Mechanics Year 2 exam questions

NOTE:
Please be aware that in the Year 13 collections, you will miss questions from the Year 13 papers. However, these questions are intentionally included because they align with Year 12 content and topics.

Ch. 4: Moments

November 2021 question 3

3.

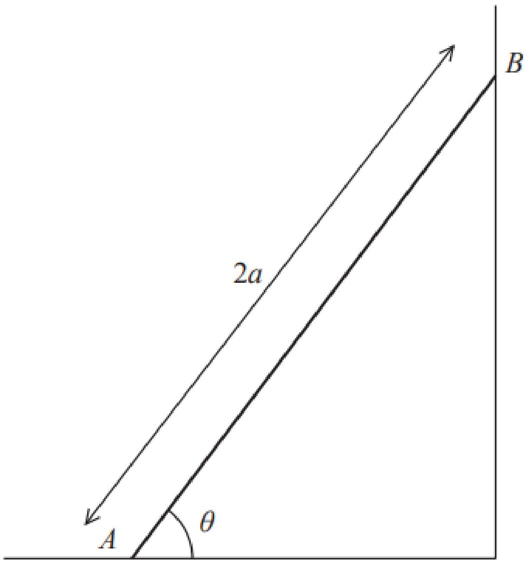


Figure 2

A beam AB has mass m and length $2a$.

The beam rests in equilibrium with A on rough horizontal ground and with B against a smooth vertical wall.

The beam is inclined to the horizontal at an angle θ, as shown in Figure 2.

The coefficient of friction between the beam and the ground is μ

The beam is modelled as a uniform rod resting in a vertical plane that is perpendicular to the wall.

Using the model,

(a) show that $\mu \geqslant \frac{1}{2}\cot\theta$

(5)

A horizontal force of magnitude kmg, where k is a constant, is now applied to the beam at A.

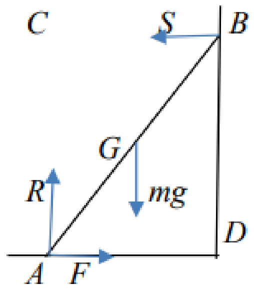
This force acts in a direction that is perpendicular to the wall and towards the wall.

Given that $\tan\theta = \frac{5}{4}$, $\mu = \frac{1}{2}$ and the beam is now in limiting equilibrium,

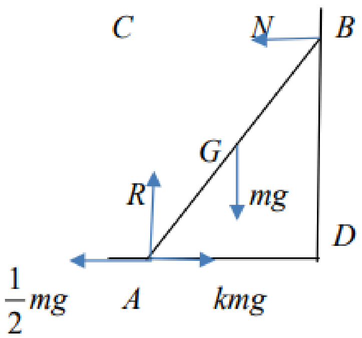
(b) use the model to find the value of k.

(5)

ANSWER

Question	Scheme	Marks	AOs
	Part (a) is a 'Show that..' so equations need to be given in full to earn A marks		
3(a)			
	Moments equation: (M1A0 for a moments inequality)	M1	3.3
	M(A), $mga\cos\theta = 2Sa\sin\theta$ M(B), $mga\cos\theta + 2Fa\sin\theta = 2Ra\cos\theta$ M(C), $F \times 2a\sin\theta = mga\cos\theta$ M(D), $2Ra\cos\theta = mga\cos\theta + 2Sa\sin\theta$ M(G), $Ra\cos\theta = Fa\sin\theta + Sa\sin\theta$.	A1	1.1b
	($\updownarrow$) $R = mg$ **OR** ($\leftrightarrow$) $F = S$	B1	3.4
	Use their equations <u>(they must have enough)</u> and $F \le \mu R$ to give an inequality in μ and θ **only** (allow DM1 for use of $F = \mu R$ to give an *equation* in μ and θ only)	DM1	2.1
	$\mu \ge \dfrac{1}{2}\cot\theta$ *	A1*	2.2a
		(5)	

Next page.

3(b)			
	Moments equation:	M1	3.4
	M(A), $mga\cos\theta = 2Na\sin\theta$ M(B), $mga\cos\theta + 2kmga\sin\theta = 2Ra\cos\theta + \frac{1}{2}mg2a\sin\theta$ M(D), $2Ra\cos\theta = mga\cos\theta + N2a\sin\theta$ M(G), $kmga\sin\theta + Na\sin\theta = \frac{1}{2}mga\sin\theta + Ra\cos\theta$	A1	1.1b
	S.C. M(C), $mga\cos\theta + \frac{1}{2}mg2a\sin\theta = kmg2a\sin\theta$ **M1A1B1** $1 + \frac{5}{4} = \frac{5k}{2}$ **M1** $k = 0.9$ **A1**		
	$N = kmg - F$ **OR** $R = mg$	B1	3.3
	Use their equations (they must have enough) to solve for k (numerical)	DM1	3.1b
	$k = 0.9$ oe	A1	1.1b
		(5)	
			(10 marks)

June 2018 Question 9

9.

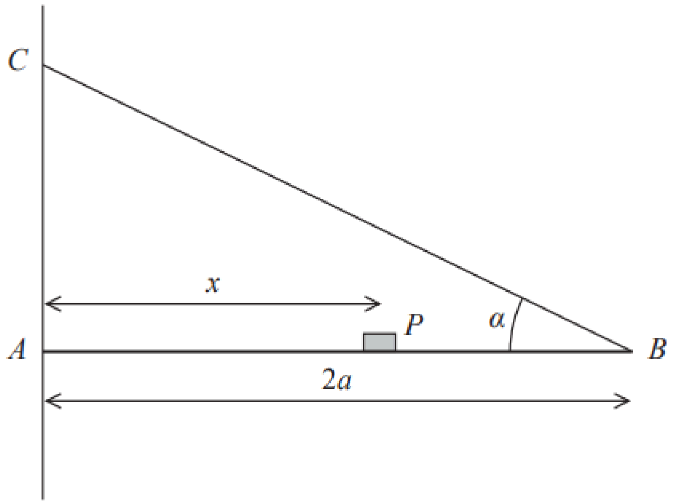


Figure 3

A plank, AB, of mass M and length $2a$, rests with its end A against a rough vertical wall. The plank is held in a horizontal position by a rope. One end of the rope is attached to the plank at B and the other end is attached to the wall at the point C, which is vertically above A.

A small block of mass $3M$ is placed on the plank at the point P, where $AP = x$. The plank is in equilibrium in a vertical plane which is perpendicular to the wall.

The angle between the rope and the plank is α, where $\tan\alpha = \frac{3}{4}$, as shown in Figure 3.

The plank is modelled as a uniform rod, the block is modelled as a particle and the rope is modelled as a light inextensible string.

(a) Using the model, show that the tension in the rope is $\dfrac{5Mg(3x + a)}{6a}$

(3)

The magnitude of the horizontal component of the force exerted on the plank at A by the wall is $2Mg$.

(b) Find x in terms of a.

(2)

The force exerted on the plank at A by the wall acts in a direction which makes an angle β with the horizontal.

(c) Find the value of $\tan\beta$

(5)

The rope will break if the tension in it exceeds $5Mg$.

(d) Explain how this will restrict the possible positions of P. You must justify your answer carefully.

(3)

ANSWER

Question	Scheme	Marks	AOs
9(a)	Moments about A (or any other complete method)	M1	3.3
	$T2a\sin\alpha = Mga + 3Mgx$	A1	1.1b
	$T = \frac{Mg(a+3x)}{2a \times \frac{3}{5}} = \frac{5Mg(3x+a)}{6a}$ * GIVEN ANSWER	A1*	2.1
		(3)	
(b)	$\frac{5Mg(3x+a)}{6a}\cos\alpha = 2Mg$ **OR** $2Mg.2a\tan\alpha = Mga + 3Mgx$	M1	3.1b
	$x = \frac{2a}{3}$	A1	2.2a
		(2)	
(c)	Resolve vertically **OR** Moments about B	M1	3.1b
	$Y = 3Mg + Mg - \frac{5Mg(3.\frac{2a}{3}+a)}{6a}\sin\alpha$ $2aY = Mga + 3Mg(2a - \frac{2a}{3})$ **Or**: $Y = 3Mg + Mg - \left(\frac{2Mg}{\cos\alpha}\right)\sin\alpha$	A1ft	1.1b
	$Y = \frac{5Mg}{2}$ **N.B.** May use $R\sin\beta$ for Y and/or $R\cos\beta$ for X throughout	A1	1.1b
	$\tan\beta = \frac{Y}{X}$ or $\frac{R\sin\beta}{R\cos\beta}$ $= \frac{\frac{5Mg}{2}}{2Mg}$	M1	3.4

	$= \frac{5}{4}$	A1	2.2a
		(5)	
(d)	$\frac{5Mg(3x+a)}{6a} \le 5Mg$ and solve for x	M1	2.4
	$x \le \frac{5a}{3}$	A1	2.4
	For rope not to break, block can't be more than $\frac{5a}{3}$ from A oe Or just: $x \le \frac{5a}{3}$, if no incorrect statement seen. **N.B. If the correct inequality is not found,** their comment must mention 'distance from A'.	B1 **A1**	2.4
		(3)	
			(13 marks)

Video solution:

https://youtu.be/NWhzZxPcpFw

June 2024 Question 3

3.

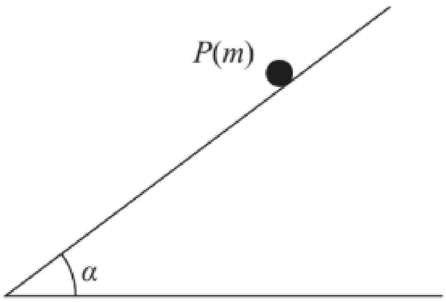


Figure 3

A particle P of mass m is held at rest at a point on a rough inclined plane, as shown in Figure 3.

It is given that

- the plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{5}{12}$
- the coefficient of friction between P and the plane is μ, where $\mu < \frac{5}{12}$

The particle P is released from rest and slides down the plane.
Air resistance is modelled as being negligible.

Using the model,

(a) find, in terms of m and g, the magnitude of the normal reaction of the plane on P, **(2)**

(b) show that, as P slides down the plane, the acceleration of P down the plane is

$$\frac{1}{13}g(5 - 12\mu)$$

(4)

(c) State what would happen to P if it is released from rest but $\mu \geqslant \frac{5}{12}$ **(1)**

ANSWER

Question	Scheme	Marks	AOs
3(a)	$(R=)mg\cos\alpha$	M1	3.4
	$=\frac{12}{13}mg$	A1	1.1b
		(2)	
3(b)	Equation of motion down the plane	M1	2.1
	$mg\sin\alpha - F = ma$ or $mg\sin\alpha - F = -ma$	A1	1.1b
	$(F=)$ $\mu\times$their R	M1	3.4
	$\frac{1}{13}g(5-12\mu)$ *	A1*	2.2a
		(4)	
3(c)	P wouldn't move	B1	2.4
		(1)	
			(7 marks)

June 2024 Question 1

1.

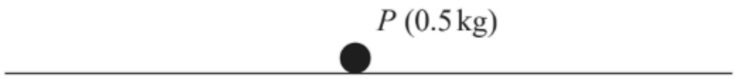


Figure 1

Figure 1 shows a particle P of mass 0.5 kg at rest on a rough horizontal plane.

(a) Find the magnitude of the normal reaction of the plane on P.

(1)

The coefficient of friction between P and the plane is $\frac{2}{7}$

A horizontal force of magnitude X newtons is applied to P.

Given that P is now in limiting equilibrium,

(b) find the value of X.

(2)

ANSWER

Question	Scheme	Marks	AOs
1(a)	$0.5g$, $\frac{1}{2}g$ or 4.9 (N) seen	B1	3.4
		(1)	
1(b)	$\frac{2}{7} \times 4.9$ oe seen	M1	3.1a
	1.4, 1.40 or $\frac{1}{7}g$	A1	1.1b
		(2)	
			(3 marks)

June 2023 Question 2

2.

Figure 1

A particle P has mass 5 kg.

The particle is pulled along a rough horizontal plane by a horizontal force of magnitude 28 N.

The only resistance to motion is a frictional force of magnitude F newtons, as shown in Figure 1.

(a) Find the magnitude of the normal reaction of the plane on P

(1)

The particle is accelerating along the plane at $1.4\,\mathrm{m\,s^{-2}}$

(b) Find the value of F

(2)

The coefficient of friction between P and the plane is μ

(c) Find the value of μ, giving your answer to 2 significant figures.

(1)

ANSWER

Question	Scheme	Marks	AOs
2(a)	Resolve vertically, $R = 5g = 49$ (N)	B1	1.1b
		(1)	
2(b)	Equation of motion: $28 - F = 5 \times 1.4$	M1	3.1a
	$F = 21$	A1	1.1b
		(2)	
2(c)	$\mu = 0.43$ (2sf required)	B1 ft	3.4
		(1)	
			(4 marks)

November 2021 question 2

2.

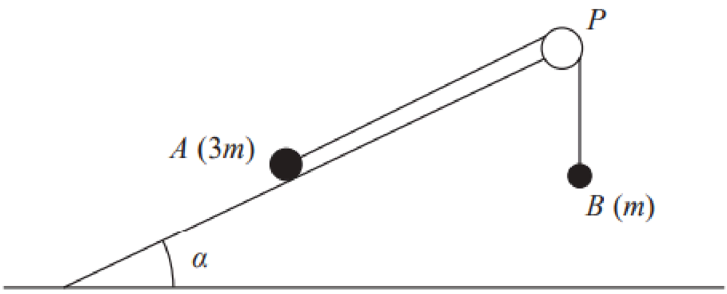


Figure 1

A small stone A of mass $3m$ is attached to one end of a string.

