Pure Year 1 Edexcel

NOTE:

Please be aware that in the Year 12 collections, you will find questions from the Year 13 papers. However, these questions are intentionally included because they align with Year 12 content and topics.

Arancha Ruiz

Ch. 2: Quadratics

June 2024 Question 9 Paper 2 (A-level)

9.

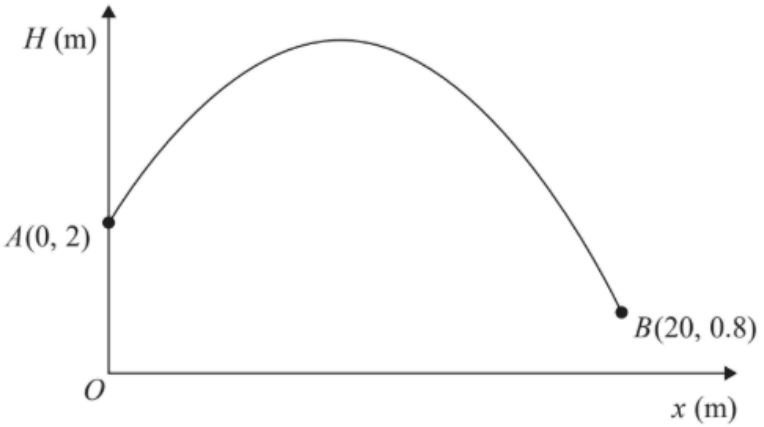

Figure 3

The graph in Figure 3 shows the path of a small ball.

The ball travels in a vertical plane above horizontal ground.

The ball is thrown from the point represented by A and caught at the point represented by B.

The height, H metres, of the ball above the ground has been plotted against the horizontal distance, x metres, measured from the point where the ball was thrown.

With respect to a fixed origin O, the point A has coordinates (0, 2) and the point B has coordinates (20, 0.8), as shown in Figure 3.

The ball reaches its maximum height when $x = 9$

A quadratic function, linking H with x, is used to model the path of the ball.

(a) Find H in terms of x.

(4)

(b) Give one limitation of the model.

(1)

Chandra is standing directly under the path of the ball at a point 16 m horizontally from O.

Chandra can catch the ball if the ball is less than 2.5 m above the ground.

(c) Use the model to determine if Chandra can catch the ball.

(2)

ANSWER

Question	Scheme	Marks	AOs
9(a) Way 1	$H = \pm ax^2 \pm bx \pm c$ $x = 0, H = 2 \Rightarrow c = 2$ **and** either $x = 20, H = 0.8 \Rightarrow 0.8 = 400a + 20b + 2$ **or** $H = ax^2 + bx + c \Rightarrow \frac{dH}{dx} = 2ax + b$ $x = 9, \frac{dH}{dx} = 0 \Rightarrow 18a + b = 0$	M1	3.3
	$H = \pm ax^2 \pm bx \pm c$ $x = 0, H = 2 \Rightarrow c = 2$ **and** $x = 20, H = 0.8 \Rightarrow 0.8 = 20^2 a + 20b + 2$ **and** $H = ax^2 + bx + c \Rightarrow \frac{dH}{dx} = 2ax + b$ $x = 9, \frac{dH}{dx} = 0 \Rightarrow 18a + b = 0$	dM1	3.1b
	$0.8 = 400a + 20b + 2, 18a + b = 0 \Rightarrow a = ..., b = ...$	ddM1	1.1b
	$H = -0.03x^2 + 0.54x + 2$	A1	2.2a
		(4)	

(b)	Examples must focus on why the model may not be appropriate or give situations where the model would break down e.g.: • H is unlikely to be a quadratic function in x • The path is unlikely to be parabolic • Wind may affect the path of the ball • Wind may affect the distance the ball travels • Air resistance has not been considered • The ball is unlikely to travel in a vertical plane (as it may spin) • The ball is not a particle so has dimensions/size • The ground is unlikely to be horizontal • There may be trees (or other hazards) that would affect the path of the ball • The shape of the ball may affect the motion Condone statements (where the link to the model is not completely made) such as • The ball will spin • Ground is not flat • The ball is not a particle Do **not** accept statements that refer to the situation outside the range of the throw e.g. • The model is not valid for all values of x • H will become negative Do **not** accept statements that do not refer to the given model or single word vague answers e.g. • The distances may have been measured incorrectly • The ball is not modelled as a particle • "Friction", "Spin", "Force", "air resistance" • It does not take into account the weight of the ball • It depends how good the thrower is • You cannot throw the ball the same way every time	B1	3.5b
		(1)	
(c)	$x=16 \Rightarrow H=-0.03(16)^2+0.54(16)+2=...$	M1	3.4
	$H=2.96$ So Chandra would not be able to catch the ball	A1	3.2a
		(2)	
			(7 marks)

June 2022 Question 5 Paper 1 (A Level)

5. The height, h metres, of a tree, t years after being planted, is modelled by the equation

$$h^2 = at + b \qquad 0 \leqslant t < 25$$

where a and b are constants.

