Pure Year 1 exam questions (AS maths)-AQA

NOTE:
Please be aware that in the Year 12 collections, you will find questions from the A-Level (Year 13) papers. However, these questions are intentionally included because they align with Year 12 content and topics.
Happy studying!

Arancha Ruiz

Ch 5 Coordinate geometry

June 2023 Question 3 Paper 3 (A-Level)

3 A curve with equation $y = f(x)$ passes through the point (3, 7)

Given that $f'(3) = 0$ find the equation of the normal to the curve at (3, 7)

Circle your answer.

[1 mark]

$y = \frac{7}{3}x$ $\quad\quad$ $y = 0$ $\quad\quad$ $x = 3$ $\quad\quad$ $x = 7$

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
3	Circles correct answer	2.2a	R1	$x = 3$
	Question 3 Total		**1**	

June 2023 Question 3 Paper 2 (A-Level)

3 A circle has equation

$$(x-5)^2+(y-13)^2=16$$

Find the radius of the circle.

Circle your answer.

[1 mark]

4 12 16 256

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
3	Circles correct answer	1.1b	B1	4
	Question 3 Total		**1**	

June 2023 Question 9 Paper 1 (A-Level)

9 The points P and Q have coordinates $(-6, 15)$ and $(12, 19)$ respectively.

9 (a) (i) Find the coordinates of the midpoint of PQ

[1 mark]

9 (a) (ii) Find the equation of the perpendicular bisector of PQ

Give your answer in the form $ax + by = c$ where a, b and c are integers.

[4 marks]

9 (b) (i) A circle passes through the points P and Q

The centre of the circle lies on the line with equation $2x - 5y = -30$

Find the equation of the circle.

[3 marks]

9 (b) (ii) The circle intersects the coordinate axes at n points.

State the value of n

[1 mark]

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ANSWER

Q	Marking instructions	AO	Marks	Typical solution
9(a)(i)	Obtains $(3,17)$ Condone position vectors, missing brackets or x = 3 and y = 17	1.1b	B1	$(3,17)$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)(ii)	Obtains gradient of PQ PI correct gradient used in equation of perpendicular bisector.	1.1b	B1	$m_{PQ}=\dfrac{19-15}{12--6}=\dfrac{4}{18}$ $y-17=-\dfrac{9}{2}(x-3)$ $2y-34=-9x+27$ $9x+2y=61$
	Forms an equation of a line Either using the negative reciprocal of their gradient **or** their midpoint	3.1a	M1	
	Forms an equation of a line using the negative reciprocal of their gradient **and** their midpoint	1.1a	M1	
	Obtains $9x+2y=61$ OE in the required form.	2.1	A1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)(i)	Solves simultaneously using their $9x+2y=61$ from (a)(ii) with $2x-5y=-30$ to obtain the centre of the circle PI by $(5,8)$ or x = 5, y = 8	3.1a	M1	Centre $(5,8)$ $(x-5)^2+(y-8)^2=r^2$ $(12-5)^2+(19-8)^2=170$ $(x-5)^2+(y-8)^2=170$
	Uses *P* or *Q* and their centre to find the radius or radius^2	3.1a	M1	
	Obtains $(x-5)^2+(y-8)^2=170$ ACF Eg $x^2-10x+y^2-16y=81$	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)(ii)	States 4 Must have the correct centre and correct radius or radius^2	2.2a	R1	4
	Subtotal		**1**	

June 2023 Question 11 Paper 2

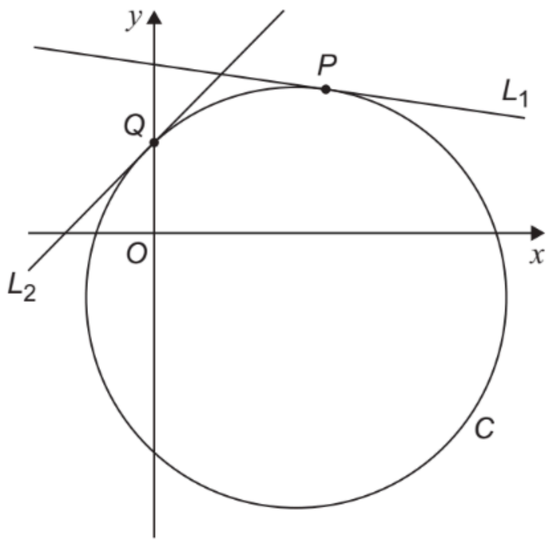
11 The line L_1 has equation $x + 7y - 41 = 0$

L_1 is a tangent to the circle C at the point $P(6, 5)$

The line L_2 has equation $y = x + 3$

L_2 is a tangent to the circle C at the point $Q(0, 3)$

The lines L_1 and L_2 and the circle C are shown in the diagram below.



