overrightarrow{OA} + \overrightarrow{PA}$	M1	3.1a
	Substitutes their $(11, 3)$ in $y = 2x + k$ to find k	M1	2.1
	$k = -19$	A1	1.1b
		(3)	
(10 marks)			

June 2018 Question 5 Core 2

5. The circle C has equation

$$x^2 + y^2 - 2x + 14y = 0$$

Find

- (a) the coordinates of the centre of C , (2)
- (b) the exact value of the radius of C , (2)
- (c) the y coordinates of the points where the circle C crosses the y -axis. (2)
- (d) Find an equation of the tangent to C at the point $(2, 0)$, giving your answer in the form $ax + by + c = 0$, where a , b and c are integers. (4)

ANSWER

Question number	Scheme	Marks
5	You may mark (a) and (b) together $x^2 + y^2 - 2x + 14y = 0$	
(a)	Obtain LHS as $\underline{(x \pm 1)^2} + \underline{(y \pm 7)^2} = \dots$ Centre is $(1, -7)$.	M1 A1 (2)
(b)	Uses $r^2 = a^2 + b^2$ or $r = \sqrt{a^2 + b^2}$ where their centre was at $(\pm a, \pm b)$ $r = \sqrt{50}$ or $5\sqrt{2}$	M1 A1 (2)
(c)	Substitute $x = 0$ in either form of equation of circle and solve resulting quadratic to give $y =$ $y^2 + 14y = 0$ so $y = 0$ and -14 or $\underline{(y \pm 7)^2 - 49} = 0$ so $y = 0$ and -14	M1 A1 (2)
(d)	Gradient of radius joining centre to $(2, 0)$ is $\frac{-7-0}{1-2} (= 7)$ Gradient of tangent is $\frac{-1}{m} (= -\frac{1}{7})$ So equation is $y - 0 = -\frac{1}{7}(x - 2)$ and so $x + 7y - 2 = 0$	M1 M1 M1, A1 (4)
		(10 marks)

Ch. 7: Algebraic Methods

June 2024 Question 1 (A-level)

1. $g(x) = 3x^3 - 20x^2 + (k + 17)x + k$

where k is a constant.

Given that $(x - 3)$ is a factor of $g(x)$, find the value of k .

(3)

ANSWER

Question	Scheme	Marks	AOs
1	$g(3) = 3(3)^3 - 20(3)^2 + 3(k + 17) + k = 0$	M1	3.1a
	$4k - 48 = 0 \Rightarrow k = \dots$	M1	1.1b
	$\{k = \} 12$	A1	1.1b
		(3)	
(3 marks)			

June 2023 Question 14

14. Prove, using algebra, that

$$n^2 + 5n$$

is even for all $n \in \mathbb{N}$

(4)

ANSWER

Question	Scheme	Marks	AOs
14	Sets up the proof by exploring when $n = 2k$ or $n = 2k + 1$ e.g. $(2k)^2 + 5(2k) = \dots k^2 + \dots k$ or $(2k + 1)^2 + 5(2k + 1) = \dots k^2 + \dots k + \dots$	M1	1.1b
	e.g. $4k^2 + 10k$ or $4k^2 + 14k + 6$ and shows or gives a reason why the expression is even (see notes)	A1	2.2a
	Explores when $n = 2k$ and $n = 2k + 1$ eg $(2k)^2 + 5(2k) = \dots k^2 + \dots k$ and $(2k + 1)^2 + 5(2k + 1) = \dots k^2 + \dots k + \dots$	dM1	2.1
	e.g. $4k^2 + 10k$ and $4k^2 + 14k + 6$ and shows or gives a reason why both of the expressions are even (see notes) hence $n^2 + 5n$ is even for all $n(\in \square)$ (or equivalent)	A1*	2.4
(4 marks)			

June 2024 Question 2

2. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = 2x^3 - 3ax^2 + bx + 8a$$

where a and b are constants.

Given that $(x - 4)$ is a factor of $f(x)$,

- (a) use the factor theorem to show that

$$10a = 32 + b \quad (2)$$

Given also that $(x - 2)$ is a factor of $f(x)$,

- (b) express $f(x)$ in the form

$$f(x) = (2x + k)(x - 4)(x - 2)$$

where k is a constant to be found. (4)

- (c) Hence,

- (i) state the number of real roots of the equation $f(x) = 0$

- (ii) write down the largest root of the equation $f\left(\frac{1}{3}x\right) = 0$ (2)

ANSWER

Question	Scheme	Marks	AOs
2(a)	$(f(4) =) 2 \times 4^3 - 3a \times 4^2 + 4b + 8a = 0$	M1	1.1b
	$128 + 4b = 40a \Rightarrow 32 + b = 10a^*$	A1*	1.1b
		(2)	
(b)	$f(2) = 2 \times 2^3 - 3a \times 2^2 + 2b + 8a = 0 \Rightarrow 8 + b = 2a$	M1	1.1b
	Solve simultaneously $\Rightarrow a = \dots$ or $\Rightarrow b = \dots$	dM1	2.1
	$a = 3$ or $b = -2$ or $k = 3$	A1	1.1b
	$(f(x) =) (2x+3)(x-4)(x-2)$	A1	1.1b
		(4)	
(c)(i) (ii)	3	B1	1.1b
	12	B1ft	2.2a
		(2)	
(8 marks)			

June 2023 Question 17

17. In this question p and q are positive integers with $q > p$

Statement 1: $q^3 - p^3$ is never a multiple of 5

(a) Show, by means of a counter example, that Statement 1 is **not** true.

(1)

Statement 2: When p and q are consecutive **even** integers $q^3 - p^3$ is a multiple of 8

(b) Prove, using algebra, that Statement 2 is true.

(4)

ANSWER

Question	Scheme	Marks	AOs
17 (a)	Provides a counter example with a reason. e.g., $6^3 - 1^3 = 215$ which is a multiple of 5	B1	2.4
		(1)	
(b)	States or uses, e.g., $2n$ and $2n+2$ or $2n+2$ and $2n+4$	M1	2.1
	Attempts $(2n+2)^3 - (2n)^3 = 8n^3 + 24n^2 + 24n + 8 - 8n^3$ leading to a quadratic.	dM1	1.1b
	$= 24n^2 + 24n + 8$	A1	1.1b
	$24n^2 + 24n + 8 = 8(3n^2 + 3n + 1)$ So $q^3 - p^3$ is a multiple of 8	A1	2.1
		(4)	
(5 marks)			

June 2023 Question 7 Paper 1 (A-Level)

14. Prove, using algebra, that

$$(n+1)^3 - n^3$$

is odd for all $n \in \mathbb{N}$

(4)

ANSWER

Question	Scheme	Marks	AOs
14	When n is even: $(2k+1)^3 - (2k)^3 = 8k^3 + 12k^2 + 6k + 1 - 8k^3 = 6k(2k+1) + 1$ $\Rightarrow \text{which is odd}$	M1	3.1a
	or		
	When n is odd: $(2k+2)^3 - (2k+1)^3 = 8(k^3 + 3k^2 + 3k + 1) - (8k^3 + 12k^2 + 6k + 1) = 6k(2k+3) + 7$ $\Rightarrow \text{which is odd}$	A1	2.2a
	When n is even: $(2k+1)^3 - (2k)^3 = 8k^3 + 12k^2 + 6k + 1 - 8k^3 = 6k(2k+1) + 1$ $\Rightarrow \text{which is odd}$ and When n is odd: $(2k+2)^3 - (2k+1)^3 = 8(k^3 + 3k^2 + 3k + 1) - (8k^3 + 12k^2 + 6k + 1) = 6k(2k+3) + 7$ $\Rightarrow \text{which is odd}$ Hence odd for all $n (\in \mathbb{N})$ *	dM1 A1*	2.1 2.4
(4 marks)			

June 2023 Question 2 Paper 1 (A-Level)

2. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = 4x^3 + 5x^2 - 10x + 4a \quad x \in \mathbb{R}$$

where a is a positive constant.

Given $(x - a)$ is a factor of $f(x)$,

(a) show that

$$a(4a^2 + 5a - 6) = 0 \quad (2)$$

(b) Hence

(i) find the value of a

(ii) use algebra to find the exact solutions of the equation

$$f(x) = 3 \quad (4)$$

ANSWER

Question	Scheme	Marks	AOs
2(a)	$(f(a) =) 4a^3 + 5a^2 - 10a + 4a = 0 \Rightarrow a(\dots) = 0$	M1	3.1a
	$a(4a^2 + 5a - 6) = 0$ *	A1*	1.1b
		(2)	
(b)(i)	$a = \frac{3}{4}$	B1	2.2a
(ii)	$4x^3 + 5x^2 - 10x + 4 \times \frac{3}{4} = 3 \Rightarrow 4x^3 + 5x^2 - 10x (= 0)$	M1	1.1b
	$x = 0$	B1	1.1b
	$x = \frac{-5 \pm \sqrt{185}}{8}$	A1	1.1b
		(4)	
(6 marks)			

June 2023 Question 17 Paper 1

17. In this question p and q are positive integers with $q > p$

Statement 1: $q^3 - p^3$ is never a multiple of 5

(a) Show, by means of a counter example, that Statement 1 is **not** true.

(1)

Statement 2: When p and q are consecutive **even** integers $q^3 - p^3$ is a multiple of 8

(b) Prove, using algebra, that Statement 2 is true.

(4)

ANSWER

Question	Scheme	Marks	AOs
17 (a)	Provides a counter example with a reason. e.g., $6^3 - 1^3 = 215$ which is a multiple of 5	B1	2.4
		(1)	
(b)	States or uses, e.g., $2n$ and $2n+2$ or $2n+2$ and $2n+4$	M1	2.1
	Attempts $(2n+2)^3 - (2n)^3 = 8n^3 + 24n^2 + 24n + 8 - 8n^3$ leading to a quadratic.	dM1	1.1b
	$= 24n^2 + 24n + 8$	A1	1.1b
	$24n^2 + 24n + 8 = 8(3n^2 + 3n + 1)$ So $q^3 - p^3$ is a multiple of 8	A1	2.1
		(4)	
(5 marks)			

June 2022 Question 11 Paper 2 (A Level)

11. Prove, using algebra, that

$$n(n^2 + 5)$$

is even for all $n \in \mathbb{N}$.

(4)

ANSWER

Question	Scheme	Marks	AOs
11	$n(n^2 + 5)$		
	Attempts even or odd numbers Sets $n = 2k$ or $n = 2k \pm 1$ oe and attempts $n(n^2 + 5)$	M1	3.1a
	Achieves $2k(4k^2 + 5)$ (for $n = 2k$) and states “even” Or achieves $(2k + 1)(4k^2 + 4k + 6) = 2(2k + 1)(2k^2 + 2k + 3)$ (for $n = 2k + 1$) and states “even” Or e.g. achieves $(2k - 1)(4k^2 - 4k + 6) = 2(2k - 1)(2k^2 - 2k + 3)$ (for $n = 2k - 1$) and states “even”	A1	2.2a
	Attempts even and odd numbers Sets $n = 2k$ and $n = 2k \pm 1$ oe and attempts $n(n^2 + 5)$	dM1	2.1
	Achieves $2k(4k^2 + 5)$ (for $n = 2k$) and states “even” and achieves $(2k \pm 1)(4k^2 \pm 4k + 6) = 2(2k \pm 1)(2k^2 \pm 2k + 3)$ (for $n = 2k \pm 1$) and states “even” Correct work and states even for both WITH a final conclusion showing that true for all $n(\in \mathbb{N})$ or e.g. true for all even and odd numbers.	A1	2.4
		(4)	
(4 marks)			

June 2022 Question 2 Paper 1 (A Level)

2. $f(x) = (x - 4)(x^2 - 3x + k) - 42$ where k is a constant

Given that $(x + 2)$ is a factor of $f(x)$, find the value of k .

(3)

ANSWER

Question	Scheme	Marks	AOs
2	Sets $f(-2) = 0 \Rightarrow (-2 - 4)((-2)^2 - 3 \times -2 + k) - 42 = 0$	M1	3.1a
	$-6(k + 10) = 42 \Rightarrow k = \dots$	M1	1.1b
	$k = -17$	A1	1.1b
		(3)	
(3 marks)			

June 2022 Question 14 Paper 1

14. (i) A student states

“if x^2 is greater than 9 then x must be greater than 3”

Determine whether or not this statement is true, giving a reason for your answer.

(1)

(ii) Prove that for all positive integers n ,

$$n^3 + 3n^2 + 2n$$

is divisible by 6

(3)

ANSWER

Question	Scheme	Marks	AOs
14(i)	The statement is not true because e.g. when $x = -4$, $x^2 = 16$ (which is > 9 but $x < 3$)	B1	2.3
		(1)	
(ii)	$n^3 + 3n^2 + 2n = n(n^2 + 3n + 2) = n(n+1)(n+2)$	M1	2.1
	$n(n+1)(n+2)$ is the product of 3 consecutive integers	A1	2.2a
	As $n(n+1)(n+2)$ is a multiple of 2 and a multiple of 3 it must be a multiple of 6 and so $n^3 + 3n^2 + 2n$ is divisible by 6 for all integers n	A1	2.4
		(3)	
(4 marks)			

Video solution:

<https://youtu.be/9GXHesA3AIM>

June 2022 Question 2

2. $f(x) = 2x^3 + 5x^2 + 2x + 15$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$.

(2)

(b) Find the constants a , b and c such that

$$f(x) = (x + 3)(ax^2 + bx + c)$$

(2)

(c) Hence show that $f(x) = 0$ has only one real root.

(2)

(d) Write down the real root of the equation $f(x - 5) = 0$

(1)

ANSWER

Question	Scheme	Marks	AOs
2(a)	$f(-3) = 2(-3)^3 + 5(-3)^2 + 2(-3) + 15$ $= -54 + 45 - 6 + 15$	M1	1.1b
	$f(-3) = 0 \Rightarrow (x + 3)$ is a factor	A1	2.4
		(2)	
(b)	At least 2 of: $a = 2, b = -1, c = 5$	M1	1.1b
	All of: $a = 2, b = -1, c = 5$	A1	1.1b
		(2)	
(c)	$b^2 - 4ac = (-1)^2 - 4(2)(5)$	M1	2.1
	$b^2 - 4ac = -39$ which is < 0 so the quadratic has no real roots so $f(x) = 0$ has only 1 real root	A1	2.4
		(2)	
(d)	$(x =) 2$	B1	2.2a
		(1)	
(7 marks)			

Video solution:

<https://youtu.be/uZaAEGT92R0>

Arancha Ruiz

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November 2021 Question 10 Paper 1

10. A student is investigating the following statement about natural numbers.

$$“n^3 - n \text{ is a multiple of } 4”$$

(a) Prove, using algebra, that the statement is true for all odd numbers.

(4)

(b) Use a counterexample to show that the statement is not always true.

(1)

ANSWER

Question	Scheme	Marks	AOs
10(a)	Selects a correct strategy. E.g uses an odd number is $2k \pm 1$	B1	3.1a
	Attempts to simplify $(2k \pm 1)^3 - (2k \pm 1) = \dots$	M1	2.1
	and factorise $8k^3 \pm 12k^2 \pm 4k = 4k(2k^2 \pm 3k \pm 1) =$	dM1	1.1b
	Correct work with statement $4 \times \dots$ is a multiple of 4	A1	2.4
		(4)	
(b)	Any counter example with correct statement. Eg. $2^3 - 2 = 6$ which is not a multiple of 4	B1	2.4
		(1)	
(5 marks)			

November 2020 Question 16 Paper 2 (A-Level)

16. Use algebra to prove that the square of any natural number is **either** a multiple of 3 **or** one more than a multiple of 3

(4)

ANSWER

Question	Scheme	Marks	AOs
16	NB any natural number can be expressed in the form: $3k, 3k + 1, 3k + 2$ or equivalent e.g. $3k - 1, 3k, 3k + 1$		
	Attempts to square any two distinct cases of the above	M1	3.1a
	Achieves accurate results and makes a valid comment for any two of the possible three cases: E.g. $(3k)^2 = 9k^2 (= 3 \times 3k^2)$ is a multiple of 3	A1 M1 on EPEN	1.1b
	$(3k + 1)^2 = 9k^2 + 6k + 1 = 3 \times (3k^2 + 2k) + 1$ is one more than a multiple of 3 $(3k + 2)^2 = 9k^2 + 12k + 4 = 3 \times (3k^2 + 4k + 1) + 1$ $(\text{or } (3k - 1)^2 = 9k^2 - 6k + 1 = 3 \times (3k^2 - 2k) + 1)$ is one more than a multiple of 3		
	Attempts to square in all 3 distinct cases. E.g. attempts to square $3k, 3k + 1, 3k + 2$ or e.g. $3k - 1, 3k, 3k + 1$	M1 A1 on EPEN	2.1
	Achieves accurate results for all three cases and gives a minimal conclusion (allow tick, QED etc.)	A1	2.4
		(4)	
(4 marks)			

June 2019 Question 1 Paper 1 (A-Level)

Answer ALL questions. Write your answers in the spaces provided.

1. $f(x) = 3x^3 + 2ax^2 - 4x + 5a$

Given that $(x + 3)$ is a factor of $f(x)$, find the value of the constant a .

(3)

ANSWER

Question	Scheme	Marks	AOs
1	Attempts $f(-3) = 3 \times (-3)^3 + 2a \times (-3)^2 - 4 \times -3 + 5a = 0$	M1	3.1a
	Solves linear equation $23a = 69 \Rightarrow a = \dots$	M1	1.1b
	$a = 3$ cso	A1	1.1b
		(3)	
(3 marks)			

June 2019 Question 10 Paper 1 (A-Level)

10. (i) Prove that for all $n \in \mathbb{N}$, $n^2 + 2$ is not divisible by 4

(4)

(ii) “Given $x \in \mathbb{R}$, the value of $|3x - 28|$ is greater than or equal to the value of $(x - 9)$.”
State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

ANSWER

Question 10 (i)	Scheme	Marks	AOs
For $n = 2m$, $n^2 + 2 = 4m^2 + 2$		M1	2.1
Concludes that this number is not divisible by 4 (as the explanation is trivial)		A1	1.1b
For $n = 2m + 1$, $n^2 + 2 = (2m + 1)^2 + 2 = \dots$ FYI $(4m^2 + 4m + 3)$		dM1	2.1
Correct working and concludes that this is a number in the 4 times table add 3 so cannot be divisible by 4 or writes $4(m^2 + m) + 3$AND stateshence true for all		A1*	2.4
		(4)	
SOMETIMES TRUE and chooses any number $x : 9.25 < x < 9.5$ and shows false Eg $x = 9.4$ $ 3x - 28 = 0.2$ and $x - 9 = 0.4$ ✗		M1	2.3
Then chooses a number where it is true Eg $x = 12$ $ 3x - 28 = 8$ $x - 9 = 3$ ✓		A1	2.4
		(2)	

June 2018 Question 2

2. (i) Show that $x^2 - 8x + 17 > 0$ for all real values of x

(3)

- (ii) “If I add 3 to a number and square the sum, the result is greater than the square of the original number.”

State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

ANSWER

Question	Scheme	Marks	AOs
2(i)	$x^2 - 8x + 17 = (x - 4)^2 - 16 + 17$	M1	3.1a
	$= (x - 4)^2 + 1$ with comment (see notes)	A1	1.1b
	As $(x - 4)^2 \geq 0 \Rightarrow (x - 4)^2 + 1 \geq 1$ hence $x^2 - 8x + 17 > 0$ for all x	A1	2.4
		(3)	
(ii)	For an explanation that it may not always be true Tests say $x = -5$ $(-5 + 3)^2 = 4$ whereas $(-5)^2 = 25$	M1	2.3
	States sometimes true and gives reasons Eg. when $x = 5$ $(5 + 3)^2 = 64$ whereas $(5)^2 = 25$ True When $x = -5$ $(-5 + 3)^2 = 4$ whereas $(-5)^2 = 25$ Not true	A1	2.4
		(2)	
		(5 marks)	

June 2018 Question 3 Core 2

3. $f(x) = 24x^3 + Ax^2 - 3x + B$

where A and B are constants.

When $f(x)$ is divided by $(2x - 1)$ the remainder is 30

(a) Show that $A + 4B = 114$

(2)

Given also that $(x + 1)$ is a factor of $f(x)$,

(b) find another equation in A and B .

(2)

(c) Find the value of A and the value of B .

(2)

(d) Hence find a quadratic factor of $f(x)$.

(2)

ANSWER

Question Number	Scheme	Marks
3. (a)	Way 1 Use $f(1/2)$ or $f(-1/2)$ and put equal to 30 Stated $\frac{24}{8} + \frac{1}{4}A - \frac{3}{2} + B = 30$ and $A + 4B = 114$ *	M1
	Way 2 Long division of $f(x)$ by $(2x - 1)$ as far as remainder put = 30 Obtains $B + \frac{1}{4}A + \frac{3}{2} = 30$ (o.e) and $A + 4B = 114$ *	A1* (2)
(b)	Way 1 Used $f(-1)$ or $f(1) = 0$ Stated $-24 + A + 3 + B = 0$ so $A + B = 21$	M1
	Way 2 Long division of $f(x)$ by $(x + 1)$ as far as remainder put = 0 Obtains $B - 21 + A = 0$	A1 (2)
(c)	Solves to obtain one of A or B Obtains both $A = -10$ and $B = 31$	M1 A1 (2)
(d)	$f(x) = (x + 1)(24x^2 - 34x + 31)$ or factor is $(24x^2 - 34x + 31)$	M1A1 (2)
		(8 marks)

Ch. 8: The Binomial Expansion

June 2024 Question 6

6. The binomial expansion of

$$(1 + ax)^{12}$$

up to and including the term in x^2 is

$$1 - \frac{15}{2}x + kx^2$$

where a and k are constants.

(a) Show that $a = -\frac{5}{8}$

(2)

(b) Hence find the value of k

(2)

Using the expansion and making your method clear,

(c) find an estimate for the value of $\left(\frac{17}{16}\right)^{12}$, giving your answer to 4 decimal places.

(2)

ANSWER

Question	Scheme	Marks	AOs
6(a)	e.g. ${}^{12}C_1 \times a = -\frac{15}{2}$	M1	1.1b
	$12a = -\frac{15}{2} \Rightarrow a = -\frac{5}{8} *$	A1*	1.1b
		(2)	
(b)	${}^{12}C_2 \times a^2 = k \Rightarrow k = \dots$	M1	1.1b
	$k = \frac{825}{32}$	A1	1.1b
		(2)	
(c)	$1 - \frac{5}{8}x = \frac{17}{16} \Rightarrow x (= -0.1) \Rightarrow 1 - \frac{15}{2} \times (-0.1) + \frac{825}{32} \times (-0.1)^2$	M1	3.1a
	= awrt 2.0078	A1	1.1b
		(2)	
(6 marks)			

June 2023 Question 14

14. Find, in simplest form, the coefficient of x^5 in the expansion of

$$(5 + 8x^2)\left(3 - \frac{1}{2}x\right)^6 \quad (5)$$

ANSWER

Question	Scheme	Marks	AOs
14	Attempts the term in x^3 or the term in x^5 of $\left(3 - \frac{1}{2}x\right)^6$ Look for ${}^6C_3 3^3 \left(-\frac{1}{2}x\right)^3$ or ${}^6C_5 3^1 \left(-\frac{1}{2}x\right)^5$	M1	3.1a
	Correct term in x^3 or correct term in x^5 of $\left(3 - \frac{1}{2}x\right)^6$ $-\frac{135}{2}x^3$ or $-\frac{9}{16}x^5$	A1	1.1b
	Attempts one of the required terms in x^5 of $(5 + 8x^2)\left(3 - \frac{1}{2}x\right)^6$ Either $5 \times {}^6C_5 3^1 \left(-\frac{1}{2}x\right)^5$ or $8x^2 \times {}^6C_3 3^3 \left(-\frac{1}{2}x\right)^3$	M1	1.1b
	Attempts the sum of $5 \times {}^6C_5 3^1 \left(-\frac{1}{2}x\right)^5$ and $8x^2 \times {}^6C_3 3^3 \left(-\frac{1}{2}x\right)^3$	dM1	2.1
	Coefficient of $x^5 = -\frac{45}{16} - 540 = -\frac{8685}{16}$	A1	1.1b
		(5)	
(5 marks)			

June 2022 Question 6

6. (a) Find the first 4 terms, in ascending powers of x , of the binomial expansion of

$$\left(3 - \frac{2x}{9}\right)^8$$

giving each term in simplest form.

(4)

$$f(x) = \left(\frac{x-1}{2x}\right)\left(3 - \frac{2x}{9}\right)^8$$

- (b) Find the coefficient of x^2 in the series expansion of $f(x)$, giving your answer as a simplified fraction.

(2)

ANSWER

Question	Scheme	Marks	AOs
6(a)	3^8 or 6561 as the constant term	B1	1.1b
	$\left(3 - \frac{2x}{9}\right)^8 = \dots + {}^8C_1(3)^7\left(-\frac{2x}{9}\right) + {}^8C_2(3)^6\left(-\frac{2x}{9}\right)^2 + {}^8C_3(3)^5\left(-\frac{2x}{9}\right)^3 + \dots$ $= \dots + 8 \times (3)^7\left(-\frac{2x}{9}\right) + 28 \times (3)^6\left(-\frac{2x}{9}\right)^2 + 56(3)^5\left(-\frac{2x}{9}\right)^3$	M1 A1	1.1b 1.1b
	$= 6561 - 3888x + 1008x^2 - \frac{448}{3}x^3 + \dots$	A1	1.1b
		(4)	
(b)	Coefficient of x^2 is $\frac{1}{2} \times "1008" - \frac{1}{2} \times " - \frac{448}{3} "$	M1	3.1a
	$= \frac{1736}{3}$ (or $578\frac{2}{3}$)	A1	1.1b
		(2)	
(6 marks)			

Video solution:

<https://youtu.be/jDXn-OKC4I8>

Arancha Ruiz

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November 2021 Question 8

8. $g(x) = (2 + ax)^8$ where a is a constant

Given that one of the terms in the binomial expansion of $g(x)$ is $3402x^5$

(a) find the value of a .

(4)

Using this value of a ,

(b) find the constant term in the expansion of

$$\left(1 + \frac{1}{x^4}\right)(2 + ax)^8$$

(3)

ANSWER

Question	Scheme	Marks	AOs
8 (a)	$(2 + ax)^8$ Attempts the term in $x^5 = {}^8C_5 2^3 (ax)^5 = 448a^5 x^5$	M1 A1	1.1a 1.1b
	Sets $448a^5 = 3402 \Rightarrow a^5 = \frac{243}{32}$	M1	1.1b
	$\Rightarrow a = \frac{3}{2}$	A1	1.1b
		(4)	
(b)	Attempts either term. So allow for 2^8 or ${}^8C_4 2^4 a^4$	M1	1.1b
	Attempts the sum of both terms $2^8 + {}^8C_4 2^4 a^4$	dM1	2.1
	$= 256 + 5670 = 5926$	A1	1.1b
		(3)	
(7 marks)			

November 2020 Question 6

6. (a) Find the first 4 terms, in ascending powers of x , in the binomial expansion of

$$(1 + kx)^{10}$$

where k is a non-zero constant. Write each coefficient as simply as possible.

(3)

Given that in the expansion of $(1 + kx)^{10}$ the coefficient of x^3 is 3 times the coefficient of x ,

- (b) find the possible values of k .

(3)

ANSWER

Question	Scheme	Marks	AOs
6 (a)	$(1 + kx)^{10} = 1 + \binom{10}{1}(kx)^1 + \binom{10}{2}(kx)^2 + \binom{10}{3}(kx)^3 \dots$	M1 A1	1.1b 1.1b
	$= 1 + 10kx + 45k^2x^2 + 120k^3x^3 \dots$	A1	1.1b
		(3)	
(b)	Sets $120k^3 = 3 \times 10k$	B1	1.2
	$4k^2 = 1 \Rightarrow k = \dots$	M1	1.1b
	$k = \pm \frac{1}{2}$	A1	1.1b
		(3)	
(6 marks)			

June 2019 Question 8

8. (a) Find the first 3 terms, in ascending powers of x , of the binomial expansion of

$$\left(2 + \frac{3x}{4}\right)^6$$

giving each term in its simplest form.

(4)

- (b) Explain how you could use your expansion to estimate the value of 1.925^6
You do not need to perform the calculation.