Given that

- the height of the tree was 2.60 m, exactly 2 years after being planted
- the height of the tree was 5.10 m, exactly 10 years after being planted

(a) find a complete equation for the model, giving the values of a and b to 3 significant figures.

(4)

Given that the height of the tree was 7 m, exactly 20 years after being planted

(b) evaluate the model, giving reasons for your answer.

(2)

ANSWER

Question	Scheme	Marks	AOs
5 (a)	Attempts to use $h^2 = at + b$ with either $t = 2, h = 2.6$ or $t = 10, h = 5.1$	M1	3.1b
	Correct equations $2a + b = 6.76$, $10a + b = 26.01$	A1	1.1b
	Solves simultaneously to find values for a and b	dM1	1.1b
	$h^2 = 2.41t + 1.95$ cao	A1	3.3
		(4)	
(b)	Substitutes $t = 20$ into their $h^2 = 2.41t + 1.95$ and finds h or h^2 Or substitutes $h = 7$ into their $h^2 = 2.41t + 1.95$ and finds t	M1	3.4
	Compares the model with the true values and concludes "good model" with a minimal reason E.g. I Finds $h = 7.08\,(\text{m})$ and states that it is a good model as 7.08 (m) is close to 7 (m) E.g II Finds t =19.5 years and states that the model is accurate as 19.5 (years) ≈ 20 (years)	A1	3.5a
		(2)	
			(6 marks)

November 2021 Question 6

6. **In this question you should show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Using algebra, find all solutions of the equation

$$3x^3 - 17x^2 - 6x = 0$$

(3)

(b) Hence find all real solutions of

$$3(y-2)^6 - 17(y-2)^4 - 6(y-2)^2 = 0$$

(3)

ANSWER

Question	Scheme	Marks	AOs
6 (a)	$3x^3 - 17x^2 - 6x = 0 \Rightarrow x\left(3x^2 - 17x - 6\right) = 0$	M1	1.1a
	$\Rightarrow x\left(3x+1\right)\left(x-6\right) = 0$	dM1	1.1b
	$\Rightarrow x = 0, -\frac{1}{3}, 6$	A1	1.1b
		(3)	
(b)	Attempts to solve $\left(y-2\right)^2 = n$ where n is any solution $\geqslant 0$ to (a)	M1	2.2a
	Two of $2, 2 \pm \sqrt{6}$	A1ft	1.1b
	All three of $2, 2 \pm \sqrt{6}$	A1	2.1
		(3)	
			(6 marks)

November 2021 Question 12 Paper 1 (A-Level)

12.

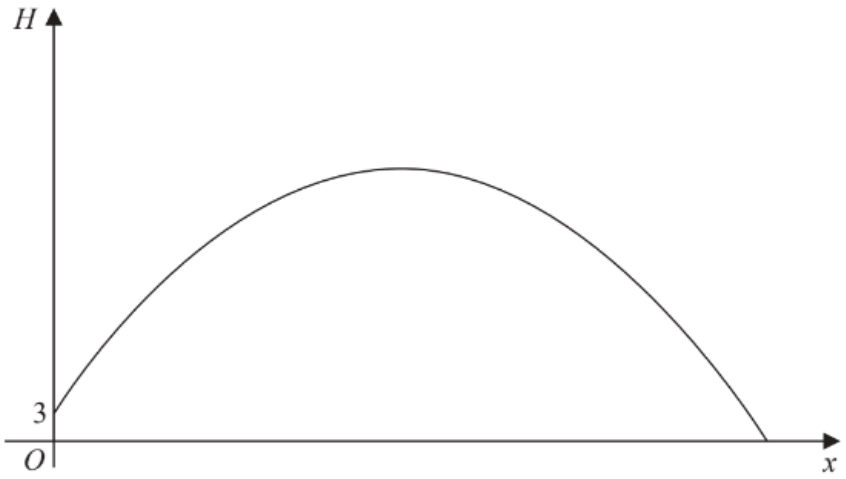

Figure 3

Figure 3 is a graph of the trajectory of a golf ball after the ball has been hit until it first hits the ground.

The vertical height, H metres, of the ball above the ground has been plotted against the horizontal distance travelled, x metres, measured from where the ball was hit.

The ball is modelled as a particle travelling in a vertical plane above horizontal ground.

Given that the ball

- is hit from a point on the top of a platform of vertical height 3 m above the ground
- reaches its maximum vertical height after travelling a horizontal distance of 90 m
- is at a vertical height of 27 m above the ground after travelling a horizontal distance of 120 m

Given also that H is modelled as a **quadratic** function in x

(a) find H in terms of x

(5)

(b) Hence find, according to the model,

(i) the maximum vertical height of the ball above the ground,

(ii) the horizontal distance travelled by the ball, from when it was hit to when it first hits the ground, giving your answer to the nearest metre.