11 (a) Show that the equation of the radius of the circle through P is $y = 7x - 37$

[3 marks]

11 (b) Find the equation of C

[4 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
11(a)	Obtains correct gradient of L_1 PI by fully rearranged equation of L_1 Condone slip in $\frac{41}{7}$ term	1.1b	B1	$y = -\frac{1}{7}x + \frac{41}{7}$
	Uses perpendicular gradients rule on their gradient of L_1	1.1a	M1	Gradient of L_1 is $\frac{-1}{7}$ Gradient of radius is 7
	Completes reasoned argument to obtain given equation AG	2.1	R1	Equation of radius is $(y-5) = 7(x-6)$ Equation of radius is $y = 7x - 37$
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
11(b)	Obtains correct equation for radius through (0, 3)	1.1b	B1	Gradient of L_2 is 1 Gradient of radius is -1 Equation of radius is $y = -x + 3$
	Solves for $y = 7x - 37$ and their **linear** radius equation for L_2 to find the coordinates of their intersection of the radii	3.1a	M1	Radii intersect when $7x - 37 = -x + 3$ $x = 5$, $y = -2$
	Calculates distance from their centre to either contact point PI by $\sqrt{50}$ or $5\sqrt{2}$ or 50 seen in equation of C	1.1a	M1	Distance from (5, –2) to (0, 3) is $\sqrt{5^2+5^2} = \sqrt{50}$
	Obtains correct equation for C	2.1	R1	Equation of C is $(x-5)^2 + (y+2)^2 = 50$
	Subtotal		**4**	

June 2023 Question 11 Paper 1

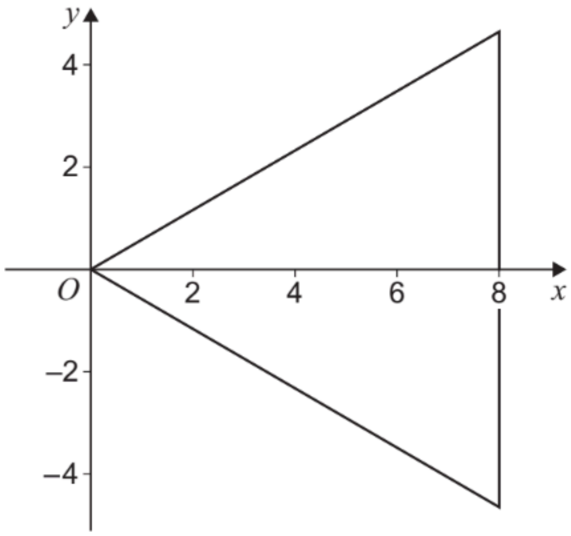
11 (a) A circle has equation

$$x^2 + y^2 - 10x - 6 = 0$$

Find the centre and the radius of the circle.

[2 marks]

11 (b) An equilateral triangle has one vertex at the origin, and one side along the line $x = 8$, as shown in the diagram below.



11 (b) (i) Show that the vertex at the origin lies inside the circle $x^2 + y^2 - 10x - 6 = 0$

[1 mark]

11 (b) (ii) Prove that the triangle lies completely within the circle $x^2 + y^2 - 10x - 6 = 0$

[4 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
11(a)	Obtains correct centre.	1.1b	B1	$x^2 - 10x + 25 + y^2 = 31$ $(x-5)^2 + y^2 = 31$ Centre is (5, 0) and radius $\sqrt{31}$
	Obtains correct radius. AWRT 5.6	1.1b	B1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
11(b)(i)	Shows that origin is inside circle	2.1	R1	Distance from centre to origin is 5 and 5 < $\sqrt{31}$ so vertex at origin is inside circle
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
11(b)(ii)	Selects an appropriate method to find the y coordinate of at least one other vertex. Condone one slip in the formation of the equation for the y coordinate.	3.1a	M1	$\tan 30° = \frac{y}{8}$ so $y = 8\tan 30°$ The other vertices are $(8, \pm\frac{8}{\sqrt{3}})$ Distance from centre to other vertices is $\sqrt{(3^2 + \frac{64}{3})} = \sqrt{(\frac{91}{3})} < \sqrt{31}$ Both vertices are inside circle Complete triangle is inside circle
	Uses distance formula for distance from their centre to at least one vertex. or Find at least one y value on the circle when x = 8 NB Correct vales are $y = \pm\sqrt{22}$	3.1a	M1	
	Compares $\sqrt{(\frac{91}{3})}$ with $\sqrt{31}$ AWRT 5.5 and 5.6 or Compares $\frac{8}{\sqrt{3}}$ with $\sqrt{22}$ AWRT 4.6 and 4.7 **and** Deduces that one other vertex is inside circle.	2.2a	A1	
	Completes proof that triangle is completely inside circle, either by proof for the third vertex or by reference to symmetry.	2.1	R1	
	Subtotal		**4**	

June 2022 Question 6 Paper 1

6 AB is a diameter of a circle where A is $(1, 4)$ and B is $(7, -2)$

6 (a) Find the coordinates of the midpoint of AB.

