Pure Year 1 exam questions Edexcel

NOTE:

Please be aware that in the Year 12 collections, you will find questions from the Year 13 papers. However, these questions are intentionally included because they align with Year 12 content and topics.

Arancha Ruiz

Ch. 6: Circles

June 2024 Question 14 Paper 2

14. The circle C_1 has equation

$$x^2 + y^2 - 6x + 14y + 33 = 0$$

(a) Find

(i) the coordinates of the centre of C_1

(ii) the radius of C_1

(3)

A different circle C_2

- has centre with coordinates (–6, –8)
- has radius k, where k is a constant

Given that C_1 and C_2 intersect at 2 distinct points,

(b) find the range of values of k, writing your answer in set notation.

(5)

ANSWER

Question	Scheme	Marks	AOs
14(a)(i)	Centre is (3, –7)	B1	1.1b
(ii)	$(x-3)^2+(y+7)^2=49+9-33 \Rightarrow r^2=...(25)$	M1	1.1b
	$r=5$	A1	1.1b
		(3)	
(b)	Distance between centres $=\sqrt{(3+6)^2+(-7+8)^2}=\sqrt{82}$	M1 A1ft	3.1a 1.1b
	"$\sqrt{82}$"–"5" **or** "$\sqrt{82}$"+"5"	dM1	3.1a
	$\sqrt{82}-5$ **and** $\sqrt{82}+5$	A1	2.2a
	$\{k:\sqrt{82}-5<k\}\cap\{k:k<\sqrt{82}+5\}$ or e.g. $\{k:\sqrt{82}-5<k<\sqrt{82}+5\}$	A1	2.5
		(5)	
			(8 marks)

June 2024 Question 10

10.

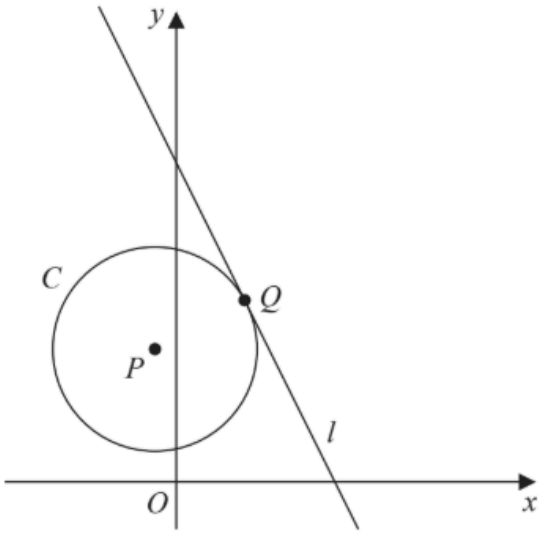


Figure 4

Figure 4 shows a sketch of the circle C

- the point P $(-1, k + 8)$ is the centre of C
- the point Q $(3, k^2 - 2k)$ lies on C
- k is a positive constant
- the line l is the tangent to C at Q

Given that the gradient of l is -2

(a) show that

$$k^2 - 3k - 10 = 0$$

(4)

(b) Hence find an equation for C

(4)

ANSWER

Question	Scheme	Marks	AOs
10(a)	tangent $= -2 \rightarrow$ normal $= \frac{1}{2}$	B1	2.2a
	$\frac{k^2-2k-k-8}{3--1}$	M1	1.1b
	$\frac{1}{2}=\frac{k^2-2k-k-8}{3--1} \Rightarrow \frac{1}{2}=\frac{k^2}{4}-\frac{2k}{4}-\frac{k}{4}-2$	dM1	1.1b
	$\Rightarrow k^2-3k-10=0$ *	A1*	2.1
		(4)	
(b)	$k=5$	B1	2.3
	y coordinate of P is 13	B1ft	1.1b
	Attempts PQ $\left(=\sqrt{(3--1)^2+("25-15-8")^2}\right)$ (or PQ^2)	M1	3.1a
	$(x+1)^2+(y-13)^2=20$	A1	1.1b
		(4)	
			(8 marks)

June 2023 Question 6

6. The circle C has equation

$$x^2 + y^2 - 6x + 10y + k = 0$$

where k is a constant.