(1)

ANSWER

Question	Scheme	Marks	AOs
8(a)	2^6 or 64 as the constant term	B1	1.1b
	$\left(2 + \frac{3x}{4}\right)^6 = \dots + {}^6C_1 2^5 \left(\frac{3x}{4}\right)^1 + {}^6C_2 2^4 \left(\frac{3x}{4}\right)^2 + \dots$	M1	1.1b
	$= \dots + 6 \times 2^5 \left(\frac{3x}{4}\right)^1 + \frac{6 \times 5}{2} \times 2^4 \left(\frac{3x}{4}\right)^2 + \dots$	A1	1.1b
	$= 64 + 144x + 135x^2 + \dots$	A1	1.1b
		(4)	
(b)	$\frac{3x}{4} = -0.075 \Rightarrow x = -0.1$ So find the value of $64 + 144x + 135x^2$ with $x = -0.1$	B1ft	2.4
		(1)	
(5 marks)			

November 2020 Question 4 Paper 2 (A-Level)

4. In the binomial expansion of

$$(a + 2x)^7 \quad \text{where } a \text{ is a constant}$$

the coefficient of x^4 is 15 120

Find the value of a .

(3)

ANSWER

Question	Scheme	Marks	AOs
4	${}^7C_4 a^3 (2x)^4$	M1	1.1b
	$\frac{7!}{4!3!} a^3 \times 2^4 = 15120 \Rightarrow a = \dots$	dM1	2.1
	$a = 3$	A1	1.1b
		(3)	
(3 marks)			

June 2018 Question 11

11. (a) Find the first 3 terms, in ascending powers of x , of the binomial expansion of

$$\left(2 - \frac{x}{16}\right)^9$$

giving each term in its simplest form.

(4)

$$f(x) = (a + bx)\left(2 - \frac{x}{16}\right)^9, \text{ where } a \text{ and } b \text{ are constants}$$

Given that the first two terms, in ascending powers of x , in the series expansion of $f(x)$ are 128 and $36x$,

- (b) find the value of a ,

(2)

- (c) find the value of b .

(2)

ANSWER

Question	Scheme	Marks	AOs
11(a)	$\left(2 - \frac{x}{16}\right)^9 = 2^9 + \binom{9}{1} 2^8 \cdot \left(-\frac{x}{16}\right) + \binom{9}{2} 2^7 \cdot \left(-\frac{x}{16}\right)^2 + \dots$	M1	1.1b
	$\left(2 - \frac{x}{16}\right)^9 = 512 + \dots$	B1	1.1b
	$\left(2 - \frac{x}{16}\right)^9 = \dots - 144x + \dots$	A1	1.1b
	$\left(2 - \frac{x}{16}\right)^9 = \dots + \dots + 18x^2 (+ \dots)$	A1	1.1b
		(4)	
(b)	Sets '512' $a = 128 \Rightarrow a = \dots$	M1	1.1b
	$(a =) \frac{1}{4}$ oe	A1 ft	1.1b
		(2)	
(c)	Sets '512' $b + ' - 144' a = 36 \Rightarrow b = \dots$	M1	2.2a
	$(b =) \frac{9}{64}$ oe	A1	1.1b
		(2)	
(8 marks)			

June 2018 Question 1 Core 2

2. (a) Find the first 4 terms, in ascending powers of x , of the binomial expansion of

$$(2 + kx)^7$$

where k is a non-zero constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^3 in this expansion is 1890

- (b) find the value of k .

(3)

ANSWER

Question Number	Scheme	Marks
2. (a)	$(2 + kx)^7$ $2^7 + {}^7C_1 2^6(kx) + {}^7C_2 2^5(kx)^2 + {}^7C_3 2^4(kx)^3 \dots$ First term of 128 $({}^7C_1 \times \dots \times x) + ({}^7C_2 \times \dots \times x^2) + ({}^7C_3 \times \dots \times x^3) \dots$ $= (128 \dots) + 448kx + 672k^2x^2 + 560k^3x^3 \dots$	B1 M1 A1, A1 (4)
(b)	$560k^3 = 1890$ $k^3 = \frac{1890}{560}$ so $k =$ $k = 1.5$ o.e.	M1 dM1 A1 (3) (7marks)

Ch. 9: Trigonometric Ratios

June 2024 Question 4

4.

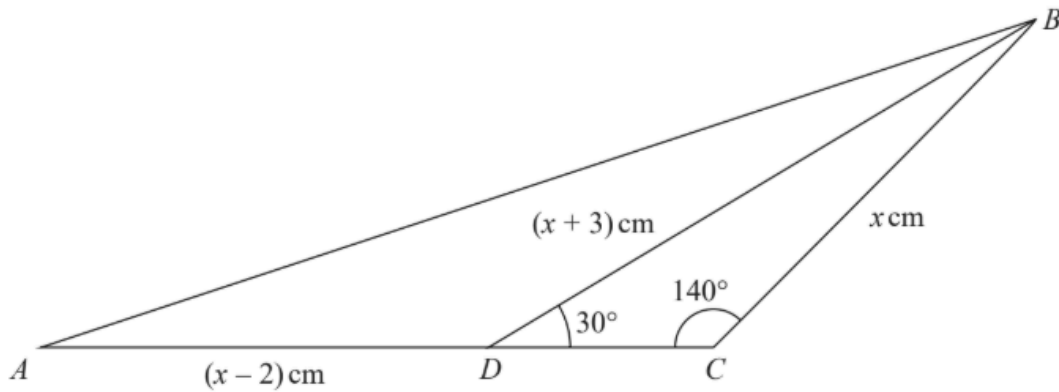


Figure 1

Figure 1 shows a sketch of triangle ABD and triangle BCD

Given that

- ADC is a straight line
- $BD = (x + 3) \text{ cm}$
- $BC = x \text{ cm}$
- $\text{angle } BDC = 30^\circ$
- $\text{angle } BCD = 140^\circ$

(a) show that $x = 10.5$ correct to 3 significant figures.

(3)

Given also that $AD = (x - 2) \text{ cm}$

(b) find the length of AB , giving your answer to 3 significant figures.

(2)

ANSWER

Question	Scheme	Marks	AOs
4(a)	$\frac{x}{\sin 30^\circ} = \frac{x+3}{\sin 140^\circ}$	M1	1.1b
	$2x \sin 140 - x = 3$	dM1	3.1a
	$x = 10.5$ *	A1*	1.1b
		(3)	
(b)	$AB^2 = 8.5^2 + 13.5^2 - 2 \times 8.5 \times 13.5 \times \cos 150^\circ$ $\Rightarrow AB = \dots$	M1	3.1a
	$AB = \text{awrt } 21.3 \text{ (cm)}$	A1	1.1b
		(2)	
(5 marks)			

June 2023 Question 3

3.

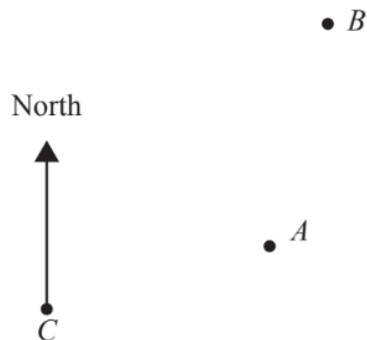


Figure 1

Figure 1 is a sketch showing the position of three phone masts, A , B and C .

The masts are identical and their bases are assumed to lie in the same horizontal plane.

From mast C

- mast A is 8.2 km away on a bearing of 072°
- mast B is 15.6 km away on a bearing of 039°

- (a) Find the distance between masts A and B , giving your answer in km to one decimal place.

(3)

An engineer needs to travel from mast A to mast B .

- (b) Give a reason why the answer to part (a) is unlikely to be an accurate value for the distance the engineer travels.

(1)

ANSWER

Question	Scheme	Marks	AOs
3 (a)	Angle $ACB = 33^\circ$	B1	1.1b
	Attempts $\{AB^2 =\} 8.2^2 + 15.6^2 - 2 \times 8.2 \times 15.6 \cos 33^\circ$	M1	1.1b
	Distance = awrt 9.8 {km}	A1	1.1b
		(3)	
(b)	- Explains that the road is not likely to be straight {and therefore the distance will be greater}. - Explains that there are likely to be objects in the way {that they must go around and therefore the distance travelled will be greater}. - The {bases of the} masts are not likely to lie in the same {horizontal} plane {and so the distance will be greater}.	B1	3.2b
		(1)	
(4 marks)			

June 2022 Question 4

4.

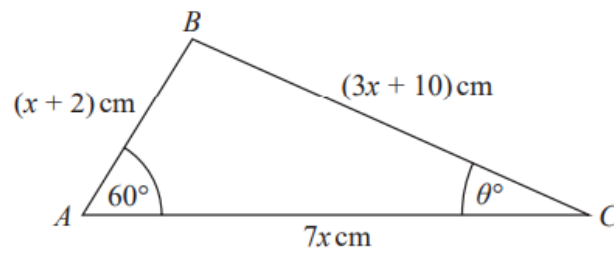


Figure 1

Figure 1 shows a sketch of triangle ABC with $AB = (x + 2)$ cm, $BC = (3x + 10)$ cm, $AC = 7x$ cm, angle $BAC = 60^\circ$ and angle $ACB = \theta^\circ$

- (a) (i) Show that $17x^2 - 35x - 48 = 0$ (3)
- (ii) Hence find the value of x . (1)
- (b) Hence find the value of θ giving your answer to one decimal place. (2)

ANSWER

Question	Scheme	Marks	AOs
4(a)(i)	$(3x+10)^2 = (x+2)^2 + (7x)^2 - 2(x+2)(7x)\cos 60^\circ$ oe	M1	3.1a
	Uses $\cos 60^\circ = \frac{1}{2}$, expands the brackets and proceeds to a 3 term quadratic equation	dM1	1.1b
	$17x^2 - 35x - 48 = 0$ *	A1*	2.1
		(3)	
	$x = 3$	B1	3.2a
		(1)	
(b)	$\frac{5}{\sin ACB} = \frac{19}{\sin 60^\circ} \Rightarrow \sin ACB = \dots \left(\frac{5\sqrt{3}}{38} \right)$ or e.g. $5^2 = 21^2 + 19^2 - 2 \times 19 \times 21 \cos ACB \Rightarrow \cos ACB = \dots \left(\frac{37}{38} \right)$	M1	1.1b
	$\theta = \text{awrt } 13.2$	A1	1.1b
		(2)	
(6 marks)			

Video solution:

<https://youtu.be/Wi1gGRu3eXQ>

November 2021 Question 7

7. A parallelogram $PQRS$ has area 50 cm^2

Given

- PQ has length 14 cm
- QR has length 7 cm
- angle SPQ is obtuse

find

(a) the size of angle SPQ , in degrees, to 2 decimal places,

(3)

(b) the length of the diagonal SQ , in cm, to one decimal place.

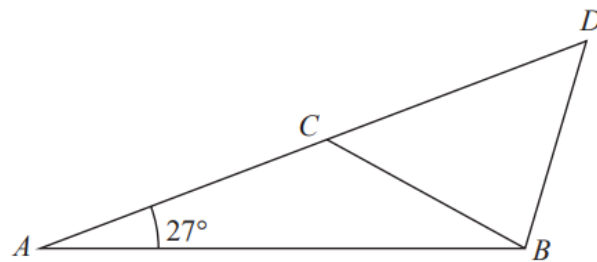
(2)

ANSWER

Question	Scheme	Marks	AOs
7 (a)	Sets $50 = 7 \times 14 \sin(SPQ)$ oe	B1	1.2
	Finds $180^\circ - \arcsin\left(\frac{50}{98}\right)$	M1	1.1b
	$= 149.32^\circ$	A1	1.1b
		(3)	
(b)	Method of finding SQ $SQ^2 = 14^2 + 7^2 - 2 \times 14 \times 7 \cos 149.32^\circ$	M1	1.1b
	$= 20.3\text{ cm}$	A1	1.1b
		(2)	
(5 marks)			

November 2020 Question 5

5.



Not to scale

Figure 1

Figure 1 shows the design for a structure used to support a roof.

The structure consists of four steel beams, AB , BD , BC and AD .

Given $AB = 12\text{ m}$, $BC = BD = 7\text{ m}$ and angle $BAC = 27^\circ$

(a) find, to one decimal place, the size of angle ACB .

(3)

The steel beams can only be bought in whole metre lengths.

(b) Find the minimum length of steel that needs to be bought to make the complete structure.

(3)

ANSWER

Question	Scheme	Marks	AOs
5 (a)	States $\frac{\sin \theta}{12} = \frac{\sin 27}{7}$	M1	1.1b
	Finds $\theta = \text{awrt } 51^\circ \text{ or awrt } 129^\circ$	A1	1.1b
	$= \text{awrt } 128.9^\circ$	A1	1.1b
		(3)	
(b)	Attempts to find part or all of AD Eg $AD^2 = 7^2 + 12^2 - 2 \times 12 \times 7 \cos 101.9 = (AD = 15.09)$ Eg $(AC)^2 = 7^2 + 12^2 - 2 \times 12 \times 7 \cos(180 - "128.9" - 27)$ Eg $12 \cos 27$ or $7 \cos "51"$	M1	1.1b
	Full method for the total length $= 12 + 7 + 7 + "15.09" =$	dM1	3.1a
	$= 42 \text{ m}$	A1	3.2a
		(3)	
(6 marks)			

June 2019 Question 6

6.

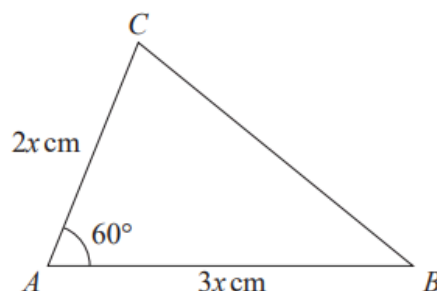


Figure 1

Figure 1 shows a sketch of a triangle ABC with $AB = 3x$ cm, $AC = 2x$ cm and angle $CAB = 60^\circ$

Given that the area of triangle ABC is $18\sqrt{3}$ cm²

(a) show that $x = 2\sqrt{3}$

(3)

(b) Hence find the exact length of BC , giving your answer as a simplified surd.

(3)

ANSWER

Question	Scheme	Marks	AOs
6 (a)	Uses $18\sqrt{3} = \frac{1}{2} \times 2x \times 3x \times \sin 60^\circ$	M1	1.1a
	Sight of $\sin 60^\circ = \frac{\sqrt{3}}{2}$ and proceeds to $x^2 = k$ oe	M1	1.1b
	$x = \sqrt{12} = 2\sqrt{3}$ *	A1*	2.1
		(3)	
(b)	Uses $BC^2 = (6\sqrt{3})^2 + (4\sqrt{3})^2 - 2 \times 6\sqrt{3} \times 4\sqrt{3} \times \cos 60^\circ$	M1	1.1b
	$BC^2 = 84$	A1	1.1b
	$BC = 2\sqrt{21}$ (cm)	A1	1.1b
		(3)	
(6 marks)			

June 2018 Question 7

7. In a triangle ABC , side AB has length 10 cm, side AC has length 5 cm, and angle $BAC = \theta$ where θ is measured in degrees. The area of triangle ABC is 15 cm^2

(a) Find the two possible values of $\cos \theta$

(4)

Given that BC is the longest side of the triangle,

(b) find the exact length of BC .

(2)

ANSWER

Question	Scheme	Marks	AOs
7 (a)	Uses $15 = \frac{1}{2} \times 5 \times 10 \times \sin \theta$	M1	1.1b
	$\sin \theta = \frac{3}{5}$ oe	A1	1.1b
	Uses $\cos^2 \theta = 1 - \sin^2 \theta$	M1	2.1
	$\cos \theta = \pm \frac{4}{5}$	A1	1.1b
		(4)	
(b)	Uses $BC^2 = 10^2 + 5^2 - 2 \times 10 \times 5 \times \frac{4}{5}$	M1	3.1a
	$BC = \sqrt{205}$	A1	1.1b
		(2)	
(6 marks)			

Ch. 10: Trigonometric Identities and Equations

June 2024 Question 13

13. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$\sin \theta (7 \sin \theta - 4 \cos \theta) = 4$$

can be written as

$$3 \tan^2 \theta - 4 \tan \theta - 4 = 0 \quad (4)$$

(b) Hence solve, for $0 < x < 360^\circ$

$$\sin x (7 \sin x - 4 \cos x) = 4$$

giving your answers to one decimal place. (4)

(c) Hence find the smallest solution of the equation

$$\sin 4\alpha (7 \sin 4\alpha - 4 \cos 4\alpha) = 4$$

in the range $720^\circ < \alpha < 1080^\circ$, giving your answer to one decimal place. (1)

June 2023 Question 12

12.

In this question you must show detailed reasoning.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$4 \tan x = 5 \cos x$$

can be written as

$$5 \sin^2 x + 4 \sin x - 5 = 0 \quad (3)$$

(b) Hence solve, for $0 < x \leq 360^\circ$

$$4 \tan x = 5 \cos x$$

giving your answers to one decimal place.

(4)

(c) Hence find the **number of solutions** of the equation

$$4 \tan 3x = 5 \cos 3x$$

in the interval $0 < x \leq 1800^\circ$, explaining briefly the reason for your answer.

(2)

ANSWER

Question	Scheme	Marks	AOs
13(a)	$7 \sin^2 \theta - 4 \sin \theta \cos \theta = 4$ $\Rightarrow 7 \tan^2 \theta - 4 \tan \theta = \frac{4}{\cos^2 \theta}$	M1	1.1b
	$\Rightarrow 3 \tan^2 \theta - 4 \tan \theta + \frac{4 \sin^2 \theta}{\cos^2 \theta} - \frac{4}{\cos^2 \theta} = 0$	dM1	1.1b
	$\frac{4 \sin^2 \theta - 4}{\cos^2 \theta} = -4 \frac{\cos^2 \theta}{\cos^2 \theta} = -4 \Rightarrow 3 \tan^2 \theta - 4 \tan \theta - 4 = 0 \quad *$	ddM1 A1*	2.1 1.1b
		(4)	
(b)	Roots: $-\frac{2}{3}, 2$	B1	1.1b
	$\tan^{-1}\left(-\frac{2}{3}\right) = \dots \text{ or } \tan^{-1}(2) = \dots$	M1	1.1b
	Two of awrt 63, awrt 146, awrt 243, awrt 326	A1	1.1b
	All four of awrt 63.4, awrt 146.3, awrt 243.4, awrt 326.3	A1	1.1b
		(4)	
(c)	awrt 735.9	B1	2.2a
		(1)	
(9 marks)			

ANSWER

Question	Scheme	Marks	AOs
12(a)	States or uses $\tan x = \frac{\sin x}{\cos x}$	B1	1.2
	$4 \sin x = 5 \cos^2 x \Rightarrow 4 \sin x = 5(1 - \sin^2 x)$	M1	1.1b
	$5 \sin^2 x + 4 \sin x - 5 = 0^*$	A1*	2.1
		(3)	
(b)	Attempts to solve $5 \sin^2 x + 4 \sin x - 5 = 0 \Rightarrow \sin x = \dots$	M1	1.1b
	$\sin x = \frac{-2 \pm \sqrt{29}}{5}$ ($\sin x = \text{awrt } 0.677$)	A1	1.1b
	Takes $\sin^{-1}$ leading to at least one answer in the range	dM1	1.1b
	$x = \text{awrt } 42.6\{\circ\}$ and $x = \text{awrt } 137.4\{\circ\}$ only	A1	1.1b
		(4)	
(c)	$15 \times "2" = 30$ following through on their "2"	B1ft	2.2a
	Explains either "mathematically" by stating $3 \times 5 \times$ their number in range 0 to 360° or 'in words' e.g., stating $3 \times "2"$ values every 360° and 5 lots of 360°	B1ft	2.4
		(2)	

June 2022 Question 13

13. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{1}{\cos \theta} + \tan \theta \equiv \frac{\cos \theta}{1 - \sin \theta} \quad \theta \neq (2n + 1)90^\circ \quad n \in \mathbb{Z} \quad (3)$$

Given that $\cos 2x \neq 0$

(b) solve for $0 < x < 90^\circ$

$$\frac{1}{\cos 2x} + \tan 2x = 3 \cos 2x$$

giving your answers to one decimal place.

(5)

ANSWER

Question	Scheme	Marks	AOs	
13(a)	$\frac{1}{\cos \theta} + \tan \theta = \frac{1 + \sin \theta}{\cos \theta} \text{ or } \frac{(1 + \sin \theta) \cos \theta}{\cos^2 \theta}$	M1	1.1b	
	$= \frac{1 + \sin \theta}{\cos \theta} \times \frac{1 - \sin \theta}{1 - \sin \theta} = \frac{1 - \sin^2 \theta}{\cos \theta (1 - \sin \theta)} = \frac{\cos^2 \theta}{\cos \theta (1 - \sin \theta)}$ or $\frac{(1 + \sin \theta) \cos \theta}{\cos^2 \theta} = \frac{(1 + \sin \theta) \cos \theta}{1 - \sin^2 \theta} = \frac{(1 + \sin \theta) \cos \theta}{(1 + \sin \theta)(1 - \sin \theta)}$	dM1	2.1	
	$= \frac{\cos \theta}{1 - \sin \theta} *$	A1*	1.1b	
		(3)		
(b)	$\frac{1}{\cos 2x} + \tan 2x = 3 \cos 2x$ $\Rightarrow 1 + \sin 2x = 3 \cos^2 2x = 3(1 - \sin^2 2x)$	$\frac{\cos 2x}{1 - \sin 2x} = 3 \cos 2x$ $\Rightarrow \cos 2x = 3 \cos 2x(1 - \sin 2x)$	M1	2.1
	$\Rightarrow 3 \sin^2 2x + \sin 2x - 2 = 0$	$\Rightarrow \cos 2x(2 - 3 \sin 2x) = 0$	A1	1.1b
	$\sin 2x = \frac{2}{3}, (-1) \Rightarrow 2x = \dots \Rightarrow x = \dots$		M1	1.1b
	$x = 20.9^\circ, 69.1^\circ$		A1	1.1b
			A1	1.1b
			(5)	
(8 marks)				

Video solution:

 AS Maths June 2022 Question 13 paper 1 Edexcel

November 2021 Question 12

12. In this question you should show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < \theta \leq 450^\circ$, the equation

$$5 \cos^2 \theta = 6 \sin \theta$$

giving your answers to one decimal place.

(5)

(ii) (a) A student's attempt to solve the question

“Solve, for $-90^\circ < x < 90^\circ$, the equation $3 \tan x - 5 \sin x = 0$ ”

is set out below.

$$\begin{aligned} 3 \tan x - 5 \sin x &= 0 \\ 3 \frac{\sin x}{\cos x} - 5 \sin x &= 0 \\ 3 \sin x - 5 \sin x \cos x &= 0 \\ 3 - 5 \cos x &= 0 \\ \cos x &= \frac{3}{5} \\ x &= 53.1^\circ \end{aligned}$$

Identify two errors or omissions made by this student, giving a brief explanation of each.

(2)

The first four positive solutions, in order of size, of the equation

$$\cos(5\alpha + 40^\circ) = \frac{3}{5}$$

are $\alpha_1, \alpha_2, \alpha_3$ and α_4

(b) Find, to the nearest degree, the value of α_4

(2)

ANSWER

Question	Scheme	Marks	AOs
12 (i)	Uses $\cos^2 \theta = 1 - \sin^2 \theta$ $5 \cos^2 \theta = 6 \sin \theta \Rightarrow 5 \sin^2 \theta + 6 \sin \theta - 5 = 0$	M1 A1	1.2 1.1b
	$\Rightarrow \sin \theta = \frac{-3 + \sqrt{34}}{5} \Rightarrow \theta = \dots$	dM1	3.1a
	$\Rightarrow \theta = 34.5^\circ, 145.5^\circ, 394.5^\circ$	A1 A1	1.1b 1.1b
		(5)	
(ii) (a)	One of - They cancel by $\sin x$ (and hence they miss the solution $\sin x = 0 \Rightarrow x = 0$) - They do not find all the solutions of $\cos x = \frac{3}{5}$ (in the given range) or they missed the solution $x = -53.1^\circ$	B1	2.3
	Both of the above	B1	2.3
		(2)	
(ii) (b)	Sets $5\alpha + 40^\circ = 720^\circ - 53.1^\circ$	M1	3.1a
	$\alpha = 125^\circ$	A1	1.1b
		(2)	
(9 marks)			

November 2020 Question 9

9.

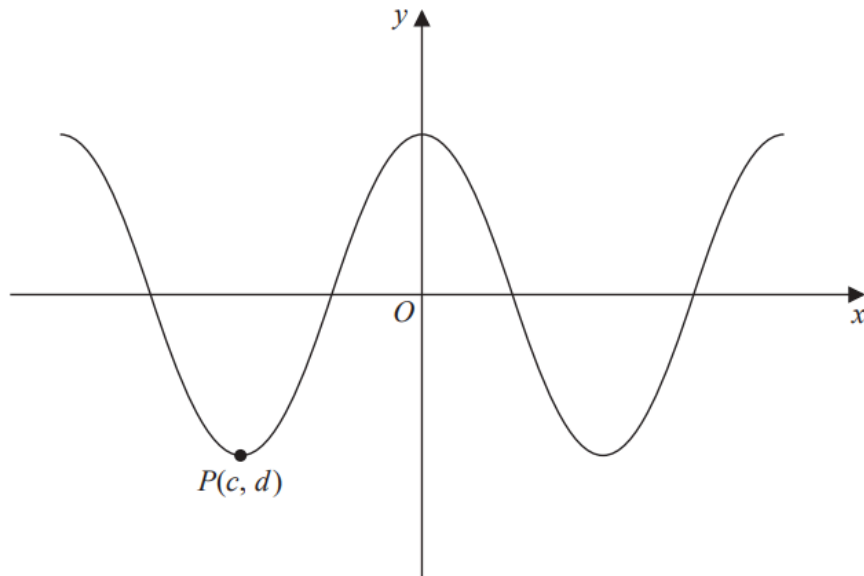


Figure 3

Figure 3 shows part of the curve with equation $y = 3 \cos x^\circ$.

The point $P(c, d)$ is a minimum point on the curve with c being the smallest negative value of x at which a minimum occurs.

(a) State the value of c and the value of d . (1)

(b) State the coordinates of the point to which P is mapped by the transformation which transforms the curve with equation $y = 3 \cos x^\circ$ to the curve with equation

(i) $y = 3 \cos \left(\frac{x^\circ}{4} \right)$

(ii) $y = 3 \cos(x - 36)^\circ$ (2)

(c) Solve, for $450^\circ \leq \theta < 720^\circ$,

$$3 \cos \theta = 8 \tan \theta$$

giving your solution to one decimal place.

In part (c) you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable. (5)

ANSWER

Question	Scheme	Marks	AOs
9 (a)	$(-180^\circ, -3)$	B1	1.1b
		(1)	
(b)	(i) $(-720^\circ, -3)$	B1ft	2.2a
	(ii) $(-144^\circ, -3)$	B1 ft	2.2a
		(2)	
(c)	Attempts to use both $\tan \theta = \frac{\sin \theta}{\cos \theta}$, $\sin^2 \theta + \cos^2 \theta = 1$ and solves a quadratic equation in $\sin \theta$ to find at least one value of θ	M1	3.1a
	$3 \cos \theta = 8 \tan \theta \Rightarrow 3 \cos^2 \theta = 8 \sin \theta$	B1	1.1b
	$3 \sin^2 \theta + 8 \sin \theta - 3 = 0$ $(3 \sin \theta - 1)(\sin \theta + 3) = 0$	M1	1.1b
	$\sin \theta = \frac{1}{3}$	A1	2.2a
	awrt 520.5° only	A1	2.1
		(5)	
(8 marks)			

June 2018 Question 12

12. (a) Show that the equation

$$4 \cos \theta - 1 = 2 \sin \theta \tan \theta$$

can be written in the form

$$6 \cos^2 \theta - \cos \theta - 2 = 0$$

(4)

(b) Hence solve, for $0 \leq x < 90^\circ$

$$4 \cos 3x - 1 = 2 \sin 3x \tan 3x$$

giving your answers, where appropriate, to one decimal place.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(4)

ANSWER

Question	Scheme	Marks	AOs
12 (a)	$4 \cos \theta - 1 = 2 \sin \theta \tan \theta \Rightarrow 4 \cos \theta - 1 = 2 \sin \theta \times \frac{\sin \theta}{\cos \theta}$	M1	1.2
	$\Rightarrow 4 \cos^2 \theta - \cos \theta = 2 \sin^2 \theta$ oe	A1	1.1b
	$\Rightarrow 4 \cos^2 \theta - \cos \theta = 2(1 - \cos^2 \theta)$	M1	1.1b
	$6 \cos^2 \theta - \cos \theta - 2 = 0$ *	A1*	2.1
		(4)	
(b)	For attempting to solve given quadratic	M1	1.1b
	$(\cos 3x) = \frac{2}{3}, -\frac{1}{2}$	B1	1.1b
	$x = \frac{1}{3} \arccos\left(\frac{2}{3}\right)$ or $\frac{1}{3} \arccos\left(-\frac{1}{2}\right)$	M1	1.1b
	$x = 40^\circ, 80^\circ$, awrt 16.1°	A1	2.2a
		(4)	
(8 marks)			

June 2018 Question 8 Core 2

- 8 *In this question solutions based entirely on graphical or numerical methods are not acceptable.*

- (i) Solve for $0 \leq x < 360^\circ$,

$$4 \cos(x + 70^\circ) = 3$$

giving your answers in degrees to one decimal place.