(3)

(c) The possible effects of wind or air resistance are two limitations of the model. Give one other limitation of this model.

(1)

ANSWER

Question	Scheme	Marks	AOs
12(a)	$H = ax^2 + bx + c$ and $x = 0,\ H = 3 \Rightarrow H = ax^2 + bx + 3$	M1	3.3
	$H = ax^2 + bx + 3$ and $x = 120, H = 27 \Rightarrow 27 = 14400a + 120b + 3$ **or** $\frac{dH}{dx} = 2ax + b = 0$ when $x = 90 \Rightarrow 180a + b = 0$	M1 A1	3.1b 1.1b
	$H = ax^2 + bx + 3$ and $x = 120, H = 27 \Rightarrow 27 = 14400a + 120b + 3$ **and** $\frac{dH}{dx} = 2ax + b = 0$ when $x = 90 \Rightarrow 180a + b = 0$ $\Rightarrow a = ..., b = ...$	dM1	3.1b
	$H = -\frac{1}{300}x^2 + \frac{3}{5}x + 3$ o.e.	A1	1.1b
		(5)	
(b)(i)	$x = 90 \Rightarrow H\left(= -\frac{1}{300}(90)^2 + \frac{3}{5}(90) + 3\right) = 30\,\text{m}$	B1	3.4
(b)(ii)	$H = 0 \Rightarrow -\frac{1}{300}x^2 + \frac{3}{5}x + 3 = 0 \Rightarrow x = ...$	M1	3.4
	$x = (-4.868...,)\ 184.868...$ $\Rightarrow x = 185\,(\text{m})$	A1	3.2a
		(3)	
(c)	Examples must focus on why the model may not be appropriate or give values/situations where the model would break down: E.g. • The ground is unlikely to be horizontal • The ball is not a particle so has dimensions/size • The ball is unlikely to travel in a vertical plane (as it will spin) • H is not likely to be a quadratic function in x	B1	3.5b
		(1)	
			(9 marks)

June 2018 Question 6

6.

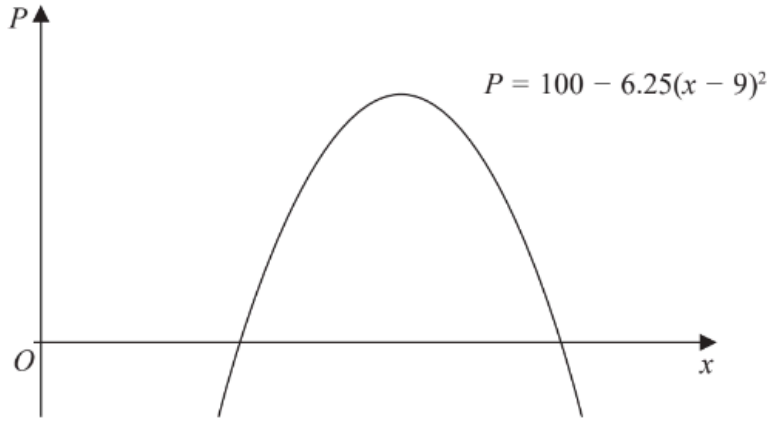

Figure 1

A company makes a particular type of children's toy.

The annual profit made by the company is modelled by the equation

$$P = 100 - 6.25(x - 9)^2$$

where P is the profit measured in thousands of pounds and x is the selling price of the toy in pounds.

A sketch of P against x is shown in Figure 1.

Using the model,

(a) explain why £15 is not a sensible selling price for the toy.

(2)

Given that the company made an annual profit of more than £80 000

(b) find, according to the model, the least possible selling price for the toy.

(3)

The company wishes to maximise its annual profit.

State, according to the model,

(c) (i) the maximum possible annual profit,

(ii) the selling price of the toy that maximises the annual profit.

(2)

ANSWER

Question	Scheme	Marks	AOs
6 (a)	Attempts $P = 100 - 6.25(15-9)^2$	M1	3.4
	$= -125$ $\therefore$ not sensible as the company would make a loss	A1	2.4
		(2)	
(b)	Uses $P > 80 \Rightarrow (x-9)^2 < 3.2$ or $P = 80 \Rightarrow (x-9)^2 = 3.2$	M1	3.1b
	$\Rightarrow 9 - \sqrt{3.2} < x < 9 + \sqrt{3.2}$	dM1	1.1b
	Minimum Price = £7.22	A1	3.2a
		(3)	
(c)	States (i) maximum profit =£ 100 000 and (ii) selling price £9	B1 B1	3.2a 2.2a
		(2)	
			(7 marks)

Video solution:

https://youtu.be/UFenaZppVpk

June 2018 Question 8 Paper 2 (A-level)

8.