(a) Find the coordinates of the centre of C.

(2)

Given that C does not cut or touch the x-axis,

(b) find the range of possible values for k.

(3)

ANSWER

Question	Scheme	Marks	AOs
6 (a)	$x^2+y^2-6x+10y+k=0$		
	$(x-3)^2+(y+5)^2 \pm ... = ...$	M1	1.1b
	Centre $(3,-5)$	A1	1.1b
		(2)	
(b)	Deduces that $k=9$ is a critical point	B1ft	2.2a
	Recognises that radius >0 $"9"+"25"-k>0$	M1	3.1a
	$9<k<34$	A1	1.1b
		(3)	
			(5 marks)

June 2023 Question 10 Paper 1 (A-Level)

10. A circle C has equation

$$x^2 + y^2 + 6kx - 2ky + 7 = 0$$

where k is a constant.

(a) Find in terms of k,

(i) the coordinates of the centre of C

(ii) the radius of C

(3)

The line with equation $y = 2x - 1$ intersects C at 2 distinct points.

(b) Find the range of possible values of k.

(6)

ANSWER

Question	Scheme	Marks	AOs
10(a)(i) (ii)	Centre $(-3k, k)$	B1	2.2a
	$(x+3k)^2 - 9k^2 + (y-k)^2 - k^2 + 7 = 0 \Rightarrow (x+3k)^2 + (y-k)^2 = ...$	M1	1.1b
	Radius $\sqrt{10k^2 - 7}$	A1ft	2.2a
		(3)	
(b)	$x^2 + (2x-1)^2 + 6kx - 2k(2x-1) + 7 = 0 \Rightarrow ...x^2 + (pk+q)x + rk + s\,(=0)$	M1	1.1a
	$5x^2 + (2k-4)x + 2k + 8 \ (=0)$	A1	1.1b
	$(2k-4)^2 - 4\times5\times(2k+8) = 0 \Rightarrow k = ...$	dM1	2.1
	Critical values $= 7 \pm \sqrt{85}$	A1	1.1b
	$k < "7 - \sqrt{85}"$ or $k > "7 + \sqrt{85}"$ o.e.	ddM1	3.1a
	$k < 7 - \sqrt{85}$ or $k > 7 + \sqrt{85}$ o.e.	A1	2.5
		(6)	
			(9 marks)

June 2022 Question 11

11.

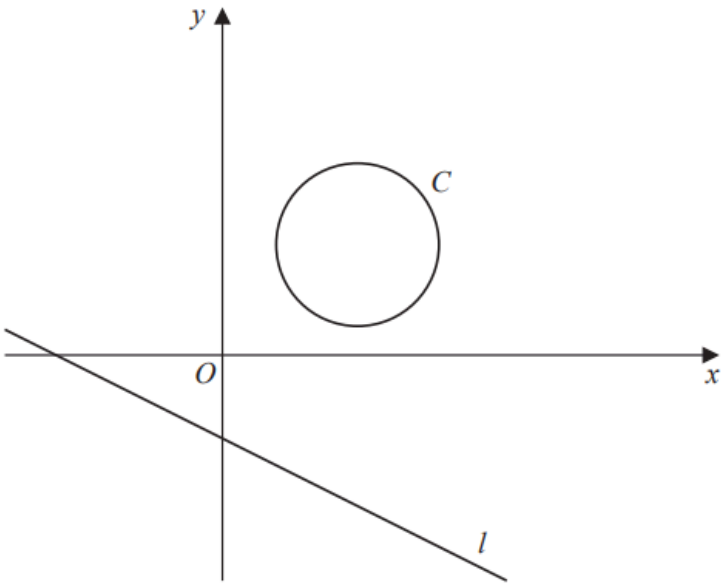


Figure 3

Figure 3 shows the circle C with equation

$$x^2 + y^2 - 10x - 8y + 32 = 0$$

and the line l with equation

$$2y + x + 6 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the radius of C.