(4)

- (ii) Find, for $0 \leq \theta < 2\pi$, all the solutions of

$$6 \cos^2 \theta - 5 = 6 \sin^2 \theta + \sin \theta$$

giving your answers in radians to 3 significant figures.

(5)

ANSWER

Question Number	Scheme	Marks
8. (i)	$4 \cos(x + 70^\circ) = 3$ $\cos(x + 70^\circ) = 0.75$, so $x + 70^\circ = 41.4(1)^\circ$ $x = 248.6^\circ$ or 331.4°	M1 A1 M1 A1 (4)
(ii)	$6 \cos^2 \theta - 5 = 6 \sin^2 \theta + \sin \theta$ so $6(1 - \sin^2 \theta) - 5 = 6 \sin^2 \theta + \sin \theta$ $12 \sin^2 \theta + \sin \theta - 1 = 0$ $(4 \sin \theta - 1)(3 \sin \theta + 1) = 0$ so $\sin \theta =$ $\theta = 0.253, 2.89, 3.48, 5.94$	M1 A1 M1 A1 A1 (5) [9]

Ch. 11: Vectors

June 2024 Question 3

3. Relative to a fixed origin O ,

- point P has position vector $9\mathbf{i} - 8\mathbf{j}$
- point Q has position vector $3\mathbf{i} - 5\mathbf{j}$

(a) Find $\overrightarrow{PQ}$

(2)

Given that R is the point such that $\overrightarrow{QR} = 9\mathbf{i} + 18\mathbf{j}$

(b) show that angle $PQR = 90^\circ$

(2)

Given also that S is the point such that $\overrightarrow{PS} = 3\overrightarrow{QR}$

(c) find the exact area of $PQRS$

(4)

ANSWER

Question	Scheme	Marks	AOs
3(a)	$\overrightarrow{PQ} = (3-9)\mathbf{i} + (-5+8)\mathbf{j}$	M1	1.1a
	$= -6\mathbf{i} + 3\mathbf{j}$	A1	1.1b
		(2)	
(b)	Gradient of $PQ = \frac{-5-8}{3-9} \left(= -\frac{1}{2} \right)$ and Gradient of $QR = \frac{18}{9} (= 2)$ or $ \overrightarrow{PQ} = \sqrt{(-6)^2 + 3^2} (= 3\sqrt{5})$ and $ \overrightarrow{QR} = \sqrt{9^2 + 18^2} (= 9\sqrt{5})$ and $ \overrightarrow{PR} = \sqrt{3^2 + 21^2} (= 15\sqrt{2})$	M1	3.1a
	e.g. shows that $-\frac{1}{2} \times 2 = -1$ and deduces angle $PQR = 90^\circ$ * or e.g. shows $ \overrightarrow{PQ} ^2 + \overrightarrow{QR} ^2 = \overrightarrow{PR} ^2$ and deduces angle $PQR = 90^\circ$ *	A1*	2.4
		(2)	
(c)	Attempts to find the length PQ and at least one of QR or PS using Pythagoras' Theorem correctly e.g. $ \overrightarrow{PQ} = \sqrt{(-6)^2 + 3^2}$ and either $ \overrightarrow{QR} = \sqrt{9^2 + 18^2}$ or $ \overrightarrow{PS} = \sqrt{27^2 + 54^2}$	M1	2.1
	$ \overrightarrow{PQ} = \sqrt{45} (= 3\sqrt{5})$ and either $ \overrightarrow{QR} = \sqrt{405} (= 9\sqrt{5})$ or $ \overrightarrow{PS} = 27\sqrt{5}$	A1ft	1.1b
	e.g. Area $= \frac{1}{2} \times (9\sqrt{5} + 27\sqrt{5}) \times \sqrt{45}$ or $\frac{1}{2} \times 4 \times 9\sqrt{5} \times 3\sqrt{5}$	dM1	3.1a
	$= 270$	A1	1.1b
		(4)	
(8 marks)			

June 2023 Question 13

13. Relative to a fixed origin O

- point A has position vector $10\mathbf{i} - 3\mathbf{j}$
- point B has position vector $-8\mathbf{i} + 9\mathbf{j}$
- point C has position vector $-2\mathbf{i} + p\mathbf{j}$ where p is a constant

(a) Find $\vec{AB}$

(2)

(b) Find $|\vec{AB}|$ giving your answer as a fully simplified surd.

(2)

Given that points A , B and C lie on a straight line,

(c) (i) find the value of p ,

(ii) state the ratio of the area of triangle AOC to the area of triangle AOB .

(3)

ANSWER

Question	Scheme	Marks	AOs
13 (a)	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (-8\mathbf{i} + 9\mathbf{j}) - (10\mathbf{i} - 3\mathbf{j})$	M1	1.1b
	$= -18\mathbf{i} + 12\mathbf{j}$	A1	1.1b
		(2)	
(b)	$ \overrightarrow{AB} = \sqrt{18^2 + 12^2} \{ = \sqrt{468} \}$	M1	1.1b
	$= 6\sqrt{13}$	A1	1.1b
		(2)	
(c)	For the key step in using the fact that BCA forms a straight line in an attempt to find " p " $\overrightarrow{AB} = \lambda \overrightarrow{BC} \Rightarrow -18\mathbf{i} + 12\mathbf{j} = 6\lambda\mathbf{i} + \lambda(p-9)\mathbf{j}$ with components equated leading to a value for λ and to $p = \dots$	M1	2.1
	(i) $p = 5$	A1	1.1b
	(ii) ratio = 2: 3	B1 (A1 on EPEN)	2.2a
		(3)	
(7 marks)			

June 2023 Question 6 Paper 2

6. Relative to a fixed origin O ,

- A is the point with position vector $12\mathbf{i}$
- B is the point with position vector $16\mathbf{j}$
- C is the point with position vector $(50\mathbf{i} + 136\mathbf{j})$
- D is the point with position vector $(22\mathbf{i} + 24\mathbf{j})$

(a) Show that AD is parallel to BC .

(2)

Points A , B , C and D are used to model the vertices of a running track in the shape of a quadrilateral.

Runners complete one lap by running along all four sides of the track.

The lengths of the sides are measured in metres.

Given that a particular runner takes exactly 5 minutes to complete 2 laps,

(b) calculate the average speed of this runner, giving the answer in kilometres per hour.

(4)

ANSWER

Question	Scheme	Marks	AOs
6(a)	$\overrightarrow{AD} = 10\mathbf{i} + 24\mathbf{j}$ and $\overrightarrow{BC} = 50\mathbf{i} + 120\mathbf{j}$	M1	1.1b
	$\overrightarrow{AD} = \frac{1}{5}\overrightarrow{BC}$ therefore AD is parallel to BC *	A1*cso	2.2a
		(2)	
(b)	Attempt to find at least two lengths between AB , BC , CD and AD $ \overrightarrow{BC} = \sqrt{50^2 + 120^2} = 130$, $ \overrightarrow{DA} = \sqrt{10^2 + 24^2} = 26$ $ \overrightarrow{AB} = \sqrt{12^2 + 16^2} = 20$, $ \overrightarrow{CD} = \sqrt{28^2 + 112^2} = 28\sqrt{17}$ (awrt 115 m)	M1	1.1b
		A1	1.1b
	Average speed = $\frac{2(26+130+20+28\sqrt{17})}{\frac{5}{60} \times 1000}$	dM1	3.1b
	awrt = 6.99 (km/h)	A1	3.2a
		(4)	
(6 marks)			

June 2022 Question 3

3. The triangle PQR is such that $\vec{PQ} = 3\mathbf{i} + 5\mathbf{j}$ and $\vec{PR} = 13\mathbf{i} - 15\mathbf{j}$

(a) Find $\vec{QR}$ (2)

(b) Hence find $|\vec{QR}|$ giving your answer as a simplified surd. (2)

The point S lies on the line segment QR so that $QS:SR = 3:2$

(c) Find $\vec{PS}$ (2)

ANSWER

Question	Scheme	Marks	AOs
3(a)	$\vec{QR} = \vec{PR} - \vec{PQ} = 13\mathbf{i} - 15\mathbf{j} - (3\mathbf{i} + 5\mathbf{j})$	M1	1.1a
	$= 10\mathbf{i} - 20\mathbf{j}$	A1	1.1b
		(2)	
(b)	$ \vec{QR} = \sqrt{10^2 + (-20)^2}$	M1	2.5
	$= 10\sqrt{5}$	A1ft	1.1b
		(2)	
(c)	$\vec{PS} = \vec{PQ} + \frac{3}{5}\vec{QR} = 3\mathbf{i} + 5\mathbf{j} + \frac{3}{5}(10\mathbf{i} - 20\mathbf{j}) = \dots$ or $\vec{PS} = \vec{PR} + \frac{2}{5}\vec{RQ} = 13\mathbf{i} - 15\mathbf{j} + \frac{2}{5}(-10\mathbf{i} + 20\mathbf{j}) = \dots$	M1	3.1a
	$= 9\mathbf{i} - 7\mathbf{j}$	A1	1.1b
		(2)	
(6 marks)			

Video solution:

<https://youtu.be/eQHKB8Vrz-c>

November 2021 Question 4

4. [In this question the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are due east and due north respectively.]

A stone slides horizontally across ice.

Initially the stone is at the point $A(-24\mathbf{i} - 10\mathbf{j})\text{m}$ relative to a fixed point O .

After 4 seconds the stone is at the point $B(12\mathbf{i} + 5\mathbf{j})\text{m}$ relative to the fixed point O .

The motion of the stone is modelled as that of a particle moving in a straight line at constant speed.

Using the model,

- (a) prove that the stone passes through O , (2)
- (b) calculate the speed of the stone. (3)

ANSWER

Question	Scheme	Marks	AOs
4(a)	Attempts to compare the two position vectors. Allow an attempt using two of $\overrightarrow{AO}$, $\overrightarrow{OB}$ or $\overrightarrow{AB}$ E.g. $(-24\mathbf{i} - 10\mathbf{j}) = -2 \times (12\mathbf{i} + 5\mathbf{j})$	M1	1.1b
	Explains that as $\overrightarrow{AO}$ is parallel to $\overrightarrow{OB}$ (and the stone is travelling in a straight line) the stone passes through the point O .	A1	2.4
		(2)	
(b)	Attempts distance $AB = \sqrt{(12+24)^2 + (10+5)^2}$	M1	1.1b
	Attempts speed = $\frac{\sqrt{(12+24)^2 + (10+5)^2}}{4}$	dM1	3.1a
	Speed = 9.75 ms^{-1}	A1	3.2a
		(3)	
(5 marks)			

November 2020 Question 2

2. [In this question the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are due east and due north respectively.]

A coastguard station O monitors the movements of a small boat.

At 10:00 the boat is at the point $(4\mathbf{i} - 2\mathbf{j})$ km relative to O .

At 12:45 the boat is at the point $(-3\mathbf{i} - 5\mathbf{j})$ km relative to O .

The motion of the boat is modelled as that of a particle moving in a straight line at constant speed.

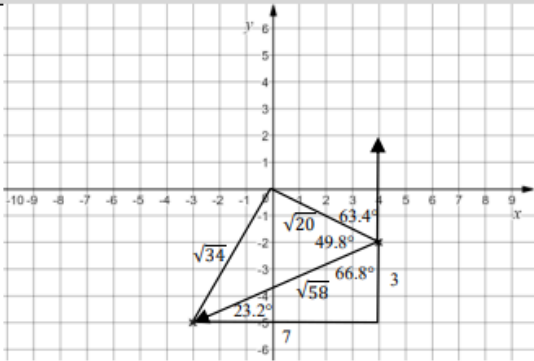
- (a) Calculate the bearing on which the boat is moving, giving your answer in degrees to one decimal place.

(3)

- (b) Calculate the speed of the boat, giving your answer in km h^{-1}

(3)

ANSWER

Question	Scheme	Marks	AOs
2(a)			
	Attempts to find an "allowable" angle Eg $\tan \theta = \frac{7}{3}$	M1	1.1b
	A full attempt to find the bearing Eg $180^\circ + "67^\circ"$	dM1	3.1b
	Bearing = awrt 246.8°	A1	1.1b
		(3)	
(b)	Attempts to find the distance travelled = $\sqrt{(4 - -3)^2 + (-2 + 5)^2} = (\sqrt{58})$	M1	1.1b
	Attempts to find the speed = $\frac{\sqrt{58}}{2.75}$	dM1	3.1b
	= awrt 2.77 km h^{-1}	A1	1.1b
		(3)	
(6 marks)			

November 2020 Question 3 Paper 1

3. Relative to a fixed origin O

- point A has position vector $2\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}$
- point B has position vector $3\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$
- point C has position vector $2\mathbf{i} - 16\mathbf{j} + 4\mathbf{k}$

(a) Find $\overrightarrow{AB}$

(2)

(b) Show that quadrilateral $OABC$ is a trapezium, giving reasons for your answer.

(2)

ANSWER

Question	Scheme	Marks	AOs
3 (a)	$\overrightarrow{AB} = (3\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}) - (2\mathbf{i} + 5\mathbf{j} - 6\mathbf{k})$	M1	1.1b
	$= \mathbf{i} - 8\mathbf{j} + 2\mathbf{k}$	A1	1.1b
		(2)	
(b)	States $\overrightarrow{OC} = 2 \times \overrightarrow{AB}$	M1	1.1b
	Explains that as OC is parallel to AB , so $OABC$ is a trapezium.	A1	2.4
		(2)	
(4 marks)			

November 2020 Question 2 Paper 2 (A-Level)

2. Relative to a fixed origin, points P , Q and R have position vectors $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ respectively.

Given that

- P , Q and R lie on a straight line
- Q lies one third of the way from P to R

show that

$$\mathbf{q} = \frac{1}{3}(\mathbf{r} + 2\mathbf{p}) \quad (3)$$

ANSWER

Question Number	Scheme	Marks	AO's
2	Attempts any one of $(\pm \overrightarrow{PQ} =) \pm (\mathbf{q} - \mathbf{p}), (\pm \overrightarrow{PR} =) \pm (\mathbf{r} - \mathbf{p}), (\pm \overrightarrow{QR} =) \pm (\mathbf{r} - \mathbf{q})$ Or e.g. $(\pm \overrightarrow{PQ} =) \pm (\overrightarrow{OQ} - \overrightarrow{OP}), (\pm \overrightarrow{PR} =) \pm (\overrightarrow{OR} - \overrightarrow{OP}), (\pm \overrightarrow{QR} =) \pm (\overrightarrow{OR} - \overrightarrow{OQ})$	M1	1.1b
	Attempts e.g. $\mathbf{r} - \mathbf{q} = 2(\mathbf{q} - \mathbf{p})$ $\mathbf{r} - \mathbf{p} = 3(\mathbf{q} - \mathbf{p})$ $\frac{2}{3}(\mathbf{q} - \mathbf{p}) = \frac{1}{3}(\mathbf{r} - \mathbf{q})$ $\mathbf{q} = \mathbf{p} + \frac{1}{3}(\mathbf{r} - \mathbf{p})$ $\mathbf{q} = \mathbf{r} + \frac{2}{3}(\mathbf{p} - \mathbf{r})$	dM1	3.1a
	E.g. $\Rightarrow \mathbf{r} - \mathbf{q} = 2\mathbf{q} - 2\mathbf{p} \Rightarrow 2\mathbf{p} + \mathbf{r} = 3\mathbf{q} \Rightarrow \mathbf{q} = \frac{1}{3}(\mathbf{r} + 2\mathbf{p})^*$	A1*	2.1
		(3)	
(3 marks)			

June 2019 Question 10 Paper 2 (A-Level)

10.

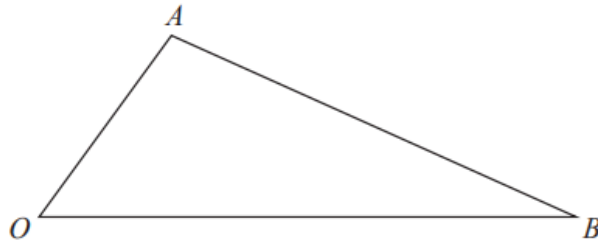


Figure 7

Figure 7 shows a sketch of triangle OAB .

The point C is such that $\vec{OC} = 2\vec{OA}$.

The point M is the midpoint of AB .

The straight line through C and M cuts OB at the point N .

Given $\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$

(a) Find $\vec{CM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$

(2)

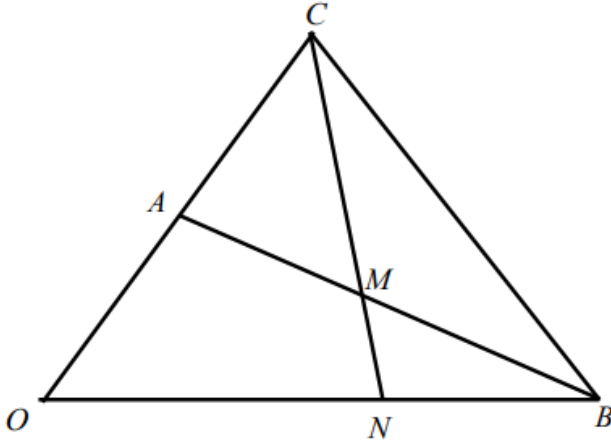
(b) Show that $\vec{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$, where λ is a scalar constant.

(2)

(c) Hence prove that $ON:NB = 2:1$

(2)

ANSWER

Question	Scheme	Marks	AOs
10	 $\vec{OA} = \mathbf{a}, \vec{OB} = \mathbf{b}$		
(a)	$\left\{ \vec{CM} = \vec{CA} + \vec{AM} = \vec{CA} + \frac{1}{2} \vec{AB} \Rightarrow \vec{CM} = -\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) \right.$ $\left\{ \vec{CM} = \vec{CB} + \vec{BM} = \vec{CB} + \frac{1}{2} \vec{BA} \Rightarrow \vec{CM} = (-2\mathbf{a} + \mathbf{b}) + \frac{1}{2}(\mathbf{a} - \mathbf{b}) \right.$ $\Rightarrow \vec{CM} = -\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} \text{ (needs to be simplified and seen in (a) only)}$	M1	3.1a
		A1	1.1b
		(2)	
(b)	$\vec{ON} = \vec{OC} + \vec{CN} \Rightarrow \vec{ON} = \vec{OC} + \lambda \vec{CM}$ $\vec{ON} = 2\mathbf{a} + \lambda \left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} \right) \Rightarrow \vec{ON} = \left(2 - \frac{3}{2}\lambda \right) \mathbf{a} + \frac{1}{2}\lambda \mathbf{b} *$	M1	1.1b
		A1*	2.1
		(2)	
(c) Way 1	$\left(2 - \frac{3}{2}\lambda \right) = 0 \Rightarrow \lambda = \dots$	M1	2.2a
	$\lambda = \frac{4}{3} \Rightarrow \vec{ON} = \frac{2}{3}\mathbf{b} \left\{ \Rightarrow \vec{NB} = \frac{1}{3}\mathbf{b} \right\} \Rightarrow ON : NB = 2 : 1 *$	A1*	2.1
		(2)	
(c) Way 2	$\vec{ON} = \mu \mathbf{b} \Rightarrow \left(2 - \frac{3}{2}\lambda \right) \mathbf{a} + \frac{1}{2}\lambda \mathbf{b} = \mu \mathbf{b}$ $\mathbf{a}: \left(2 - \frac{3}{2}\lambda \right) = 0 \Rightarrow \lambda = \dots \left\{ \mathbf{b}: \frac{1}{2}\lambda = \mu \text{ \& } \lambda = \frac{4}{3} \Rightarrow \mu = \frac{2}{3} \right\}$ $\lambda = \frac{4}{3} \text{ or } \mu = \frac{2}{3} \Rightarrow \vec{ON} = \frac{2}{3}\mathbf{b} \left\{ \Rightarrow \vec{NB} = \frac{1}{3}\mathbf{b} \right\} \Rightarrow ON : NB = 2 : 1 *$	M1	2.2a
		A1*	2.1
		(2)	
(6 marks)			

June 2018 Question 3

3. Given that the point A has position vector $4\mathbf{i} - 5\mathbf{j}$ and the point B has position vector $-5\mathbf{i} - 2\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$, (2)

(b) find $|\overrightarrow{AB}|$.

Give your answer as a simplified surd.

(2)

ANSWER

Question	Scheme	Marks	AOs
3(a)	Attempts $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ or similar	M1	1.1b
	$\overrightarrow{AB} = -9\mathbf{i} + 3\mathbf{j}$	A1	1.1b
		(2)	
(b)	Finds length using 'Pythagoras' $ \overrightarrow{AB} = \sqrt{(-9)^2 + (3)^2}$	M1	1.1b
	$ \overrightarrow{AB} = 3\sqrt{10}$	A1ft	1.1b
		(2)	
(4 marks)			

Ch. 12: Differentiation

June 2024 Question 15 Paper 2

15. The curve C has equation

$$(x + y)^3 = 3x^2 - 3y - 2$$

- (a) Find an expression for $\frac{dy}{dx}$ in terms of x and y .

(5)

The point $P(1, 0)$ lies on C .

- (b) Show that the normal to C at P has equation

$$y = -2x + 2$$

(2)

- (c) Prove that the normal to C at P does **not** meet C again.

You should use algebra for your proof and make your reasoning clear.

(5)

ANSWER

Question	Scheme	Marks	AOs
15(a)	$3(x+y)^2 \left(1 + \frac{dy}{dx}\right) = 6x - 3 \frac{dy}{dx}$	M1 A1 A1	3.1a 1.1b 1.1b
	$(3(x+y)^2 + 3) \frac{dy}{dx} = 6x - 3(x+y)^2 \Rightarrow \frac{dy}{dx} = \dots$	M1	2.1
	$\frac{dy}{dx} = \frac{6x - 3(x+y)^2}{3(x+y)^2 + 3} \left(\text{oe e.g. } \frac{2x - (x+y)^2}{(x+y)^2 + 1} \right)$	A1	1.1b
		(5)	
(b)	$\frac{dy}{dx} = \frac{6(1) - 3(0+1)^2}{3(0+1)^2 + 3} = \frac{1}{2}$ or e.g. $\frac{dy}{dx} = \frac{6(1) - 3(1)^2 - 6(1)(0) - 3(0)^2}{3(1)^2 + 6(1)(0) + 3(0)^2 + 3} = \frac{1}{2}$ $\Rightarrow y - 0 = -2(x - 1)$ or $\Rightarrow y = -2x + c \Rightarrow 0 = -2 + c \Rightarrow c = \dots$	M1	2.1
	$y = -2x + 2^*$	A1*	1.1b
		(2)	

June 2024 Question 1 Paper 2 (A-level)

1. $y = 4x^3 - 7x^2 + 5x - 10$

(a) Find in simplest form

(i) $\frac{dy}{dx}$

(ii) $\frac{d^2y}{dx^2}$

(3)

(b) Hence find the exact value of x when $\frac{d^2y}{dx^2} = 0$

(2)

ANSWER

Question	Scheme	Marks	AOs
1(a)(i)	$y = 4x^3 - 7x^2 + 5x - 10 \Rightarrow \left(\frac{dy}{dx} = \right) 12x^2 - 14x + 5$	M1 A1	1.1b 1.1b
(ii)	$\left(\frac{d^2y}{dx^2} = \right) 24x - 14$	A1ft	1.1b
		(3)	
(b)	$24x - 14 = 0 \Rightarrow x = \dots$	M1	1.1b
	$x = \frac{7}{12}$ oe e.g. $x = \frac{14}{24}$	A1	1.1b
		(2)	
(5 marks)			

June 2024 Question 4 Paper 1 (A-level)

4. Given that $y = x^2$, use differentiation from first principles to show that $\frac{dy}{dx} = 2x$ (3)

ANSWER

Question	Scheme	Marks	AOs
4	$\frac{(x+h)^2 - x^2}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h}$	M1	2.1
	$= \frac{2xh + h^2}{h}$	A1	1.1b
	$\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} = \lim_{h \rightarrow 0} (2x + h) = 2x^*$	A1*	2.5
		(3)	
(3 marks)			

June 2024 Question 12

12.

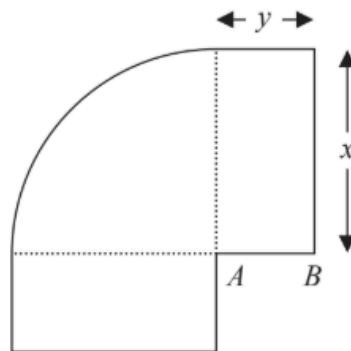


Figure 5

Figure 5 shows the plan view of the design for a swimming pool.

The pool is modelled as a quarter of a circle joined to two equal sized rectangles as shown.

Given that

- the quarter circle has radius x metres
- the rectangles each have length x metres and width y metres
- the total surface area of the swimming pool is 100 m^2

- (a) show that, according to the model, the perimeter P metres of the swimming pool is given by

$$P = 2x + \frac{200}{x} \quad (5)$$

- (b) Use calculus to find the value of x for which P has a stationary value.

(4)

- (c) Prove, by further calculus, that this value of x gives a minimum value for P

(2)

Access to the pool is by side AB shown in Figure 5.

Given that AB must be at least one metre,

- (d) determine, according to the model, whether the swimming pool with the minimum perimeter would be suitable.

(2)

ANSWER

Question	Scheme	Marks	AOs
12(a)	(Surface area $\Rightarrow 2xy + \frac{\pi x^2}{4}$	B1	1.1b
	$2xy + \frac{\pi x^2}{4} = 100 \Rightarrow y = \frac{400 - \pi x^2}{8x} = \frac{50}{x} - \frac{\pi x}{8}$	M1	3.4
	(P =) $2x + 4y + \frac{2\pi x}{4}$	B1	1.1b
	(P =) $2x + 4\left(\frac{400 - \pi x^2}{8x}\right) + \frac{2\pi x}{4}$	M1	3.4
	$P = 2x + \frac{200}{x} \quad *$	A1*	2.1
		(5)	
(b)	$\left(\frac{dP}{dx} =\right) 2 - 200x^{-2}$	M1 A1	1.1b 1.1b
	$2 - 200x^{-2} = 0 \Rightarrow x = \dots$	dM1	3.1b
	$x = 10$	A1	1.1b
		(4)	
(c)	$\left(\frac{d^2P}{dx^2} =\right) "400" x^{-3} \Rightarrow "400" \times 10^{-3} > 0$	M1	1.1b
	e.g. $\frac{d^2P}{dx^2} (= 0.4) > 0$ hence minimum (perimeter)	A1	2.4
		(2)	
(d)	e.g. $y = \frac{400 - \pi \times "10" ^2}{8 \times "10"}$	M1	3.4
	e.g. $y = 1.07$ (m) so yes this would be suitable	A1	2.2a
		(2)	
(13 marks)			

June 2023 Question 1

1. A curve has equation

$$y = \frac{2}{3}x^3 - \frac{7}{2}x^2 - 4x + 5$$

(a) Find $\frac{dy}{dx}$ writing your answer in simplest form.

(2)

(b) Hence find the range of values of x for which y is decreasing.

(4)

ANSWER

Question	Scheme	Marks	AOs
1 (a)	$\left\{ \frac{dy}{dx} = \right\} 2x^2 - 7x - 4$	M1 A1	1.1b 1.1b
		(2)	
(b)	Attempts to solve $\left\{ \frac{dy}{dx} = \right\} 2x^2 - 7x - 4 \dots 0$ e.g., $(2x+1)(x-4) = 0$ leading to $x = \dots$ and $x = \dots$	M1	1.1b
	Correct critical values $x = -\frac{1}{2}, 4$	A1	1.1b
	Chooses inside region for their critical values	dM1	1.1b
	Accept either $-\frac{1}{2} < x < 4$ or $-\frac{1}{2} \leq x \leq 4$	A1	1.1b
		(4)	
(6 marks)			

June 2022 Question 4 Paper 2 (A Level)

4. Given that

$$y = 2x^2$$

use differentiation from first principles to show that

$$\frac{dy}{dx} = 4x$$

(3)

ANSWER

Question	Scheme	Marks	AOs
4	$\frac{2(x+h)^2 - 2x^2}{h} = \dots$	M1	2.1
	$\frac{2(x+h)^2 - 2x^2}{h} = \frac{4xh + 2h^2}{h}$	A1	1.1b
	$\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{4xh + 2h^2}{h} = \lim_{h \rightarrow 0} (4x + 2h) = 4x^*$	A1*	2.5
		(3)	
(3 marks)			

June 2022 Question 15 Paper 1 (A Level)

15.