A small stone B of mass m is attached to the other end of the string.

Initially A is held at rest on a fixed rough plane.

The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

The string passes over a pulley P that is fixed at the top of the plane.

The part of the string from A to P is parallel to a line of greatest slope of the plane.

Stone B hangs freely below P, as shown in Figure 1.

The coefficient of friction between A and the plane is $\frac{1}{6}$

Stone A is released from rest and begins to move down the plane.

The stones are modelled as particles.

The pulley is modelled as being small and smooth.

The string is modelled as being light and inextensible.

Using the model for the motion of the system before B reaches the pulley,

(a) write down an equation of motion for A

(2)

Continues in next page

(b) show that the acceleration of A is $\frac{1}{10}g$

(7)

(c) sketch a velocity-time graph for the motion of B, from the instant when A is released from rest to the instant just before B reaches the pulley, explaining your answer.

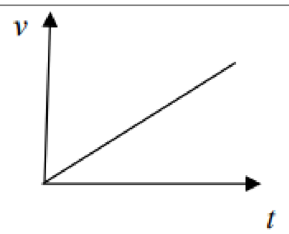
(2)

In reality, the string is not light.

(d) State how this would affect the working in part (b).

(1)

ANSWER

Question	Scheme	Marks	AOs
	Mark parts (a) and (b) together		
2(a)	Equation of motion for A	M1	3.3
	$3mg\sin\alpha - F - T = 3ma$	A1	1.1b
		(2)	
2(b)	Resolve perpendicular to the plane	M1	3.4
	$R = 3mg\cos\alpha$	A1	1.1b
	$F = \frac{1}{6}R$	B1	1.2
	Equation of motion for B **OR** for whole system	M1	3.3
	$T - mg = ma$ **OR** $3mg\sin\alpha - F - mg = 3ma + ma$	A1	1.1b
	Complete method to solve for a	DM1	3.1b
	$a = \frac{1}{10}g$ *	A1*	2.2a
		(7)	
2(c)		B1	1.1b
	e.g. acceleration (of B) is constant; dependent on first B1	DB1	2.4
		(2)	
2(d)	e.g. the tensions in the two equations of motion would be different. Tension on A would be different to tension on B	B1	3.5a
		(1)	
			(12 marks)

Video solution:

https://youtu.be/_LKDrLRhhnE

June 2018 Question 7

7.

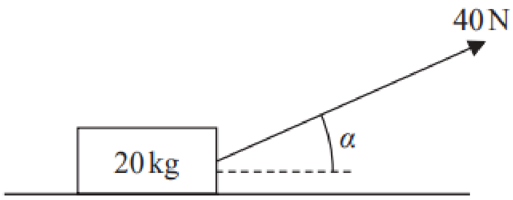


Figure 1

A wooden crate of mass 20 kg is pulled in a straight line along a rough horizontal floor using a handle attached to the crate.

The handle is inclined at an angle α to the floor, as shown in Figure 1, where $\tan\alpha = \frac{3}{4}$

The tension in the handle is 40 N.
The coefficient of friction between the crate and the floor is 0.14
The crate is modelled as a particle and the handle is modelled as a light rod.

Using the model,

(a) find the acceleration of the crate.

(6)

The crate is now pushed along the same floor using the handle. The handle is again inclined at the same angle α to the floor, and the thrust in the handle is 40 N as shown in Figure 2 below.

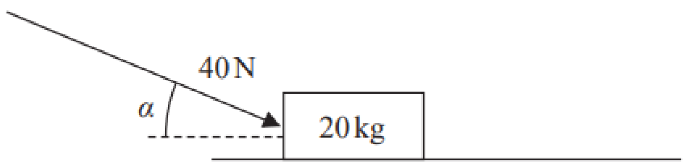


Figure 2

(b) Explain briefly why the acceleration of the crate would now be less than the acceleration of the crate found in part (a).

(2)

ANSWER

Question	Scheme	Marks	AOs
7(a)	Resolve vertically	M1	3.1b
	$R + 40\sin\alpha = 20g$	A1	1.1b
	Resolve horizontally	M1	3.1b
	$40\cos\alpha - F = 20a$	A1	1.1b
	$F = 0.14R$	B1	1.2
	$a = 0.396$ or 0.40 (m s^{-2})	A1	2.2a
		(6)	
(b)	Pushing will increase R which will increase available F	B1	2.4
	Increasing F will <u>decrease a</u> * GIVEN ANSWER	B1*	2.4
		(2)	
			(8 marks)

Ch. 6: Projectiles

June 2024 Question 5

5.

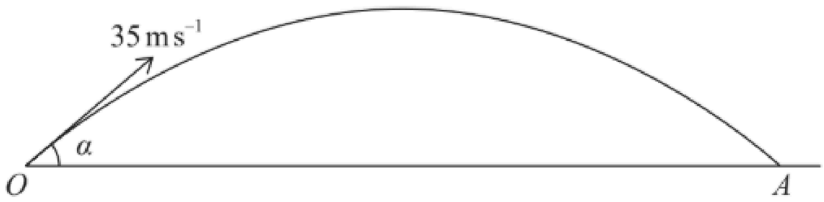


Figure 4

At time $t = 0$, a small stone is projected with velocity 35 m s^{-1} from a point O on horizontal ground.

The stone is projected at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$

In an initial model

- the stone is modelled as a particle P moving freely under gravity
- the stone hits the ground at the point A

Figure 4 shows the path of P from O to A.

For the motion of P from O to A

- at time t seconds, the horizontal distance of P from O is x metres
- at time t seconds, the vertical distance of P above the ground is y metres

(a) Using the model, show that

$$y = \frac{3}{4}x - \frac{1}{160}x^2$$

(6)

(b) Use the answer to (a), or otherwise, to find the length OA.

(2)

Using the model, the greatest height of the stone above the ground is found to be H metres.

(c) Use the answer to (a), or otherwise, to find the value of H.

(2)

- The model is refined to include air resistance.

Using this new model, the greatest height of the stone above the ground is found to be K metres.

(d) State which is greater, H or K, justifying your answer.

(1)

(e) State one limitation of this refined model.

(1)

ANSWER

Question	Scheme	Marks	AOs
5(a)	Using horizontal motion, $s = ut$, with 35 resolved	M1	3.3
	$x = 35\cos\alpha \times t$	A1	1.1b
	Using vertical motion, $s = ut + \frac{1}{2}at^2$, with 35 resolved	M1	3.4
	$y = 35\sin\alpha \times t - \frac{1}{2}gt^2$	A1	1.1b
	Eliminate t : $y = 35\sin\alpha \times \frac{x}{35\cos\alpha} - \frac{1}{2}g\left(\frac{x}{35\cos\alpha}\right)^2$	DM1	3.1b
	$y = \frac{3}{4}x - \frac{1}{160}x^2$ *	A1*	1.1b
	N.B. No marks available if they just quote the equation of the path.		

	ALTERNATIVE: they do (b) and/or (c) first <u>using a suvat method</u> Assume $y = ax^2 + bx + c$ Use any three of (0,0), (120,0) from part (b), (60,22.5) from part (c) or $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{4}$ at $x = 0$ to find a, b, and c. **M1A1, M1A1, DM1A1*** for finding each of a, b and c and stating final answer in correct form. **N.B.** If they realise that $c = 0$, and just use $y = ax^2 + bx$, that could score M1A1. Enter marks on ePEN in the order in which a, b and c are found: e.g. $x = 0, y = 0 \Rightarrow c = 0$ M1A1 $x = 0, \frac{\mathrm{d}y}{\mathrm{d}x} = 2ax + b = \frac{3}{4} \Rightarrow b = \frac{3}{4}$ M1 A1 $x = 120, y = 0 \Rightarrow 0 = 120^2 a + 120 \times \frac{3}{4} \Rightarrow a = \frac{-1}{160}$ DM1 so, $y = \frac{3}{4}x - \frac{1}{160}x^2$ A1*		
		(6)	

5(b)	**ALT 1** $0 = \frac{3}{4}x - \frac{1}{160}x^2$ and solve for x **ALT 2** $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{4} - \frac{x}{80} = 0 \Rightarrow x = 60$ **and** $OA = 2 \times 60$ **ALT 3**	M1	3.1b

	A complete *suvat* method to find OA: e.g. $0 = 35\sin\alpha \times t - \frac{1}{2}gt^2$ or $0 = 35\sin\alpha - g\frac{t}{2}$ or $-35\sin\alpha = 35\sin\alpha - gt$ to find t $\left(= \frac{70\sin\alpha}{g} = \frac{30}{7}\right)$ **AND** $(OA =)35\cos\alpha \times t = 35\cos\alpha \times \frac{70\sin\alpha}{g}$ **N.B. OR** use the calculator to input the equation of the path which then gives $y_{max} = 45/2$ when $x = 60$ **with no working**, so $OA = 2 \times 60$		
	$(OA =)120$ (m)	A1	1.1b
		(2)	

5(c)	**ALT 1** $H = \frac{3}{4}\times 60 - \frac{1}{160}\times 60^2$ **ALT 2** $y = \frac{-1}{160}(x^2 - 120x) = 22.5 - \frac{1}{160}(x-60)^2$ so max $y = 45/2$ or 22.5 **ALT 3** $\frac{dy}{dx} = \frac{3}{4} - \frac{2x}{160} = 0 \Rightarrow x = 60$ then find y when $x = 60$ **ALT 4** A complete *suvat* method: e.g. $H = \frac{(35\sin\alpha)^2}{2g}$ **or** $0 = 35\sin\alpha - gt$ to find the time to top, $t = \frac{35\sin\alpha}{g}$, or use half their time they found in (b) **AND** $H = 35\sin\alpha \times \frac{35\sin\alpha}{g} - \frac{1}{2}g\left(\frac{35\sin\alpha}{g}\right)^2$ or $\left(\frac{35\sin\alpha + 0}{2}\right)\times\frac{35\sin\alpha}{g}$ **N.B. OR** use the calculator to input the equation of the path which then gives $y_{max} = 45/2$ (when $x = 60$) with no working.	M1	3.1b
	$(H =)$ 22.5 Accept 23	A1	1.1b
		(2)	
5(d)	H is greater (or K is smaller), as air resistance would slow the particle down oe.	B1	3.5a
		(1)	
5(e)	e.g. the inaccuracy of using 9.8 m s^{-2} for g	B1	3.5b
		(1)	
			(12 marks)

June 2023 Question 5

5.

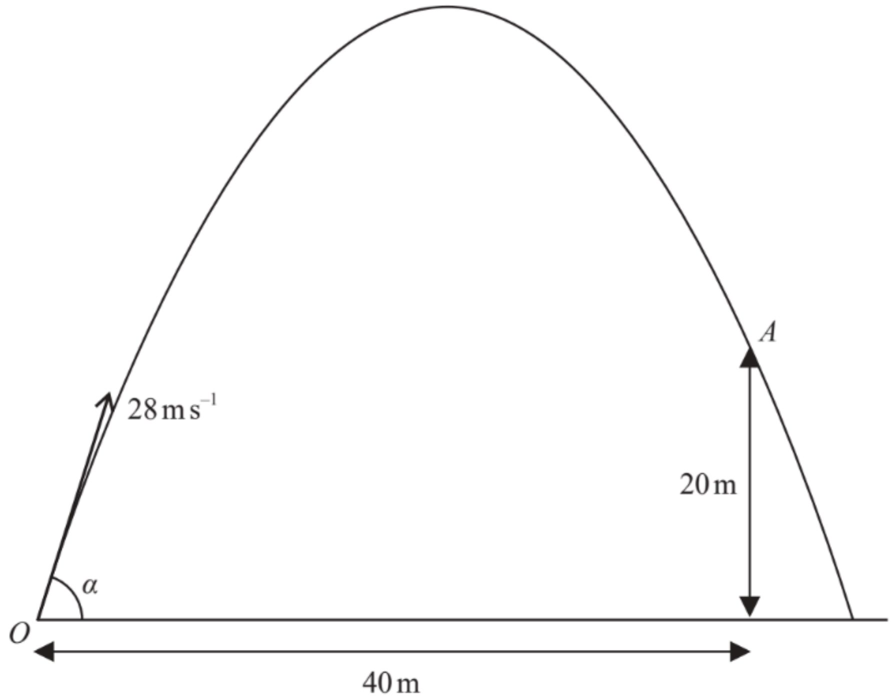


Figure 2

A small ball is projected with speed $28\,\mathrm{m\,s^{-1}}$ from a point O on horizontal ground.

After moving for T seconds, the ball passes through the point A.

The point A is 40 m horizontally and 20 m vertically from the point O, as shown in Figure 2.

The motion of the ball from O to A is modelled as that of a particle moving freely under gravity.

Given that the ball is projected at an angle α to the ground, use the model to

(a) show that $T = \dfrac{10}{7\cos\alpha}$

(2)

(b) show that $\tan^2\alpha - 4\tan\alpha + 3 = 0$

(5)

(c) find the greatest possible height, in metres, of the ball above the ground as the ball moves from O to A.

(3)

The model does not include air resistance.