(3)

(b) Find the shortest distance between C and l.

(5)

ANSWER

Question	Scheme	Marks	AOs
11(a)	$(x\pm5)^2+(y\pm4)^2$	M1	1.1b
	(i) Centre is (5, 4)	A1	1.1b
	(ii) Radius is 3	A1	1.1b
		(3)	
(b)	$2y+x+6=0\Rightarrow y=-\frac{1}{2}x+...\Rightarrow -\frac{1}{2}\rightarrow 2$	B1	2.2a
	$m_N=2\Rightarrow y-4=2(x-5)$ $y-4=2(x-5),\ 2y+x+6=0\Rightarrow x=...,\ y=...$	M1	3.1a
	Intersection is at $\left(\frac{6}{5},\ -\frac{18}{5}\right)$ oe	A1	1.1b
	Distance from centre to intersection is $\sqrt{\left(5-\frac{6}{5}\right)^2+\left(4+\frac{18}{5}\right)^2}$ So distance required is $\sqrt{\left("5"-"\frac{6}{5}"\right)^2+\left("4"+"\frac{18}{5}"\right)^2}-"3"$	dM1	3.1a
	$=\frac{19\sqrt{5}}{5}-3$ (or awrt 5.50)	A1	1.1b
		(5)	
			(8 marks)

Video solution:

https://youtu.be/a5vglR4UYmA

June 2022 Question 3 Paper 1 (A Level)

3. A circle has equation

$$x^2 + y^2 - 10x + 16y = 80$$

(a) Find

(i) the coordinates of the centre of the circle,

(ii) the radius of the circle.

(3)

Given that P is the point on the circle that is furthest away from the origin O,

(b) find the exact length OP

(2)

ANSWER

Question	Scheme	Marks	AOs
3 (a)	(i) $x^2+y^2-10x+16y=80 \Rightarrow (x-5)^2+(y+8)^2=\ldots$	M1	1.1b
	Centre $(5,-8)$	A1	1.1b
	(ii) Radius 13	A1	1.1b
		(3)	
(b)	Attempts $\sqrt{"5"^2+"8"^2}+"13"$	M1	3.1a
	$13+\sqrt{89}$ but ft on their centre and radius	A1ft	1.1b
		(2)	
			(5 marks)

November 2021 Question 7 Paper 1 (A-Level)

7. The circle C has equation

$$x^2 + y^2 - 10x + 4y + 11 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C, giving your answer as a simplified surd.

(4)

The line l has equation $y = 3x + k$ where k is a constant.

Given that l is a tangent to C,

(b) find the possible values of k, giving your answers as simplified surds.

(5)

ANSWER

Question	Scheme	Marks	AOs
7(a)(i)	$(x-5)^2+(y+2)^2=\ldots$	M1	1.1b
	$(5,-2)$	A1	1.1b
(ii)	$r=\sqrt{"5"^2+"-2"^2-11}$	M1	1.1b
	$r=3\sqrt{2}$	A1	1.1b
		(4)	
(b)	$y=3x+k \Rightarrow x^2+(3x+k)^2-10x+4(3x+k)+11=0$ $\Rightarrow x^2+9x^2+6kx+k^2-10x+12x+4k+11=0$	M1	2.1
	$\Rightarrow 10x^2+(6k+2)x+k^2+4k+11=0$	A1	1.1b
	$b^2-4ac=0 \Rightarrow (6k+2)^2-4\times10\times(k^2+4k+11)=0$	M1	3.1a
	$\Rightarrow 4k^2+136k+436=0 \Rightarrow k=\ldots$	M1	1.1b
	$k=-17\pm6\sqrt{5}$	A1	2.2a
		(5)	
			(9 marks)

November 2021 Question 15

15.

