Pure Year 1 exam questions Edexcel

NOTE:
Please be aware that in the Year 12 collections, you will find questions from the Year 13 papers. However, these questions are intentionally included because they align with Year 12 content and topics.

Ch. 7: Algebraic Methods

June 2024 Question 1 (A-level)

1. $$g(x) = 3x^3 - 20x^2 + (k+17)x + k$$

where k is a constant.

Given that $(x - 3)$ is a factor of $g(x)$, find the value of k.

(3)

ANSWER

Question	Scheme	Marks	AOs
1	$g(3) = 3(3)^3 - 20(3)^2 + 3(k+17) + k = 0$	M1	3.1a
	$4k - 48 = 0 \Rightarrow k = \ldots$	M1	1.1b
	$\{k =\}12$	A1	1.1b
		(3)	
			(3 marks)

June 2024 Question 2

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = 2x^3 - 3ax^2 + bx + 8a$$

where a and b are constants.

Given that $(x - 4)$ is a factor of $f(x)$,

(a) use the factor theorem to show that

$$10a = 32 + b$$

(2)

Given also that $(x - 2)$ is a factor of $f(x)$,

(b) express $f(x)$ in the form

$$f(x) = (2x + k)(x - 4)(x - 2)$$

where k is a constant to be found.

(4)

(c) Hence,

(i) state the number of real roots of the equation $f(x) = 0$

(ii) write down the largest root of the equation $f\left(\frac{1}{3}x\right) = 0$

(2)

ANSWER

Question	Scheme	Marks	AOs
2(a)	$(\mathrm{f}(4)=)\ 2\times4^3-3a\times4^2+4b+8a=0$	M1	1.1b
	$128+4b=40a \Rightarrow 32+b=10a$ *	A1*	1.1b
		(2)	
(b)	$\mathrm{f}(2)=2\times2^3-3a\times2^2+2b+8a=0 \Rightarrow 8+b=2a$	M1	1.1b
	Solve simultaneously $\Rightarrow a=\ldots$ or $\Rightarrow b=\ldots$	dM1	2.1
	$a=3$ or $b=-2$ or $k=3$	A1	1.1b
	$(\mathrm{f}(x)=)\ (2x+3)(x-4)(x-2)$	A1	1.1b
		(4)	
(c)(i)	3	B1	1.1b
(ii)	12	B1ft	2.2a
		(2)	
			(8 marks)

June 2023 Question 17

17. In this question p and q are positive integers with $q > p$

Statement 1: $q^3 - p^3$ is never a multiple of 5

(a) Show, by means of a counter example, that Statement 1 is **not** true.

(1)

Statement 2: When p and q are consecutive **even** integers $q^3 - p^3$ is a multiple of 8

(b) Prove, using algebra, that Statement 2 is true.

(4)

ANSWER

Question	Scheme	Marks	AOs
17 (a)	Provides a counter example with a reason. e.g., $6^3 - 1^3 = 215$ which is a multiple of 5	B1	2.4
		(1)	
(b)	States or uses, e.g., $2n$ and $2n+2$ **or** $2n+2$ and $2n+4$	M1	2.1
	Attempts $(2n+2)^3 - (2n)^3 = 8n^3 + 24n^2 + 24n + 8 - 8n^3$ leading to a quadratic.	dM1	1.1b
	$= 24n^2 + 24n + 8$	A1	1.1b
	$24n^2 + 24n + 8 = 8(3n^2 + 3n + 1)$ So $q^3 - p^3$ is a multiple of 8	A1	2.1
		(4)	
(5 marks)			

June 2023 Question 7 Paper 1 (A-Level)

14. Prove, using algebra, that

$$(n+1)^3 - n^3$$

is odd for all $n \in \mathbb{N}$

(4)

ANSWER

Question	Scheme	Marks	AOs
14	When n is even: $(2k+1)^3 - (2k)^3 = 8k^3 + 12k^2 + 6k + 1 - 8k^3 = 6k(2k+1) + 1$ $\Rightarrow$ which is odd **or**	M1	3.1a
	When n is odd: $(2k+2)^3 - (2k+1)^3 = 8(k^3 + 3k^2 + 3k + 1) - (8k^3 + 12k^2 + 6k + 1) = 6k(2k+3) + 7$ $\Rightarrow$ which is odd	A1	2.2a
	When n is even: $(2k+1)^3 - (2k)^3 = 8k^3 + 12k^2 + 6k + 1 - 8k^3 = 6k(2k+1) + 1$ $\Rightarrow$ which is odd **and** When n is odd: $(2k+2)^3 - (2k+1)^3 = 8(k^3 + 3k^2 + 3k + 1) - (8k^3 + 12k^2 + 6k + 1) = 6k(2k+3) + 7$ $\Rightarrow$ which is odd	dM1	2.1
	Hence odd for all $n(\in \mathbb{N})$ *	A1*	2.4
			(4 marks)