(d) State one other limitation of the model.

(1)

ANSWER

Question	Scheme	Marks	AOs
	N.B. In this question, allow misread of α for *a*.		
5(a)	Use horizontal motion to give an equation in *T* and α only: $28\cos\alpha \times T = 40$	M1	3.4
	$T = \dfrac{10}{7\cos\alpha}$ *	A1*	1.1b
		(2)	

5(b)	Use vertical motion to give an equation in *T* and α only	M1	3.3
	$20 = (28\sin\alpha)T - \frac{1}{2}gT^2$	A1	1.1b
	Eliminate *T* to give an unsimplified equation in α only: $20 = (28\sin\alpha)\times\dfrac{10}{7\cos\alpha} - \dfrac{1}{2}g\left(\dfrac{10}{7\cos\alpha}\right)^2$	M1	1.1b
	Use $\sec^2\alpha = 1+\tan^2\alpha$ oe to give an unsimplified equation in **tan** α **only** : $20 = 40\tan\alpha - \frac{1}{2}g\times\frac{100}{49}(1+\tan^2\alpha)$	M1	3.1b
	$\tan^2\alpha - 4\tan\alpha + 3 = 0$ * (allow $0 = \tan^2\alpha - 4\tan\alpha + 3$)	A1*	2.2a
		(5)	

5(c)	Solve and use of $\tan\alpha = 3$ or $\sin\alpha = \frac{3}{\sqrt{10}}$ or $\alpha = 71.565..^{o}$ to find an equation in *H* only.	M1	3.1b
	$0 = (28\sin\alpha)^2 - 2gH$ **where** $\tan\alpha = 3 \ \ (\alpha = 71.565..^{o})$	M1	3.4
	H = 36 or 36.0 (m)	A1	1.1b
		(3)	
5(d)	e.g. spin of the ball, the wind, the dimensions or shape of the ball, ball is modelled as a particle, uses an inaccurate value of *g*, motion takes place in 3D not in 2D, *g* could be variable. B0 if mass or weight are mentioned. B0 for ground may not be horizontal.	B1	3.5b
		(1)	

June 2022 Question 5

5.

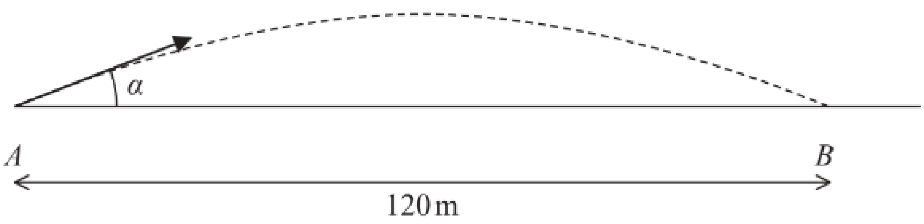


Figure 3

A golf ball is at rest at the point A on horizontal ground.

The ball is hit and initially moves at an angle α to the ground.

The ball first hits the ground at the point B, where $AB = 120$ m, as shown in Figure 3.

The motion of the ball is modelled as that of a particle, moving freely under gravity, whose initial speed is U m s^{-1}

Using this model,

(a) show that $U^2 \sin\alpha\cos\alpha = 588$

(6)

The ball reaches a maximum height of 10 m above the ground.

(b) Show that $U^2 = 1960$

(4)

In a refinement to the model, the effect of air resistance is included.

The motion of the ball, from A to B, is now modelled as that of a particle whose initial speed is V m s^{-1}

This refined model is used to calculate a value for V

(c) State which is greater, U or V, giving a reason for your answer.

(1)

(d) State one further refinement to the model that would make the model more realistic.

(1)

ANSWER

Question	Scheme	Marks	AOs
5(a)	Using horizontal motion	M1	3.3
	Whole Motion **Half way**		
	$U\cos\alpha \times t = 120$ $\quad$ $U\cos\alpha \times t = 60$	A1	1.1b
	Using vertical motion **OR**	M1	3.4
	$U\sin\alpha \times t - \frac{1}{2}gt^2 = 0$ $\quad$ $0 = U\sin\alpha - gt$	A1	1.1b
	Attempt to solve problem by eliminating t	DM1	3.1b
	$U^2 \sin\alpha\cos\alpha = 588$ *	A1*	2.2a
		(6)	
	N.B. No credit given if they use the given answer from (b).		
5(b)	Using vertical motion **OR** conservation of energy	M1	3.4
	$0^2 = (U\sin\alpha)^2 - 2g \times 10$ $\quad$ $\frac{1}{2}mU^2 - \frac{1}{2}m(U\cos\alpha)^2 = mg \times 10$	A1	1.1b
	Attempt to solve problem by eliminating α : e.g. $U\sin\alpha = 14$ => $U\cos\alpha = 42$, from part (a) or from using $t = \frac{10}{7}$, then square and add to give result **OR:** $U^2\sin^2\alpha = 20g = 196$ and $U^2\sin\alpha\cos\alpha = 588$, divide to give $\tan\alpha = \frac{1}{3}$ then $\sin^2\alpha = \frac{1}{10}$, hence result **OR in ALTERNATIVE 2**: sub for U^2 using part (a), to give $\tan\alpha = \frac{1}{3}$ then $\sin^2\alpha = \frac{1}{10}$, hence result	DM1	3.1b

	N.B. Just stating that $\sin^2\alpha = \frac{1}{10}$, with no working is DM0A0.		
	$U^2 = 1960$ *	A1*	2.2a
	N.B. Verification (i.e. starting with $U^2 = 1960$ and trying to work backwards) is not an acceptable method for this question.		
		(4)	
5(c)	V, since air resistance has to be overcome, or just 'because of air resistance' isw	B1	3.5a
		(1)	
5(d)	e.g. wind effects, more accurate value of *g*, spin of ball, size of ball, shape of ball, dimensions of ball, not a particle, variable acceleration, surface area of ball, humidity. Allow wind resistance and rotational resistance (Ignore any mention of air resistance or drag)	B1	3.5c
		(1)	
			(12 marks)

November 2021 question 4

4.

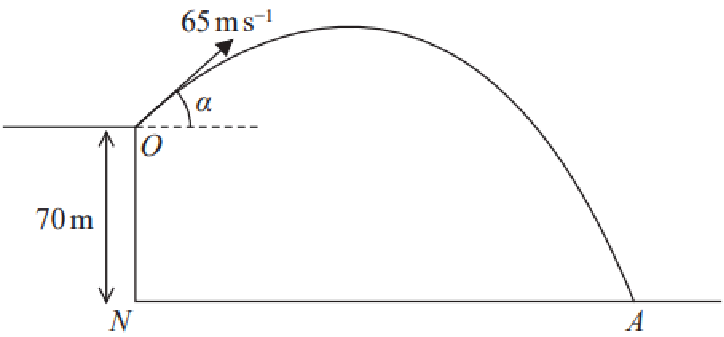


Figure 3

A small stone is projected with speed $65\,\text{m s}^{-1}$ from a point O at the top of a vertical cliff.

Point O is 70 m vertically above the point N.

Point N is on horizontal ground.

The stone is projected at an angle α above the horizontal, where $\tan\alpha = \dfrac{5}{12}$

The stone hits the ground at the point A, as shown in Figure 3.

The stone is modelled as a particle moving freely under gravity.

The acceleration due to gravity is modelled as having magnitude $10\,\text{m s}^{-2}$

Using the model,

(a) find the time taken for the stone to travel from O to A,

(4)

(b) find the speed of the stone at the instant just before it hits the ground at A.

(5)

One limitation of the model is that it ignores air resistance.

(c) State one other limitation of the model that could affect the reliability of your answers.

(1)

ANSWER

Question	Scheme	Marks	AOs
	Note that $g = 10$; penalise once for whole question if $g = 9.8$		
4(a)	Use $s = ut + \frac{1}{2}at^2$ vertically or any complete method to give an equation in t only	M1	3.4
	$-70 = 65\sin\alpha \times t - \frac{1}{2} \times g \times t^2$	A1	1.1b
		M (A)1	1.1b
	$t = 7$ (s)	A1	1.1b
		(4)	
4(b)	Horizontal velocity component at A = $65\cos\alpha$ (60)	B1	3.4
	Complete method to find vertical velocity component at A	M1	3.4
	$65\sin\alpha - g \times 7$ **OR** $\sqrt{(-25)^2 + 2g \times 70}$ (45)	A1**ft**	1.1b
	Sub for trig and square, add and square root : $\sqrt{60^2 + (-45)^2}$	M1	3.1b
	75 Accept 80 (m s^{-1})	A1	1.1b
		(5)	
4(c)	e.g. an approximate value of g has been used, the dimensions of the stone could affect its motion, spin of the stone, $g = 10$ instead of 9.8 has been used, g has been assumed to be constant, wind effect, shape of the stone	B1	3.5b
		(1)	
		(10 marks)	

October 2020 question 5

5.

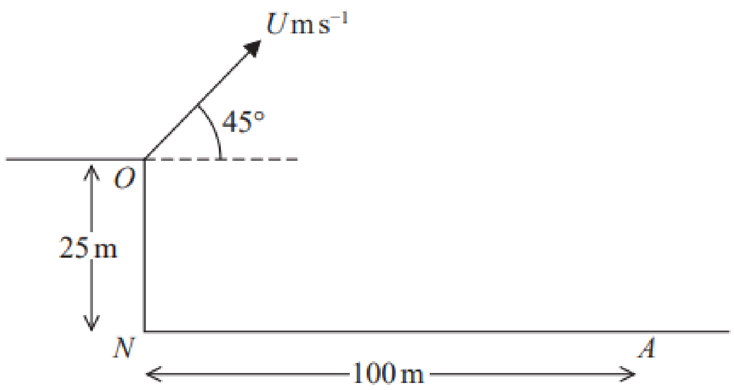


Figure 2

A small ball is projected with speed U m s^{-1} from a point O at the top of a vertical cliff.

The point O is 25 m vertically above the point N which is on horizontal ground.

The ball is projected at an angle of 45° above the horizontal.

The ball hits the ground at a point A, where $AN = 100$ m, as shown in Figure 2.

The motion of the ball is modelled as that of a particle moving freely under gravity.

Using this initial model,

(a) show that $U = 28$

(6)

(b) find the greatest height of the ball above the horizontal ground NA.

(3)

In a refinement to the model of the motion of the ball from O to A, the effect of air resistance is included.

This refined model is used to find a new value of U.

(c) How would this new value of U compare with 28, the value given in part (a)?

(1)

(d) State one further refinement to the model that would make the model more realistic.

(1)

ANSWER

Question	Scheme	Marks	AOs
5(a)	Using horizontal motion	M1	3.3
	$U\cos 45^\circ t = 100$	A1	1.1b
	Using vertical motion	M1	3.4
	$U\sin 45^\circ t - \frac{1}{2}gt^2 = -25$	A1	1.1b
	Solve problem by eliminating t and solving for U	M1	3.1b
	$U = 28$*	A1*	1.1b
		(6)	
5(b)	Using vertical motion	M1	3.4
	$0^2 = (28\sin 45^\circ)^2 - 2gh$	A1	1.1b
	Greatest height = 45 m	A1	1.1b
		(3)	
5(c)	New value > 28	B1	3.5a
		(1)	
5(d)	e.g. wind effects, more accurate value of g, spin of ball, include size of the ball, not model as a particle, shape of ball	B1	3.5c
		(1)	
			(11 marks)

June 2019 question 5

5.

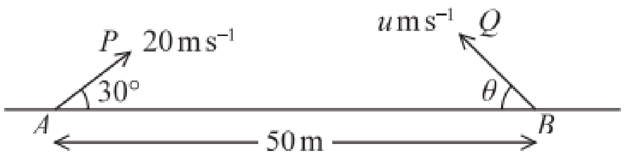


Figure 3

The points A and B lie 50 m apart on horizontal ground.

At time $t = 0$ two small balls, P and Q, are projected in the vertical plane containing AB.

Ball P is projected from A with speed $20\,\text{m s}^{-1}$ at $30°$ to AB.

Ball Q is projected from B with speed $u\,\text{m s}^{-1}$ at angle θ to BA, as shown in Figure 3.

At time $t = 2$ seconds, P and Q collide.

Until they collide, the balls are modelled as particles moving freely under gravity.

(a) Find the velocity of P at the instant before it collides with Q.

(6)

(b) Find

(i) the size of angle θ,

(ii) the value of u.

(6)

(c) State one limitation of the model, other than air resistance, that could affect the accuracy of your answers.