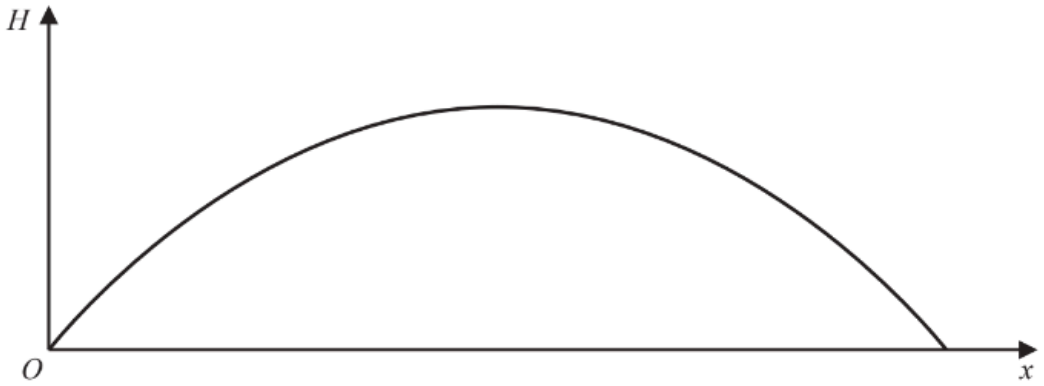

Figure 1

Figure 1 is a graph showing the trajectory of a rugby ball.

The height of the ball above the ground, H metres, has been plotted against the horizontal distance, x metres, measured from the point where the ball was kicked.

The ball travels in a vertical plane.

The ball reaches a maximum height of 12 metres and hits the ground at a point 40 metres from where it was kicked.

(a) Find a quadratic equation linking H with x that models this situation.

(3)

The ball passes over the horizontal bar of a set of rugby posts that is perpendicular to the path of the ball. The bar is 3 metres above the ground.

(b) Use your equation to find the greatest horizontal distance of the bar from O.

(3)

(c) Give one limitation of the model.

(1)

ANSWER

Question	Scheme	Marks	AOs
8 (a) Way 1	$H = Ax(40-x)$ {or $H = Ax(x-40)$}	M1	3.3
	$x=20,\ H=12 \Rightarrow 12 = A(20)(40-20) \Rightarrow A = \frac{3}{100}$	dM1	3.1b
	$H = \frac{3}{100}x(40-x)$ or $H = -\frac{3}{100}x(x-40)$	A1	1.1b
		(3)	
(a) Way 2	$H = 12-\lambda(x-20)^2$ {or $H = 12+\lambda(x-20)^2$}	M1	3.3
	$x=40,\ H=0 \Rightarrow 0 = 12-\lambda(40-20)^2 \Rightarrow \lambda = \frac{3}{100}$	dM1	3.1b
	$H = 12 - \frac{3}{100}(x-20)^2$	A1	1.1b
		(3)	
(a) Way 3	$H = ax^2+bx+c$ (or deduces $H = ax^2+bx$) **Both** $x=0,\ H=0 \Rightarrow 0 = 0+0+c \Rightarrow c=0$ **and either** $x=40,\ H=0 \Rightarrow 0 = 1600a+40b$ **or** $x=20,\ H=12 \Rightarrow 12 = 400a+20b$ **or** $\frac{-b}{2a} = 20$ $\{\Rightarrow b = -40a\}$	M1	3.3
	$b=-40a \Rightarrow 12 = 400a+20(-40a) \Rightarrow a = -0.03$ so $b = -40(-0.03) = 1.2$	dM1	3.1b
	$H = -0.03x^2+1.2x$	A1	1.1b
		(3)	

(b)	$\{H=3 \Rightarrow\}\ 3=\frac{3}{100}x(40-x) \Rightarrow x^2-40x+100=0$ **or** $\{H=3 \Rightarrow\}\ 3=12-\frac{3}{100}(x-20)^2 \Rightarrow (x-20)^2=300$	M1	3.4
	e.g. $x=\frac{40\pm\sqrt{1600-4(1)(100)}}{2(1)}$ or $x=20\pm\sqrt{300}$	dM1	1.1b
	$\left\{\text{chooses } 20+\sqrt{300} \Rightarrow\right\}$ greatest distance = awrt 37.3 m	A1	3.2a
		(3)	
(c)	Gives a limitation of the model. Accept e.g. • the ground is horizontal • the ball needs to be kicked from the ground • the ball is modelled as a particle • the horizontal bar needs to be modelled as a line • there is no wind or air resistance on the ball • there is no spin on the ball • no obstacles in the trajectory (or path) of the ball • the trajectory of the ball is a perfect parabola	B1	3.5b
		(1)	
			(7 marks)