June 2023 Question 2 Paper 1 (A-Level)

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = 4x^3 + 5x^2 - 10x + 4a \qquad x \in \mathbb{R}$$

where a is a positive constant.

Given $(x - a)$ is a factor of $f(x)$,

(a) show that

$$a(4a^2 + 5a - 6) = 0$$

(2)

(b) Hence

(i) find the value of a

(ii) use algebra to find the exact solutions of the equation

$$f(x) = 3$$

(4)

ANSWER

Question	Scheme	Marks	AOs
2(a)	$(\mathrm{f}(a)=)\ 4a^3+5a^2-10a+4a=0 \Rightarrow a(....)=0$	M1	3.1a
	$a(4a^2+5a-6)=0$ *	A1*	1.1b
		(2)	
(b)(i)	$a=\dfrac{3}{4}$	B1	2.2a
(ii)	$4x^3+5x^2-10x+4\times"\dfrac{3}{4}"=3 \Rightarrow 4x^3+5x^2-10x\ (=0)$	M1	1.1b
	$x=0$	B1	1.1b
	$x=\dfrac{-5\pm\sqrt{185}}{8}$	A1	1.1b
		(4)	
			(6 marks)

June 2023 Question 17 Paper 1

17. In this question p and q are positive integers with $q > p$

Statement 1: $q^3 - p^3$ is never a multiple of 5

(a) Show, by means of a counter example, that Statement 1 is **not** true.

(1)

Statement 2: When p and q are consecutive **even** integers $q^3 - p^3$ is a multiple of 8

(b) Prove, using algebra, that Statement 2 is true.

(4)

ANSWER

Question	Scheme	Marks	AOs
17 (a)	Provides a counter example with a reason. e.g., $6^3 - 1^3 = 215$ which is a multiple of 5	B1	2.4
		(1)	
(b)	States or uses, e.g., $2n$ and $2n+2$ **or** $2n+2$ and $2n+4$	M1	2.1
	Attempts $(2n+2)^3 - (2n)^3 = 8n^3 + 24n^2 + 24n + 8 - 8n^3$ leading to a quadratic.	dM1	1.1b
	$= 24n^2 + 24n + 8$	A1	1.1b
	$24n^2 + 24n + 8 = 8(3n^2 + 3n + 1)$ So $q^3 - p^3$ is a multiple of 8	A1	2.1
		(4)	
			(5 marks)

June 2022 Question 11 Paper 2 (A Level)

11. Prove, using algebra, that

$$n(n^2 + 5)$$

is even for all $n \in \mathbb{N}$.

(4)

ANSWER

Question	Scheme	Marks	AOs
11	$n\left(n^2+5\right)$		
	Attempts even **or** odd numbers Sets $n=2k$ **or** $n=2k\pm1$ oe and attempts $n\left(n^2+5\right)$	M1	3.1a
	Achieves $2k\left(4k^2+5\right)$ (for $n=2k$) and states "even" **Or** achieves $(2k+1)\left(4k^2+4k+6\right)=2(2k+1)\left(2k^2+2k+3\right)$ (for $n=2k+1$) and states "even" Or e.g. achieves $(2k-1)\left(4k^2-4k+6\right)=2(2k-1)\left(2k^2-2k+3\right)$ (for $n=2k-1$) and states "even"	A1	2.2a
	Attempts even **and** odd numbers Sets $n=2k$ **and** $n=2k\pm1$ oe and attempts $n\left(n^2+5\right)$	dM1	2.1
	Achieves $2k\left(4k^2+5\right)$ (for $n=2k$) and states "even" **and** achieves $(2k\pm1)\left(4k^2\pm4k+6\right)=2(2k\pm1)\left(2k^2\pm2k+3\right)$ (for $n=2k\pm1$) and states "even" Correct work and states even for both WITH a final conclusion showing that true for all $n\,(\in\mathbb{N})$ or e.g. true for all even and odd numbers.	A1	2.4
		(4)	
			(4 marks)

June 2022 Question 2 Paper 1 (A Level)

2. $\mathrm{f}(x) = (x-4)(x^2 - 3x + k) - 42$ where k is a constant

Given that $(x+2)$ is a factor of $\mathrm{f}(x)$, find the value of k.