(1)

ANSWER

Question	Scheme	Marks	AO
	In this question mark parts (a) and (b) together.		
5(a)	Horizontal speed $=20\cos 30°$	B1	3.4
	Vertical velocity at $t=2$	M1	3.4
	$=20\sin 30° - 2g$	A1	1.1b
	$\theta = \tan^{-1}\left(\pm\frac{9.6}{10\sqrt{3}}\right)$	M1	1.1b
	Speed $=\sqrt{100\times 3+9.6^2}$ or e.g. speed $=\frac{9.6}{\sin\theta}$	M1	1.1b
	19.8 or 20 (m s^{-1}) at 29.0° or 29° to the horizontal oe	A1	2.2a
		(6)	
(b)	Using sum of horizontal distances $=50$ at $t=2$	M1	3.3
	$(u\cos\theta)\times 2 + (20\cos 30°)\times 2 = 50$ $(u\cos\theta = 25 - 20\cos 30°)$	A1	1.1b
	Vertical distances equal	M1	3.4
	$\Rightarrow (20\sin 30°)\times 2 - \frac{g}{2}\times 4 = (u\sin\theta)\times 2 - \frac{g}{2}\times 4$ $(20\sin 30° = u\sin\theta)$	A1	1.1b
	Solving for both θ and u	M1	3.1b
	$\theta = 52°$ or better (52.47756849….°) $u = 13$ or better (12.6085128…)	A1	2.2a
		(6)	
(c)	It does not take account of the fact that they are not particles (moving freely under gravity) It does not take account of the size(s) of the balls It does not take account of the spin of the balls It does not take account of the wind g is not exactly 9.8 m s^{-2} **N.B.** If they refer to the mass or weight of the balls give B0	B1	3.5b
		(1)	
		(13)	

June 2018 Question 10

10.

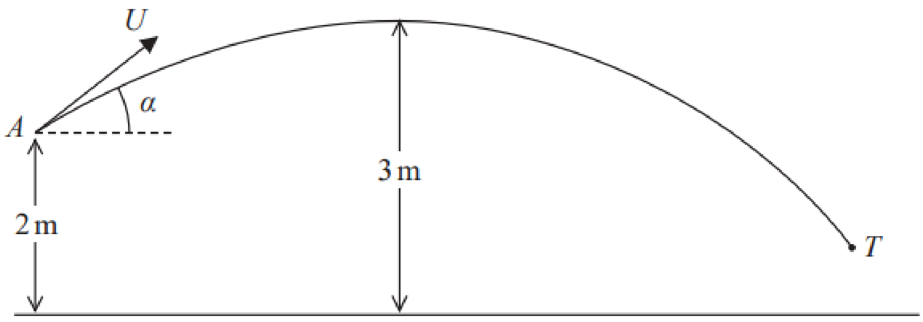


Figure 4

A boy throws a ball at a target. At the instant when the ball leaves the boy's hand at the point A, the ball is 2 m above horizontal ground and is moving with speed U at an angle α above the horizontal.

In the subsequent motion, the highest point reached by the ball is 3 m above the ground. The target is modelled as being the point T, as shown in Figure 4.
The ball is modelled as a particle moving freely under gravity.

Using the model,

(a) show that $U^2 = \dfrac{2g}{\sin^2 \alpha}$.

(2)

The point T is at a horizontal distance of 20 m from A and is at a height of 0.75 m above the ground. The ball reaches T without hitting the ground.

(b) Find the size of the angle α

(9)

(c) State one limitation of the model that could affect your answer to part (b).

(1)

(d) Find the time taken for the ball to travel from A to T.

(3)

ANSWER

Question	Scheme	Marks	AOs
10(a)	Using the model and vertical motion: $0^2 = (U\sin\alpha)^2 - 2g \times (3-2)$	M1	3.3
	$U^2 = \dfrac{2g}{\sin^2\alpha}$ * GIVEN ANSWER	A1*	2.2a
		(2)	
(b)	Using the model and horizontal motion: $s = ut$	M1	3.4
	$20 = Ut\cos\alpha$	A1	1.1b
	Using the model and vertical motion: $s = ut + \frac{1}{2}at^2$	M1	3.4
	$-\frac{5}{4} = Ut\sin\alpha - \frac{1}{2}gt^2$	A1	1.1b
	sub for t: $-\frac{5}{4} = U\sin\alpha\left(\dfrac{20}{U\cos\alpha}\right) - \frac{1}{2}g\left(\dfrac{20}{U\cos\alpha}\right)^2$	M1 (I)	3.1b
	sub for U^2	M1(II)	3.1b
	$-\frac{5}{4} = 20\tan\alpha - 100\tan^2\alpha$	A1(I)	1.1b
	$(4\tan\alpha - 1)(100\tan\alpha + 5) = 0$	M1(III)	1.1b
	$\tan\alpha = \frac{1}{4} \Rightarrow \alpha = 14^0$ or better	A1(II)	2.2a
		(9)	

(c)	The target will have dimensions so in practice there would be a range of possible values of α **Or** There will be air resistance **Or** The ball will have dimensions **Or** Wind effects **Or** Spin of the ball	B1	3.5b
		(1)	
(d)	Find U using their α e.g. $U=\sqrt{\dfrac{2g}{\sin^2\alpha}}$	M1	3.1b
	Use $20 = Ut\cos\alpha$ (or use vertical motion equation)	A1 **M1**	1.1b
	$t=\dfrac{5}{\sqrt{2g}}$ or 1.1 or 1.13	B1 **A1**	1.1b
		(3)	

Ch. 7: Applications of Forces

June 2024 Question 6

6.

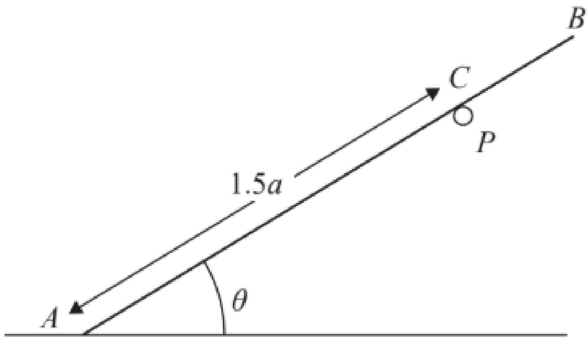


Figure 5

Figure 5 shows a uniform rod AB of mass M and length $2a$.

- the rod has its end A on rough horizontal ground
- the rod rests in equilibrium against a small smooth fixed horizontal peg P
- the point C on the rod, where $AC = 1.5a$, is the point of contact between the rod and the peg
- the rod is at an angle θ to the ground, where $\tan\theta = \frac{4}{3}$

The rod lies in a vertical plane perpendicular to the peg.

The magnitude of the normal reaction of the peg on the rod at C is S.

(a) Show that $S = \frac{2}{5}Mg$

(3)

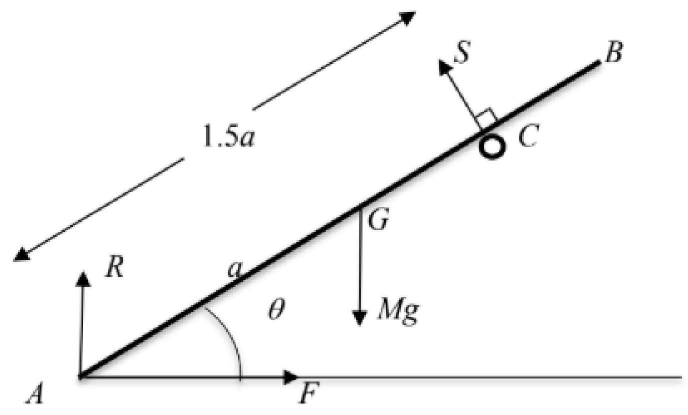
The coefficient of friction between the rod and the ground is μ.

Given that the rod is in limiting equilibrium,

(b) find the value of μ.

(6)

ANSWER

Question	Scheme	Marks	AOs
6.			
6(a)	Take moments about A	M1	3.1a
	$S \times 1.5a = Mga\cos\theta = (Mga \times \frac{3}{5})$	A1	1.1b
	$S = \frac{2}{5}Mg$ *	A1*	2.2a
		(3)	

6(b)	**N.B. Marks for the equations should be awarded in the order in which they appear on the script.**		
	Resolve horizontally:	M1	3.4
	$F = S\sin\theta$	A1	1.1b
	Resolve vertically:	M1	3.3
	$R = Mg - S\cos\theta$	A1	1.1b
	Other possible equations: (any of which is worth max M1A1) (parallel to the rod): $F\cos\theta + R\sin\theta = Mg\sin\theta$ (perp to the rod): $F\sin\theta + Mg\cos\theta = S + R\cos\theta$ M(B): $(S \times 0.5a) + (R \times 2a\cos\theta) = (Mg \times a\cos\theta) + (F \times 2a\sin\theta)$ M(C): $(R \times 1.5a\cos\theta) = (Mg \times 0.5a\cos\theta) + (F \times 1.5a\sin\theta)$ M(G): $(R \times a\cos\theta) = (S \times 0.5a) + (F \times a\sin\theta)$		

	N.B. If they have more than two equations, mark only those that they use to try to find μ		
	$F = \mu R$ and two of their equations used to solve for μ	DM1	3.1a
	$\mu = \frac{8}{19} = 0.42105\ldots$	A1	2.2a

		(6)	
			(9 marks)

June 2023 Question 6

6.

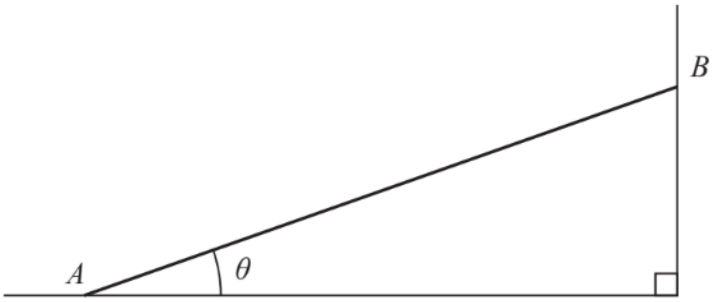


Figure 3

A rod AB has mass M and length $2a$.

The rod has its end A on rough horizontal ground and its end B against a smooth vertical wall.

The rod makes an angle θ with the ground, as shown in Figure 3.

The rod is at rest in limiting equilibrium.

(a) State the direction (left or right on Figure 3 above) of the frictional force acting on the rod at A. **Give a reason for your answer.**

(1)

The magnitude of the normal reaction of the wall on the rod at B is S.

In an initial model, the rod is modelled as being **uniform**.

Use this initial model to answer parts (b), (c) and (d).

(b) By taking moments about A, show that

$$S = \frac{1}{2}Mg\cot\theta$$

(3)

The coefficient of friction between the rod and the ground is μ

Given that $\tan\theta = \frac{3}{4}$

(c) find the value of μ

(5)

(d) find, in terms of M and g, the magnitude of the resultant force acting on the rod at A.

(3)

In a new model, the rod is modelled as being **non-uniform**, with its centre of mass closer to B than it is to A.

A new value for S is calculated using this new model, with $\tan\theta = \frac{3}{4}$

(e) State whether this new value for S is larger, smaller or equal to the value that S would take using the initial model. **Give a reason for your answer.**

(1)

ANSWER

Question	Scheme	Marks	AOs
6(a)	The normal reaction at *B* is acting to the left so it must act to the right, right as it needs to balance (oppose, counter) the force at *B*, right as it prevents the rod from sliding (slipping, falling), right as the weight (mass) of the rod will mean the rod tends to slip left, mass or weight will be pushing the rod to the left so friction will oppose that. **N.B.** You may see an arrow on the diagram at *A*, instead of 'right'. B0 if they say the rod is moving oe Accept towards the wall instead of to the right.	B1	2.4
		(1)	

Question	Scheme	Marks	AOs
6(b)	Take moments about *A*	M1	3.4
	$S\times 2a\sin\theta = Mga\cos\theta$	A1	1.1b
	$S=\frac{1}{2}Mg\cot\theta$ *	A1*	2.2a
		(3)	

Question	Scheme	Marks	AOs
6(c)	Resolve vertically, $R = Mg$	B1	3.3
	Resolve horizontally, $F = S$	B1	3.3
	Other possible equations: Resolve along the rod, $F\cos\theta + R\sin\theta = S\cos\theta + Mg\sin\theta$ Resolve perp to the rod, $R\cos\theta + S\sin\theta = F\sin\theta + Mg\cos\theta$ M(*B*), $R\times 2a\cos\theta = F\times 2a\sin\theta + Mga\cos\theta$ M(*G*), $Ra\cos\theta = Fa\sin\theta + Sa\sin\theta$ **N.B.** When entering these two B marks on ePEN**,** **First** B1 is for a **vertical** resolution, **second** B1 is for a **horizontal** resolution, and if either is replaced by a different equation, enter appropriately. If both are replaced by other equations, enter in the order in which they appear in their working.		
	$F=\mu R$	B1	1.2

	$\frac{1}{2}Mg\times\frac{4}{3}=\mu Mg$	**dM**1	2.1
	$\mu=\frac{2}{3}$ oe Accept 0.67 or better	A1	2.2a
	S.C. For $F \leqslant \mu R$, B0 $\frac{1}{2}Mg\times\frac{4}{3} \leqslant \mu Mg$ M1		
	$\frac{2}{3} \leqslant \mu$ A0 **N.B.** If $\mu=\frac{2}{3}$ follows this, they could score all the marks.		
		(5)	
6(d)	$\sqrt{F^2+R^2}$	M1	3.1a
	$\sqrt{\left(\frac{2}{3}Mg\right)^2+(Mg)^2}$	M1	1.1b
	$\frac{1}{3}Mg\sqrt{13}$ or 1.2Mg or better	A1	2.2a
		(3)	
6(e)	New value of *S* would be **larger** as the **moment** of the **weight** about ***A*** would be larger	B1	3.5a
		(1)	
			(13 marks)

June 2022 Question 4

4.