[1 mark]

6 (b) Show that the equation of the circle may be written as

$$x^2 + y^2 - 8x - 2y = 1$$

[4 marks]

6 (c) The circle has centre C and crosses the x-axis at points D and E.

Find the exact area of triangle DEC.

[4 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Finds correct midpoint of *AB*	1.1b	B1	(4,1)
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)	Calculates length of radius, *AC*, *BC* or half *AB* using 'their' centre.	3.1a	M1	$r = \sqrt{(4-1)^2 + (1-4)^2}$ $= \sqrt{18}$ $(x-4)^2 + (y-1)^2 = 18$ $x^2 - 8x + 16 + y^2 - 2y + 1 = 18$ $x^2 + y^2 - 8x - 2y = 1$
	Obtains correct value for the radius or square of the radius.	1.1b	A1	
	Derives circle equation in any form using their centre and radius. Condone sign error in brackets. Or Completes the square on given equation to obtain centre and radius. Condone sign error in brackets.	1.1a	M1	
	Completes reasoned argument to obtain equation in given form Or Justifies that the centre and radius obtained from completing the square on the given equation corresponds to the midpoint from part (a) and the radius from part (b) AG	2.1	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
6(c)	Substitutes $y = 0$ into the given formula and solves the quadratic equation. Or Uses Pythagoras on the triangle CDM where M(4 , 0) to find DM or EM	3.1a	M1	$x^2 - 8x - 1 = 0$ $x = 4 + \sqrt{17}$ and $4 - \sqrt{17}$ DE = $2\sqrt{17}$ Area = $\frac{1}{2} \times 1 \times 2\sqrt{17}$ $= \sqrt{17}$
	Obtains 2 correct values for x Condone decimal equivalents AWRT 8.1 and – 0.1 Or Obtains DM or EM = $\sqrt{17}$	1.1b	A1	
	Deduces length of DE as the difference between their two values of x Or	2.2a	M1	
	Obtains DM or EM = $\sqrt{17}$			DE = $2\sqrt{17}$ Area = $\frac{1}{2} \times 1 \times 2\sqrt{17}$ $= \sqrt{17}$
	Deduces length of DE as the difference between their two values of x Or Deduces the length of DE as twice the length of DM or EM PI	2.2a	M1	
	Obtains $\sqrt{17}$ CAO	1.1b	A1	
	Subtotal		**4**	

	Question 6 Total		**9**	

June 2022 Question 8 Paper 1 (A-Level)

8 The lines L_1 and L_2 are parallel.

L_1 has equation

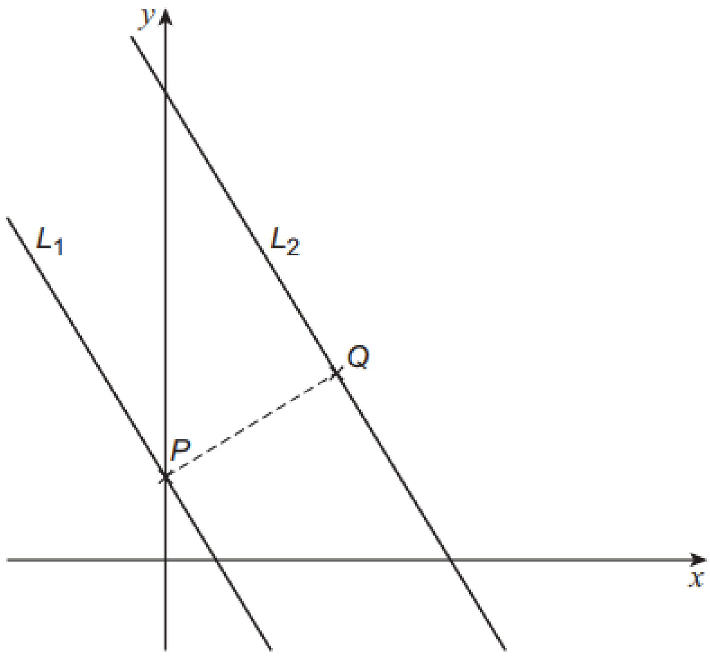
$$5x + 3y = 15$$

and L_2 has equation

$$5x + 3y = 83$$

L_1 intersects the y-axis at the point P.