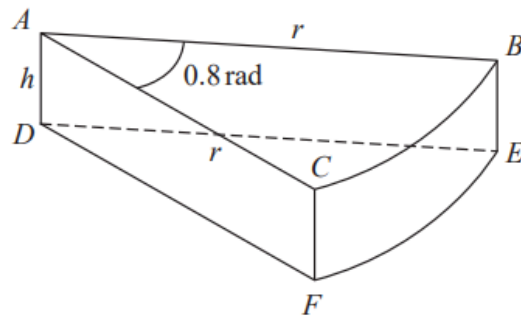


Figure 5

A company makes toys for children.

Figure 5 shows the design for a solid toy that looks like a piece of cheese.

The toy is modelled so that

- face ABC is a sector of a circle with radius r cm and centre A
- angle $BAC = 0.8$ radians
- faces ABC and DEF are congruent
- edges AD , CF and BE are perpendicular to faces ABC and DEF
- edges AD , CF and BE have length h cm

Given that the volume of the toy is 240 cm^3

(a) show that the surface area of the toy, $S \text{ cm}^2$, is given by

$$S = 0.8r^2 + \frac{1680}{r}$$

making your method clear.

(4)

Using algebraic differentiation,

(b) find the value of r for which S has a stationary point.

(4)

(c) Prove, by further differentiation, that this value of r gives the minimum surface area of the toy.

(2)

ANSWER

Question	Scheme	Marks	AOs
15 (a)	Sets up an allowable equation using volume = 240 E.g. $\frac{1}{2}r^2 \times 0.8h = 240 \Rightarrow h = \frac{600}{r^2}$ o.e.	M1 A1	3.4 1.1b
	Attempts to substitute their $h = \frac{600}{r^2}$ into $(S =) \frac{1}{2}r^2 \times 0.8 + \frac{1}{2}r^2 \times 0.8 + 2rh + 0.8rh$	dM1	3.4
	$S = 0.8r^2 + 2.8rh = 0.8r^2 + 2.8 \times \frac{600}{r} = 0.8r^2 + \frac{1680}{r}$ *	A1*	2.1
		(4)	
(b)	$\left(\frac{dS}{dr}\right) = 1.6r - \frac{1680}{r^2}$	M1 A1	3.1a 1.1b
	Sets $\frac{dS}{dr} = 0 \Rightarrow r^3 = 1050$ $r = \text{awrt } 10.2$	dM1 A1	2.1 1.1b
		(4)	
(c)	Attempts to substitute their positive r into $\left(\frac{d^2S}{dr^2}\right) = 1.6 + \frac{3360}{r^3}$ and considers its value or sign	M1	1.1b
	E.g. Correct $\frac{d^2S}{dr^2} = 1.6 + \frac{3360}{r^3}$ with $\frac{d^2S}{dr^2} \bigg _{r=10.2} = 5 > 0$ proving a minimum value of S	A1	1.1b
		(2)	
(10 marks)			

Video solution:

<https://youtu.be/XQcAU5FCr0w>

June 2022 Question 6 Paper 1 (A Level)

6.

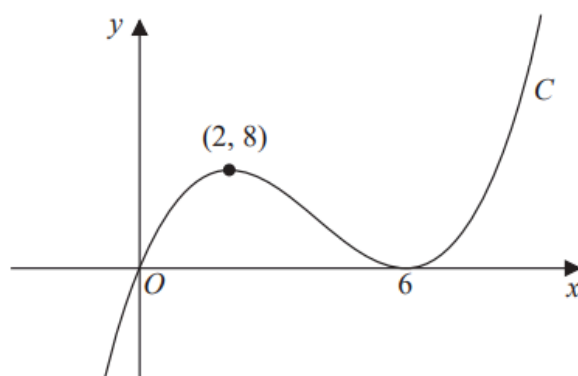


Figure 1

Figure 1 shows a sketch of a curve C with equation $y = f(x)$ where $f(x)$ is a cubic expression in x .

The curve

- passes through the origin
- has a maximum turning point at $(2, 8)$
- has a minimum turning point at $(6, 0)$

(a) Write down the set of values of x for which

$$f'(x) < 0$$

(1)

The line with equation $y = k$, where k is a constant, intersects C at only one point.

(b) Find the set of values of k , giving your answer in set notation.

(2)

(c) Find the equation of C . You may leave your answer in factorised form.

(3)

ANSWER

Question	Scheme	Marks	AOs
6 (a)	$2 < x < 6$	B1	1.1b
		(1)	
(b)	States either $k > 8$ or $k < 0$	M1	3.1a
	States e.g. $\{k : k > 8\} \cup \{k : k < 0\}$	A1	2.5
		(2)	
(c)	Please see notes for alternatives		
	States $y = ax(x-6)^2$ or $f(x) = ax(x-6)^2$	M1	1.1b
	Substitutes (2,8) into $y = ax(x-6)^2$ and attempts to find a	dM1	3.1a
	$y = \frac{1}{4}x(x-6)^2$ or $f(x) = \frac{1}{4}x(x-6)^2$ o.e	A1	2.1
		(3)	
(6 marks)			

June 2022 Question 12

12. A company makes drinks containers out of metal.

The containers are modelled as closed cylinders with base radius r cm and height h cm and the capacity of each container is 355 cm^3

The metal used

- for the circular base and the curved side costs 0.04 pence/cm^2
- for the circular top costs 0.09 pence/cm^2

Both metals used are of negligible thickness.

(a) Show that the total cost, C pence, of the metal for one container is given by

$$C = 0.13\pi r^2 + \frac{28.4}{r} \quad (4)$$

(b) Use calculus to find the value of r for which C is a minimum, giving your answer to 3 significant figures.

(4)

(c) Using $\frac{d^2C}{dr^2}$ prove that the cost is minimised for the value of r found in part (b).

(2)

(d) Hence find the minimum value of C , giving your answer to the nearest integer.

(2)

ANSWER

Question	Scheme	Marks	AOs
12(a)	$V = \pi r^2 h = 355 \Rightarrow h = \frac{355}{\pi r^2}$ $\left(\text{or } rh = \frac{355}{\pi r} \text{ or } \pi rh = \frac{355}{r} \right)$	B1	1.1b
	$C = 0.04(\pi r^2 + 2\pi rh) + 0.09(\pi r^2)$	M1	3.4
	$C = 0.13\pi r^2 + 0.08\pi rh = 0.13\pi r^2 + 0.08\pi r \left(\frac{355}{\pi r^2} \right)$	dM1	2.1
	$C = 0.13\pi r^2 + \frac{28.4}{r} *$	A1*	1.1b
		(4)	
(b)	$\frac{dC}{dr} = 0.26\pi r - \frac{28.4}{r^2}$	M1 A1	3.4 1.1b
	$\frac{dC}{dr} = 0 \Rightarrow r^3 = \frac{28.4}{0.26\pi} \Rightarrow r = \dots$	M1	1.1b
	$r = \sqrt[3]{\frac{1420}{13\pi}} = 3.26\dots$	A1	1.1b
		(4)	
(c)	$\left(\frac{d^2C}{dr^2} = \right) 0.26\pi + \frac{56.8}{r^3} = 0.26\pi + \frac{56.8}{"3.26"}^3$	M1	1.1b
	$\left(\frac{d^2C}{dr^2} = \right) (2.45\dots) > 0$ Hence minimum (cost)	A1	2.4
		(2)	
(d)	$C = 0.13\pi ("3.26")^2 + \frac{28.4}{"3.26"}$	M1	3.4
	$(C =) 13$	A1	1.1b
		(2)	
(12 marks)			

Video Solution:

<https://youtu.be/MbHVtg0QoWk>

November 2021 Question 5

5.

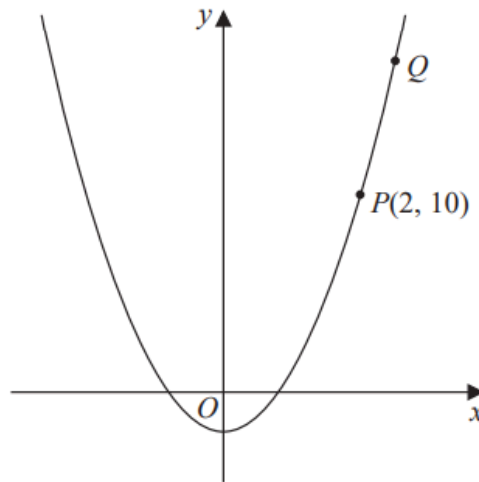


Figure 1

Figure 1 shows part of the curve with equation $y = 3x^2 - 2$

The point $P(2, 10)$ lies on the curve.

(a) Find the gradient of the tangent to the curve at P .

(2)

The point Q with x coordinate $2 + h$ also lies on the curve.

(b) Find the gradient of the line PQ , giving your answer in terms of h in simplest form.

(3)

(c) Explain briefly the relationship between part (b) and the answer to part (a).

(1)

ANSWER

Question	Scheme	Marks	AOs
5(a)	Attempts to find the value of $\frac{dy}{dx}$ at $x = 2$	M1	1.1b
	$\frac{dy}{dx} = 6x \Rightarrow$ gradient of tangent at P is 12	A1	1.1b
		(2)	
(b)	Gradient $PQ = \frac{3(2+h)^2 - 2 - 10}{(2+h) - 2}$ oe	B1	1.1b
	$= \frac{3(2+h)^2 - 12}{(2+h) - 2} = \frac{12h + 3h^2}{h}$	M1	1.1b
	$= 12 + 3h$	A1	2.1
		(3)	
(c)	Explains that as $h \rightarrow 0$, $12 + 3h \rightarrow 12$ and states that the gradient of the chord tends to the gradient of (the tangent to) the curve	B1	2.4
		(1)	
(6 marks)			

November 2021 Question 5 Paper 2 (A-Level)

5. The curve C has equation

$$y = 5x^4 - 24x^3 + 42x^2 - 32x + 11 \quad x \in \mathbb{R}$$

(a) Find

(i) $\frac{dy}{dx}$

(ii) $\frac{d^2y}{dx^2}$

(3)

(b) (i) Verify that C has a stationary point at $x = 1$

(ii) Show that this stationary point is a point of inflection, giving reasons for your answer.

(4)

ANSWER

Question	Scheme	Marks	AOs
5(a)(i)	$\frac{dy}{dx} = 20x^3 - 72x^2 + 84x - 32$	M1 A1	1.1b 1.1b
(ii)	$\frac{d^2y}{dx^2} = 60x^2 - 144x + 84$	A1ft	1.1b
		(3)	
(b)(i)	$x = 1 \Rightarrow \frac{dy}{dx} = 20 - 72 + 84 - 32$	M1	1.1b
	$\frac{dy}{dx} = 0$ so there is a stationary point at $x = 1$	A1	2.1
	Alternative for (b)(i)		
	$20x^3 - 72x^2 + 84x - 32 = 4(x-1)^2(5x-8) = 0 \Rightarrow x = \dots$	M1	1.1b
	When $x = 1$, $\frac{dy}{dx} = 0$ so there is a stationary point	A1	2.1
(b)(ii)	Note that in (b)(ii) there are no marks for <u>just</u> evaluating $\left(\frac{d^2y}{dx^2}\right)_{x=1}$		
	E.g. $\left(\frac{d^2y}{dx^2}\right)_{x=0.8} = \dots \left(\frac{d^2y}{dx^2}\right)_{x=1.2} = \dots$	M1	2.1
	$\left(\frac{d^2y}{dx^2}\right)_{x=0.8} > 0, \left(\frac{d^2y}{dx^2}\right)_{x=1.2} < 0$ Hence point of inflection	A1	2.2a
		(4)	
	Alternative 1 for (b)(ii)		
	$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 60x^2 - 144x + 84 = 0$ (is inconclusive) $\left(\frac{d^3y}{dx^3}\right) = 120x - 144 \Rightarrow \left(\frac{d^3y}{dx^3}\right)_{x=1} = \dots$	M1	2.1
	$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 0$ and $\left(\frac{d^3y}{dx^3}\right)_{x=1} \neq 0$ Hence point of inflection	A1	2.2a
	Alternative 2 for (b)(ii)		
	E.g. $\left(\frac{dy}{dx}\right)_{x=0.8} = \dots \left(\frac{dy}{dx}\right)_{x=1.2} = \dots$	M1	2.1
	$\left(\frac{dy}{dx}\right)_{x=0.8} < 0, \left(\frac{dy}{dx}\right)_{x=1.2} < 0$ Hence point of inflection	A1	2.2a

November 2021 Question 14 Paper 2 (A-Level)

14.

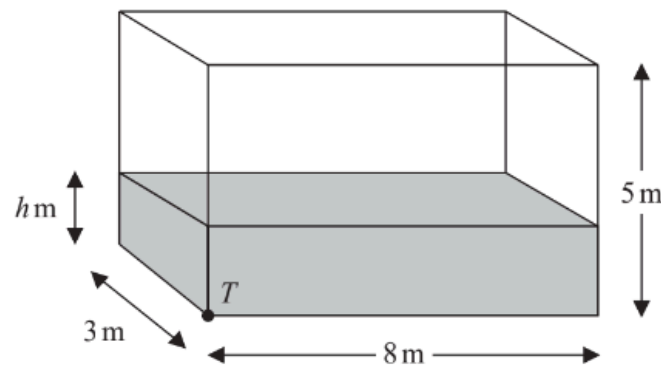


Figure 5

Water flows at a constant rate into a large tank.

The tank is a cuboid, with all sides of negligible thickness.

The base of the tank measures 8 m by 3 m and the height of the tank is 5 m.

There is a tap at a point T at the bottom of the tank, as shown in Figure 5.

At time t minutes after the tap has been opened

- the depth of water in the tank is h metres
- water is flowing into the tank at a constant rate of 0.48 m^3 per minute
- water is modelled as leaving the tank through the tap at a rate of $0.1h \text{ m}^3$ per minute

(a) Show that, according to the model,

$$1200 \frac{dh}{dt} = 24 - 5h \quad (4)$$

Given that when the tap was opened, the depth of water in the tank was 2 m,

(b) show that, according to the model,

$$h = A + Be^{-kt}$$

where A , B and k are constants to be found. (6)

Given that the tap remains open,

(c) determine, according to the model, whether the tank will ever become full, giving a reason for your answer. (2)

ANSWER

Question	Scheme	Marks	AOs
14(a)	$\frac{dV}{dt} = 0.48 - 0.1h$	B1	3.1b
	$V = 24h \Rightarrow \frac{dV}{dh} = 24 \text{ or } \frac{dh}{dV} = \frac{1}{24}$	B1	3.1b
	$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV} = \frac{0.48 - 0.1h}{24}$ or e.g. $\frac{dV}{dt} = \frac{dV}{dh} \frac{dh}{dt} \Rightarrow 0.48 - 0.1h = 24 \frac{dh}{dt}$	M1	2.1
	$1200 \frac{dh}{dt} = 24 - 5h^*$	A1*	1.1b
		(4)	
(b)	$1200 \frac{dh}{dt} = 24 - 5h \Rightarrow \int \frac{1200}{24 - 5h} dh = \int dt$ $\Rightarrow \text{e.g. } \alpha \ln(24 - 5h) = t(+c) \text{ oe}$ or $1200 \frac{dh}{dt} = 24 - 5h \Rightarrow \frac{dt}{dh} = \frac{1200}{24 - 5h}$ $\Rightarrow \text{e.g. } t(+c) = \alpha \ln(24 - 5h) \text{ oe}$	M1	3.1a
	$t = -240 \ln(24 - 5h)(+c) \text{ oe}$	A1	1.1b
	$t = 0, h = 2 \Rightarrow 0 = -240 \ln(24 - 10) + c \Rightarrow c = \dots (240 \ln 14)$	M1	3.4

	$t = 240 \ln(14) - 240 \ln(24 - 5h)$	A1	1.1b
	$t = 240 \ln \frac{14}{24 - 5h} \Rightarrow \frac{t}{240} = \ln \frac{14}{24 - 5h} \Rightarrow e^{\frac{t}{240}} = \frac{14}{24 - 5h}$ $\Rightarrow 14e^{-\frac{t}{240}} = 24 - 5h \Rightarrow h = \dots$	ddM1	2.1
	$h = 4.8 - 2.8e^{-\frac{t}{240}}$ oe e.g. $h = \frac{24}{5} - \frac{14}{5}e^{-\frac{t}{240}}$	A1	3.3
		(6)	
(c)	Examples: - As $t \rightarrow \infty, e^{-\frac{t}{240}} \rightarrow 0$ - When $h > 4.8, \frac{dV}{dt} < 0$ - Flow in = flow out at max h so $0.1h = 4.8 \rightarrow h = 4.8$ - As $e^{-\frac{t}{240}} > 0, h < 4.8$ $h = 5 \Rightarrow \frac{dV}{dt} = -0.02$ or $\frac{dh}{dt} = -\frac{1}{1200}$ $\frac{dh}{dt} = 0 \Rightarrow h = 4.8$ $h = 5 \Rightarrow 4.8 - 2.8e^{-\frac{t}{240}} = 5 \Rightarrow e^{-\frac{t}{240}} < 0$	M1	3.1b
	- The limit for h (according to the model) is 4.8m and the tank is 5m high so the tank will never become full - If $h = 5$ the tank would be emptying so can never be full - The equation can't be solved when $h = 5$	A1	3.2a

November 2021 Question 16

16. The curve C has equation $y = f(x)$ where

$$f(x) = ax^3 + 15x^2 - 39x + b$$

and a and b are constants.

Given

- the point $(2, 10)$ lies on C
- the gradient of the curve at $(2, 10)$ is -3

(a) (i) show that the value of a is -2

(ii) find the value of b .

(4)

(b) Hence show that C has no stationary points.

(3)

(c) Write $f(x)$ in the form $(x - 4)Q(x)$ where $Q(x)$ is a quadratic expression to be found.

(2)

(d) Hence deduce the coordinates of the points of intersection of the curve with equation

$$y = f(0.2x)$$

and the coordinate axes.

(2)

ANSWER

Question	Scheme	Marks	AOs
16 (a) (i)	Uses $\frac{dy}{dx} = -3$ at $x = 2 \Rightarrow 12a + 60 - 39 = -3$	M1	1.1b
	Solves a correct equation and shows one correct intermediate step $12a + 60 - 39 = -3 \Rightarrow 12a = -24 \Rightarrow a = -2$ *	A1*	2.1
(a) (ii)	Uses the fact that $(2, 10)$ lies on C $10 = 8a + 60 - 78 + b$	M1	3.1a
	Subs $a = -2$ into $10 = 8a + 60 - 78 + b \Rightarrow b = 44$	A1	1.1b
		(4)	
(b)	$f(x) = -2x^3 + 15x^2 - 39x + 44 \Rightarrow f'(x) = -6x^2 + 30x - 39$	B1	1.1b
	Attempts to show that $-6x^2 + 30x - 39$ has no roots Eg. calculates $b^2 - 4ac = 30^2 - 4 \times -6 \times -39 = -36$	M1	3.1a
	States that as $f'(x) \neq 0 \Rightarrow$ hence $f(x)$ has no turning points *	A1*	2.4
		(3)	
(c)	$-2x^3 + 15x^2 - 39x + 44 \equiv (x - 4)(-2x^2 + 7x - 11)$	M1 A1	1.1b 1.1b
		(2)	
(d)	Deduces either intercept. $(0, 44)$ or $(20, 0)$	B1 ft	1.1b
	Deduces both intercepts $(0, 44)$ and $(20, 0)$	B1 ft	2.2a
		(2)	
(11 marks)			

Video solution:

<https://youtu.be/iJGi0JletYA>

November 2020 Question 1

1. A curve has equation

$$y = 2x^3 - 4x + 5$$

Find the equation of the tangent to the curve at the point $P(2, 13)$.

Write your answer in the form $y = mx + c$, where m and c are integers to be found.

Solutions relying on calculator technology are not acceptable.

(5)

ANSWER

Question	Scheme	Marks	AOs
1	Attempts to differentiate $x^n \rightarrow x^{n-1}$ seen once	M1	1.1b
	$y = 2x^3 - 4x + 5 \Rightarrow \frac{dy}{dx} = 6x^2 - 4$	A1	1.1b
	For substituting $x = 2$ into their $\frac{dy}{dx} = 6x^2 - 4$	dM1	1.1b
	For a correct method of finding a tangent at $P(2,13)$. Score for $y - 13 = "20"(x - 2)$	ddM1	1.1b
	$y = 20x - 27$	A1	1.1b
		(5)	
(5 marks)			

June 2019 Question 5

5. A curve has equation

$$y = 3x^2 + \frac{24}{x} + 2 \quad x > 0$$

(a) Find, in simplest form, $\frac{dy}{dx}$

(3)

(b) Hence find the exact range of values of x for which the curve is increasing.

(2)

ANSWER

Question	Scheme	Marks	AOs
5(a)	$x^n \rightarrow x^{n-1}$	M1	1.1b
	$\left(\frac{dy}{dx}\right) = 6x - \frac{24}{x^2}$	A1 A1	1.1b 1.1b
		(3)	
(b)	Attempts $6x - \frac{24}{x^2} > 0 \Rightarrow x >$	M1	1.1b
	$x > \sqrt[3]{4} \text{ or } x \geq \sqrt[3]{4}$	A1	2.5
		(2)	
(5 marks)			

June 2019 Question 13 Paper 2 (A-Level)

13.

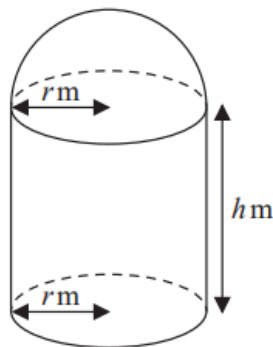


Figure 9

[A sphere of radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$]

A manufacturer produces a storage tank.

The tank is modelled in the shape of a hollow circular cylinder closed at one end with a hemispherical shell at the other end as shown in Figure 9.

The walls of the tank are assumed to have negligible thickness.

The cylinder has radius r metres and height h metres and the hemisphere has radius r metres.

The volume of the tank is 6 m^3 .

(a) Show that, according to the model, the surface area of the tank, in m^2 , is given by

$$\frac{12}{r} + \frac{5}{3}\pi r^2 \quad (4)$$

The manufacturer needs to minimise the surface area of the tank.

(b) Use calculus to find the radius of the tank for which the surface area is a minimum. (4)

(c) Calculate the minimum surface area of the tank, giving your answer to the nearest integer. (2)

ANSWER

Question	Scheme	Marks	AOs
13 (a)	States or uses $6 = \pi r^2 h + \frac{2}{3} \pi r^3$	B1	1.1a
	$\Rightarrow h = \frac{6}{\pi r^2} - \frac{2}{3} r, \pi h = \frac{6}{r^2} - \frac{2}{3} \pi r, \pi r h = \frac{6}{r} - \frac{2}{3} \pi r^2, r h = \frac{6}{\pi r} - \frac{2}{3} r^2$		
	$A = \pi r^2 + 2\pi r h + 2\pi r^2 \{\Rightarrow A = 3\pi r^2 + 2\pi r h\}$		
	$A = 2\pi r^2 + 2\pi r \left(\frac{6}{\pi r^2} - \frac{2}{3} r \right) + \pi r^2$	M1	3.1a
		A1	1.1b
	$A = 3\pi r^2 + \frac{12}{r} - \frac{4}{3} \pi r^2 \Rightarrow A = \frac{12}{r} + \frac{5}{3} \pi r^2 *$	A1*	2.1
		(4)	
(b)	$\left\{ A = 12r^{-1} + \frac{5}{3} \pi r^2 \Rightarrow \right\} \frac{dA}{dr} = -12r^{-2} + \frac{10}{3} \pi r$	M1	3.4
		A1	1.1b
	$\left\{ \frac{dA}{dr} = 0 \Rightarrow \right\} -\frac{12}{r^2} + \frac{10}{3} \pi r = 0 \Rightarrow -36 + 10\pi r^3 = 0 \Rightarrow r^{\pm 3} = \dots \left\{ = \frac{18}{5\pi} \right\}$	M1	2.1
	$r = 1.046447736... \Rightarrow r = 1.05 \text{ (m) (3 sf) or awrt 1.05 (m)}$	A1	1.1b
	Note: Give final A1 for correct exact values for r	(4)	
(c)	$A_{\min} = \frac{12}{(1.046...)} + \frac{5}{3} \pi (1.046...)^2$	M1	3.4
	$\{ A_{\min} = 17.20... \Rightarrow \} A = 17 \text{ (m}^2\text{) or } A = \text{awrt } 17 \text{ (m}^2\text{)}$	A1ft	1.1b
		(2)	
(10 marks)			

June 2018 Question 8

8. A lorry is driven between London and Newcastle.

In a simple model, the cost of the journey £ C when the lorry is driven at a steady speed of v kilometres per hour is

$$C = \frac{1500}{v} + \frac{2v}{11} + 60$$

- (a) Find, according to this model,

(i) the value of v that minimises the cost of the journey,

(ii) the minimum cost of the journey.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(6)

- (b) Prove by using $\frac{d^2C}{dv^2}$ that the cost is minimised at the speed found in (a)(i).

(2)

- (c) State one limitation of this model.

(1)

ANSWER

Question	Scheme	Marks	AOs
8 (a)(i)	$C = \frac{1500}{v} + \frac{2v}{11} + 60 \Rightarrow \frac{dC}{dv} = -\frac{1500}{v^2} + \frac{2}{11}$	M1 A1	3.1b 1.1b
	Sets $\frac{dC}{dv} = 0 \Rightarrow v^2 = 8250$	M1	1.1b
	$\Rightarrow v = \sqrt{8250} \Rightarrow v = 90.8 \text{ (km h}^{-1}\text{)}$	A1	1.1b
(ii)	For substituting their $v = 90.8$ in $C = \frac{1500}{v} + \frac{2v}{11} + 60$	M1	3.4
	Minimum cost = awrt (£) 93	A1 ft	1.1b
		(6)	
(b)	Finds $\frac{d^2C}{dv^2} = +\frac{3000}{v^3}$ at $v = 90.8$	M1	1.1b
	$\frac{d^2C}{dv^2} = (+0.004) > 0$ hence minimum (cost)	A1 ft	2.4
		(2)	
(c)	It would be impossible to drive at this speed over the whole journey	B1	3.5b
		(1)	
(9 marks)			

June 2018 Question 10

10. Prove, from first principles, that the derivative of x^3 is $3x^2$

(4)

ANSWER

Question	Scheme	Marks	AOs
10	Considers $\frac{(x+h)^3 - x^3}{h}$	B1	2.1
	Expands $(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$	M1	1.1b
	so gradient (of chord) = $\frac{3x^2h + 3xh^2 + h^3}{h} = 3x^2 + 3xh + h^2$	A1	1.1b
	States as $h \rightarrow 0$, $3x^2 + 3xh + h^2 \rightarrow 3x^2$ so derivative = $3x^2$ *	A1*	2.5
(4 marks)			

June 2018 Question 2 Paper 1

2. A curve C has equation

$$y = x^2 - 2x - 24\sqrt{x}, \quad x > 0$$

(a) Find (i) $\frac{dy}{dx}$

(ii) $\frac{d^2y}{dx^2}$

(3)

(b) Verify that C has a stationary point when $x = 4$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

ANSWER

Question	Scheme	Marks	AOs
2(a)	(i) $\frac{dy}{dx} = 2x - 2 - 12x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	(ii) $\frac{d^2y}{dx^2} = 2 + 6x^{-\frac{3}{2}}$	B1ft	1.1b
		(3)	
(b)	Substitutes $x = 4$ into their $\frac{dy}{dx} = 2 \times 4 - 2 - 12 \times 4^{-\frac{1}{2}} = \dots$	M1	1.1b
	Shows $\frac{dy}{dx} = 0$ and states "hence there is a stationary point" oe	A1	2.1
		(2)	
(c)	Substitutes $x = 4$ into their $\frac{d^2y}{dx^2} = 2 + 6 \times 4^{-\frac{3}{2}} = (2.75)$	M1	1.1b
	$\frac{d^2y}{dx^2} = 2.75 > 0$ and states "hence minimum"	A1ft	2.2a
		(2)	
(7 marks)			

June 2018 Question 11 Paper 2 (A-Level)

11.

$$\frac{1 + 11x - 6x^2}{(x - 3)(1 - 2x)} \equiv A + \frac{B}{(x - 3)} + \frac{C}{(1 - 2x)}$$

(a) Find the values of the constants A , B and C .