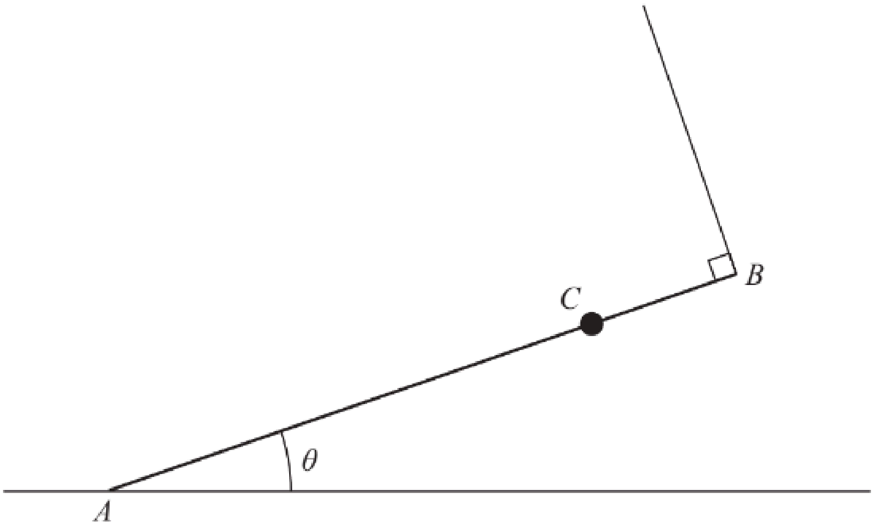


Figure 2

A uniform rod AB has mass M and length $2a$

A particle of mass $2M$ is attached to the rod at the point C, where $AC = 1.5a$

The rod rests with its end A on rough horizontal ground.

The rod is held in equilibrium at an angle θ to the ground by a light string that is attached to the end B of the rod.

The string is perpendicular to the rod, as shown in Figure 2.

(a) Explain why the frictional force acting on the rod at A acts horizontally to the right on the diagram.

(1)

The tension in the string is T

(b) Show that $T = 2Mg\cos\theta$

(3)

Given that $\cos\theta = \frac{3}{5}$

(c) show that the magnitude of the vertical force exerted by the ground on the rod at A is $\frac{57Mg}{25}$

(3)

The coefficient of friction between the rod and the ground is μ

Given that the rod is in limiting equilibrium,

(d) show that $\mu = \frac{8}{19}$

(4)

ANSWER

Question	Scheme	Marks	AOs
4(a)	The horizontal component of T acts to the left and since the **only** other horizontal force is friction, it must act to the right oe	B1	2.4
		(1)	
4(b)	Take moments about A or any other complete method to obtain **an equation in T, M and θ only.** (see possible equations below that they may use)	M1	3.1b
	$T.2a = Mga\cos\theta + 2Mg \times 1.5a\cos\theta$ (A0 if a's missing)	A1	1.1b
	Other possible equations but F and R would need to be eliminated. $(\nwarrow)$, $R\cos\theta + T = F\sin\theta + Mg\cos\theta + 2Mg\cos\theta$ $(\nearrow)$, $R\sin\theta + F\cos\theta = Mg\sin\theta + 2Mg\sin\theta$ $(\rightarrow)$, $F = T\sin\theta$ M(B), $R.2a\cos\theta = Mga\cos\theta + 2Mg \times 0.5a\cos\theta + F.2a\sin\theta$ M(G), $Fa\sin\theta + Ta = Ra\cos\theta + 2Mg \times 0.5a\cos\theta$ M(C), $R \times 1.5a\cos\theta = T \times 0.5a + Mg \times 0.5a\cos\theta + F \times 1.5a\sin\theta$		
	$T = 2Mg\cos\theta$ *	A1*	1.1b
		(3)	

	$(\nwarrow), R\cos\theta + T = F\sin\theta + Mg\cos\theta + 2Mg\cos\theta$ $(\nearrow), R\sin\theta + F\cos\theta = Mg\sin\theta + 2Mg\sin\theta$ $(\rightarrow),\ F = T\sin\theta$ $\text{M}(B),\ R.2a\cos\theta = Mga\cos\theta + 2Mg\times 0.5a\cos\theta + F.2a\sin\theta$ $\text{M}(G),\ Fa\sin\theta + Ta = Ra\cos\theta + 2Mg\times 0.5a\cos\theta$ $\text{M}(C),\ R\times 1.5a\cos\theta = T\times 0.5a + Mg\times 0.5a\cos\theta + F\times 1.5a\sin\theta$		
	$F = \mu R$ used i.e. both F and R are substituted.	M1	3.1b
	$\mu = \frac{8}{19}$ *	A1*	2.2a
		(4)	
			(11 marks)

4(c)	e.g. Resolve vertically	M1	3.4
	$(\uparrow), R + T\cos\theta = Mg + 2Mg$	A1	1.1b
	$R = \frac{57Mg}{25}$ *	A1*	1.1b
		(3)	
	Other possible equations but F would need to be eliminated. $(\nwarrow), R\cos\theta + T = F\sin\theta + Mg\cos\theta + 2Mg\cos\theta$ $(\nearrow), R\sin\theta + F\cos\theta = Mg\sin\theta + 2Mg\sin\theta$ $(\rightarrow),\ F = T\sin\theta$ $\text{M}(B),\ R.2a\cos\theta = Mga\cos\theta + 2Mg\times 0.5a\cos\theta + F.2a\sin\theta$ $\text{M}(G),\ Fa\sin\theta + Ta = Ra\cos\theta + 2Mg\times 0.5a\cos\theta$ $\text{M}(C),\ R\times 1.5a\cos\theta = T\times 0.5a + Mg\times 0.5a\cos\theta + F\times 1.5a\sin\theta$		
4(d)	Find an equation containing F e.g. Resolve horizontally	M1	3.4
	$(\rightarrow),\ F = T\sin\theta$	A1	1.1b
	Other possible equations		

Video solution:

https://youtu.be/tR8tNGHPYCI

June 2022 Question 2

2.

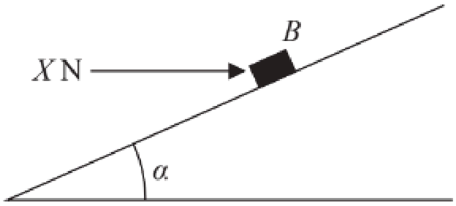


Figure 1

A rough plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

A small block B of mass 5 kg is held in equilibrium on the plane by a horizontal force of magnitude X newtons, as shown in Figure 1.

The force acts in a vertical plane which contains a line of greatest slope of the inclined plane.

The block B is modelled as a particle.

The magnitude of the normal reaction of the plane on B is 68.6 N.

Using the model,

(a) (i) find the magnitude of the frictional force acting on B,

(3)

(ii) state the direction of the frictional force acting on B.

(1)

The horizontal force of magnitude X newtons is now removed and B moves down the plane.

Given that the coefficient of friction between B and the plane is 0.5

(b) find the acceleration of B down the plane.

(6)

ANSWER

Question	Scheme	Marks	AOs
2(a)(i)	Resolve vertically	M1	3.1b
	F acting UP the plane: **OR** F acting DOWN the plane: $(\uparrow)\ F\sin\alpha + 68.6\cos\alpha = 5g$ $\quad -F\sin\alpha + 68.6\cos\alpha = 5g$ Other possible equations from which X would need to be eliminated to give an equation in F **only** to earn the M mark are shown below. The equation in F **only** must then be correct to earn the A mark. Possible equations: $(\nwarrow)\ 68.6 = X\sin\alpha + 5g\cos\alpha$ (leads to $X = 49$ with $g = 9.8$) F acting UP the plane: **OR** F acting DOWN the plane: $(\nearrow)\ F + X\cos\alpha = 5g\sin\alpha$ $\quad -F + X\cos\alpha = 5g\sin\alpha$ $(\rightarrow)\ F\cos\alpha + X = 68.6\sin\alpha$ $\quad -F\cos\alpha + X = 68.6\sin\alpha$	A1	1.1b
	9.8 (N) (49/5 is A0) **N.B.** If sin and cos are interchanged in all equations, this leads to an answer of 9.8 in the wrong direction and can only score (a) (i)M1A0A0 (ii) A0	A1	1.1b
		(3)	

Video solution:

https://youtu.be/3l6D-YY5jKw

2(a)(ii)	Down the plane (Allow down or downwards or an arrow ↙, but must appear as the answer to (a) (ii) not just on the diagram.)	A1	2.2a
		(1)	
2(b)	**N.B.** If they use $R = 68.6$ in this part, the maximum they can score is M1A1M0A0M0A0 If they use $F = 9.8$ or their F from (a) in this part, the maximum they can score is M1A1M0A0M0A0		
	Equation of motion down the plane	M1	2.1
	$5g\sin\alpha - F = 5a$ Allow ($-a$) instead of a	A1	1.1b
	Resolve perpendicular to the plane	M1	3.1b
	$R = 5g\cos\alpha$	A1	1.1b
	$F = 0.5R$ seen	M1	3.4
	$a = 1.96$ or 2.0 or 2 (m s^{-2}) or $\frac{1}{5}g$	A1	1.1b
		(6)	
			(10 marks)

October 2020 question 1

1. A rough plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

A brick P of mass m is placed on the plane.

The coefficient of friction between P and the plane is μ

Brick P is in equilibrium and on the point of sliding down the plane.

Brick P is modelled as a particle.

Using the model,

(a) find, in terms of m and g, the magnitude of the normal reaction of the plane on brick P

(2)

(b) show that $\mu = \frac{3}{4}$

(4)

For parts (c) and (d), you are not required to do any further calculations.

Brick P is now removed from the plane and a much heavier brick Q is placed on the plane.

The coefficient of friction between Q and the plane is also $\frac{3}{4}$

(c) Explain briefly why brick Q will remain at rest on the plane.

(1)

Brick Q is now projected with speed $0.5\,\text{m s}^{-1}$ down a line of greatest slope of the plane.

Brick Q is modelled as a particle.

Using the model,

(d) describe the motion of brick Q, giving a reason for your answer.

(2)

ANSWER

Question	Scheme	Marks	AOs
1.(a)	Resolve perpendicular to the plane	M1	3.4
	$R = mg\cos\alpha = \frac{4}{5}mg$	A1	1.1b
		(2)	
1(b)	Resolve parallel to the plane or horizontally or vertically	M1	3.4
	$F = mg\sin\alpha$ or $R\sin\alpha = F\cos\alpha$	A1	1.1b
	Use $F = \mu R$ and solve for μ	M1	2.1
	$\mu = \frac{3}{4}$ *	A1*	2.2a
		(4)	
1(c)	The forces acting on Q will still balance as the m's cancel oe Other possibilities: e.g. the friction will increase in the same proportion as the weight component or force down the plane. The force pulling the brick down the plane increases by the same amount as the friction oe This mark can be scored if they do the calculation.	B1	2.4
		(1)	
1(d)	Brick Q slides down the plane with **constant** speed.	B1	2.4
	No resultant force down the plane (so no acceleration) oe	B1	2.4
	These marks can be scored if they do the calculation.	**(2)**	
		(9 marks)	

October 2020 question 4

4.

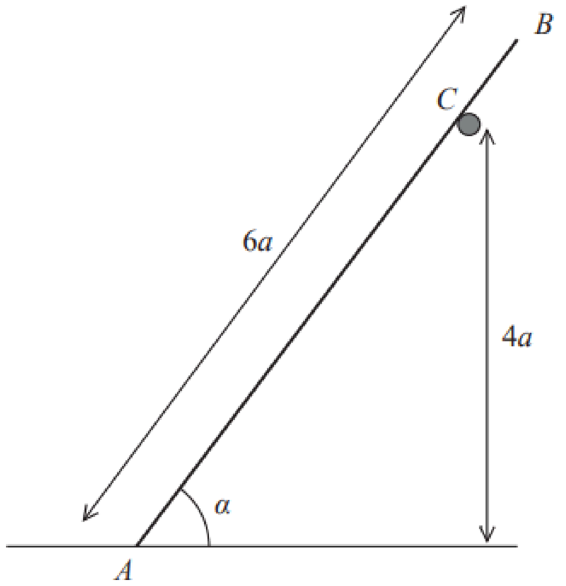


Figure 1

A ladder AB has mass M and length $6a$.

The end A of the ladder is on rough horizontal ground.

The ladder rests against a fixed smooth horizontal rail at the point C.

The point C is at a vertical height $4a$ above the ground.

The vertical plane containing AB is perpendicular to the rail.

The ladder is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{4}{5}$, as shown in Figure 1.

The coefficient of friction between the ladder and the ground is μ.

The ladder rests in limiting equilibrium.

The ladder is modelled as a uniform rod.

Using the model,

(a) show that the magnitude of the force exerted on the ladder by the rail at C is $\frac{9Mg}{25}$

(3)

(b) Hence, or otherwise, find the value of μ.