(3)

ANSWER

Question	Scheme	Marks	AOs
2	Sets $\mathrm{f}(-2)=0 \Rightarrow (-2-4)\left((-2)^2-3\times-2+k\right)-42=0$	M1	3.1a
	$-6(k+10)=42 \Rightarrow k=\ldots$	M1	1.1b
	$k=-17$	A1	1.1b
		(3)	
			(3 marks)

June 2022 Question 14 Paper 1

14. (i) A student states

"if x^2 is greater than 9 then x must be greater than 3"

Determine whether or not this statement is true, giving a reason for your answer.

(1)

(ii) Prove that for all positive integers n,

$$n^3 + 3n^2 + 2n$$

is divisible by 6

(3)

ANSWER

Question	Scheme	Marks	AOs
14(i)	The statement is **not true** because e.g. when $x = -4$, $x^2 = 16$ (which is > 9 but $x < 3$)	B1	2.3
		(1)	
(ii)	$n^3 + 3n^2 + 2n = n\left(n^2 + 3n + 2\right) = n(n+1)(n+2)$	M1	2.1
	$n(n + 1)(n + 2)$ is the product of 3 consecutive integers	A1	2.2a
	As $n(n + 1)(n + 2)$ is a multiple of 2 **and** a multiple of 3 it must be a multiple of 6 and so $n^3 + 3n^2 + 2n$ is divisible by 6 for all integers n	A1	2.4
		(3)	
			(4 marks)

Video solution:

https://youtu.be/9GXHesA3AIM

June 2022 Question 2

2. $$f(x) = 2x^3 + 5x^2 + 2x + 15$$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$.

(2)

(b) Find the constants a, b and c such that

$$f(x) = (x + 3)(ax^2 + bx + c)$$

(2)

(c) Hence show that $f(x) = 0$ has only one real root.

(2)

(d) Write down the real root of the equation $f(x - 5) = 0$

(1)

ANSWER

Question	Scheme	Marks	AOs
2(a)	$f(-3) = 2(-3)^3 + 5(-3)^2 + 2(-3) + 15$ $= -54 + 45 - 6 + 15$	M1	1.1b
	$f(-3) = 0 \Rightarrow (x + 3)$ is a factor	A1	2.4
		(2)	
(b)	At least 2 of: $a = 2,\ b = -1,\ c = 5$	M1	1.1b
	All of: $a = 2,\ b = -1,\ c = 5$	A1	1.1b
		(2)	
(c)	$b^2 - 4ac = (-1)^2 - 4(2)(5)$	M1	2.1
	$b^2 - 4ac = -39$ which is < 0 so the quadratic has no real roots so $f(x) = 0$ has only 1 real root	A1	2.4
		(2)	
(d)	$(x =)\ 2$	B1	2.2a
		(1)	
			(7 marks)

Video solution:
https://youtu.be/uZaAEGT92R0

November 2021 Question 10 Paper 1

10. A student is investigating the following statement about natural numbers.

"$n^3 - n$ is a multiple of 4"

(a) Prove, using algebra, that the statement is true for all odd numbers. **(4)**

(b) Use a counterexample to show that the statement is not always true. **(1)**

ANSWER

Question	Scheme	Marks	AOs
10(a)	Selects a correct strategy. E.g uses an odd number is $2k \pm 1$	B1	3.1a
	Attempts to simplify $(2k \pm 1)^3 - (2k \pm 1) = ...$	M1	2.1
	..........and factorise $8k^3 \pm 12k^2 \pm 4k = 4k(2k^2 \pm 3k \pm 1) =$	dM1	1.1b
	Correct work with statement $4\times..$ is a multiple of 4	A1	2.4
		(4)	
(b)	Any counter example with correct statement. Eg. $2^3 - 2 = 6$ which is not a multiple of 4	B1	2.4
		(1)	
			(5 marks)

November 2020 Question 16 Paper 2 (A-Level)

16. Use algebra to prove that the square of any natural number is **either** a multiple of 3 **or** one more than a multiple of 3

(4)