The point Q is the point on L_2 closest to P, as shown in the diagram.



8 (a) (i) Find the coordinates of Q.

[5 marks]

8 (a) (ii) Hence show that $PQ = k\sqrt{34}$, where k is an integer to be found.

[2 marks]

8 (b) A circle, C, has centre $(a, -17)$.

L_1 and L_2 are both tangents to C.

8 (b) (i) Find a.

[2 marks]

8 (b) (ii) Find the equation of C.

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
8(a)(i)	Obtains correct y-intercept at P (0,5) or y = 5 seen anywhere PI by correct equation for line PQ	3.1a	B1	
	Obtains equation of PQ with correct gradient. For example $y=\frac{3}{5}x+c$ or $5y-3x=k$ or Forms an equation for the distance or distance squared from (0,5) to a point on L_2 For example, $d^2 = x^2 + \left(-\frac{5}{3}x+\frac{68}{3}\right)^2$	3.1a	M1	At P $x = 0 \Rightarrow y = 5$
	Obtains correct equation ACF	1.1b	A1	Line PQ $3x - 5y = -25$ $5x + 3y = 83$
	Solves simultaneous equations for their PQ and L_2 to obtain values for x and y Their PQ must not be a horizontal or vertical line Condone errors in rearrangement of the equation(s) or Minimises their distance or distance squared equation to find one coordinate	3.1a	M1	(10, 11)
	Obtains (10, 11) or $x=10,\ y=11$	1.1b	A1	
	Subtotal		**5**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)(ii)	Uses the distance formula to find the value of PQ or PQ^2 or Uses Pythagoras theorem with 10^2+6^2 seen If the coordinates of P and Q are **incorrect**, differences in x and y must be clearly shown for M1	1.1a	M1	$PQ^2=(10-0)^2+(11-5)^2$ $PQ=\sqrt{136}=2\sqrt{34}$
	Completes demonstration to show that k = 2 Must have shown clear use of distance formula Condone not seeing x = 0 substituted in the distance formula Answer of $2\sqrt{34}$ and no working shown scores M1 R0	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)(i)	Uses a valid method to find a. Evidence could be: Forming the equation of the line mid-way between L_1 and L_2 $5x+3y=49$ seen or Using (a , -17) as the mid-point of a line segment from L_1 to L_2 For example: $5x+3(-17)=15 \quad x=13.2$ $5x+3(-17)=83 \quad x=26.8$ $a=\frac{(26.8+13.2)}{2}=20$ or Finding the mid-point of PQ their (5 , 8) and using the gradient of L_1 and L_2 $=-\frac{5}{3}$ For example: $8+5(-5)=-17$ $a=5+5(3)=20$ or Substitutes y = 17 and x = a into $y=\frac{3}{5}x+5$	3.1a	M1	$5x+3y=49$ $5a+3(-17)=49$ $a=20$
	Deduces a = 20	2.2a	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)(ii)	Forms expression of the form $(x\pm a)^2+(y\pm 17)^2$ using a or their value of a	1.1a	M1	
	Obtains correct equation for their value of a and their radius² = $\frac{17k^2}{2}$ from part (a)(ii) for an integer value of k Condone $\left(\sqrt{34}\right)^2$	1.1b	A1F	$(x-20)^2+(y+17)^2=34$
	Subtotal		**2**	

	Question 8 Total		**11**	

June 2022 Question 1 Paper 2 (A-Level)

1 A circle has centre $(4, -5)$ and radius 6

Find the equation of the circle.

Tick (✓) **one** box.

[1 mark]

$(x-4)^2+(y+5)^2=6$ ☐

$(x+4)^2+(y-5)^2=6$ ☐

$(x-4)^2+(y+5)^2=36$ ☐

$(x+4)^2+(y-5)^2=36$ ☐

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
1	Ticks correct box	1.1b	B1	$(x-4)^2+(y+5)^2=36$
	Question 1 Total		**1**	

November 2021 Question 5 Paper 1 (A-Level)

5 The line L has equation

$$3y - 4x = 21$$

The point P has coordinates (15, 2)

5 (a) Find the equation of the line perpendicular to L which passes through P.

[2 marks]

5 (b) Hence, find the shortest distance from P to L.