(7)

ANSWER

Question	Scheme	Marks	AOs
4(a)	Take moments about A	M1	3.3
	$N\times\dfrac{4a}{\sin\alpha}=Mg\times 3a\cos\alpha$	A1	1.1b
	$\dfrac{9Mg}{25}$ *	A1*	1.1b
		(3)	
4(b)	Resolve horizontally	M1	3.4
	$(\rightarrow)\ F=\dfrac{9Mg}{25}\sin\alpha$	A1	1.1b
	Resolve vertically	M1	3.4
	$(\uparrow)\ R+\dfrac{9Mg}{25}\cos\alpha=Mg$	A1	1.1b
	Other possible equations: $(\nwarrow),\ R\cos\alpha+\dfrac{9Mg}{25}=Mg\cos\alpha+F\sin\alpha$ $(\nearrow),\ Mg\sin\alpha=F\cos\alpha+R\sin\alpha$ $\text{M}(C),\ Mg.2a\cos\alpha+F.5a\sin\alpha=R.5a\cos\alpha$ $\text{M}(G),\ \dfrac{9Mg}{25}.2a+F.3a\sin\alpha=R.3a\cos\alpha$ $\text{M}(B),\ Mg.3a\cos\alpha+F.6a\sin\alpha=R.6a\cos\alpha+\dfrac{9Mg}{25}a$ $\left(F=\dfrac{36Mg}{125}\ ,\ R=\dfrac{98Mg}{125}\right)$		
	$F=\mu R$ used	M1	3.4
	Eliminate R and F and solve for μ	M1	3.1b
	$\mu=\dfrac{18}{49}$ (0.3673…..accept 0.37 or better)	A1	2.2a
		(7)	
			(10 marks)

June 2019 question 3

3.

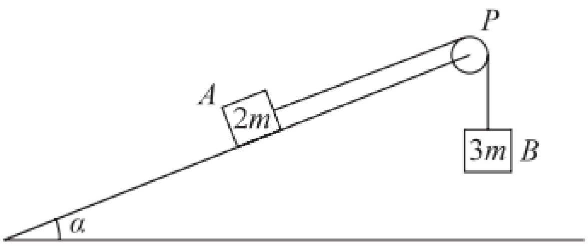


Figure 1

Two blocks, A and B, of masses $2m$ and $3m$ respectively, are attached to the ends of a light string.

Initially A is held at rest on a fixed rough plane.

The plane is inclined at angle α to the horizontal ground, where $\tan\alpha = \frac{5}{12}$

The string passes over a small smooth pulley, P, fixed at the top of the plane.

The part of the string from A to P is parallel to a line of greatest slope of the plane. Block B hangs freely below P, as shown in Figure 1.

The coefficient of friction between A and the plane is $\frac{2}{3}$

The blocks are released from rest with the string taut and A moves up the plane.

The tension in the string immediately after the blocks are released is T.

The blocks are modelled as particles and the string is modelled as being inextensible.

(a) Show that $T = \frac{12mg}{5}$

(8)

After B reaches the ground, A continues to move up the plane until it comes to rest before reaching P.

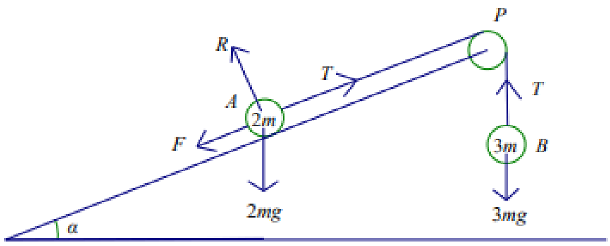
(b) Determine whether A will remain at rest, carefully justifying your answer.

(2)

(c) Suggest two refinements to the model that would make it more realistic.

(2)

ANSWER

Question	Scheme	Marks	AO
3(a)			
	$R = 2mg\cos\alpha$	B1	3.4
	$F = \frac{2}{3}R$	B1	1.2
	Equation of motion for A:	M1	3.3
	$T - F - 2mg\sin\alpha = 2ma$	A1	1.1b
	Equation of motion for B:	M1	3.3
	$3mg - T = 3ma$	A1	1.1b
	Complete strategy to find an equation in T, m and g only.	M1	3.1b
	$T = \frac{12mg}{5}$ *	A1*	2.2a
		(8)	
(b)	$(F_{max} =)\frac{16mg}{13} > \frac{10mg}{13}$	M1	2.1
	…… so A will not move.	A1	2.2a
		(2)	
(c)	• Extensible string • Weight of string • Friction at pulley e.g. rough pulley • Allow for the dimensions of the blocks e.g. "Do not model blocks as particles"; "(include) air resistance";"include rotational effects of forces on blocks i.e. spin"	B1 B1	3.5c 3.5c
		(2)	
		(12)	

June 2019 question 4

4.

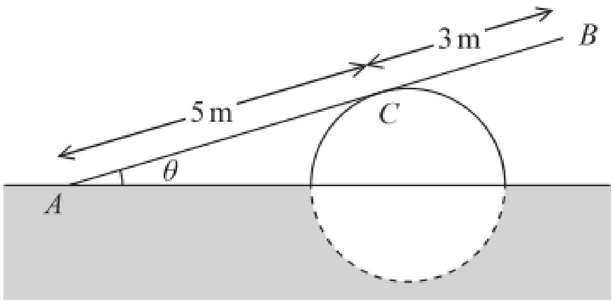


Figure 2

A ramp, AB, of length 8 m and mass 20 kg, rests in equilibrium with the end A on rough horizontal ground.

The ramp rests on a smooth solid cylindrical drum which is partly under the ground. The drum is fixed with its axis at the same horizontal level as A.

The point of contact between the ramp and the drum is C, where $AC = 5$ m, as shown in Figure 2.

The ramp is resting in a vertical plane which is perpendicular to the axis of the drum, at an angle θ to the horizontal, where $\tan\theta = \dfrac{7}{24}$

The ramp is modelled as a uniform rod.

(a) Explain why the reaction from the drum on the ramp at point C acts in a direction which is perpendicular to the ramp.

(1)

(b) Find the magnitude of the resultant force acting on the ramp at A.

(9)

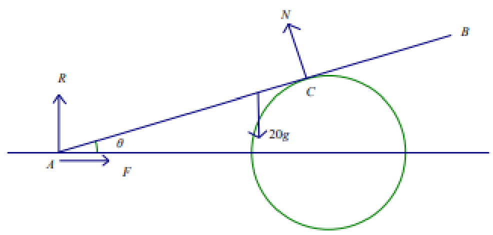
The ramp is still in equilibrium in the position shown in Figure 2 but the ramp is not now modelled as being uniform.

Given that the centre of mass of the ramp is assumed to be closer to A than to B,

(c) state how this would affect the magnitude of the normal reaction between the ramp and the drum at C.

(1)

ANSWER

Question	Scheme	Marks	AO
4(a)	Drum **smooth**, or no friction, (therefore reaction is perpendicular to the ramp)	B1	2.4
		(1)	
(b)	**N.B.** In (b), for a moments equation, if there is an extra $\sin\theta$ or $\cos\theta$ on a length, give M0 for the equation e.g. $\mathrm{M}(A)$: $20g \times 4\cos\theta = 5N\sin\theta$ would be given M0A0		
			
	Possible equns $(\nearrow): F\cos\theta + R\sin\theta = 20g\sin\theta$ $(\nwarrow): N + R\cos\theta = 20g\cos\theta + F\sin\theta$ $(\uparrow) R + N\cos\theta = 20g$ $(\rightarrow): F = N\sin\theta$ $\mathrm{M}(A)$: $20g \times 4\cos\theta = 5N$ $\mathrm{M}(B)$: $3N + R \times 8\cos\theta = F \times 8\sin\theta + 20g \times 4\cos\theta$ $\mathrm{M}(C)$: $R \times 5\cos\theta = F \times 5\sin\theta + 20g \times \cos\theta$ $\mathrm{M}(G)$: $R \times 4\cos\theta = F \times 4\sin\theta + N$	M1 A1 M1 A1 M1 A1	3.3 1.1b 3.4 1.1b 3.4 1.1b

(The values of the 3 unknowns are: $N = 150.528$; $F = 42.14784$; $R = 51.49312$)		

	Solve their 3 equations for F and R **OR** X and Y **OR** H and S	M1	1.1b
	$\lvert\text{Force}\rvert = \sqrt{R^2 + F^2}$ Main scheme **OR** $= \sqrt{X^2 + Y^2}$ Alternative 1 **OR** $= \sqrt{(H^2 + S^2 - 2HS\cos(90^o - \theta)}$ Alternative 2	M1	3.1b
	Magnitude = 67 or 66.5 (N)	A1	2.2a
		(9)	
(c)	Magnitude of the normal reaction (at C) will **decrease**.	B1	3.5a
		(1)	
		(11)	

June 2018 Question 9

9.

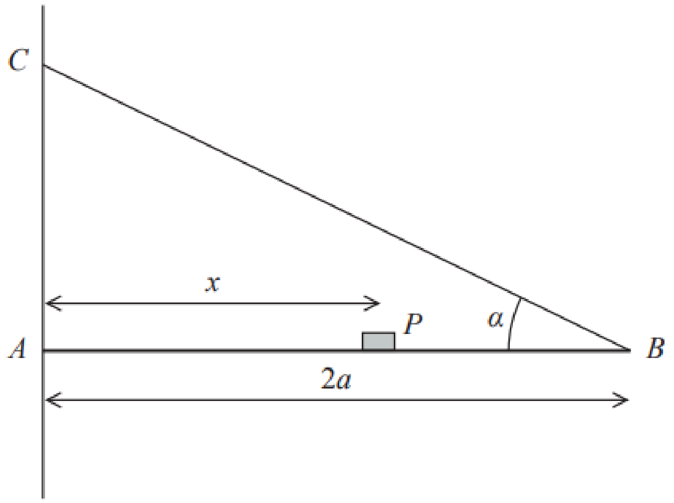


Figure 3

A plank, AB, of mass M and length $2a$, rests with its end A against a rough vertical wall. The plank is held in a horizontal position by a rope. One end of the rope is attached to the plank at B and the other end is attached to the wall at the point C, which is vertically above A.

A small block of mass $3M$ is placed on the plank at the point P, where $AP = x$. The plank is in equilibrium in a vertical plane which is perpendicular to the wall.

The angle between the rope and the plank is α, where $\tan\alpha = \frac{3}{4}$, as shown in Figure 3.

The plank is modelled as a uniform rod, the block is modelled as a particle and the rope is modelled as a light inextensible string.

(a) Using the model, show that the tension in the rope is $\dfrac{5Mg(3x + a)}{6a}$

(3)

The magnitude of the horizontal component of the force exerted on the plank at A by the wall is $2Mg$.

(b) Find x in terms of a.

(2)

The force exerted on the plank at A by the wall acts in a direction which makes an angle β with the horizontal.

(c) Find the value of $\tan\beta$

(5)

The rope will break if the tension in it exceeds $5Mg$.

(d) Explain how this will restrict the possible positions of P. You must justify your answer carefully.

(3)

ANSWER

Question	Scheme	Marks	AOs
9(a)	Moments about A (or any other complete method)	M1	3.3
	$T2a\sin\alpha = Mga + 3Mgx$	A1	1.1b
	$T = \dfrac{Mg(a+3x)}{2a \times \frac{3}{5}} = \dfrac{5Mg(3x+a)}{6a}$ * GIVEN ANSWER	A1*	2.1
		(3)	
(b)	$\dfrac{5Mg(3x+a)}{6a}\cos\alpha = 2Mg$ **OR** $2Mg.2a\tan\alpha = Mga + 3Mgx$	M1	3.1b
	$x = \dfrac{2a}{3}$	A1	2.2a
		(2)	

(c)	Resolve vertically **OR** Moments about B	M1	3.1b
	$Y = 3Mg + Mg - \dfrac{5Mg(3.\frac{2a}{3}+a)}{6a}\sin\alpha$ $\quad 2aY = Mga + 3Mg(2a - \frac{2a}{3})$ **Or**: $Y = 3Mg + Mg - \left(\dfrac{2Mg}{\cos\alpha}\right)\sin\alpha$	A1ft	1.1b
	$Y = \dfrac{5Mg}{2}$ **N.B.** May use $R\sin\beta$ for Y and/or $R\cos\beta$ for X throughout	A1	1.1b
	$\tan\beta = \dfrac{Y}{X}$ or $\dfrac{R\sin\beta}{R\cos\beta}$ $= \dfrac{\frac{5Mg}{2}}{2Mg}$	M1	3.4
	$= \dfrac{5}{4}$	A1	2.2a
		(5)	
(d)	$\dfrac{5Mg(3x+a)}{6a} \le 5Mg$ and solve for x	M1	2.4
	$x \le \dfrac{5a}{3}$	A1	2.4
	For rope not to break, block can't be more than $\dfrac{5a}{3}$ from A oe Or just: $x \le \dfrac{5a}{3}$, if no incorrect statement seen. **N.B. If the correct inequality is not found,** their comment must mention 'distance from A'.	B1 **A1**	2.4
		(3)	
		(13 marks)	

Video solution:

https://youtu.be/NWhzZxPcpFw

Ch. 8: Further Kinematics

June 2024 Question 4

4. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

[*In this question,* **i** *is a unit vector due east and* **j** *is a unit vector due north. Position vectors are given relative to a fixed origin O.*]

At time t seconds, $t \geqslant 1$, the position vector of a particle P is $\mathbf{r}$ metres, where

$$\mathbf{r} = ct^{\frac{1}{2}}\mathbf{i} - \frac{3}{8}t^2\mathbf{j}$$

and c is a constant.

When $t = 4$, the bearing of P from O is 135°

(a) Show that $c = 3$

(3)

(b) Find the speed of P when $t = 4$

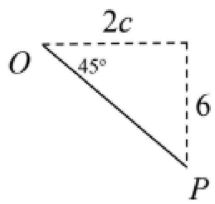
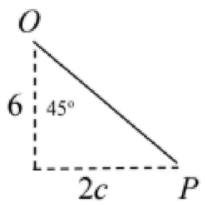
(4)

When $t = T$, P is accelerating in the direction of $(-\mathbf{i} - 27\mathbf{j})$.