ANSWER

Question	Scheme	Marks	AOs
16	NB any natural number can be expressed in the form: $3k$, $3k+1$, $3k+2$ or equivalent e.g. $3k-1$, $3k$, $3k+1$		
	Attempts to square **any two** distinct cases of the above	M1	3.1a
	Achieves accurate results and makes a valid comment for **any two** of the possible three cases: E.g. $(3k)^2 = 9k^2 \left(= 3\times 3k^2\right)$ is a multiple of 3	A1 **M1 on EPEN**	1.1b
	$(3k+1)^2 = 9k^2+6k+1 = 3\times\left(3k^2+2k\right)+1$ is one more than a multiple of 3 $(3k+2)^2 = 9k^2+12k+4 = 3\times\left(3k^2+4k+1\right)+1$ $\left(\text{or } (3k-1)^2 = 9k^2-6k+1 = 3\times\left(3k^2-2k\right)+1\right)$ is one more than a multiple of 3		
	Attempts to square in all 3 distinct cases. E.g. attempts to square $3k$, $3k+1$, $3k+2$ or e.g. $3k-1$, $3k$, $3k+1$	M1 **A1 on EPEN**	2.1
	Achieves accurate results for all three cases and gives a minimal conclusion (allow tick, QED etc.)	A1	2.4
		(4)	
			(4 marks)

June 2019 Question 1 Paper 1 (A-Level)

Answer ALL questions. Write your answers in the spaces provided.

1. $$f(x) = 3x^3 + 2ax^2 - 4x + 5a$$

Given that $(x + 3)$ is a factor of $f(x)$, find the value of the constant a.

(3)

ANSWER

Question	Scheme	Marks	AOs
1	Attempts $f(-3) = 3\times(-3)^3 + 2a\times(-3)^2 - 4\times -3 + 5a = 0$	M1	3.1a
	Solves linear equation $23a = 69 \Rightarrow a = \ldots$	M1	1.1b
	$a = 3$ cso	A1	1.1b
		(3)	
			(3 marks)

June 2019 Question 10 Paper 1 (A-Level)

10. (i) Prove that for all $n \in \mathbb{N}$, $n^2 + 2$ is not divisible by 4

(4)

(ii) "Given $x \in \mathbb{R}$, the value of $|3x - 28|$ is greater than or equal to the value of $(x - 9)$."
State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

ANSWER

Question 10 (i)	Scheme	Marks	AOs

For $n = 2m$, $\quad n^2 + 2 = 4m^2 + 2$	M1	2.1
Concludes that this number is not divisible by 4 (as the explanation is trivial)	A1	1.1b
For $n = 2m+1$, $\quad n^2 + 2 = (2m+1)^2 + 2 = \ldots$ FYI $\left(4m^2 + 4m + 3\right)$	dM1	2.1
Correct working and concludes that this is a number in the 4 times table add 3 so cannot be divisible by 4 or writes $4\left(m^2 + m\right) + 3$ ....AND states ......hence true for all	A1*	2.4
	(4)	

SOMETIMES TRUE and chooses any number $x : 9.25 < x < 9.5$ and shows false Eg $x = 9.4$ $\quad |3x - 28| = 0.2$ and $x - 9 = 0.4$ ×	M1	2.3
Then chooses a number where it is true Eg $x = 12$ $\quad |3x - 28| = 8$ $\quad x - 9 = 3$ ✓	A1	2.4
	(2)	

June 2018 Question 2

2. (i) Show that $x^2 - 8x + 17 > 0$ for all real values of x

(3)

(ii) "If I add 3 to a number and square the sum, the result is greater than the square of the original number."

State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

ANSWER

Question	Scheme	Marks	AOs
2(i)	$x^2 - 8x + 17 = (x-4)^2 - 16 + 17$	M1	3.1a
	$= (x-4)^2 + 1$ with comment (see notes)	A1	1.1b
	As $(x-4)^2 \geqslant 0 \Rightarrow (x-4)^2 + 1 \geqslant 1$ hence $x^2 - 8x + 17 > 0$ for all x	A1	2.4
		(3)	
(ii)	For an explanation that it may not always be true Tests say $x = -5$ $(-5+3)^2 = 4$ whereas $(-5)^2 = 25$	M1	2.3
	States sometimes true and gives reasons Eg. when $x = 5$ $(5+3)^2 = 64$ whereas $(5)^2 = 25$ True When $x = -5$ $(-5+3)^2 = 4$ whereas $(-5)^2 = 25$ Not true	A1	2.4
		(2)	
			(5 marks)