(4)

$$f(x) = \frac{1 + 11x - 6x^2}{(x - 3)(1 - 2x)} \quad x > 3$$

(b) Prove that $f(x)$ is a decreasing function.

(3)

ANSWER

Question	Scheme	Marks	AOs
11	$\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv A + \frac{B}{(x-3)} + \frac{C}{(1-2x)}$		
(a) Way 1	$1+11x-6x^2 \equiv A(1-2x)(x-3) + B(1-2x) + C(x-3) \Rightarrow B = \dots, C = \dots$	M1	2.1
	$A = 3$	B1	1.1b
	Uses substitution or compares terms to find either $B = \dots$ or $C = \dots$	M1	1.1b
	$B = 4$ and $C = -2$ which have been found using a correct identity	A1	1.1b
		(4)	
(a) Way 2	{long division gives} $\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv 3 + \frac{-10x+10}{(x-3)(1-2x)}$		
	$-10x+10 \equiv B(1-2x) + C(x-3) \Rightarrow B = \dots, C = \dots$	M1	2.1
	$A = 3$	B1	1.1b
	Uses substitution or compares terms to find either $B = \dots$ or $C = \dots$	M1	1.1b
	$B = 4$ and $C = -2$ which have been found using $-10x+10 \equiv B(1-2x) + C(x-3)$	A1	1.1b
		(4)	
(b)	$f(x) = 3 + \frac{4}{(x-3)} - \frac{2}{(1-2x)} \quad \{ = 3 + 4(x-3)^{-1} - 2(1-2x)^{-1} \}; x > 3$		
	$f'(x) = -4(x-3)^{-2} - 4(1-2x)^{-2} \left\{ = -\frac{4}{(x-3)^2} - \frac{4}{(1-2x)^2} \right\}$	M1	2.1
		A1ft	1.1b
	Correct $f'(x)$ and as $(x-3)^2 > 0$ and $(1-2x)^2 > 0$, then $f'(x) = -(+ve) - (+ve) < 0$, so $f(x)$ is a decreasing function	A1	2.4
		(3)	
(7 marks)			

June 2018 Question 10 Core 1

10.

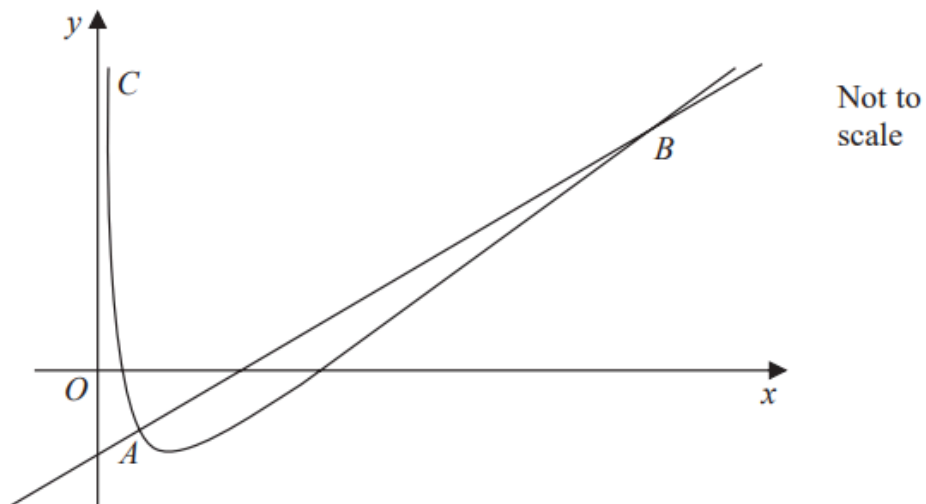


Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = \frac{1}{2}x + \frac{27}{x} - 12, \quad x > 0$$

The point A lies on C and has coordinates $\left(3, -\frac{3}{2}\right)$.

- (a) Show that the equation of the normal to C at A can be written as $10y = 4x - 27$ (5)

The normal to C at A meets C again at the point B , as shown in Figure 3.

- (b) Use algebra to find the coordinates of B . (5)

ANSWER

Question Number	Scheme		Marks
10(a)	$\frac{dy}{dx} = \frac{1}{2} - \frac{27}{x^2}$	M1: $\frac{1}{2}$ or $-\frac{27}{x^2}$	M1A1
		A1: $\frac{dy}{dx} = \frac{1}{2} - \frac{27}{x^2}$ oe e.g. $\frac{1}{2}x^0 - 27x^{-2}$	
	$x = 3 \Rightarrow \frac{dy}{dx} = \frac{1}{2} - \frac{27}{9} = \left(-\frac{5}{2}\right)$	Substitutes $x = 3$ into their $\frac{dy}{dx}$ to obtain a numerical gradient	M1
	$m_T = -\frac{5}{2} \Rightarrow m_N = -1 \div -\frac{5}{2}$ $\Rightarrow y - \left(-\frac{3}{2}\right) = \frac{2}{5}(x - 3)$	The correct method to find the equation of a normal. Uses $-\frac{1}{m_T}$ with $\left(3, -\frac{3}{2}\right)$ where m_T has come from calculus. If using $y = mx + c$ must reach as far as $c = \dots$	M1
	$10y = 4x - 27^*$	Cso (correct equation must be seen in (a))	A1*
			(5)

(b)	$\frac{1}{2}x + \frac{27}{x} - 12 = \frac{4x - 27}{10}$ or $y = \frac{10y + 27}{8} + \frac{108}{10y + 27} - 12$	Equate equations to produce an equation just in x or just in y . Do not allow e.g. $\frac{1}{2}x^2 + 27 - 12x = \frac{4x - 27}{10}$ Unless $\frac{1}{2}x + \frac{27}{x} - 12 = \frac{4x - 27}{10}$ was seen previously. Allow sign slips only.	M1
	$x^2 - 93x + 270 = 0$ or $20y^2 - 636y - 999 = 0$	Correct 3 term quadratic equation (or any multiple of). Allow terms on both sides e.g. $x^2 - 93x = -270$ (The “= 0” may be implied by their attempt to solve)	A1
	$(x - 90)(x - 3) = 0 \Rightarrow x = \dots$ or $x = \frac{93 \pm \sqrt{93^2 - 4 \times 270}}{2}$ or $(10y - 333)(2y + 3) = 0 \Rightarrow y = \dots$ or $y = \frac{636 \pm \sqrt{636^2 - 4 \times 20 \times (-999)}}{2 \times 20}$	Attempt to solve a 3TQ (see general guidance) leading to at least one for x or y . Dependent on the first method mark.	dM1
	$x = 90$ or $y = 33.3$ oe	Cso. The x must be 90 and the y an equivalent number such as e.g. $\frac{333}{10}$	A1
	$x = 90$ and $y = 33.3$ oe	Cso. The x must be 90 and the y an equivalent number such as e.g. $\frac{333}{10}$	A1
			(5)
			(10 marks)

Ch. 13: Integration

June 2024 Question 10 Paper 1 (A-level)

10.

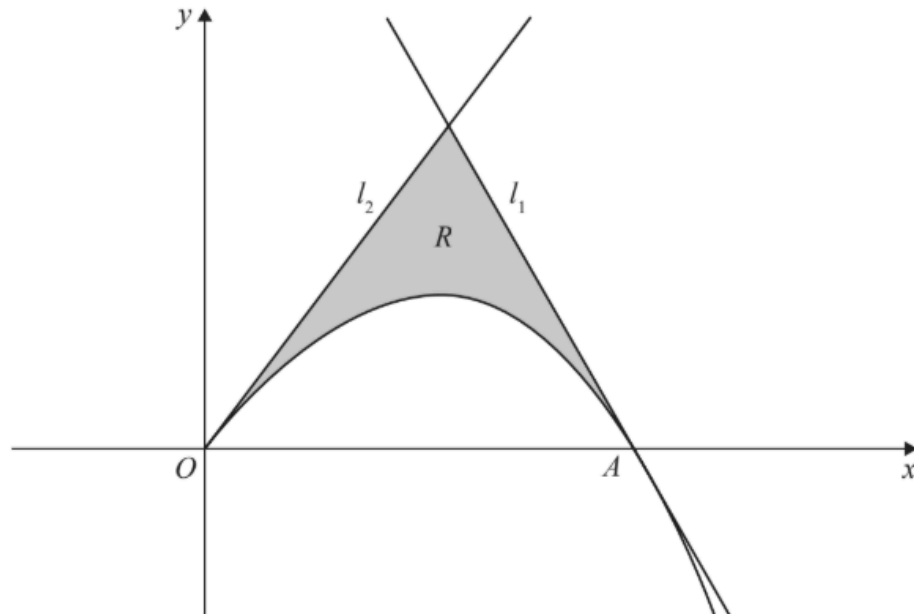


Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = 8x - x^{\frac{5}{2}} \quad x \geq 0$$

The curve crosses the x -axis at the point A .

(a) Verify that the x coordinate of A is 4

(1)

The line l_1 is the tangent to the curve at A .

(b) Use calculus to show that an equation of line l_1 is

$$12x + y = 48$$

(3)

The line l_2 has equation $y = 8x$

The region R , shown shaded in Figure 3, is bounded by the curve, the line l_1 and the line l_2

(c) Use algebraic integration to find the exact area of R .

(5)

ANSWER

Question	Scheme	Marks	AOs
10(a)	$8(4) - 4^{\frac{5}{2}} = 32 - 32 = 0$	B1	1.1b
		(1)	
(b)	$8 - \frac{5}{2}x^{\frac{3}{2}}$	B1	1.1b
	$x = 4 \Rightarrow \left\{ \frac{dy}{dx} \right\} = 8 - \frac{5}{2} \times 8 = -12$ $\Rightarrow y\{-0\} = "-12"(x - 4)$	M1	1.1b
	$12x + y = 48$ *	A1*	1.1b
		(3)	
(c)	Attempts to find one of the coordinates of the point of intersection $y = 8x, 12x + y = 48 \Rightarrow y = 19.2$ (or $x = 2.4$)	M1	1.1b
	Triangle area is $\frac{1}{2} \times 4 \times "19.2" \left(= 38.4 \text{ or } \frac{192}{5} \right)$ or $\int_0^{2.4} 8x \, dx + \int_{2.4}^4 "(48 - 12x)" \, dx$	dM1	3.1a
	$\int \left(8x - x^{\frac{5}{2}} \right) dx = 4x^2 - \frac{2}{7}x^{\frac{7}{2}}$	B1	1.1b
	$A = 38.4 - \left["4x^2 - \frac{2}{7}x^{\frac{7}{2}}" \right]_0^4 = 38.4 - 64 + \frac{256}{7}$	ddM1	3.1a
	$= \frac{384}{35}$	A1	1.1b
		(5)	

June 2024 Question 8

8.

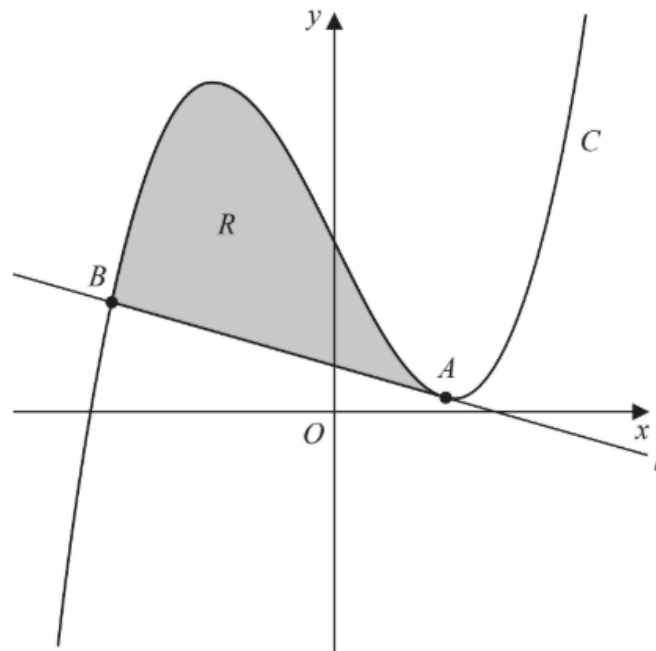


Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of the curve C with equation

$$y = x^3 - 14x + 23$$

The line l is the tangent to C at the point A , also shown in Figure 3.

Given that l has equation $y = -2x + 7$

(a) show, using calculus, that the x coordinate of A is 2

(3)

The line l cuts C again at the point B .

(b) Verify that the x coordinate of B is -4

(2)

The finite region, R , shown shaded in Figure 3, is bounded by C and l .

Using algebraic integration,

(c) show that the area of R is 108

(5)

ANSWER

Question	Scheme	Marks	AOs
8(a)	$\left(\frac{dy}{dx} = \right) 3x^2 - 14 = -2$ or $\frac{d}{dx}(\pm(\text{curve-line})) = \pm(3x^2 - 12) = 0$	M1	3.1a
	e.g. $3x^2 - 14 = -2 \Rightarrow 3x^2 = 12 \Rightarrow x = \dots$	dM1	1.1b
	$x = 2$ only *	A1*	2.2a
		(3)	
(b)	e.g. Substitutes $x = -4$ into $y = x^3 - 14x + 23$ and $y = -2x + 7$	M1	1.1b
	Correct solution + conclusion * (see notes)	A1*	2.4
		(2)	
(c)	$\int (x^3 - 14x + 23) dx = \frac{x^4}{4} - 7x^2 + 23x \quad (+c)$	M1 A1	1.1b 1.1b
	$= \left(\frac{2^4}{4} - 7 \times 2^2 + 23 \times 2 \right) - \left(\frac{(-4)^4}{4} - 7(-4)^2 + 23(-4) \right)$	dM1	1.1b
	Area $= \int_{-4}^2 (x^3 - 14x + 23) dx - \frac{1}{2} \times (15 + 3) \times (2 - (-4))$		
	$= \left(\frac{2^4}{4} - 7 \times 2^2 + 23 \times 2 \right) - \left(\frac{(-4)^4}{4} - 7(-4)^2 + 23(-4) \right) - 54$	dM1	3.1a
	Area = 108 *	A1*	2.1
		(5)	

June 2024 Question 1

1. Find

$$\int \frac{2\sqrt{x} - 3}{x^2} dx$$

giving your answer in simplest form.

(4)

ANSWER

Question	Scheme	Marks	AOs
1	$\frac{2\sqrt{x}}{x^2} \rightarrow \dots x^{-\frac{3}{2}}$ or $\frac{-3}{x^2} \rightarrow \dots x^{-2}$	M1	1.1b
	$\int \frac{2\sqrt{x} - 3}{x^2} dx = \int 2x^{-\frac{3}{2}} - 3x^{-2} dx$	A1	1.1b
	$\dots x^{-\frac{3}{2}} \rightarrow \dots x^{-\frac{1}{2}}$ or $\dots x^{-2} \rightarrow \dots x^{-1}$	dM1	1.1b
	$\int 2x^{-\frac{3}{2}} - 3x^{-2} dx = -4x^{-\frac{1}{2}} + 3x^{-1} + c$	A1	1.1b
		(4)	
(4 marks)			

June 2023 Question 16

16. A curve has equation $y = f(x)$, $x \geq 0$

Given that

- $f'(x) = 4x + a\sqrt{x} + b$, where a and b are constants
- the curve has a stationary point at $(4, 3)$
- the curve meets the y -axis at -5

find $f(x)$, giving your answer in simplest form.

(6)

ANSWER

Question	Scheme	Marks	AOs
16	Sets $f'(4) = 0 \Rightarrow 16 + 2a + b = 0$	M1	2.1
	Integrates $f'(x) = 4x + a\sqrt{x} + b \Rightarrow \{f(x) = \} 2x^2 + \frac{2}{3}ax^{\frac{3}{2}} + bx \{+c\}$	M1 A1ft	1.1b 1.1b
	Deduces that $c = -5$	B1	2.2a
	Full and complete method using the given information $f'(4) = 0$ and $f(4) = 3$ in order to find values for a and b Note: $a = -15$ and $b = 14$	ddM1	3.1a
	$\{f(x) = \} 2x^2 - 10x^{\frac{3}{2}} + 14x - 5$	A1	1.1b
		(6)	
(6 marks)			

June 2023 Question 5

5.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

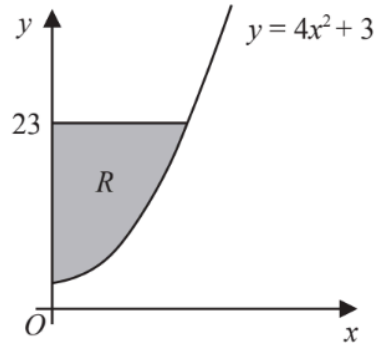


Figure 2

The finite region R , shown shaded in Figure 2, is bounded by the curve with equation $y = 4x^2 + 3$, the y -axis and the line with equation $y = 23$

Show that the exact area of R is $k\sqrt{5}$ where k is a rational constant to be found.

(5)

ANSWER

Question	Scheme	Marks	AOs
5	States or uses the upper limit is $\sqrt{5}$	B1	1.1b
	$\int 4x^2 + 3 \, dx = \frac{4}{3}x^3 + 3x$	M1 A1	1.1b 1.1b
	Full method of finding the area of R e.g. $23\sqrt{5} - \left[\frac{4}{3}x^3 + 3x \right]_0^{\sqrt{5}} = \dots$ e.g. $\left[20x - \frac{4}{3}x^3 \right]_0^{\sqrt{5}} = \dots$	M1	2.1
	$\Rightarrow \text{Area } R = \frac{40}{3}\sqrt{5}$	A1	1.1b
		(5)	
(5 marks)			

June 2023 Question 5 Paper 2 (A-Level)

5. The curve C has equation $y = f(x)$

The curve

- passes through the point $P(3, -10)$
- has a turning point at P

Given that

$$\frac{dy}{dx} = 2x^3 - 9x^2 + 5x + k$$

where k is a constant,

(a) show that $k = 12$

(2)

(b) Hence find the coordinates of the point where C crosses the y -axis.

(3)

ANSWER

Question	Scheme	Marks	AOs
5(a)	$2(3)^3 - 9(3)^2 + 5(3) + k = 0 \Rightarrow k = \dots$	M1	1.1b
	$54 - 81 + 15 + k = 0 \Rightarrow k = 12^* \text{ or } -12 + k = 0 \Rightarrow k = 12^*$	A1*	1.1b
		(2)	
	(a) Alternative by verification:		
	$2(3)^3 - 9(3)^2 + 5(3) + 12 = 0$	M1	1.1b
	$54 - 81 + 15 + 12 = 0 \text{ Hence } k = 12^*$	A1*	1.1b
		(2)	
(b)	$\int (2x^3 - 9x^2 + 5x + 12) dx \dots x^4 \pm \dots x^3 \pm \dots x^2 \pm \dots x \pm \dots$	M1	3.1a
	$\frac{1}{2}(3)^4 - 3(3)^3 + \frac{5}{2}(3)^2 + 12(3) + c = -10 \Rightarrow c = \dots$	dM1	1.1b
	(0, -28)	A1	2.2a
		(3)	
(5 marks)			

June 2023 Question 1 Paper 1 (A-Level)

1. Find

$$\int \frac{x^{\frac{1}{2}}(2x-5)}{3} dx$$

writing each term in simplest form.

(4)

ANSWER

Question	Scheme	Marks	AOs
1	$x^{\frac{1}{2}}(2x-5) = \dots x^{\frac{3}{2}} + \dots x^{\frac{1}{2}} \text{ or } \frac{x^{\frac{1}{2}}(2x-5)}{3} = \frac{\dots x^{\frac{3}{2}} + \dots x^{\frac{1}{2}}}{3}$	M1	1.1b
	$\frac{2x^{\frac{3}{2}}}{3} - \frac{5x^{\frac{1}{2}}}{3}$	A1	1.1b
	$\int \frac{2x^{\frac{3}{2}}}{3} - \frac{5x^{\frac{1}{2}}}{3} dx = \dots x^{\frac{5}{2}} \pm \dots x^{\frac{3}{2}} (+c)$	dM1	1.1b
	$\frac{4}{15}x^{\frac{5}{2}} - \frac{10}{9}x^{\frac{3}{2}} + c$	A1	1.1b
		(4)	
(4 marks)			

June 2022 Question 8 Paper 2 (A Level)

8. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

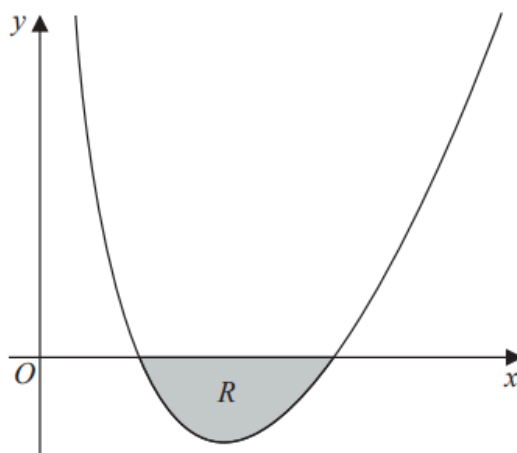


Figure 3

Figure 3 shows a sketch of part of a curve with equation

$$y = \frac{(x-2)(x-4)}{4\sqrt{x}} \quad x > 0$$

The region R , shown shaded in Figure 3, is bounded by the curve and the x -axis.

Find the exact area of R , writing your answer in the form $a\sqrt{2} + b$, where a and b are constants to be found.

(6)

ANSWER

Question	Scheme	Marks	AOs
8	$y = \frac{(x-2)(x-4)}{4\sqrt{x}} = \frac{x^2 - 6x + 8}{4\sqrt{x}} = \frac{1}{4}x^{\frac{3}{2}} - \frac{3}{2}x^{\frac{1}{2}} + 2x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	$\int \frac{1}{4}x^{\frac{3}{2}} - \frac{3}{2}x^{\frac{1}{2}} + 2x^{-\frac{1}{2}} dx = \frac{1}{10}x^{\frac{5}{2}} - x^{\frac{3}{2}} + 4x^{\frac{1}{2}} (+c)$	dM1 A1	3.1a 1.1b
	Deduces limits of integral are 2 and 4 and applies to their $\frac{1}{10}x^{\frac{5}{2}} - x^{\frac{3}{2}} + 4x^{\frac{1}{2}}$	M1	2.2a
	$\left(\frac{32}{10} - 8 + 8\right) - \left(\frac{2}{5}\sqrt{2} - 2\sqrt{2} + 4\sqrt{2}\right) = \frac{16}{5} - \frac{12}{5}\sqrt{2}$ $\text{Area } R = \frac{12}{5}\sqrt{2} - \frac{16}{5} \left(\text{or } \frac{16}{5} - \frac{12}{5}\sqrt{2}\right)$	A1	2.1
		(6)	
(6 marks)			

June 2022 Question 10

10.

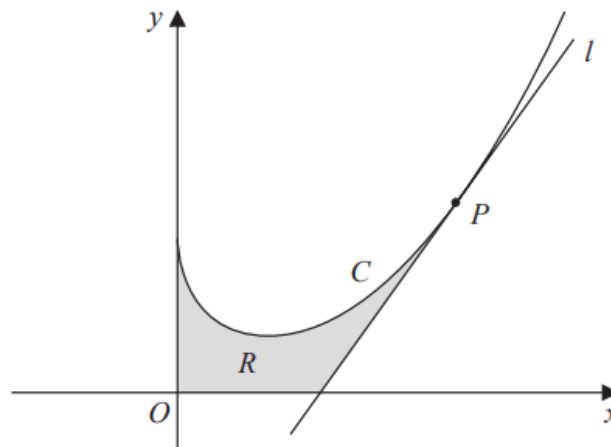


Figure 2

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 2 shows a sketch of part of the curve C with equation

$$y = \frac{1}{3}x^2 - 2\sqrt{x} + 3 \quad x \geq 0$$

The point P lies on C and has x coordinate 4

The line l is the tangent to C at P .

(a) Show that l has equation

$$13x - 6y - 26 = 0 \quad (5)$$

The region R , shown shaded in Figure 2, is bounded by the y -axis, the curve C , the line l and the x -axis.

(b) Find the exact area of R . (5)

ANSWER

Question	Scheme	Marks	AOs
10(a)	$y = \frac{1}{3}x^2 - 2\sqrt{x} + 3 \Rightarrow \frac{dy}{dx} = \frac{2}{3}x - x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	$x = 4 \Rightarrow y = \frac{13}{3}$	B1	1.1b
	$\left(\frac{dy}{dx}\right)_{x=4} = \frac{2}{3} \times 4 - 4^{-\frac{1}{2}} \left(= \frac{13}{6} \right) \therefore y - \frac{13}{3} = \frac{13}{6}(x - 4)$	M1	2.1
	$13x - 6y - 26 = 0^*$	A1*	1.1b
		(5)	
(b)	$\int \left(\frac{x^2}{3} - 2\sqrt{x} + 3 \right) dx = \frac{x^3}{9} - \frac{4}{3}x^{\frac{3}{2}} + 3x (+c)$	M1 A1	1.1b 1.1b
	$y = 0 \Rightarrow x = 2$	B1	2.2a
	Area of R is $\left[\frac{x^3}{9} - \frac{4}{3}x^{\frac{3}{2}} + 3x \right]_0^4 - \frac{1}{2} \times (4 - 2) \times \frac{13}{3} = \frac{76}{9} - \frac{13}{3}$	M1	3.1a
	$= \frac{37}{9}$	A1	1.1b
		(5)	
(10 marks)			

Video solution:

 AS level maths June 2022 Question 10 Paper 1 Edexcel

June 2022 Question 1

1. Find

$$\int \left(8x^3 - \frac{3}{2\sqrt{x}} + 5 \right) dx$$

giving your answer in simplest form.

(4)

ANSWER

Question	Scheme	Marks	AOs
1	$x^n \rightarrow x^{n+1}$	M1	1.1b
	$\int \left(8x^3 - \frac{3}{2\sqrt{x}} + 5 \right) dx = \frac{8x^4}{4} \dots + 5x$	A1	1.1b
	$= \dots - 2 \times \frac{3}{2} x^{\frac{1}{2}} + \dots$	A1	1.1b
	$= 2x^4 - 3x^{\frac{1}{2}} + 5x + c$	A1	1.1b
		(4)	
(4 marks)			

November 2021 Question 3

3. Find

$$\int \frac{3x^4 - 4}{2x^3} dx$$

writing your answer in simplest form.

(4)

ANSWER

Question	Scheme	Marks	AOs
3	$\int \frac{3x^4 - 4}{2x^3} dx = \int \frac{3}{2}x - 2x^{-3} dx$	M1 A1	1.1b 1.1b
	$= \frac{3}{2} \times \frac{x^2}{2} - 2 \times \frac{x^{-2}}{-2} (+c)$	dM1	3.1a
	$= \frac{3}{4}x^2 + \frac{1}{x^2} + c \quad \text{o.e}$	A1	1.1b
		(4)	
(4 marks)			

November 2021 Question 7 Paper 2 (A-Level)

7.

In this question you should show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

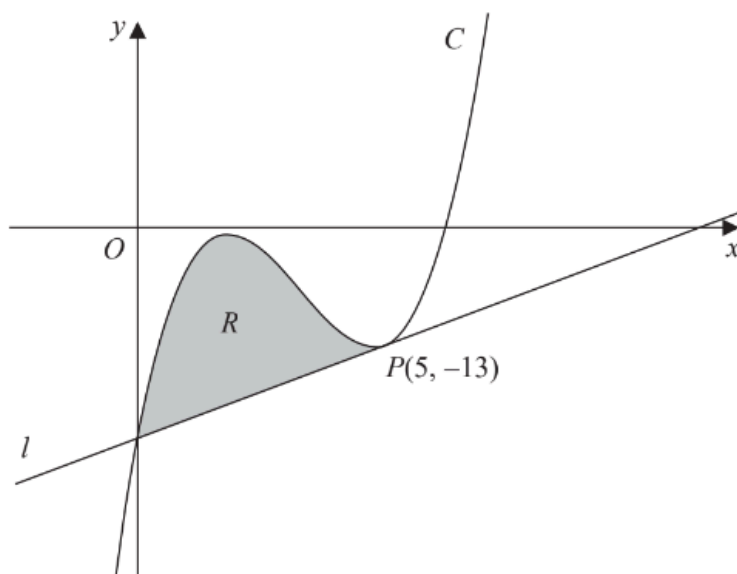


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + 27x - 23$$

The point $P(5, -13)$ lies on C The line l is the tangent to C at P

- (a) Use differentiation to find the equation of l , giving your answer in the form $y = mx + c$ where m and c are integers to be found.