(c) Find the value of T.

(4)

ANSWER

Question	Scheme	Marks	AOs
4(a)	**ALTERNATIVES** when $t = 4$ is substituted at the beginning.		
	$2c\mathbf{i} - 6\mathbf{j}$ or as a column vector, seen or implied.	B1	1.1b
	ALT 1  **or**  **AND** **either** $\tan 45^o = \frac{2c}{6} \Rightarrow 2c = 6$ **or** **states** isosceles triangle so $2c = 6$ **N.B.** In both of the above,we must see the justification for the equation. **ALT 2** $\tan 135^o = \frac{2c}{-6} \Rightarrow 2c = 6$ **N.B.** M0 if they are using the wrong bearing.	M1	3.1a
	$c = 3$ *	A1*	2.2a

4(b)	Differentiate **r** wrt t to obtain **v**	M1	2.1
	$\mathbf{v} = 3\times\frac{1}{2}t^{-\frac{1}{2}}\mathbf{i} - \frac{3}{8}\times 2t\mathbf{j} = \frac{3}{2}t^{-\frac{1}{2}}\mathbf{i} - \frac{3}{4}t\mathbf{j}$ oe	A1	1.1b
	Put $t = 4$ into **both** components and use Pythagoras: $\sqrt{\left(\frac{3}{4}\right)^2 + (-3)^2}$	M1	3.1a
	$\sqrt{\frac{153}{16}}$ or $\frac{\sqrt{153}}{4}$ or $\frac{3\sqrt{17}}{4}$ or $3\sqrt{\frac{17}{16}} = 3.0923..$ (m s^{-1})	A1	1.1b
		(4)	
4(c)	Differentiate their **v** wrt t to obtain **a**	M1	3.4
	$\mathbf{a} = -\frac{3}{4}t^{-\frac{3}{2}}\mathbf{i} - \frac{3}{4}\mathbf{j}$	A1	1.1b
	$\frac{-\frac{3}{4}T^{-\frac{3}{2}}}{-\frac{3}{4}} = \frac{-1}{-27}$ oe	M1	2.1
	$(T =)\ 9$	A1	1.1b
		(4)	

June 2023 Question 4

4. [*In this question,* **i** *and* **j** *are horizontal unit vectors and position vectors are given relative to a fixed origin O*]

A particle P is moving on a smooth horizontal plane.

The particle has constant acceleration $(2.4\mathbf{i} + \mathbf{j})\,\mathrm{m\,s^{-2}}$

At time $t = 0$, P passes through the point A.

At time $t = 5\,\mathrm{s}$, P passes through the point B.

The velocity of P as it passes through A is $(-16\mathbf{i} - 3\mathbf{j})\,\mathrm{m\,s^{-1}}$

(a) Find the speed of P as it passes through B.

(4)

The position vector of A is $(44\mathbf{i} - 10\mathbf{j})\,\mathrm{m}$.

At time $t = T$ seconds, where $T > 5$, P passes through the point C.

The position vector of C is $(4\mathbf{i} + c\mathbf{j})\,\mathrm{m}$.

(b) Find the value of T.

(3)

(c) Find the value of c.

(3)

ANSWER

Question	Scheme	Marks	AOs
4(a)	$\mathbf{v}_B = (-16\mathbf{i} - 3\mathbf{j}) + 5(2.4\mathbf{i} + \mathbf{j})$	M1	3.4
	$\mathbf{v}_B = (-4\mathbf{i} + 2\mathbf{j})$	A1	1.1b
	$\sqrt{(-4)^2 + 2^2}$	M1	3.1a
	$\sqrt{20}$ = $2\sqrt{5}$, 4.5 or better ($\mathrm{m\ s^{-1}}$)	A1	1.1b
		(4)	
4(b)	Using *A* as the initial position: $\mathbf{r}_C = \mathbf{v}_A t + \frac{1}{2}\mathbf{a}t^2 + \mathbf{r}_A$ where *t* = *T* $(4\mathbf{i} + c\mathbf{j}) = (-16\mathbf{i} - 3\mathbf{j})T + \frac{1}{2}(2.4\mathbf{i} + \mathbf{j})T^2 + (44\mathbf{i} - 10\mathbf{j})$ **OR** $\begin{pmatrix}4\\c\end{pmatrix} = \begin{pmatrix}-16\\-3\end{pmatrix}T + \frac{1}{2}\begin{pmatrix}2.4\\1\end{pmatrix}T^2 + \begin{pmatrix}44\\-10\end{pmatrix}$ Equating **i**-components, to give a **quadratic** equation in *T* only. Allow *t* instead of *T*. **N.B.** Allow omission of 44 for this M mark. Also allow ± 4 but M0 if 4 is not used at all i.e. $4 = -16T + \frac{1}{2} \times 2.4T^2$ scores M1A0A0	M1	3.1a
	$4 = -16T + \frac{1}{2} \times 2.4T^2 + 44$	A1	1.1b
	(*T* =) 10	A1	1.1b

4(c)	Equating **j**-components, with <u>their value of *T* or *t* substituted</u>, to give an equation, which must have a square term, in *c* only. **N.B.** Allow $\pm c$ in their equation. **(N.B.** Allow omission of -10 or their -12.5 for this M mark i.e. if using *A* as initial position $c=(-3\times 10)+\frac{1}{2}\times 1\times 10^2$ scores M1M0A0 **OR** if using *B* as initial position $c=(2\times 5)+\frac{1}{2}\times 1\times 5^2$ scores M1M0A0)	M1	2.1
	if using *A* as initial position $c=(-3\times 10)+\frac{1}{2}\times 1\times 10^2+(-10)$ **N.B.** Allow $\pm c$ and/or $\pm(-10)$ in their equation **OR** if using *B* as initial position $c=(2\times 5)+\frac{1}{2}\times 1\times 5^2+(-12.5)$ **N.B.** Allow $\pm c$ and/or $\pm(-12.5)$ in their equation	M1	1.1b
	c = 10	A1	1.1b
		(3)	
			(10 marks)

June 2023 Question 3

3. At time t seconds, where $t \geqslant 0$, a particle P has velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$ where

$$\mathbf{v} = (t^2 - 3t + 7)\mathbf{i} + (2t^2 - 3)\mathbf{j}$$

Find

(a) the speed of P at time $t = 0$

(3)

(b) the value of t when P is moving parallel to $(\mathbf{i} + \mathbf{j})$

(2)

(c) the acceleration of P at time t seconds

(2)

(d) the value of t when the direction of the acceleration of P is perpendicular to $\mathbf{i}$

(2)

ANSWER

Question	Scheme	Marks	AOs
3(a)	$7\mathbf{i} - 3\mathbf{j}$ seen or implied by Pythagoras	B1	1.1b
	Use Pythagoras: $\sqrt{7^2 + (-3)^2}$	M1	3.1a
	$\sqrt{58}$, 7.6 or better ($\mathrm{m\ s^{-1}}$)	A1	1.1b
		(3)	
3(b)	$t^2 - 3t + 7 = 2t^2 - 3$ **OR** $\frac{t^2 - 3t + 7}{2t^2 - 3} = \frac{1}{1} = 1$	M1	2.1
	$t = 2$ only	A1	1.1b
		(2)	
3(c)	Differentiate **v** wrt t to give a vector.	M1	3.1a
3(c)	Differentiate **v** wrt t to give a vector.	M1	3.1a
	$(2t - 3)\mathbf{i} + 4t\mathbf{j}$	A1	1.1b
		(2)	
3(d)	$2t - 3 = 0$	M1	3.1a
	$t = 1.5$	A1	1.1b
		(2)	
			(9 marks)

June 2022 Question 1

1. *[In this question, position vectors are given relative to a fixed origin.]*

At time t seconds, where $t > 0$, a particle P has velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$ where

$$\mathbf{v} = 3t^2\mathbf{i} - 6t^{\frac{1}{2}}\mathbf{j}$$

(a) Find the speed of P at time $t = 2$ seconds.

(2)

(b) Find an expression, in terms of t, $\mathbf{i}$ and $\mathbf{j}$, for the acceleration of P at time t seconds, where $t > 0$

(2)

At time $t = 4$ seconds, the position vector of P is $(\mathbf{i} - 4\mathbf{j})$ m.

(c) Find the position vector of P at time $t = 1$ second.

(4)

ANSWER

Question	Scheme	Marks	AOs
1(a)	Put $t = 2$ in **v** and use Pythagoras: $\sqrt{12^2 + (-6\sqrt{2})^2}$	M1	3.1a
	$\sqrt{216}, 6\sqrt{6}$ or 15 or better (m s^{-1})	A1	1.1b
		(2)	
1(b)	Differentiate **v** wrt t to obtain **a**	M1	3.4
	$6t\mathbf{i} - 3t^{-\frac{1}{2}}\mathbf{j}$ oe (m s^{-2}) isw	A1	1.1b
		(2)	
1(c)	Integrate **v** wrt t to obtain **r**	M1	3.4
	$\mathbf{r} = t^3\mathbf{i} - 4t^{\frac{3}{2}}\mathbf{j}\ (+\mathbf{C})$	A1	1.1b
	$(\mathbf{i} - 4\mathbf{j}) = 4^3\mathbf{i} - 4 \times 4^{\frac{3}{2}}\mathbf{j} + \mathbf{C}$	M1	3.1a
	$(-62\mathbf{i} + 24\mathbf{j})$ (m) isw e.g. if they go on to find the distance.	A1	1.1b
		(4)	
			(8 marks)

Video solution:

https://youtu.be/hqMrE749ZIU

June 2022 Question 3

3. [*In this question,* **i** *and* **j** *are horizontal unit vectors.*]

A particle P of mass 4 kg is at rest at the point A on a smooth horizontal plane.

At time $t = 0$, two forces, $\mathbf{F}_1 = (4\mathbf{i} - \mathbf{j})\,\text{N}$ and $\mathbf{F}_2 = (\lambda\mathbf{i} + \mu\mathbf{j})\,\text{N}$, where λ and μ are constants, are applied to P

Given that P moves in the direction of the vector $(3\mathbf{i} + \mathbf{j})$

(a) show that

$$\lambda - 3\mu + 7 = 0$$

(4)

At time $t = 4$ seconds, P passes through the point B.

Given that $\lambda = 2$

(b) find the length of AB.

(5)

ANSWER

Question	Scheme	Marks	AOs
3(a)	$(4\mathbf{i}-\mathbf{j})+(\lambda\mathbf{i}+\mu\mathbf{j}) = (4+\lambda)\mathbf{i}+(-1+\mu)\mathbf{j}$	M1	3.4
	Use **ratios** to obtain an equation in λ and μ *only*	M1	2.1
	$\dfrac{(4+\lambda)}{(-1+\mu)} = \dfrac{3}{1}$ or $\dfrac{\frac{1}{4}(4+\lambda)}{\frac{1}{4}(-1+\mu)} = \dfrac{3}{1}$	A1	1.1b
	$\lambda - 3\mu + 7 = 0$ * Allow $0 = \lambda - 3\mu + 7$ but nothing else.	A1*	1.1b
		(4)	
(b)	$\lambda = 2 \Rightarrow \mu = 3$; Resultant force $= (6\mathbf{i}+2\mathbf{j})$ (N)	M1	3.1a
	$(6\mathbf{i}+2\mathbf{j}) = 4\mathbf{a}$ **OR** $\lvert(6\mathbf{i}+2\mathbf{j})\rvert = 4a$	M1	1.1b
	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with $\mathbf{u} = \mathbf{0}$, their **a** and $t = 4$: **Or** they may integrate their **a** twice with $\mathbf{u} = \mathbf{0}$ and put $t = 4$: $\mathbf{r} = \frac{1}{2} \times \frac{(6\mathbf{i}+2\mathbf{j})}{4} 4^2 = (12\mathbf{i}+4\mathbf{j})$	DM1	2.1
	$\sqrt{12^2+4^2}$	M1	1.1b

	ALTERNATIVE 1 for last two M marks: Use of $s = ut + \frac{1}{2}at^2$, with $u = 0$, their a and $t = 4$: DM1 $s = \frac{1}{2} \times \sqrt{1.5^2 + 0.5^2} \times 4^2$ Use of Pythagoras to find mag of **a** : $a = \sqrt{1.5^2 + 0.5^2}$ M1		
	ALTERNATIVE 2 for last two M marks: Use of $s = ut + \frac{1}{2}at^2$, with $u = 0$, their a and $t = 4$: DM1 $s = \frac{1}{2} \times \left(\frac{\sqrt{6^2 + 2^2}}{4} \right) \times 4^2$ Use of Pythagoras to find $\lvert(6\mathbf{i} + 2\mathbf{j})\rvert$: $= \sqrt{6^2 + 2^2}$ M1		
	$\sqrt{160}$, $2\sqrt{40}$, $4\sqrt{10}$ oe or 13 or better (m)	A1	1.1b
		(5)	
			(9 marks)

November 2021 question 1

1. A particle P moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j})\,\mathrm{m\,s^{-2}}$

At time $t = 0$, P is moving with velocity $4\mathbf{i}\,\mathrm{m\,s^{-1}}$

(a) Find the velocity of P at time $t = 2$ seconds.