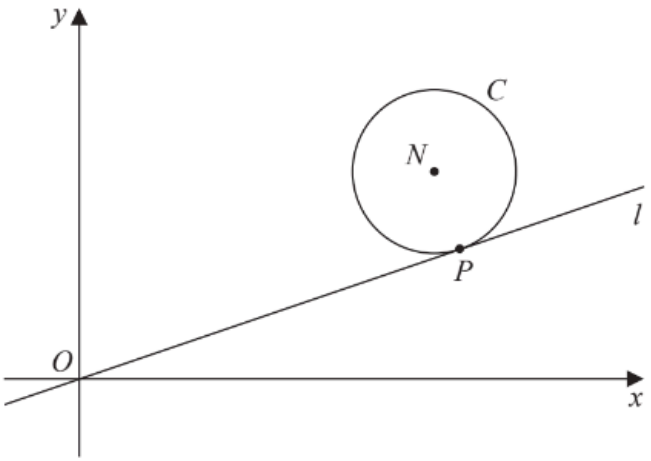


Figure 4

Figure 4 shows a sketch of a circle C with centre $N(7, 4)$

The line l with equation $y = \frac{1}{3}x$ is a tangent to C at the point P.

Find

(a) the equation of line PN in the form $y = mx + c$, where m and c are constants,

(2)

(b) an equation for C.

(4)

The line with equation $y = \frac{1}{3}x + k$, where k is a non-zero constant, is also a tangent to C.

(c) Find the value of k.

(3)

ANSWER

Question	Scheme	Marks	AOs
15 (a)	Deduces the line has gradient "-3" and point $(7,4)$ Eg $y-4=-3(x-7)$	M1	2.2a
	$y=-3x+25$	A1	1.1b
		(2)	
(b)	Solves $y=-3x+25$ and $y=\frac{1}{3}x$ simultaneously	M1	3.1a
	$P=\left(\frac{15}{2},\frac{5}{2}\right)$ oe	A1	1.1b
	Length $PN=\sqrt{\left(\frac{15}{2}-7\right)^2+\left(4-\frac{5}{2}\right)^2}=\left(\sqrt{\frac{5}{2}}\right)$	M1	1.1b
	Equation of C is $(x-7)^2+(y-4)^2=\frac{5}{2}$ o.e.	A1	1.1b
		(4)	
(c)	Attempts to find where $y=\frac{1}{3}x+k$ meets C using vectors Eg: $\begin{pmatrix}7.5\\2.5\end{pmatrix}+2\times\begin{pmatrix}-0.5\\1.5\end{pmatrix}$	M1	3.1a
	Substitutes their $\left(\frac{13}{2},\frac{11}{2}\right)$ in $y=\frac{1}{3}x+k$ to find k	M1	2.1
	$k=\frac{10}{3}$	A1	1.1b
		(3)	
			(9 marks)

November 2020 Question 14 Paper 2 (A-Level)

14. A circle C with radius r

- lies only in the 1st quadrant
- touches the x-axis and touches the y-axis

The line l has equation $2x + y = 12$

(a) Show that the x coordinates of the points of intersection of l with C satisfy

$$5x^2 + (2r - 48)x + (r^2 - 24r + 144) = 0$$

(3)

Given also that l is a tangent to C,

(b) find the two possible values of r, giving your answers as fully simplified surds.

(4)

ANSWER

Question	Scheme	Marks	AOs
14 (a)	C is $(x-r)^2+(y-r)^2=r^2$ or $x^2+y^2-2rx-2ry+r^2=0$	B1	2.2a
	$y=12-2x,\ x^2+y^2-2rx-2ry+r^2=0$ $\Rightarrow x^2+(12-2x)^2-2rx-2r(12-2x)+r^2=0$ or	M1	1.1b
	$y=12-2x,\ (x-r)^2+(y-r)^2=r^2$ $\Rightarrow (x-r)^2+(12-2x-r)^2=r^2$		
	$x^2+144-48x+4x^2-2rx-24r+4rx+r^2=0$ $\Rightarrow 5x^2+(2r-48)x+(r^2-24r+144)=0$ *	A1*	2.1
		(3)	
(b)	$b^2-4ac=0\Rightarrow(2r-48)^2-4\times5\times(r^2-24r+144)=0$	M1	3.1a
	$r^2-18r+36=0$ or any multiple of this equation	A1	1.1b
	$\Rightarrow(r-9)^2-81+36=0\Rightarrow r=...$	dM1	1.1b
	$r=9\pm3\sqrt{5}$	A1	1.1b
		(4)	
			(7 marks)