(4)

- (b) Hence verify that l meets C again on the y -axis.

(1)

The finite region R , shown shaded in Figure 2, is bounded by the curve C and the line l .

- (c) Use algebraic integration to find the exact area of R .

(4)

ANSWER

Question	Scheme	Marks	AOs
7(a)	$y = x^3 - 10x^2 + 27x - 23 \Rightarrow \frac{dy}{dx} = 3x^2 - 20x + 27$	B1	1.1b
	$\left(\frac{dy}{dx}\right)_{x=5} = 3 \times 5^2 - 20 \times 5 + 27 (= 2)$	M1	1.1b
	$y + 13 = 2(x - 5)$	M1	2.1
	$y = 2x - 23$	A1	1.1b
	(4)		
(b)	Both C and l pass through $(0, -23)$ and so C meets l again on the y -axis	B1	2.2a
		(1)	
(c)	$\pm \int (x^3 - 10x^2 + 27x - 23 - (2x - 23)) dx$ $= \pm \left(\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{25}{2}x^2 \right)$	M1 A1ft	1.1b 1.1b
	$\left[\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{25}{2}x^2 \right]_0^5$ $= \left(\frac{625}{4} - \frac{1250}{3} + \frac{625}{2} \right) (-0)$	dM1	2.1
	$= \frac{625}{12}$	A1	1.1b
		(4)	

November 2021 Question 14

14. A curve C has equation $y = f(x)$ where

$$f(x) = -3x^2 + 12x + 8$$

(a) Write $f(x)$ in the form

$$a(x + b)^2 + c$$

where a , b and c are constants to be found.

(3)

The curve C has a maximum turning point at M .

(b) Find the coordinates of M .

(2)

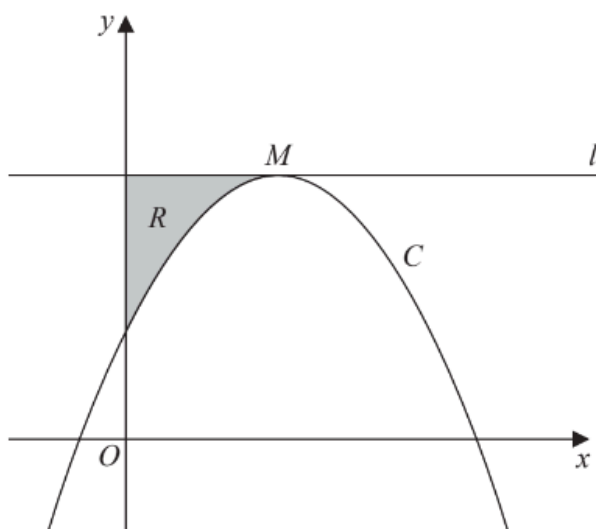


Figure 3

Figure 3 shows a sketch of the curve C .

The line l passes through M and is parallel to the x -axis.

The region R , shown shaded in Figure 3, is bounded by C , l and the y -axis.

(c) Using algebraic integration, find the area of R .

(5)

ANSWER

Question	Scheme	Marks	AOs
14 (a)	$f(x) = -3x^2 + 12x + 8 = -3(x \pm 2)^2 + \dots$	M1	1.1b
	$= -3(x - 2)^2 + \dots$	A1	1.1b
	$= -3(x - 2)^2 + 20$	A1	1.1b
		(3)	
(b)	Coordinates of $M = (2, 20)$	B1ft B1ft	1.1b 2.2a
		(2)	
(c)	$\int -3x^2 + 12x + 8 \, dx = -x^3 + 6x^2 + 8x$	M1 A1	1.1b 1.1b
	Method to find $R =$ their $2 \times 20 - \int_0^2 (-3x^2 + 12x + 8) \, dx$	M1	3.1a
	$R = 40 - [-2^3 + 24 + 16]$	dM1	1.1b
	$= 8$	A1	1.1b
		(5)	
(10 marks)			

November 2020 Question 7

7. Given that k is a positive constant and $\int_1^k \left(\frac{5}{2\sqrt{x}} + 3 \right) dx = 4$

(a) show that $3k + 5\sqrt{k} - 12 = 0$

(4)

(b) Hence, using algebra, find any values of k such that

$$\int_1^k \left(\frac{5}{2\sqrt{x}} + 3 \right) dx = 4$$

(4)

ANSWER

Question	Scheme	Marks	AOs
7 (a)	$x^n \rightarrow x^{n+1}$	M1	1.1b
	$\int \left(\frac{5}{2\sqrt{x}} + 3 \right) dx = 5\sqrt{x} + 3x$	A1	1.1b
	$[5\sqrt{x} + 3x]_1^k = 4 \Rightarrow 5\sqrt{k} + 3k - 8 = 4$	dM1	1.1b
	$3k + 5\sqrt{k} - 12 = 0$ *	A1*	2.1
		(4)	
(b)	$3k + 5\sqrt{k} - 12 = 0 \Rightarrow (3\sqrt{k} - 4)(\sqrt{k} + 3) = 0$	M1	3.1a
	$\sqrt{k} = \frac{4}{3}, (-3)$	A1	1.1b
	$\sqrt{k} = \dots \Rightarrow k = \dots$ oe	dM1	1.1b
	$k = \frac{16}{9}, \cancel{\times}$	A1	2.3
		(4)	
(8 marks)			

November 2020 Question 10

10.

$$g(x) = 2x^3 + x^2 - 41x - 70$$

(a) Use the factor theorem to show that $g(x)$ is divisible by $(x - 5)$.

(2)

(b) Hence, showing all your working, write $g(x)$ as a product of three linear factors.

(4)

The finite region R is bounded by the curve with equation $y = g(x)$ and the x -axis, and lies below the x -axis.

(c) Find, using algebraic integration, the exact value of the area of R .

(4)

ANSWER

Question	Scheme	Marks	AOs
10 (a)	$g(5) = 2 \times 5^3 + 5^2 - 41 \times 5 - 70 = \dots$	M1	1.1a
	$g(5) = 0 \Rightarrow (x - 5)$ is a factor, hence $g(x)$ is divisible by $(x - 5)$.	A1	2.4
		(2)	
(b)	$2x^3 + x^2 - 41x - 70 = (x - 5)(2x^2 \dots x \pm 14)$	M1	1.1b
	$= (x - 5)(2x^2 + 11x + 14)$	A1	1.1b
	Attempts to factorise quadratic factor	dM1	1.1b
	$(g(x)) = (x - 5)(2x + 7)(x + 2)$	A1	1.1b
		(4)	
(c)	$\int 2x^3 + x^2 - 41x - 70 \, dx = \frac{1}{2}x^4 + \frac{1}{3}x^3 - \frac{41}{2}x^2 - 70x$	M1 A1	1.1b 1.1b
	Deduces the need to use $\int_{-2}^5 g(x) \, dx$	M1	2.2a
	$-\frac{1525}{3} - \frac{190}{3}$		
	Area = $571\frac{2}{3}$	A1	2.1
		(4)	
(10 marks)			

November 2020 Question 8 Paper 2 (A-Level)

8. A curve C has equation $y = f(x)$

Given that

- $f'(x) = 6x^2 + ax - 23$ where a is a constant
- the y intercept of C is -12
- $(x + 4)$ is a factor of $f(x)$

find, in simplest form, $f(x)$

(6)

ANSWER

8	$f'(x) = 6x^2 + ax - 23 \Rightarrow f(x) = 2x^3 + \frac{1}{2}ax^2 - 23x + c$	M1 A1	1.1b 1.1b
	"c" = -12	B1	2.2a
	$f(-4) = 0 \Rightarrow 2 \times (-4)^3 + \frac{1}{2}a(-4)^2 - 23(-4) - 12 = 0$	dM1	3.1a
	$a = \dots (6)$	dM1	1.1b
	$(f(x) =) 2x^3 + 3x^2 - 23x - 12$ Or Equivalent e.g. $(f(x) =) (x+4)(2x^2 - 5x - 3) \quad (f(x) =) (x+4)(2x+1)(x-3)$	A1 cso	2.1
		(6)	
(6 marks)			

June 2019 Question 3

3. (a) Given that k is a constant, find

$$\int \left(\frac{4}{x^3} + kx \right) dx$$

simplifying your answer.

(3)

- (b) Hence find the value of k such that

$$\int_{0.5}^2 \left(\frac{4}{x^3} + kx \right) dx = 8$$

(3)

ANSWER

Question	Scheme	Marks	AOs
3(a)	$x^n \rightarrow x^{n+1}$	M1	1.1b
	$\int \left(\frac{4}{x^3} + kx \right) dx = -\frac{2}{x^2} + \frac{1}{2} kx^2 + c$	A1 A1	1.1b 1.1b
		(3)	
(b)	$\left[-\frac{2}{x^2} + \frac{1}{2} kx^2 \right]_{0.5}^2 = \left(-\frac{2}{2^2} + \frac{1}{2} k \times 4 \right) - \left(-\frac{2}{(0.5)^2} + \frac{1}{2} k \times (0.5)^2 \right) = 8$	M1	1.1b
	$7.5 + \frac{15}{8} k = 8 \Rightarrow k = \dots$	dM1	1.1b
	$k = \frac{4}{15}$ oe	A1	1.1b
		(3)	
(6 marks)			

June 2019 Question 8 Paper 1 (A-Level)

8.

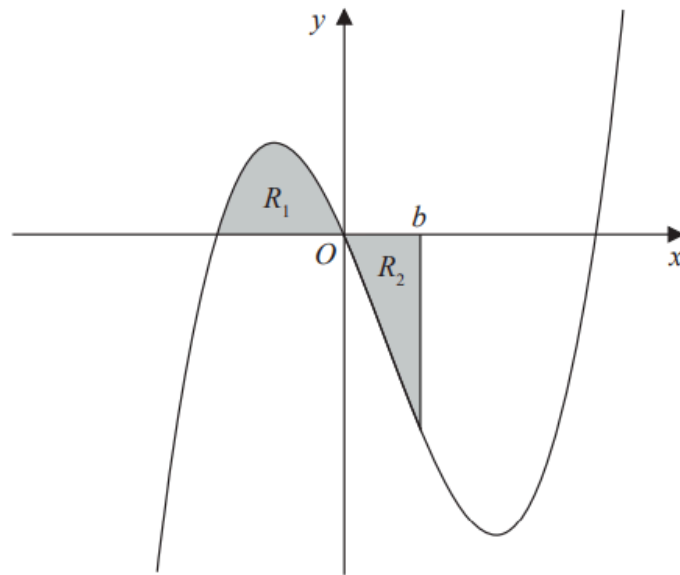


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = x(x + 2)(x - 4)$.

The region R_1 shown shaded in Figure 2 is bounded by the curve and the negative x -axis.

- (a) Show that the exact area of R_1 is $\frac{20}{3}$ (4)

The region R_2 also shown shaded in Figure 2 is bounded by the curve, the positive x -axis and the line with equation $x = b$, where b is a positive constant and $0 < b < 4$

Given that the area of R_1 is equal to the area of R_2

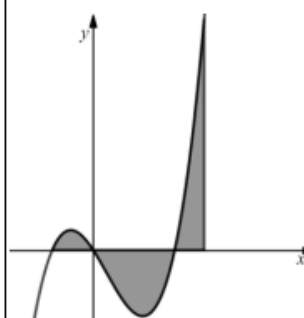
- (b) verify that b satisfies the equation

$$(b + 2)^2 (3b^2 - 20b + 20) = 0 \quad (4)$$

The roots of the equation $3b^2 - 20b + 20 = 0$ are 1.225 and 5.442 to 3 decimal places.
The value of b is therefore 1.225 to 3 decimal places.

- (c) Explain, with the aid of a diagram, the significance of the root 5.442 (2)

ANSWER

Question	Scheme	Marks	AOs
8 (a)	$y = x(x+2)(x-4) = x^3 - 2x^2 - 8x$	B1	1.1b
	$\int x^3 - 2x^2 - 8x \, dx \rightarrow \frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2$	M1	1.1b
	Attempts area using the correct strategy $\int_{-2}^0 y \, dx$	dM1	2.2a
	$\left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2 \right]_{-2}^0 = (0) - \left(4 - \frac{-16}{3} - 16 \right) = \frac{20}{3} *$	A1*	2.1
		(4)	
(b)	For setting 'their' $\frac{1}{4}b^4 - \frac{2}{3}b^3 - 4b^2 = \pm \frac{20}{3}$	M1	1.1b
	For correctly deducing that $3b^4 - 8b^3 - 48b^2 + 80 = 0$	A1	2.2a
	Attempts to factorise $3b^4 - 8b^3 - 48b^2 \pm 80 = (b+2)(b+2)(3b^2 \dots b \dots 20)$	M1	1.1b
	Achieves $(b+2)^2(3b^2 - 20b + 20) = 0$ with no errors	A1*	2.1
		(4)	
(c)	 States that between $x = -2$ and $x = 5.442$ the area above the x-axis = area below the x-axis	B1	1.1b
		B1	2.4
		(2)	
(10 marks)			

June 2018 Question 1

1. Find

$$\int \left(\frac{2}{3}x^3 - 6\sqrt{x} + 1 \right) dx$$

giving your answer in its simplest form.

(4)

ANSWER

Question	Scheme	Marks	AOs
1	$\int \left(\frac{2}{3}x^3 - 6\sqrt{x} + 1 \right) dx$		
	Attempts to integrate awarded for any correct power	M1	1.1a
	$\int \left(\frac{2}{3}x^3 - 6\sqrt{x} + 1 \right) dx = \frac{2}{3} \times \frac{x^4}{4} + \dots + x$	A1	1.1b
	$= \dots - 6 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \dots$	A1	1.1b
	$= \frac{1}{6}x^4 - 4x^{\frac{3}{2}} + x + c$	A1	1.1b
(4 marks)			

June 2018 Question 15

15.

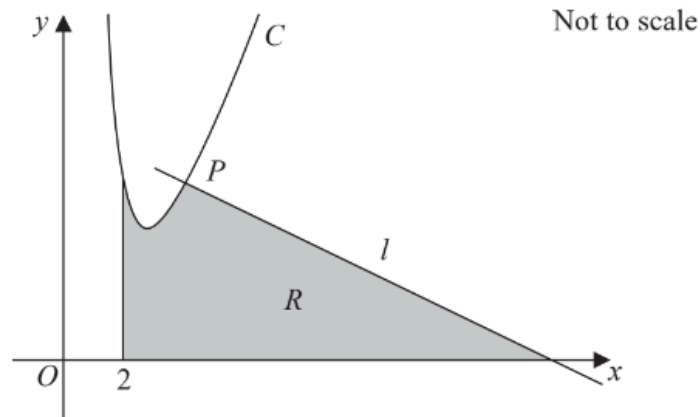


Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{32}{x^2} + 3x - 8, \quad x > 0$$

The point $P(4, 6)$ lies on C .

The line l is the normal to C at the point P .

The region R , shown shaded in Figure 4, is bounded by the line l , the curve C , the line with equation $x = 2$ and the x -axis.

Show that the area of R is 46

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(10)

ANSWER

Question	Scheme	Marks	AOs
15.	For the complete strategy of finding where the normal cuts the x -axis. Key points that must be seen are Attempt at differentiation	M1	3.1a
	- Attempt at using a changed gradient to find equation of normal - Correct attempt to find where normal cuts the x - axis		
	$y = \frac{32}{x^2} + 3x - 8 \Rightarrow \frac{dy}{dx} = -\frac{64}{x^3} + 3$	M1 A1	1.1b 1.1b
	For a correct method of attempting to find Either the equation of the normal: this requires substituting $x = 4$ in their $\frac{dy}{dx} = -\frac{64}{x^3} + 3 = (2)$, then using the perpendicular gradient rule to find the equation of normal $y - 6 = -\frac{1}{2}(x - 4)$ Or where the equation of the normal at (4,6) cuts the x - axis. As above but may not see equation of normal. Eg $0 - 6 = -\frac{1}{2}(x - 4) \Rightarrow x = \dots$ or an attempt using just gradients $-\frac{1}{2} = \frac{6}{a - 4} \Rightarrow a = \dots$	dM1	2.1
	Normal cuts the x -axis at $x = 16$	A1	1.1b

For the complete strategy of finding the values of the two key areas. Points that must be seen are		
- There must be an attempt to find the area under the curve by integrating between 2 and 4 - There must be an attempt to find the area of a triangle using $\frac{1}{2} \times (16 - 4) \times 6$ or $\int_4^{16} \left(-\frac{1}{2}x + 8\right) dx$	M1	3.1a
$\int \frac{32}{x^2} + 3x - 8 dx = -\frac{32}{x} + \frac{3}{2}x^2 - 8x$	M1 A1	1.1b 1.1b
Area under curve = $\left[-\frac{32}{x} + \frac{3}{2}x^2 - 8x\right]_2^4 = (-16) - (-26) = (10)$	dM1	1.1b
Total area = $10 + 36 = 46$ *	A1*	2.1
	(10)	
(10 marks)		

Video solution:

<https://youtu.be/OPmmw6wUOFc>

June 2018 Question 2 Core 1

2. Given

$$y = 3\sqrt{x} - 6x + 4, \quad x > 0$$

(a) find $\int y dx$, simplifying each term.

(3)

(b) (i) Find $\frac{dy}{dx}$ (ii) Hence find the value of x such that $\frac{dy}{dx} = 0$

(4)

ANSWER

Question Number	Scheme		Marks
2.(a)	$2x^{1.5} - 3x^2 + 4x + c$	M1: For $x^n \rightarrow x^{n+1}$ i.e. $x^{1.5}$ or x^2 or x seen (not for “+ c”)	M1A1A1
		A1: For two out of three terms correct un-simplified or simplified (Ignore + c for this mark)	
		A1: cao $2x^{1.5} - 3x^2 + 4x + c$. All correct and simplified and on one line including “+ c”.	
		Allow $\sqrt{x^3}$ for $x^{1.5}$ but not x^1 for x .	
Ignore any spurious integral signs.			
			(3)
(b)(i)	Mark (b)(i) and (ii) together and must be differentiating the original function not their answer to part (a)		
	$\frac{3}{2}x^{-0.5} - 6$	M1: For $x^n \rightarrow x^{n-1}$ i.e. $x^{0.5} \rightarrow x^{-0.5}$ or $6x \rightarrow 6$	M1A1
		A1: For $\frac{3}{2}x^{-0.5} - 6$ or equivalent. May be un-simplified.	
		Allow $\frac{3/2}{\sqrt{x}} - 6$.	
			(2)
(ii)	$\frac{3}{2}x^{-0.5} - 6 = 0 \Rightarrow x^n = \dots$	Sets their $\frac{dy}{dx} = 0$ (may be implied by their working) and reaches $x^n = C$ (including $n = 1$) with correct processing allowing sign errors only – this may be implied by e.g. $\sqrt{x} = \frac{1}{4}$ or $\frac{1}{\sqrt{x}} = 4$.	M1
		Allow equivalent fractions e.g. $\frac{9}{144}$ or 0.0625. If other solutions are given (e.g. likely to be $x = 0$ or $x = -1/16$) then this mark should be withheld.	A1
	$x = \frac{1}{16}$ cso		
			(7 marks)

June 2018 Question 9 Core 1

9. The curve C has equation $y = f(x)$, where

$$f'(x) = (x - 3)(3x + 5)$$

Given that the point $P(1, 20)$ lies on C ,

- (a) find $f(x)$, simplifying each term.

(5)

- (b) Show that

$$f(x) = (x - 3)^2(x + A)$$

where A is a constant to be found.

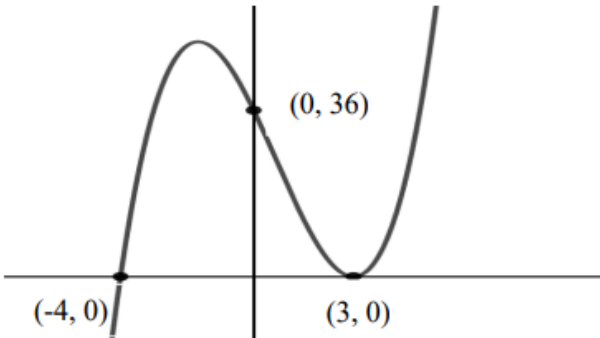
(3)

- (c) Sketch the graph of C . Show clearly the coordinates of the points where C cuts or meets the x -axis and where C cuts the y -axis.

(4)

ANSWER

Question Number	Scheme		Marks
9.(a)	$(x-3)(3x+5) = 3x^2 - 4x - 15$ Allow $3x^2 + 5x - 9x - 15$	Correct expansion simplified or unsimplified.	B1
	$f(x) = x^3 - 2x^2 - 15x + c$	M1: $x^n \rightarrow x^{n+1}$ for any term. Follow through on incorrect indices but not for “+ c” A1: All terms correct. Need not be simplified. No need for + c here.	M1A1
	$x = 1, y = 20 \Rightarrow 20 = 1 - 2 - 15 + c$ $\Rightarrow c = 36$	Substitutes $x = 1$ and $y = 20$ into their $f(x)$ to find c . Must have + c at this stage. Dependent on the first method mark.	dM1
	$(f(x) =) x^3 - 2x^2 - 15x + 36$	Cao $(f(x) =) x^3 - 2x^2 - 15x + 36$ (All together and on one line)	A1
			(5)
(b) Way 1	$A = 4$	Correct value (may be implied)	B1
	$f(x) = (x-3)^2(x+A) = (x^2 - 6x + 9)(x+A)$ $f(x) = x^3 + (A-6)x^2 + (9-6A)x + 9A$ $A-6 = -2 \Rightarrow A = 4 \quad 9-6A = -15 \Rightarrow A = 4 \quad 9A = 36 \Rightarrow A = 4$ M1: Expands $(x-3)^2(x+A)$ and compares coefficients with their $f(x)$ from part (a) to form 3 equations and attempts to solve at least two of them in an attempt to show that A is the same in each case or substitutes their A to show that the coefficients are the same. A1: Fully correct proof – must use all 3 coefficients		M1A1
			(3)

9(c)		
	A positive cubic shape. Its position is not important but must be a curve and not straight lines and the “ends” must not clearly turn back in on themselves.	B1
	Touches at the point $(3, 0)$ (could be a maximum). Accept 3 marked on the x -axis and accept $(0, 3)$ as long as it is in the correct place. Allow $(3, 0)$ in the body of the script but it must correspond with the sketch. If ambiguous, the sketch takes precedence.	B1
	Crosses or reaches the x -axis at $(-4, 0)$. Accept -4 marked on the x -axis and accept $(0, -4)$ as long as it is in the correct place. FT on their $-A$ from part (b) and allow “ $-A$ ” and allow a “made up” A . Allow $(-4, 0)$ in the body of the script but it must correspond with the sketch. If ambiguous, the sketch takes precedence.	B1ft
	Crosses the y -axis at $(0, 36)$ and with a maximum in the second quadrant. Accept 36 marked on the y – axis and accept $(36, 0)$ as long as it is in the correct place. FT on their numerical 'c' from part (a) only. Allow $(0, 36)$ in the body of the script but it must correspond with the sketch. If ambiguous, the sketch takes precedence.	B1ft
		(4)
		(12 marks)

June 2018 Question 9 Core 2

9.

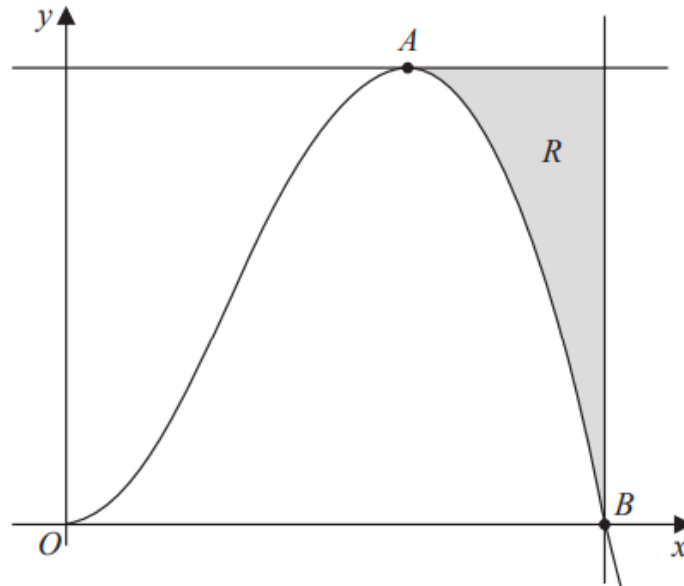


Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 7x^2(5 - 2\sqrt{x}), \quad x \geq 0$$

The curve has a turning point at the point A , where $x > 0$, as shown in Figure 3.

(a) Using calculus, find the coordinates of the point A .

(5)

The curve crosses the x -axis at the point B , as shown in Figure 3.

(b) Use algebra to find the x coordinate of the point B .

(2)

The finite region R , shown shaded in Figure 3, is bounded by the curve, the line through A parallel to the x -axis and the line through B parallel to the y -axis.

(c) Use integration to find the area of the region R , giving your answer to 2 decimal places.

(5)

ANSWER

Question Number	Scheme	Marks
9. (a)	$\frac{dy}{dx} = 70x - 35x^{\frac{3}{2}}$ Put $\frac{dy}{dx} = 0$ to give $70x - 35x^{\frac{3}{2}} = 0$ so $x^{\frac{1}{2}} = 2$ $x = 4$ $y = 112$	M1A1 M1 A1 A1 (5)
(b) (Way 1)	When $y = 0$, $35x^2 = 14x^{\frac{5}{2}}$ and $x^{\frac{1}{2}} = \frac{35}{14}$ or $5 = 2\sqrt{x}$ so $\sqrt{x} = \frac{5}{2}$ $x = \frac{25}{4}$	M1 A1 (2)
(b) (Way 2)	When $y = 0$, $35x^2 = 14x^{\frac{5}{2}}$ so $1225x^4 = 196x^5$ or $5 = 2\sqrt{x}$ so $25 = 4x$ $x = \frac{25}{4}$ or $x = \frac{1225}{196}$	M1 A1 (2)
(c) Way 1	$\int 35x^2 - 14x^{\frac{5}{2}} dx = \frac{35}{3}x^3 - \frac{14x^{\frac{7}{2}}}{\frac{7}{2}} (+c)$ $\left[\frac{35}{3}x^3 - 4x^{\frac{7}{2}} \right]_4^{\frac{25}{4}} = 406.901... - 234.667 = 172.23$ Hence Area = "their $112 \times (6\frac{1}{4} - 4)$ " - "172.23" or "252" - "172.23" 79.77	M1A1ft dM1 ddM1 A1 (5)
(c) Way 2	$\int "112" - \{35x^2 - 14x^{\frac{5}{2}}\} dx = (112x) - \frac{35}{3}x^3 + \frac{14x^{\frac{7}{2}}}{\frac{7}{2}} (+c)$ $\left[(112x) - \left(\frac{35}{3}x^3 - 4x^{\frac{7}{2}} \right) \right]_{"4"}^{"\frac{25}{4}"}$ with correct use of limits Integrated their 112 to give 112x with correct use of limits 79.77	M1A1ft dM1 ddM1 A1 (5) [12]

Ch. 14: Exponentials and Logarithms

June 2024 Question 13 Paper 2 (A-level)

13. The world human population, P billions, is modelled by the equation

$$P = ab^t$$

where a and b are constants and t is the number of years after 2004

Using the estimated population figures for the years from 2004 to 2007, a graph is plotted of $\log_{10} P$ against t .

The points lie approximately on a straight line with

- gradient 0.0054
- intercept 0.81 on the $\log_{10} P$ axis

(a) Estimate, to 3 decimal places, the value of a and the value of b .

(4)

In the context of the model,

(b) (i) interpret the value of the constant a ,

(ii) interpret the value of the constant b .

(2)

(c) Use the model to estimate the world human population in 2030

(2)

(d) Comment on the reliability of the answer to part (c).