(2)

At time $t = 0$, the position vector of P relative to a fixed origin O is $(\mathbf{i} + \mathbf{j})\,\mathrm{m}$.

(b) Find the position vector of P relative to O at time $t = 3$ seconds.

(2)

ANSWER

Question	Scheme	Marks	AOs
1(a)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ with $t = 2$: $\mathbf{v} = 4\mathbf{i} + 2(2\mathbf{i} - 3\mathbf{j})$ **OR integration:** $\mathbf{v} = (2\mathbf{i} - 3\mathbf{j})t + 4\mathbf{i}$,with $t = 2$	M1	3.1a
	$\mathbf{v} = 8\mathbf{i} - 6\mathbf{j}$	A1	1.1b
		(2)	
1(b)	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ at $t = 3$: $(\mathbf{i} + \mathbf{j}) + \left[3 \times 4\mathbf{i} + \frac{1}{2} \times (2\mathbf{i} - 3\mathbf{j}) \times 3^2\right]$ **OR**: find $\mathbf{v}$ at $t = 3$: $4\mathbf{i} + 3(2\mathbf{i} - 3\mathbf{j}) = (10\mathbf{i} - 9\mathbf{j})$ then use $\mathbf{r} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$ $(\mathbf{i} + \mathbf{j}) + \left[\frac{1}{2}[4\mathbf{i} + (10\mathbf{i} - 9\mathbf{j})] \times 3\right]$ or $\mathbf{r} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$ $(\mathbf{i} + \mathbf{j}) + \left[3 \times (10\mathbf{i} - 9\mathbf{j}) - \frac{1}{2} \times (2\mathbf{i} - 3\mathbf{j}) \times 3^2\right]$ **OR integration:** $\mathbf{r} = (\mathbf{i} + \mathbf{j}) + \left[(2\mathbf{i} - 3\mathbf{j})\frac{1}{2}t^2 + 4t\mathbf{i}\right]$, with $t = 3$	M1	3.1a
	$\mathbf{r} = 22\mathbf{i} - 12.5\mathbf{j}$	A1	2.2a
		(2)	
			(4 marks)

November 2021 question 5

5. At time t seconds, a particle P has velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$, where

$$\mathbf{v} = 3t^{\frac{1}{2}}\,\mathbf{i} - 2t\,\mathbf{j} \qquad t > 0$$

(a) Find the acceleration of P at time t seconds, where $t > 0$

(2)

(b) Find the value of t at the instant when P is moving in the direction of $\mathbf{i} - \mathbf{j}$

(3)

At time t seconds, where $t > 0$, the position vector of P, relative to a fixed origin O, is $\mathbf{r}$ metres.

When $t = 1$, $\mathbf{r} = -\mathbf{j}$

(c) Find an expression for $\mathbf{r}$ in terms of t.

(3)

(d) Find the exact distance of P from O at the instant when P is moving with speed $10\,\mathrm{m\,s^{-1}}$

(6)

ANSWER

Question	Scheme	Marks	AOs
	Allow column vectors throughout this question		
5(a)	Differentiate **v** wrt t	M1	3.1a
	$\frac{3}{2}t^{-\frac{1}{2}}\mathbf{i}-2\mathbf{j}$ isw	A1	1.1b
		(2)	
5(b)	$3t^{\frac{1}{2}}=2t$	M1	2.1
	Solve for t	DM1	1.1b
	$t=\frac{9}{4}$	A1	1.1b
		(3)	
5(c)	Integrate **v** wrt t	M1	3.1a
	$\mathbf{r}=2t^{\frac{3}{2}}\mathbf{i}-t^2\mathbf{j}(+\mathbf{C})$	A1	1.1b
	$t=1$, $\mathbf{r}=-\mathbf{j}$ => $\mathbf{C}=-2\mathbf{i}$ so $\mathbf{r}=2t^{\frac{3}{2}}\mathbf{i}-t^2\mathbf{j}-2\mathbf{i}$	A1	2.2a
		(3)	
5(d)	$\sqrt{(3t^{\frac{1}{2}})^2+(2t)^2}=10$ or $(3t^{\frac{1}{2}})^2+(2t)^2=10^2$	M1	2.1
	$9t+4t^2=100$	M(A)1	1.1b
	$t=4$	A1	1.1b
	$\mathbf{r}=14\mathbf{i}-16\mathbf{j}$	M1	1.1b
	$\sqrt{14^2+(-16)^2}$	M1	3.1a
	$\sqrt{452}$ $(2\sqrt{113})$ (m)	A1	1.1b
		(6)	
			(14 marks)

October 2020 question 2

2. A particle P moves with acceleration $(4\mathbf{i} - 5\mathbf{j})\,\text{m s}^{-2}$

At time $t = 0$, P is moving with velocity $(-2\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$

(a) Find the velocity of P at time $t = 2$ seconds.

(2)

At time $t = 0$, P passes through the origin O.

At time $t = T$ seconds, where $T > 0$, the particle P passes through the point A.

The position vector of A is $(\lambda\mathbf{i} - 4.5\mathbf{j})$m relative to O, where λ is a constant.

(b) Find the value of T.

(4)

(c) Hence find the value of λ

(2)

ANSWER

Question	Scheme	Marks	AOs
2(a)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ or integrate to give: $\mathbf{v} = (-2\mathbf{i} + 2\mathbf{j}) + 2(4\mathbf{i} - 5\mathbf{j})$	M1	3.1a
	$(6\mathbf{i} - 8\mathbf{j})\ (\text{m s}^{-1})$	A1	1.1b
		(2)	
2(b)	Solve problem through use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ or integration (M0 if $\mathbf{u} = \mathbf{0}$) **Or** any other complete method e.g use $\mathbf{v} = \mathbf{u} + \mathbf{a}T$ and $\mathbf{r} = \frac{(\mathbf{u}+\mathbf{v})T}{2}$:	M1	3.1a
	$-4.5\mathbf{j} = 2t\mathbf{j} - \frac{1}{2}t^2 5\mathbf{j}$ ($\mathbf{j}$ terms only)	A1	1.1b
	The first two marks could be implied if they go straight to an algebraic equation.		
	Attempt to equate $\mathbf{j}$ components to give equation in T only $(-4.5 = 2T - \frac{5}{2}T^2)$	M1	2.1
	$T = 1.8$	A1	1.1b
		(4)	
2(c)	Solve problem by substituting <u>their</u> T value (M0 if $T < 0$) into the $\mathbf{i}$ component equation to give an equation in λ only: $\lambda = -2T + \frac{1}{2}T^2 \times 4$	M1	3.1a
	$\lambda = 2.9$ or 2.88 or $\frac{72}{25}$ oe	A1	1.1b
		(2)	
Notes: Accept column vectors throughout		**(8 marks)**	

October 2020 question 3

3. (i) At time t seconds, where $t \geqslant 0$, a particle P moves so that its acceleration $\mathbf{a}\,\mathrm{m\,s^{-2}}$ is given by

$$\mathbf{a} = (1 - 4t)\mathbf{i} + (3 - t^2)\mathbf{j}$$

At the instant when $t = 0$, the velocity of P is $36\mathbf{i}\,\mathrm{m\,s^{-1}}$

(a) Find the velocity of P when $t = 4$

(3)

(b) Find the value of t at the instant when P is moving in a direction perpendicular to $\mathbf{i}$

(3)

(ii) At time t seconds, where $t \geqslant 0$, a particle Q moves so that its position vector $\mathbf{r}$ metres, relative to a fixed origin O, is given by

$$\mathbf{r} = (t^2 - t)\mathbf{i} + 3t\mathbf{j}$$

Find the value of t at the instant when the speed of Q is $5\,\mathrm{m\,s^{-1}}$

(6)

ANSWER

Question	Scheme	Marks	AOs
3(i)(a)	Integrate **a** wrt t to obtain velocity	M1	3.4
	$\mathbf{v} = (t - 2t^2)\mathbf{i} + \left(3t - \frac{1}{3}t^3\right)\mathbf{j}\ (+\mathrm{C})$	A1	1.1b
	$8\mathbf{i} - \frac{28}{3}\mathbf{j}\ (\mathrm{m\,s^{-1}})$	A1	1.1b
		(3)	
3(i)(b)	Equate **i** component of **v** to zero	M1	3.1a
	$t - 2t^2 + 36 = 0$	A1ft	1.1b
	$t = 4.5$ (ignore an incorrect second solution)	A1	1.1b
		(3)	
3(ii)	Differentiate **r** wrt to t to obtain velocity	M1	3.4
	$\mathbf{v} = (2t-1)\mathbf{i} + 3\mathbf{j}$	A1	1.1b
	Use magnitude to give an equation in t only	M1	2.1
	$(2t-1)^2 + 3^2 = 5^2$	A1	1.1b
	Solve problem by solving this equation for t	M1	3.1a
	$t = 2.5$	A1	1.1b
		(6)	
		(12 marks)	

June 2019 question 1

1. [*In this question position vectors are given relative to a fixed origin O*]

At time t seconds, where $t \geqslant 0$, a particle, P, moves so that its velocity $\mathbf{v}$ m s^{-1} is given by

$$\mathbf{v} = 6t\mathbf{i} - 5t^{\frac{3}{2}}\mathbf{j}$$

When $t = 0$, the position vector of P is $(-20\mathbf{i} + 20\mathbf{j})$ m.

(a) Find the acceleration of P when $t = 4$

(3)

(b) Find the position vector of P when $t = 4$

(3)

ANSWER

Question	Scheme	Marks	AO
1(a)	Differentiate $\mathbf{v}$	M1	1.1a
	$(\mathbf{a} =) 6\mathbf{i} - \frac{15}{2}t^{\frac{1}{2}}\mathbf{j}$	A1	1.1b
	$= 6\mathbf{i} - 15\mathbf{j}$ (m s^{-2})	A1	1.1b
		(3)	
1(b)	Integrate $\mathbf{v}$	M1	1.1a
	$(\mathbf{r} =)(\mathbf{r}_0) + 3t^2\mathbf{i} - 2t^{\frac{5}{2}}\mathbf{j}$	A1	1.1b
	$= (-20\mathbf{i} + 20\mathbf{j}) + (48\mathbf{i} - 64\mathbf{j}) = 28\mathbf{i} - 44\mathbf{j}$ (m)	A1	2.2a
		(3)	
		(6)	

June 2019 question 2

2. A particle, P, moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-2}$

At time $t = 0$, the particle is at the point A and is moving with velocity $(-\mathbf{i} + 4\mathbf{j})\,\text{m s}^{-1}$

At time $t = T$ seconds, P is moving in the direction of vector $(3\mathbf{i} - 4\mathbf{j})$

(a) Find the value of T.

(4)

At time $t = 4$ seconds, P is at the point B.

(b) Find the distance AB.

(4)

ANSWER

Question	Scheme	Marks	AO
2(a)	$(\mathbf{v} =)\mathbf{C} + (2\mathbf{i} - 3\mathbf{j})t$	M1	3.1a
	$(\mathbf{v} =)(-\mathbf{i} + 4\mathbf{j}) + (2\mathbf{i} - 3\mathbf{j})t$	A1	1.1b
	$\dfrac{4-3T}{-1+2T} = \dfrac{-4}{3}$ oe	M1	3.1a
	$T = 8$	A1	1.1b
		(4)	
(b)	$(\mathbf{s} =)\mathbf{C}t + (2\mathbf{i} - 3\mathbf{j})\dfrac{1}{2}t^2 \quad (+\,\mathbf{D})$	M1	3.1a
	$(\mathbf{s} =)(-\mathbf{i} + 4\mathbf{j})t + \dfrac{1}{2}(2\mathbf{i} - 3\mathbf{j})t^2 \quad (+\,\mathbf{D})$	A1	1.1b
	$AB = \sqrt{12^2 + 8^2}$ **N.B. Beware you may see 4(2i – 3j) which leads to** $\sqrt{(8^2 + 12^2)}$ this is M0A0M0A0.	M1	3.1a
	$= 4\sqrt{13}\,(= 14.422051....)$ (m)	A1cso	1.1b
		(4)	
		(8)	

June 2018 Question 6

6. At time t seconds, where $t \geqslant 0$, a particle P moves in the x-y plane in such a way that its velocity $\mathbf{v}\,\text{m}\,\text{s}^{-1}$ is given by

$$\mathbf{v} = t^{-\frac{1}{2}}\mathbf{i} - 4t\mathbf{j}$$

When $t = 1$, P is at the point A and when $t = 4$, P is at the point B.

Find the exact distance AB.

(6)

ANSWER

Question	Scheme	Marks	AOs
6.	Integrate $\mathbf{v}$ w.r.t. time	M1	1.1a
	$\mathbf{r} = 2t^{\frac{1}{2}}\mathbf{i} - 2t^2\mathbf{j}\ \ (+\ \mathbf{C})$	A1	1.1b
	Substitute $t = 4$ and $t = 1$ into their $\mathbf{r}$	M1	1.1b
	$t = 4, \mathbf{r} = 4\mathbf{i} - 32\mathbf{j}(+\ \mathbf{C});\ t = 1, \mathbf{r} = 2\mathbf{i} - 2\mathbf{j}(+\ \mathbf{C})$ or $(4, -32); (2, -2)$	A1	1.1b
	$\sqrt{2^2 + (-30)^2}$	M1	1.1b
	$\sqrt{904} = 2\sqrt{226}$	A1	1.1b
		(6)	
			(6 marks)