Video solution:
https://youtu.be/DZBSLH6mEgw

June 2018 Question 14

14. The circle C has equation

$$x^2 + y^2 - 6x + 10y + 9 = 0$$

(a) Find

(i) the coordinates of the centre of C

(ii) the radius of C

(3)

The line with equation $y = kx$, where k is a constant, cuts C at two distinct points.

(b) Find the range of values for k.

(6)

ANSWER

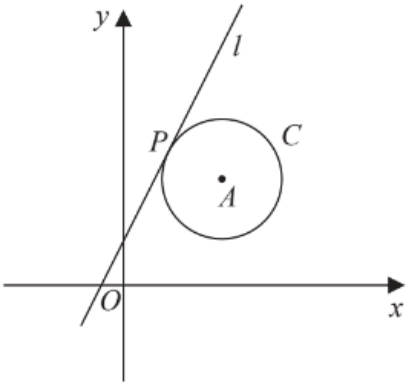
Question	Scheme	Marks	AOs
14 (a)	Attempts to complete the square $(x\pm3)^2+(y\pm5)^2=...$	M1	1.1b
	(i) Centre $(3,-5)$	A1	1.1b
	(ii) Radius 5	A1	1.1b
		(3)	
(b)	Uses a sketch or otherwise to deduce $k=0$ is a critical value	B1	2.2a
	Substitute $y=kx$ in $x^2+y^2-6x+10y+9=0$	M1	3.1a
	Collects terms to form correct 3TQ $(1+k^2)x^2+(10k-6)x+9=0$	A1	1.1b
	Attempts $b^2-4ac...0$ for their a, b and c leading to values for k $"(10k-6)^2-36(1+k^2)...0"\rightarrow k=...,...$ $\left(0 \text{ and } \frac{15}{8}\right)$	M1	1.1b
	Uses $b^2-4ac>0$ and chooses the outside region (see note) for **their** critical values (Both a and b must have been expressions in k)	dM1	3.1a
	Deduces $k<0, k>\frac{15}{8}$ oe	A1	2.2a
		(6)	
			(9 marks)

Video solution:

https://youtu.be/mdhsyR5xWdo

June 2018 Question 6 Paper 1 (A-Level)

6.



Not to scale

Figure 3

The circle C has centre A with coordinates $(7, 5)$.

The line l, with equation $y = 2x + 1$, is the tangent to C at the point P, as shown in Figure 3.

(a) Show that an equation of the line PA is $2y + x = 17$

(3)

(b) Find an equation for C.

(4)

The line with equation $y = 2x + k$, $k \neq 1$ is also a tangent to C.

(c) Find the value of the constant k.

(3)

ANSWER

Question	Scheme	Marks	AOs
6 (a)	Deduces that gradient of PA is $-\frac{1}{2}$	M1	2.2a
	Finding the equation of a line with gradient "$-\frac{1}{2}$" and point $(7,5)$ $y-5=-\frac{1}{2}(x-7)$	M1	1.1b
	Completes proof $2y+x=17$ *	A1*	1.1b
		(3)	
(b)	Solves $2y+x=17$ and $y=2x+1$ simultaneously	M1	2.1
	$P=(3,7)$	A1	1.1b
	Length $PA=\sqrt{(3-7)^2+(7-5)^2}=(\sqrt{20})$	M1	1.1b
	Equation of C is $(x-7)^2+(y-5)^2=20$	A1	1.1b
		(4)	
(c)	Attempts to find where $y=2x+k$ meets C using $\overrightarrow{OA}+\overrightarrow{PA}$	M1	3.1a
	Substitutes their $(11,3)$ in $y=2x+k$ to find k	M1	2.1
	$k=-19$	A1	1.1b
		(3)	
			(10 marks)