(1)

ANSWER

Question	Scheme	Marks	AOs
13(a)	$\log_{10} b = 0.0054 \Rightarrow b = 10^{0.0054}$ or $\log_{10} a = 0.81 \Rightarrow a = 10^{0.81}$	M1	3.1a
	$b = 1.01$ or $a = 6.46$	A1	1.1b
	$\log_{10} b = 0.0054 \Rightarrow b = 10^{0.0054}$ and $\log_{10} a = 0.81 \Rightarrow a = 10^{0.81}$	M1	2.1
	$b = 1.013$ and $a = 6.457$	A1	1.1b
		(4)	
(b)(i)	e.g. The world population in billions in 2004	B1ft	3.2a
(ii)	$b = 1.013$ represents the scale factor of the <u>yearly increase</u> in the world population	B1ft	3.2a
		(2)	
(c)	$P = 6.457 \dots (1.013 \dots)^{26}$ or e.g. $\log P = 0.81 + 26 \times 0.0054 \Rightarrow P = \dots$	M1	3.4
	awrt 9 billion	A1	2.2b
		(2)	
(d)	Not reliable since the data used for the model covered the years 2004 – 2007 and it would not be sensible to assume that the model still holds in 2030	B1	3.2b
		(1)	
(9 marks)			

June 2024 Question 11

11. The prices of two precious metals are being monitored.

The price per gram of metal A , £ V_A , is modelled by the equation

$$V_A = 100 + 20e^{0.04t}$$

where t is the number of months after monitoring began.

The price per gram of metal B , £ V_B , is modelled by the equation

$$V_B = pe^{-0.02t}$$

where p is a positive constant and t is the number of months after monitoring began.

Given that $V_B = 2V_A$ when $t = 0$

(a) find the value of p

(2)

When $t = T$, the rate of **increase** in the price per gram of metal A was equal to the rate of **decrease** in the price per gram of metal B

(b) Find the value of T , giving your answer to one decimal place.

(Solutions based entirely on calculator technology are not acceptable.)

(4)

ANSWER

Question	Scheme	Marks	AOs
11(a)	At $t = 0$, $V_A = 100 + 20 = 120 \Rightarrow p = 2 \times "120"$	M1	1.1b
	$(p =) 240$	A1	1.1b
		(2)	
(b)	$\dots e^{0.04T} = \dots e^{-0.02T}$	M1	3.1b
	$0.8e^{0.04T} = "4.8" e^{-0.02T}$	A1ft	1.1b
	$\dots e^{0.04T} = \dots e^{-0.02T} \Rightarrow e^{\pm 0.06T} = \dots$	dM1	3.1a
	$(T =) \text{awrt } 29.9 \text{ (months)}$	A1cso	1.1b
		(4)	
(6 marks)			

June 2024 Question 9

9.

$$p = \log_a 16$$

$$q = \log_a 25$$

where a is a constant.

Find in terms of p and/or q ,

(a) $\log_a 256$

(1)

(b) $\log_a 100$

(2)

(c) $\log_a 80 \times \log_a 3.2$

(2)

ANSWER

Question	Scheme	Marks	AOs
9(a)	$2p$	B1	1.1b
		(1)	
(b)	$\log_a 100 = \log_a 4 + \log_a 25$	M1	1.2
	$\log_a 16^{\frac{1}{2}} + \log_a 25 = \frac{1}{2}p + q$	A1	1.1b
		(2)	
(c)	e.g. $\log_a 80 \times \log_a 3.2 = (\log_a 16 + \log_a 5) \times (\log_a 16 - \log_a 5)$	M1	3.1a
	$\left(p + \frac{1}{2}q\right) \times \left(p - \frac{1}{2}q\right)$ or $p^2 - \frac{1}{4}q^2$	A1	1.1b
		(2)	
(5 marks)			

June 2024 Question 7

7.

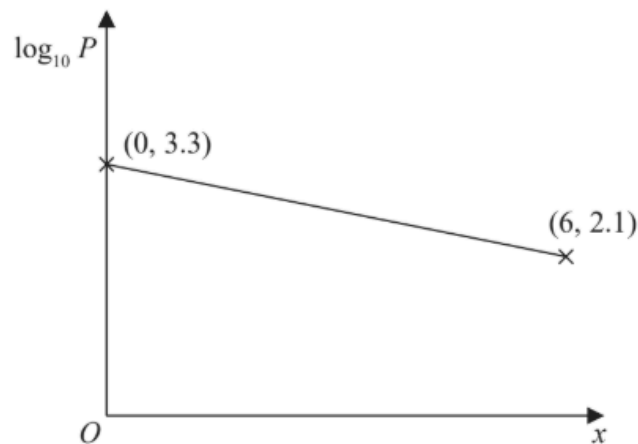


Figure 2

A chimney emits smoke particles.

On a particular day, the concentration of smoke particles in the air emitted by this chimney, P parts per million, is measured at various distances, x km, from the chimney.

Figure 2 shows a sketch of the linear relationship between $\log_{10} P$ and x that is used to model this situation.

The line passes through the point $(0, 3.3)$ and the point $(6, 2.1)$

- (a) Find a complete equation for the model in the form

$$P = ab^x$$

where a and b are constants. Give the value of a and the value of b each to 4 significant figures.

(4)

- (b) With reference to the model, interpret the value of ab

(1)

ANSWER

Question	Scheme		Marks	AOs
7(a)	$\log_{10} a = 3.3$ or $\log_{10} b = \frac{2.1-3.3}{6} (= -0.2)$	e.g. $\log_{10} P = 3.3 - 0.2x$ or $\log_{10} P = 3.3 + \frac{2.1-3.3}{6}x$	M1	1.1b
	$a = 10^{3.3}$ or $b = 10^{-0.2}$		A1	1.1b
	$a = 10^{3.3}$ and $b = 10^{-0.2}$	$P = 10^{3.3-0.2x} = 10^{3.3} \times 10^{-0.2x}$	dM1	3.1a
	$P = 1995 \times 0.6310^x$		A1	3.3
			(4)	
(b)	The concentration (in parts per million) 1 km from the chimney		B1	3.2a
			(1)	
(5 marks)				

June 2023 Question 11

11. The height, h metres, of a plant, t years after it was first measured, is modelled by the equation

$$h = 2.3 - 1.7e^{-0.2t} \quad t \in \mathbb{R} \quad t \geq 0$$

Using the model,

- (a) find the height of the plant when it was first measured, (2)
- (b) show that, exactly 4 years after it was first measured, the plant was growing at approximately 15.3 cm per year. (3)
- According to the model, there is a limit to the height to which this plant can grow.
- (c) Deduce the value of this limit. (1)

ANSWER

Question	Scheme	Marks	AOs
11(a)	$h = 2.3 - 1.7e^0$	M1	3.4
	Either 0.6 {m} or 60 cm	A1	1.1b
		(2)	
(b)	$\left\{ \frac{dh}{dt} \right\} = 0.34e^{-0.2t}$	M1	3.1b
	At $t = 4 \Rightarrow$ Rate of growth is $0.34e^{-0.2 \times 4} = 0.15277... \{m / year\}$	dM1	3.4
	$0.153 \{m \text{ per year}\} = 15.3 \text{ cm } \{per \text{ year}\} *$	A1*	1.1b
		(3)	
(c)	2.3 (m)	B1	2.2a
		(1)	
(6 marks)			

June 2023 Question 9

9. Using the laws of logarithms, solve the equation

$$2 \log_5 (3x - 2) - \log_5 x = 2$$

(5)

ANSWER

Question	Scheme	Marks	AOs
9	$2 \log_5 (3x - 2) - \log_5 x = 2$		
	Uses one correct law e.g. $2 \log_5 (3x - 2) \rightarrow \log_5 (3x - 2)^2$ or $2 \rightarrow \log_5 25$ or $\log_5 \dots = 2 \rightarrow \dots = 5^2$	B1	1.1a
	Uses two correct log laws: either $2 \log_5 (3x - 2) \rightarrow \log_5 (3x - 2)^2$ and $2 \rightarrow \log_5 25$ or $2 \log_5 (3x - 2) - \log_5 x \rightarrow \log_5 \frac{(3x - 2)^2}{x}$ leading to an equation without logs	M1	3.1a
	Correct equation without logs, usually $\frac{(3x - 2)^2}{x} = 25$	A1	1.1b
	$\frac{(3x - 2)^2}{x} = 25 \Rightarrow 9x^2 - 37x + 4 = 0 \Rightarrow (9x - 1)(x - 4) = 0 \Rightarrow x = \dots$	dM1	1.1b
	$x = 4$ only	A1 cso	3.2a
		(5)	
(5 marks)			

June 2023 Question 4 Paper 2 (A-Level)

4. Coffee is poured into a cup.

The temperature of the coffee, H °C, t minutes after being poured into the cup is modelled by the equation

$$H = Ae^{-Bt} + 30$$

where A and B are constants.

Initially, the temperature of the coffee was 85 °C.

- (a) State the value of A .

(1)

Initially, the coffee was cooling at a rate of 7.5 °C per minute.

- (b) Find a complete equation linking H and t , giving the value of B to 3 decimal places.

(3)

ANSWER

Question	Scheme	Marks	AOs
4(a)	$(A =) 55$	B1	3.4
		(1)	
(b)	$\left\{ \frac{dH}{dt} = \right\} -ABe^{-Bt}$ or $\left\{ \frac{dH}{dt} = \right\} -"55" Be^{-Bt}$	M1	3.1b
	$-B \times "55" = -7.5 \Rightarrow B = \dots \left(\frac{3}{22} = \text{awrt } 0.136 \right)$	M1	1.1b
	$H = 55e^{-0.136t} + 30$	A1cso	3.3
		(3)	
(4 marks)			

June 2023 Question 3 Paper 2 (A-Level)

3. Given that

$$\log_2(x+3) + \log_2(x+10) = 2 + 2\log_2 x$$

(a) show that

$$3x^2 - 13x - 30 = 0$$

(3)

(b) (i) Write down the roots of the equation

$$3x^2 - 13x - 30 = 0$$

(ii) Hence state which of the roots in part (b)(i) is not a solution of

$$\log_2(x+3) + \log_2(x+10) = 2 + 2\log_2 x$$

giving a reason for your answer.

(2)

ANSWER

Question	Scheme	Marks	AOs
3(a)	Uses one correct log law e.g. $\log_2(x+3) + \log_2(x+10) = \log_2(x+3)(x+10)$ $2 = \log_2 4, 2\log_2 x = \log_2 x^2$	M1	1.1b
	$(x+3)(x+10) = 4x^2$ oe	dM1	2.1
	$\Rightarrow 3x^2 - 13x - 30 = 0$ *	A1*	1.1b
		(3)	
(b)(i)	$(x =) 6, -\frac{5}{3}$	B1	1.1b
(ii)	$x \neq -\frac{5}{3}$ because $\log_{(2)}\left(-\frac{5}{3}\right)$ is not real	B1	2.4
		(2)	
(5 marks)			

June 2023 Question 11 Paper 1 (A-Level)

11.

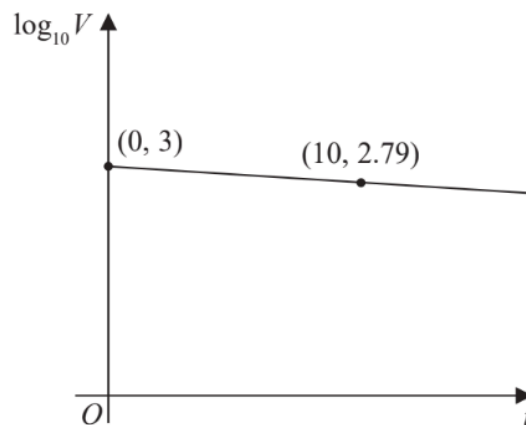


Figure 2

The value, V pounds, of a mobile phone, t months after it was bought, is modelled by

$$V = ab^t$$

where a and b are constants.

Figure 2 shows the linear relationship between $\log_{10} V$ and t .

The line passes through the points $(0, 3)$ and $(10, 2.79)$ as shown.

Using these points,

- (a) find the initial value of the phone, (2)

- (b) find a complete equation for V in terms of t , giving the exact value of a and giving the value of b to 3 significant figures. (3)

Exactly 2 years after it was bought, the value of the phone was £320

- (c) Use this information to evaluate the reliability of the model. (2)

ANSWER

Question	Scheme	Marks	AOs
11(a)	$\log_{10} V = 3 \Rightarrow V = 10^3$	M1	1.1b
	$(V =) \text{£}1000$	A1	3.4
		(2)	
(b)	e.g. $(\log_{10} b =) \frac{2.79-3}{10-0} = -0.021$ or $\log_{10} V = 3 - 0.021t$ or $10^{2.79} = "1000"b^{10}$	M1	1.1b
	e.g. $b = 10^{-0.021} (= 0.952796....)$ or $V = 10^3 \times 10^{-0.021t}$ or $b = \sqrt[10]{0.61659...}$	M1	3.1b
	$V = 1000 \times 0.953^t$	A1ft	3.3
		(3)	
(c)	e.g. $V = 1000 \times "0.953"^{24} (= \text{£}315)$ or e.g. $\log_{10} V = 3 - "0.021" \times 24 \Rightarrow V = ... (= \text{£}313)$ which is close (to £320) so it is a suitable model	M1	3.4
		A1	3.2b
		(2)	
(7 marks)			

June 2023 Question 6 Paper 1 (A-Level)

6. $a = \log_2 x$ $b = \log_2(x + 8)$

Express in terms of a and/or b

(a) $\log_2 \sqrt{x}$ (1)

(b) $\log_2(x^2 + 8x)$ (2)

(c) $\log_2\left(8 + \frac{64}{x}\right)$

Give your answer in simplest form. (3)

ANSWER

Question	Scheme	Marks	AOs
6(a)	$\frac{1}{2}a$	B1	2.2a
		(1)	
(b)	$\log_2 x(x+8) \Rightarrow \log_2 x + \log_2(x+8)$	M1	1.2
	$= a + b$	A1	2.2a
		(2)	
(c)	e.g. $8 + \frac{64}{x} = \frac{8x+64}{x}$	B1	1.1b
	$\log_2 \frac{8}{x}(x+8) = 3 - \log_2 x + \log_2(x+8)$	M1	1.1b
	$3 + b - a$	A1	2.2a
		(3)	
(6 marks)			

June 2022 Question 2 Paper 2 (A Level)

2. (a) Sketch the curve with equation

$$y = 4^x$$

stating any points of intersection with the coordinate axes.

(2)

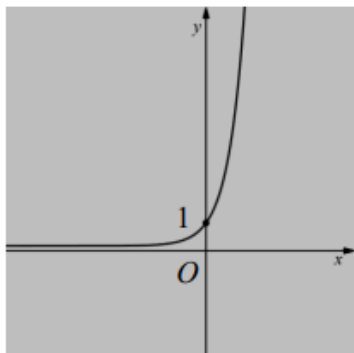
- (b) Solve

$$4^x = 100$$

giving your answer to 2 decimal places.

(2)

ANSWER

Question	Scheme	Marks	AOs	
2(a)		Correct shape or correct intercept – see notes	B1	1.2
		Fully correct – see notes	B1	1.1b
			(2)	
(b)	$4^x = 100 \Rightarrow x = \log_4 100$ or $\text{e.g. } x \log 4 = \log 100 \Rightarrow x = \frac{\log 100}{\log 4}$	M1	1.1b	
	$\Rightarrow (x =) \text{awrt } 3.32$	A1	1.1b	
			(2)	
(4 marks)				

June 2022 Question 10 Paper 1 (A Level)

10. A scientist is studying the number of bees and the number of wasps on an island.

The number of bees, measured in thousands, N_b , is modelled by the equation

$$N_b = 45 + 220e^{0.05t}$$

where t is the number of years from the start of the study.

According to the model,

(a) find the number of bees at the start of the study, (1)

(b) show that, exactly 10 years after the start of the study, the number of bees was increasing at a **rate** of approximately 18 thousand per year. (3)

The number of wasps, measured in thousands, N_w , is modelled by the equation

$$N_w = 10 + 800e^{-0.05t}$$

where t is the number of years from the start of the study.

When $t = T$, according to the models, there are an equal number of bees and wasps.

(c) Find the value of T to 2 decimal places. (4)

ANSWER

Question	Scheme	Marks	AOs
10 (a)	265 thousand	B1	3.4
		(1)	
(b)	Attempts $\frac{dN_b}{dt} = 11e^{0.05t}$	M1	1.1b
	Substitutes $t = 10$ into their $\frac{dN_b}{dt}$	M1	3.4
	$\frac{dN_b}{dt} = \text{awrt } 18.1$ which is approximately 18 thousand per year *	A1*	2.1
		(3)	
(c)	Sets $45 + 220e^{0.05t} = 10 + 800e^{-0.05t} \Rightarrow 220e^{0.05t} + 35 - 800e^{-0.05t} = 0$	M1	3.1b
	Correct quadratic equation $\Rightarrow 220(e^{0.05t})^2 + 35e^{0.05t} - 800 = 0$	A1	1.1b
	$e^{0.05t} = 1.829, (-1.988) \Rightarrow 0.05t = \ln(1.829)$	M1	2.1
	$T = 12.08$	A1	1.1b
		(4)	
(8 marks)			

Video solution:

<https://youtu.be/Qc-RPb6Z1OU>

June 2022 Question 9

9. (a) Given that $p = \log_3 x$, where $x > 0$, find in simplest form in terms of p ,

(i) $\log_3 \left(\frac{x}{9} \right)$

(ii) $\log_3 (\sqrt{x})$

(2)

(b) Hence, or otherwise, solve

$$2\log_3 \left(\frac{x}{9} \right) + 3\log_3 (\sqrt{x}) = -11$$

giving your answer as a simplified fraction.

Solutions relying on calculator technology are not acceptable.

(4)

ANSWER

Question	Scheme	Marks	AOs
9(a)(i)	$\log_3 \left(\frac{x}{9} \right) = \log_3 x - \log_3 9 = p - 2$	B1	1.2
(ii)	$\log_3 (\sqrt{x}) = \frac{1}{2} p$	B1	1.1b
		(2)	
(b)	$2\log_3 \left(\frac{x}{9} \right) + 3\log_3 (\sqrt{x}) = -11 \Rightarrow 2p - 4 + \frac{3}{2} p = -11 \Rightarrow p = \dots$	M1	1.1b
	$p = -2$	A1	1.1b
	$\log_3 x = -2 \Rightarrow x = 3^{-2}$	M1	1.1b
	$x = \frac{1}{9}$	A1	1.1b
		(4)	

Video solution:

<https://youtu.be/-IE3ybfqRvg>

June 2022 Question 8

8. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

The air pressure, P kg/cm², inside a car tyre, t minutes from the instant when the tyre developed a puncture is given by the equation

$$P = k + 1.4e^{-0.5t} \quad t \in \mathbb{R} \quad t \geq 0$$

where k is a constant.

Given that the initial air pressure inside the tyre was 2.2 kg/cm²

(a) state the value of k .

(1)

From the instant when the tyre developed the puncture,

(b) find the time taken for the air pressure to fall to 1 kg/cm²

Give your answer in minutes to one decimal place.

(3)

(c) Find the rate at which the air pressure in the tyre is decreasing exactly 2 minutes from the instant when the tyre developed the puncture.

Give your answer in kg/cm² per minute to 3 significant figures.

(2)

ANSWER

Question	Scheme	Marks	AOs
8(a)	$(k =) 0.8$	B1	1.1b
		(1)	
(b)	$1 = 0.8 + 1.4e^{-0.5t} \Rightarrow 1.4e^{-0.5t} = 0.2$	M1	3.1b
	$-0.5t = \ln\left(\frac{0.2}{1.4}\right) \Rightarrow t = \dots$	M1	1.1b
	awrt 3.9 minutes	A1	1.1b
		(3)	
(c)	$\left(\frac{dP}{dt}\right) = -0.7e^{-0.5t}$ $\left(\frac{dP}{dt}\right)_{t=2} = -0.7e^{-0.5 \times 2}$	M1	3.1b
	= awrt 0.258 (kg/cm ² per minute)	A1	1.1b
		(2)	
(6 marks)			

Video solution:

<https://youtu.be/Dnw1ESD5AhQ>

June 2022 Question 5

5. The mass, A kg, of algae in a small pond, is modelled by the equation

$$A = pq^t$$

where p and q are constants and t is the number of weeks after the mass of algae was first recorded.

Data recorded indicates that there is a linear relationship between t and $\log_{10} A$ given by the equation

$$\log_{10} A = 0.03t + 0.5$$

- (a) Use this relationship to find a complete equation for the model in the form

$$A = pq^t$$

giving the value of p and the value of q each to 4 significant figures.

(4)

- (b) With reference to the model, interpret

(i) the value of the constant p ,

(ii) the value of the constant q .

(2)

- (c) Find, according to the model,

(i) the mass of algae in the pond when $t = 8$, giving your answer to the nearest 0.5 kg,

(ii) the number of weeks it takes for the mass of algae in the pond to reach 4 kg.

(3)

- (d) State one reason why this may not be a realistic model in the long term.

(1)

ANSWER

Question	Scheme	Marks	AOs
5(a)	$p = 10^{0.5}$ (or $\log_{10} p = 0.5$) or $q = 10^{0.03}$ (or $\log_{10} q = 0.03$)	M1	1.1b
	$p = \text{awrt } 3.162$ or $q = \text{awrt } 1.072$	A1	1.1b
	$p = 10^{0.5}$ (or $\log_{10} p = 0.5$) and $q = 10^{0.03}$ (or $\log_{10} q = 0.03$)	dM1	3.1a
	$A = 3.162 \times 1.072^t$	A1	3.3
		(4)	
(b)(i)	The initial mass (in kg) of algae (in the pond).	B1	3.4
(b)(ii)	The ratio of algae from one week to the next.	B1	3.4
		(2)	
(c)(i)	5.5 kg	B1	2.2a
(c)(ii)	$4 = 3.162 \times 1.072^t$ or $\log_{10} 4 = 0.03t + 0.5$	M1	3.4
	awrt 3.4 (weeks)	A1	1.1b
		(3)	
(d)	- The model predicts unlimited growth. - The weather may affect the rate of growth	B1	3.5b
		(1)	
(10 marks)			

Video solution:

<https://youtu.be/BxMnlf4QtqE>

November 2021 Question 11

11. The owners of a nature reserve decided to increase the area of the reserve covered by trees.

Tree planting started on 1st January 2005.

The area of the nature reserve covered by trees, $A \text{ km}^2$, is modelled by the equation

$$A = 80 - 45e^{ct}$$

where c is a constant and t is the number of years after 1st January 2005.

Using the model,

- (a) find the area of the nature reserve that was covered by trees just before tree planting started.

(1)

On 1st January 2019 an area of 60 km^2 of the nature reserve was covered by trees.

- (b) Use this information to find a complete equation for the model, giving your value of c to 3 significant figures.

(4)

On 1st January 2020, the owners of the nature reserve announced a long-term plan to have 100 km^2 of the nature reserve covered by trees.

- (c) State a reason why the model is not appropriate for this plan.

(1)

ANSWER

Question	Scheme	Marks	AOs
11 (a)	$35 (\text{km}^2)$	B1	3.4
		(1)	
(b)	Sets their $60 = 80 - 45e^{14c} \Rightarrow 45e^{14c} = 20$	M1 A1	1.1b 1.1b
	$\Rightarrow c = \frac{1}{14} \ln\left(\frac{20}{45}\right) = \dots - 0.0579$	dM1	3.1b
	$A = 80 - 45e^{-0.0579t}$	A1	3.3
		(4)	
(c)	Gives a suitable answer - The maximum area covered by trees is only 80 km^2 - The "80" would need to be "100" - Substitutes 100 into the equation of the model and shows that the formula fails with a reason eg. you cannot take a log of a negative number	B1	3.5b
		(1)	
(6 marks)			

November 2021 Question 13

13.

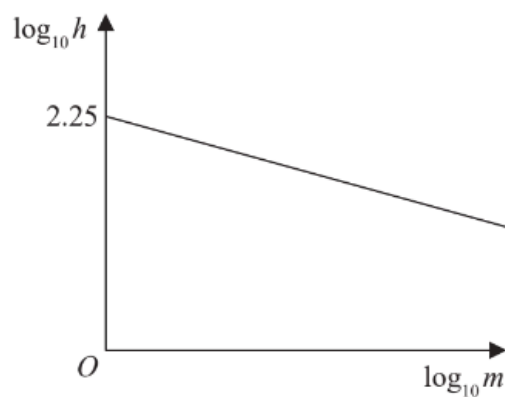


Figure 2

The resting heart rate, h , of a mammal, measured in beats per minute, is modelled by the equation

$$h = pm^q$$

where p and q are constants and m is the mass of the mammal measured in kg.

Figure 2 illustrates the linear relationship between $\log_{10} h$ and $\log_{10} m$

The line meets the vertical $\log_{10} h$ axis at 2.25 and has a gradient of -0.235

(a) Find, to 3 significant figures, the value of p and the value of q . (3)

A particular mammal has a mass of 5 kg and a resting heart rate of 119 beats per minute.

(b) Comment on the suitability of the model for this mammal. (3)

(c) With reference to the model, interpret the value of the constant p . (1)

ANSWER

Question	Scheme		Marks	AOs
13 (a)	$\log_{10} h = 2.25 - 0.235 \log_{10} m$ $\Rightarrow h = 10^{2.25 - 0.235 \log_{10} m}$ $\Rightarrow h = 10^{2.25} \times m^{-0.235}$	$h = pm^q$ $\Rightarrow \log_{10} h = \log_{10} p + \log_{10} m^q$ $\Rightarrow \log_{10} h = \log_{10} p + q \log_{10} m$	M1	1.1b
	Either one of $p = 10^{2.25}$ $q = -0.235$	Or either one of $\log_{10} p = 2.25$ $q = -0.235$	A1	1.1b
	$\Rightarrow p = 178$ and $q = -0.235$		A1	2.2a
			(3)	
(b)	$h = "178" \times 5^{-0.235}"$	$\log_{10} h = "2.25" - "0.235" \log_{10} 5$	M1	3.1b
	$h = 122$	$h = 122$	A1	1.1b
	Reasonably accurate (to 2 sf) so suitable		A1ft	3.2b
			(3)	
(c)	"p" would be the (resting) heart rate (in bpm) of a mammal with a mass of 1 kg		B1	3.4
			(1)	
(7 marks)				

November 2021 Question 8 Paper 1 (A-Level)

8. A scientist is studying the growth of two different populations of bacteria.

The number of bacteria, N , in the **first** population is modelled by the equation

$$N = Ae^{kt} \quad t \geq 0$$

where A and k are positive constants and t is the time in hours from the start of the study.

Given that

- there were 1000 bacteria in this population at the start of the study
- it took exactly 5 hours from the start of the study for this population to double

(a) find a complete equation for the model.

(4)

(b) Hence find the rate of increase in the number of bacteria in this population exactly 8 hours from the start of the study. Give your answer to 2 significant figures.

(2)

The number of bacteria, M , in the **second** population is modelled by the equation

$$M = 500e^{1.4kt} \quad t \geq 0$$

where k has the value found in part (a) and t is the time in hours from the start of the study.

Given that T hours after the start of the study, the number of bacteria in the two different populations was the same,

(c) find the value of T .

(3)

ANSWER

Question	Scheme	Marks	AOs
8(a)	$A = 1000$	B1	3.4
	$2000 = 1000e^{5k}$ or $e^{5k} = 2$	M1	1.1b
	$e^{5k} = 2 \Rightarrow 5k = \ln 2 \Rightarrow k = \dots$	M1	2.1
	$N = 1000e^{\left(\frac{1}{5}\ln 2\right)t}$ or $N = 1000e^{0.139t}$	A1	3.3
		(4)	
(b)	$\frac{dN}{dt} = 1000 \times \left(\frac{1}{5}\ln 2\right)e^{\left(\frac{1}{5}\ln 2\right)t}$ or $\frac{dN}{dt} = 1000 \times 0.139e^{0.139t}$ $\left(\frac{dN}{dt}\right)_{t=8} = 1000 \times \left(\frac{1}{5}\ln 2\right)e^{8 \times \frac{1}{5}\ln 2}$ or $\left(\frac{dN}{dt}\right)_{t=8} = 1000 \times 0.139e^{0.139 \times 8}$	M1	3.1b
	= awrt 420	A1	1.1b
		(2)	
(c)	$500e^{1.4 \times \left(\frac{1}{5}\ln 2\right)T} = 1000e^{\left(\frac{1}{5}\ln 2\right)T}$ or $500e^{1.4 \times "0.139" \times T} = 1000e^{0.139 \times T}$	M1	3.4
	Correct method of getting a linear equation in T E.g. $0.08T \ln 2 = \ln 2$ or $1.4 \times "0.139" \times T = \ln 2 + "0.139" \times T$	M1	2.1
	$T = 12.5$ hours	A1	1.1b
		(3)	
(9 marks)			

November 2021 Question 3 Paper 2 (A-Level)

3. Using the laws of logarithms, solve the equation

$$\log_3 (12y + 5) - \log_3 (1 - 3y) = 2$$

(3)

ANSWER

Question	Scheme	Marks	AOs
3	$\log_3 (12y + 5) - \log_3 (1 - 3y) = 2 \Rightarrow \log_3 \frac{12y + 5}{1 - 3y} = 2$ or e.g. $2 = \log_3 9$	B1 M1 on EPEN	1.1b
	$\log_3 \frac{12y + 5}{1 - 3y} = 2 \Rightarrow \frac{12y + 5}{1 - 3y} = 3^2 \Rightarrow 9 - 27y = 12y + 5 \Rightarrow y = \dots$ or e.g. $\log_3 (12y + 5) = \log_3 (3^2 (1 - 3y)) \Rightarrow (12y + 5) = 3^2 (1 - 3y) \Rightarrow y = \dots$	M1	2.1
	$y = \frac{4}{39}$	A1	1.1b
		(3)	
(3 marks)			

November 2021 Question 10 Paper 2 (A-Level)

10. The time, T seconds, that a pendulum takes to complete one swing is modelled by the formula

$$T = al^b$$

where l metres is the length of the pendulum and a and b are constants.

(a) Show that this relationship can be written in the form

$$\log_{10} T = b \log_{10} l + \log_{10} a \quad (2)$$

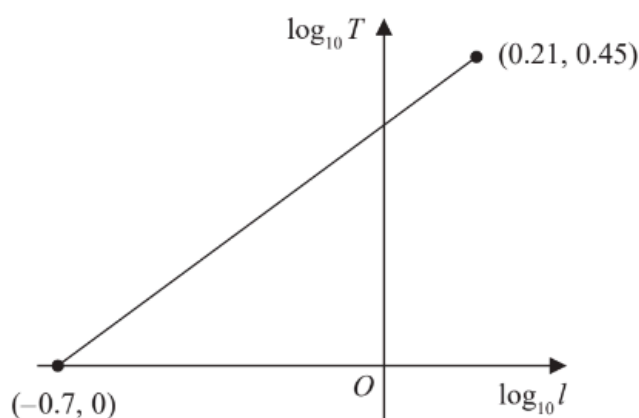


Figure 3

A student carried out an experiment to find the values of the constants a and b .

The student recorded the value of T for different values of l .

Figure 3 shows the linear relationship between $\log_{10} l$ and $\log_{10} T$ for the student's data.

The straight line passes through the points $(-0.7, 0)$ and $(0.21, 0.45)$

Using this information,

(b) find a complete equation for the model in the form

$$T = al^b$$

giving the value of a and the value of b , each to 3 significant figures. (3)

(c) With reference to the model, interpret the value of the constant a . (1)

ANSWER

Question	Scheme	Marks	AOs
10(a)	$T = al^b \Rightarrow \log_{10} T = \log_{10} a + \log_{10} l^b$	M1	2.1
	$\Rightarrow \log_{10} T = \log_{10} a + b \log_{10} l^*$ or $\Rightarrow \log_{10} T = b \log_{10} l + \log_{10} a^*$	A1*	1.1b
		(2)	
(b)	$b = 0.495$ or $b = \frac{45}{91}$	B1	2.2a
	$0 = "0.495" \times -0.7 + \log_{10} a \Rightarrow a = 10^{0.346...}$ or $0.45 = "0.495" \times 0.21 + \log_{10} a \Rightarrow a = 10^{0.346...}$	M1	3.1a
	$T = 2.22l^{0.495}$	A1	3.3
		(3)	
(c)	The time taken for one swing of a pendulum of length 1 m	B1	3.2a
		(1)	
(6 marks)			

November 2020 Question 8

8. The temperature, $\theta^\circ\text{C}$, of a cup of tea t minutes after it was placed on a table in a room, is modelled by the equation

$$\theta = 18 + 65e^{-\frac{t}{8}} \quad t \geq 0$$

Find, according to the model,

- the temperature of the cup of tea when it was placed on the table, (1)
- the value of t , to one decimal place, when the temperature of the cup of tea was 35°C . (3)
- Explain why, according to this model, the temperature of the cup of tea could not fall to 15°C . (1)

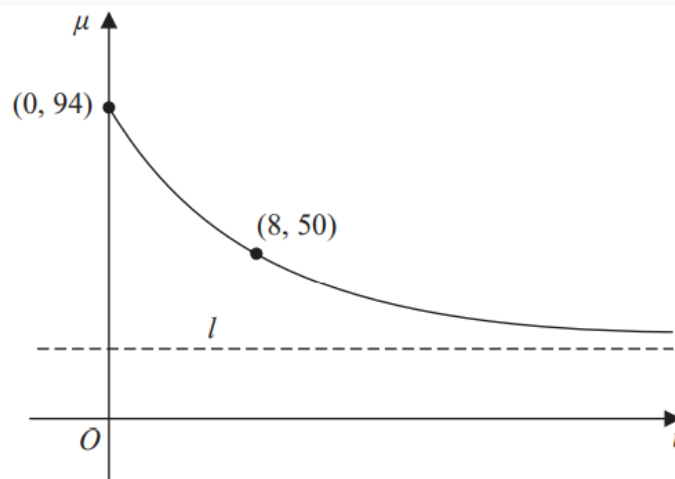


Figure 2

The temperature, $\mu^\circ\text{C}$, of a second cup of tea t minutes after it was placed on a table in a different room, is modelled by the equation

$$\mu = A + Be^{-\frac{t}{8}} \quad t \geq 0$$

where A and B are constants.

Figure 2 shows a sketch of μ against t with two data points that lie on the curve.

The line l , also shown on Figure 2, is the asymptote to the curve.

Using the equation of this model and the information given in Figure 2

- find an equation for the asymptote l . (4)

ANSWER

Question	Scheme	Marks	AOs
8 (a)	Temperature = 83°C	B1	3.4
		(1)	
(b)	$18 + 65e^{-\frac{t}{8}} = 35 \Rightarrow 65e^{-\frac{t}{8}} = 17$	M1	1.1b
	$t = -8 \ln\left(\frac{17}{65}\right)$ $\ln 65 - \frac{t}{8} = \ln 17 \Rightarrow t = \dots$	dM1	1.1b
	$t = 10.7$	A1	1.1b
		(3)	
(c)	States a suitable reason - As $t \rightarrow \infty, \theta \rightarrow 18$ from above. - The minimum temperature is 18°C	B1	2.4
		(1)	
(d)	$A + B = 94$ or $A + Be^{-1} = 50$	M1	3.4
	$A + B = 94$ and $A + Be^{-1} = 50$	A1	1.1b
	Full method to find at least a value for A	dM1	2.1
	Deduces $\mu = \frac{50e - 94}{e - 1}$ or accept $\mu = \text{awrt } 24.4$	A1	2.2a
		(4)	
(9 marks)			

November 2020 Question 2 Paper 1 (A-Level)

2. By taking logarithms of both sides, solve the equation

$$4^{3p-1} = 5^{210}$$

giving the value of p to one decimal place.

(3)

ANSWER

Question	Scheme	Marks	AOs
2	$4^{3p-1} = 5^{210} \Rightarrow (3p-1)\log 4 = 210\log 5$	M1	1.1b
	$\Rightarrow 3p = \frac{210\log 5}{\log 4} + 1 \Rightarrow p = \dots$	dM1	2.1
	$p = \text{awrt } 81.6$	A1	1.1b
		(3)	
(3 marks)			

November 2020 Question 3 Paper 2 (A-Level)

3. (a) Given that

$$2 \log(4 - x) = \log(x + 8)$$

show that

$$x^2 - 9x + 8 = 0$$

(3)

(b) (i) Write down the roots of the equation

$$x^2 - 9x + 8 = 0$$

(ii) State which of the roots in (b)(i) is **not** a solution of

$$2 \log(4 - x) = \log(x + 8)$$

giving a reason for your answer.

(2)

ANSWER

3(a)	$2 \log(4-x) = \log(4-x)^2$	B1	1.2
	$2 \log(4-x) = \log(x+8) \Rightarrow \log(4-x)^2 = \log(x+8)$ $(4-x)^2 = (x+8)$ or $2 \log(4-x) = \log(x+8) \Rightarrow \log(4-x)^2 - \log(x+8) = 0$ $\frac{(4-x)^2}{(x+8)} = 1$	M1	1.1b
	$16 - 8x + x^2 = x + 8 \Rightarrow x^2 - 9x + 8 = 0 *$	A1*	2.1
		(3)	
	(a) Alternative - working backwards:		
	$x^2 - 9x + 8 = 0 \Rightarrow (4-x)^2 - x - 8 = 0$	B1	1.2
	$\Rightarrow (4-x)^2 = x + 8$ $\Rightarrow \log(4-x)^2 = \log(x+8)$	M1	1.1b
	$\Rightarrow 2 \log(4-x) = \log(x+8) *$ Hence proved.	A1	2.1
(b)	(i) $(x=) 1, 8$	B1	1.1b
	(ii) 8 is not a solution as $\log(4-8)$ cannot be found	B1	2.3
		(2)	
(5 marks)			

November 2020 Question 5 Paper 2 (A-Level)

5. The curve with equation $y = 3 \times 2^x$ meets the curve with equation $y = 15 - 2^{x+1}$ at the point P .

Find, using algebra, the exact x coordinate of P .

(4)

ANSWER

Question	Scheme	Marks	AOs
5	$15 - 2^{x+1} = 3 \times 2^x$	B1	1.1b
	$\Rightarrow 15 - 2 \times 2^x = 3 \times 2^x \Rightarrow 2^x = 3$ or e.g. $\Rightarrow \frac{15}{2^x} - 2 = 3 \Rightarrow 2^x = 3$	M1	1.1b
	$2^x = 3 \Rightarrow x = \dots$	dM1	1.1b
	$x = \log_2 3$	A1cso	1.1b
		(4)	
	Alternative		
	$y = 3 \times 2^x \Rightarrow 2^x = \frac{y}{3} \Rightarrow y = 15 - 2 \times \frac{y}{3}$	B1	1.1b
	$3y + 2y = 45 \Rightarrow y = 9 \Rightarrow 3 \times 2^x = 9 \Rightarrow 2^x = 3$	M1	1.1b
	$2^x = 3 \Rightarrow x = \dots$	dM1	1.1b
	$x = \log_2 3$	A1cso	1.1b
(4 marks)			

November 2020 Question 9 Paper 2 (A-Level)

9. A quantity of ethanol was heated until it reached boiling point.

The temperature of the ethanol, $\theta^{\circ}\text{C}$, at time t seconds after heating began, is modelled by the equation

$$\theta = A - Be^{-0.07t}$$

where A and B are positive constants.

Given that

- the initial temperature of the ethanol was 18°C
- after 10 seconds the temperature of the ethanol was 44°C

- (a) find a complete equation for the model, giving the values of A and B to 3 significant figures.

(4)

Ethanol has a boiling point of approximately 78°C

- (b) Use this information to evaluate the model.

(2)

ANSWER

Question	Scheme	Marks	AOs
9(a)	$t = 0, \theta = 18 \Rightarrow 18 = A - B$ or $t = 10, \theta = 44 \Rightarrow 44 = A - Be^{-0.7}$	M1	3.1b
	$t = 0, \theta = 18 \Rightarrow 18 = A - B$ and $t = 10, \theta = 44 \Rightarrow 44 = A - Be^{-0.7}$ and $\Rightarrow A = \dots, B = \dots$	M1	3.1a
	At least one of: $A = 69.6, B = 51.6$ but allow awrt 70/awrt 52	A1 M1 on EPEN	1.1b
	$\theta = 69.6 - 51.6e^{-0.07t}$	A1	3.3
		(4)	
(b)	The maximum temperature is “69.6”(°C) (according to the model) (The model has an) upper limit of “69.6”(°C) (The model suggests that) the boiling point is “69.6”(°C)	B1ft	3.4
	Model is not appropriate as 69.6(°C) is much lower than 78(°C)	B1ft	3.5a
		(2)	
(6 marks)			

June 2019 Question 7 Paper 1 (A-Level)

7. In a simple model, the value, $\pounds V$, of a car depends on its age, t , in years.

The following information is available for car A

- its value when new is $\pounds 20\,000$
- its value after one year is $\pounds 16\,000$

(a) Use an exponential model to form, for car A , a possible equation linking V with t . (4)

The value of car A is monitored over a 10-year period.
Its value after 10 years is $\pounds 2\,000$

(b) Evaluate the reliability of your model in light of this information. (2)

The following information is available for car B

- it has the same value, when new, as car A
- its value depreciates more slowly than that of car A

(c) Explain how you would adapt the equation found in (a) so that it could be used to model the value of car B . (1)

ANSWER

Question	Scheme	Marks	AOs
7 (a)	Uses a model $V = Ae^{\pm kt}$ or (See next page for other suitable models)	M1	3.3
	Eg. Substitutes $t = 0, V = 20\,000 \Rightarrow A = 20\,000$	M1	1.1b
	Eg. Substitutes $t = 1, V = 16\,000 \Rightarrow 16\,000 = 20\,000e^{-1k} \Rightarrow k = ..$	dM1	3.1b
	$V = 20\,000e^{-0.223t}$	A1	1.1b
		(4)	
(b)	Substitutes $t = 10$ in their $V = 20\,000e^{-0.223t} \Rightarrow V = (£\,2150)$	M1	3.4
	Eg. The model is reliable as $£2150 \approx £2000$	A1	3.5a
		(2)	
(c)	Make the "-0.223" less negative. Alt: Adapt model to for example $V = 18\,000e^{-0.223t} + 2000$	B1ft	3.3
		(1)	
(7 marks)			

June 2019 Question 9 Paper 1 (A-Level)

9. Given that $a > b > 0$ and that a and b satisfy the equation

$$\log a - \log b = \log(a - b)$$

(a) show that

$$a = \frac{b^2}{b-1} \quad (3)$$

(b) Write down the full restriction on the value of b , explaining the reason for this restriction. (2)

ANSWER

Question	Scheme	Marks	AOs
9 (a)	States $\log a - \log b = \log \frac{a}{b}$	B1	1.2
	Proceeds from $\frac{a}{b} = a - b \rightarrow \dots \rightarrow ab - a = b^2$	M1	1.1b
	$ab - a = b^2 \rightarrow a(b-1) = b^2 \Rightarrow a = \frac{b^2}{b-1} *$	A1*	2.1
		(3)	
(b)	States either $b > 1$ or $b \neq 1$ with reason $\frac{b^2}{b-1}$ is not defined at $b = 1$ or	B1	2.2a
	States $b > 1$ and explains that as $a > 0 \Rightarrow \frac{b^2}{b-1} > 0 \Rightarrow b > 1$	B1	2.4
		(2)	
(5 marks)			

June 2019 Question 9 Paper 2 (A-Level)

9. A research engineer is testing the effectiveness of the braking system of a car when it is driven in wet conditions.

The engineer measures and records the braking distance, d metres, when the brakes are applied from a speed of $V \text{ km h}^{-1}$.

Graphs of d against V and $\log_{10} d$ against $\log_{10} V$ were plotted.

The results are shown below together with a data point from each graph.

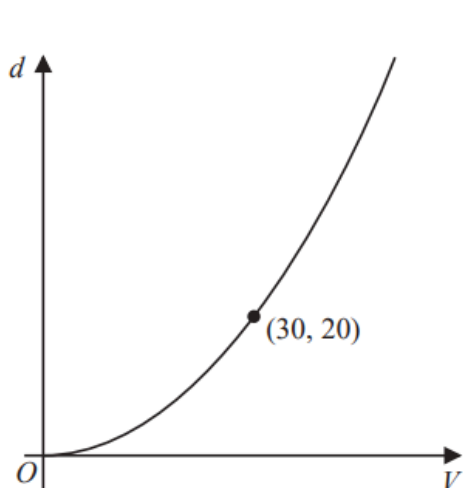


Figure 5

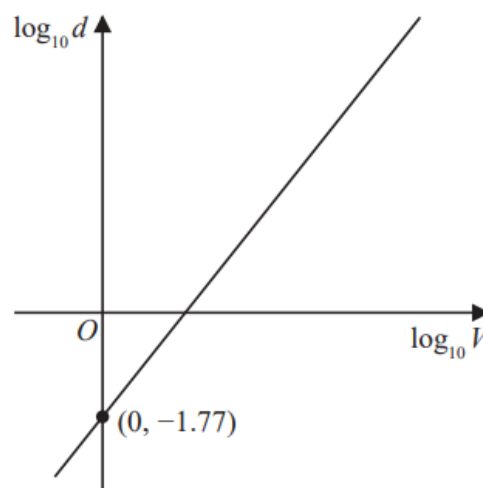


Figure 6

- (a) Explain how Figure 6 would lead the engineer to believe that the braking distance should be modelled by the formula

$$d = kV^n \quad \text{where } k \text{ and } n \text{ are constants}$$

with $k \approx 0.017$

(3)

Using the information given in Figure 5, with $k = 0.017$

- (b) find a complete equation for the model giving the value of n to 3 significant figures.

(3)

Sean is driving this car at 60 km h^{-1} in wet conditions when he notices a large puddle in the road 100 m ahead. It takes him 0.8 seconds to react before applying the brakes.

- (c) Use your formula to find out if Sean will be able to stop before reaching the puddle.

(3)

ANSWER

Question	Scheme	Marks	AOs
9 (a) Way 1	$\{d = kV^n \Rightarrow\} \log_{10} d = \log_{10} k + n \log_{10} V$ or $\log_{10} d = m \log_{10} V + c$ or $\log_{10} d = m \log_{10} V - 1.77$ seen or used as part of their argument	M1	2.1
	Alludes to $d = kV^n$ and gives a full explanation by comparing their result with a linear model e.g. $Y = MX + C$	A1	2.4
	$\{k =\} 10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
	(3)		
9 (a) Way 2	$\log_{10} d = m \log_{10} V + c$ or $\log_{10} d = m \log_{10} V - 1.77$ or $\log_{10} d = \log_{10} k + n \log_{10} V$ seen or used as part of their argument	M1	2.1
	$\{d = kV^n \Rightarrow\} \log_{10} d = \log_{10} (kV^n)$ $\Rightarrow \log_{10} d = \log_{10} k + \log_{10} V^n \Rightarrow \log_{10} d = \log_{10} k + n \log_{10} V$	A1	2.4
	$\{k =\} 10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
	(3)		
(b)	$\{d = 20, V = 30 \Rightarrow\} 20 = k(30)^n$ or $\log_{10} 20 = \log_{10} k + n \log_{10} 30$	M1	3.4
	$20 = k(30)^n \Rightarrow \log 20 = \log k + n \log 30 \Rightarrow n = \frac{\log 20 - \log k}{\log 30} \Rightarrow n = \dots$	M1	1.1b
	$\log_{10} 20 = \log_{10} k + n \log_{10} 30 \Rightarrow n = \frac{\log_{10} 20 - \log_{10} k}{\log_{10} 30} \Rightarrow n = \dots$		
	$\{n = \text{awrt } 2.08 \Rightarrow\} d = (0.017)V^{2.08}$ or $\log_{10} d = -1.77 + 2.08 \log_{10} V$	A1	1.1b
	Note: You can recover the A1 mark for a correct model equation given in part (c)	(3)	
(c)	$d = (0.017)(60)^{2.08}$	M1	3.4
	• $13.333\dots + 84.918\dots = 98.251\dots \Rightarrow$ Sean stops in time	M1	3.1b
	• $100 - 13.333\dots = 86.666\dots$ & $d = 84.918 \Rightarrow$ Sean stops in time	A1ft	3.2a
	(3)		

(9 marks)

June 2018 Question 5

5. A student's attempt to solve the equation $2\log_2 x - \log_2 \sqrt{x} = 3$ is shown below.

$$2\log_2 x - \log_2 \sqrt{x} = 3$$

$$2\log_2 \left(\frac{x}{\sqrt{x}} \right) = 3$$

using the subtraction law for logs

$$2\log_2 (\sqrt{x}) = 3$$

simplifying

$$\log_2 x = 3$$

using the power law for logs

$$x = 3^2 = 9$$

using the definition of a log

(a) Identify two errors made by this student, giving a brief explanation of each.

(2)

(b) Write out the correct solution.

(3)

ANSWER

Question	Scheme		Marks	AOs
5 (a)	Identifies one of the two errors "You cannot use the subtraction law without dealing with the 2 first" " They undo the logs incorrectly. It should be $x = 2^3 = 8$ "		B1	2.3
	Identifies both errors. See above.		B1	2.3
			(2)	
(b)	$\log_2\left(\frac{x^2}{\sqrt{x}}\right) = 3$	$\frac{3}{2}\log_2(x) = 3$	M1	1.1b
	$x^{\frac{3}{2}} = 2^3$ or $\frac{x^2}{\sqrt{x}} = 2^3$	$x = 2^2$	M1	1.1b
	$x = \left(2^3\right)^{\frac{2}{3}} = 4$	$x = 4$	A1	1.1b
			(3)	
(5 marks)				

June 2018 Question 13

13.

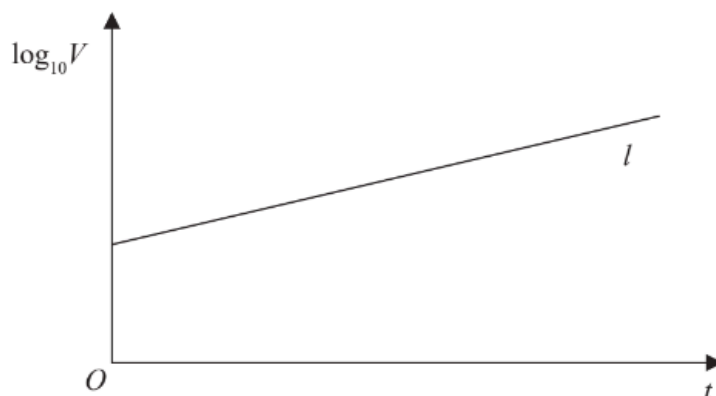


Figure 3

The value of a rare painting, £ V , is modelled by the equation $V = pq^t$, where p and q are constants and t is the number of years since the value of the painting was first recorded on 1st January 1980.

The line l shown in Figure 3 illustrates the linear relationship between t and $\log_{10} V$ since 1st January 1980.

The equation of line l is $\log_{10} V = 0.05t + 4.8$

- (a) Find, to 4 significant figures, the value of p and the value of q . (4)
- (b) With reference to the model interpret
- (i) the value of the constant p ,
 - (ii) the value of the constant q . (2)
- (c) Find the value of the painting, as predicted by the model, on 1st January 2010, giving your answer to the nearest hundred thousand pounds. (2)

ANSWER

Question	Scheme	Marks	AOs
13(a)	For a correct equation in p or q $p = 10^{4.8}$ or $q = 10^{0.05}$	M1	1.1b
	For $p = \text{awrt } 63100$ or $q = \text{awrt } 1.122$	A1	1.1b
	For correct equations in p and q $p = 10^{4.8}$ and $q = 10^{0.05}$	dM1	3.1a
	For $p = \text{awrt } 63100$ and $q = \text{awrt } 1.122$	A1	1.1b
		(4)	
(b)	(i) The value of the painting on 1st January 1980	B1	3.4
	(ii) The proportional increase in value each year	B1	3.4
		(2)	
(c)	Uses $V = 63100 \times 1.122^{30}$ or $\log V = 0.05 \times 30 + 4.8$ leading to $V =$	M1	3.4
	$= \text{awrt } (\pounds) 2000000$	A1	1.1b
		(2)	
(8 marks)			

June 2018 Question 12 Paper 1

12. The value, £ V , of a vintage car t years after it was first valued on 1st January 2001, is modelled by the equation

$$V = Ap^t \quad \text{where } A \text{ and } p \text{ are constants}$$

Given that the value of the car was £32 000 on 1st January 2005 and £50 000 on 1st January 2012

- (a) (i) find p to 4 decimal places,
- (ii) show that A is approximately 24 800 (4)
- (b) With reference to the model, interpret
- (i) the value of the constant A ,
- (ii) the value of the constant p . (2)

Using the model,

- (c) find the year during which the value of the car first exceeds £100 000 (4)

ANSWER

Question	Scheme	Marks	AOs
12 (a)	(i) Method to find p Eg. Divides $32000 = Ap^4$ by $50000 = Ap^{11}$ $p^7 = \frac{50000}{32000} \Rightarrow p = \sqrt[7]{\frac{50000}{32000}} = \dots$	M1	3.1a
	$p = 1.0658$	A1	1.1b
	(ii) Substitutes their $p = 1.0658$ into either equation and finds A $A = \frac{32000}{1.0658^4} \text{ or } A = \frac{50000}{1.0658^{11}}$	M1	1.1b
	$A = 24795 \rightarrow 24805 \approx 24\,800 *$	A1*	1.1b
		(4)	
(b)	A / (£)24 800 is the value of the car on 1st January 2001	B1	3.4
	$p / 1.0658$ is the factor by which the value rises each year. Accept that the value rises by 6.6 % a year (ft on their p)	B1	3.4
		(2)	
(c)	Attempts $100000 = 24800 \times 1.0658^t$		
	$1.0658^t = \frac{100000}{24800}$	M1	3.4
	$t = \log_{1.0658} \left(\frac{100000}{24800} \right)$	dM1	1.1b
	$t = 21.8 \text{ or } 21.9$	A1	1.1b
	cs0 2022	A1	3.2a
		(4)	
(10 marks)			

June 2018 Question 7 Core 2

7. (i) Find the value of y for which

$$1.01^{y-1} = 500$$

Give your answer to 2 decimal places.

(2)

- (ii) Given that

$$2\log_4(3x+5) = \log_4(3x+8) + 1, \quad x > -\frac{5}{3}$$

- (a) show that

$$9x^2 + 18x - 7 = 0$$

(4)

- (b) Hence solve the equation

$$2\log_4(3x+5) = \log_4(3x+8) + 1, \quad x > -\frac{5}{3}$$

(2)

ANSWER

Question Number	Scheme	Marks
7. (i)	Use of power rule so $(y-1)\log 1.01 = \log 500$ or $(y-1) = \log_{1.01} 500$	M1
	625.56	A1
		(2)
(ii) (a)	Ignore labels (a) and (b) in part ii and mark work as seen	
	$\log_4(3x+5)^2 =$ Applies power law of logarithms	M1
	Uses $\log_4 4 = 1$ or $4^1 = 4$	M1
	Uses quotient or product rule so e.g. $\log(3x+5)^2 = \log 4(3x+8)$ or $\log \frac{(3x+5)^2}{(3x+8)} = 1$	M1
	Obtains with no errors $9x^2 + 18x - 7 = 0$ *	A1* cso
		(4)
(b)	Solves given or "their" quadratic equation by any of the standard methods	M1
	Obtains $x = \frac{1}{3}$ and $-\frac{7}{3}$ and rejects $-\frac{7}{3}$ to give just $\frac{1}{3}$	A1
		(2)
		[8]

June 2018 Question 3 Core 3

3. The value of a car is modelled by the formula

$$V = 16\,000e^{-kt} + A, \quad t \geq 0, t \in \mathbb{R}$$

where V is the value of the car in pounds, t is the age of the car in years, and k and A are positive constants.

Given that the value of the car is £17 500 when new and £13 500 two years later,

- (a) find the value of A ,

(1)

- (b) show that $k = \ln\left(\frac{2}{\sqrt{3}}\right)$

(4)

- (c) Find the age of the car, in years, when the value of the car is £6000

Give your answer to 2 decimal places.

(4)

ANSWER

Question Number	Scheme	Marks
3(a)	$A = 1500$	B1 (1)
(b)	Sub $t = 2, V = 13500 \Rightarrow 16000e^{-2k} = 12000$ $\Rightarrow e^{-2k} = \frac{3}{4} \quad 0.75 \quad \text{oe}$ $\Rightarrow k = -\frac{1}{2} \ln \frac{3}{4}, = \ln \sqrt{\frac{4}{3}} = \ln \left(\frac{2}{\sqrt{3}} \right)$	M1 A1 dM1, A1* (4)
(c)	Sub $6000 = 16000e^{-\ln\left(\frac{2}{\sqrt{3}}\right)T} + '1500' \Rightarrow e^{-\ln\left(\frac{2}{\sqrt{3}}\right)T} = C$ $\Rightarrow e^{-\ln\left(\frac{2}{\sqrt{3}}\right)T} = \frac{45}{160} = 0.28125$ $\Rightarrow T = -\frac{\ln\left(\frac{45}{160}\right)}{\ln\left(\frac{2}{\sqrt{3}}\right)} = 8.82$	M1 A1 M1 A1 (4)
		(9 marks)