[4 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Uses negative reciprocal to obtain an equation with the correct gradient.	1.1a	M1	$4y+3x=c$ $4\times 2+3\times 15=53$ $4y+3x=53$
	Obtains correct equation ACF ISW once ACF seen Eg $y=-\frac{3}{4}x+\frac{53}{4}$ $y-2=-\frac{3}{4}(x-15)$	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)	Begins to solve $3y-4x=21$ and their $4y+3x=53$ with elimination of one variable or better. Or obtains correct point of intersection for $3y-4x=21$ and their $4y+3x=53$	1.1a	M1	$3y-4x=21$ $4y+3x=53$ $y=11$ $x=3$ $(3-15)^2+(11-2)^2=12^2+9^2$ $=225$ $\text{Distance}=15$
	Uses distance formula to find the distance between (15, 2) and another point other than the origin. PI correct distance or square of correct distance	1.1a	M1	
	Uses distance formula for (15, 2) and their point of intersection to find distance or distance^2	3.1a	M1	
	Obtains 15 CAO	1.1b	A1	
	Subtotal		**4**	

November 2021 Question 7 Paper 2 (A-Level)

7 A circle has equation

$$x^2 + y^2 - 6x - 8y = p$$

7 (a) (i) State the coordinates of the centre of the circle.

[1 mark]

7 (a) (ii) Find the radius of the circle in terms of p.

[3 marks]

7 (b) The circle intersects the coordinate axes at exactly three points.

Find the **two** possible values of p.

[4 marks]

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ANSWER

Q	Marking instructions	AO	Marks	Typical solution
7(a)(i)	Obtains centre $= (3,4)$ Accept a = 3 , b = 4	1.1b	B1	Centre $(3,4)$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)(ii)	Rearranges into a standard form with $(x \pm 3)^2 + (y \pm 4)^2$ seen or Forms an expression for radius of the form $\sqrt{(\pm 3)^2 + (\pm 4)^2 \pm p}$ This working can be seen in part (a)(i)	1.1a	M1	$x^2 + y^2 = 6x + 8y + p$ $x^2 + y^2 - 6x - 8y - p = 0$ $(x-3)^2 + (y-4)^2 - 9 - 16 - p = 0$ $(x-3)^2 + (y-4)^2 = 25 + p$
	Obtains $(x-3)^2 + (y-4)^2 = 25 + p$ Or Obtains $\sqrt{3^2 + 4^2 + p}$	1.1b	A1	Radius = $\sqrt{25+p}$
	States radius = $\sqrt{25+p}$	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)	Begins to solve the problem by either Sketching (part of) a circle which goes through the origin or Sketching (part of) a circle that touches one of the axes or Substituting either x = 0 or y = 0 into their circle equation – must involve p PI by correctly formed equation involving p	3.1a	M1	Circle passes through the origin $\sqrt{25+p}$ = 5 $\Rightarrow p = 0$ Circle just touches x-axis $\sqrt{25+p}$ = 4 $\Rightarrow p = -9$

	Forms an equation to find p By either Equating their expression for the radius to 5 or the greater value of their a and b or Substituting both $x = 0$ and $y = 0$ into their circle equation to form an equation in terms of p only or Substituting $y = 0$ and using $b^2 - 4ac = 0$ to form an equation in terms of p only	3.1a	M1	

	Deduces $p = 0$	2.2a	R1	
	Deduces $p = -9$	2.2a	R1	
	Subtotal		**4**	

	Question Total		**8**	

Video solution:

https://youtu.be/_geLVXvTYs0

November 2021 Question 4 Paper 1

4 *ABCD* is a trapezium where A is the point $(1, -2)$, B is the point $(7, 1)$ and C is the point $(3, 4)$

DC is parallel to *AB*.

AD is perpendicular to *AB*.

4 (a) (i) Find the equation of the line *CD*.

[2 marks]

4 (a) (ii) Show that point D has coordinates $(-1, 2)$

[3 marks]

4 (b) (i) Find the sum of the length of *AB* and the length of *CD* in simplified surd form.

[2 marks]

4 (b) (ii) Hence, find the area of the trapezium *ABCD*.

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
4(a)(i)	Uses coordinates of *A* and *B* to find gradient of *AB*	3.1a	M1	Grad $AB = \frac{3}{6} = \frac{1}{2}$ = Grad DC Equation is $y - 4 = \frac{1}{2}(x - 3)$ $2y = x + 5$
	Obtains correct equation of *CD* (any form)	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
4(a)(ii)	Uses perpendicular gradients property.	1.1a	M1	Grad $DA = \frac{-1}{\text{Grad AB}} = -2$ Equation is $y + 2 = -2(x - 1)$ $y = -2x$ Intersect at $(-1, 2) = D$
	Obtains correct equation of *AD* using their gradient (any form) Or shows that A to (-1, 2) has required gradient of -2	1.1b	A1F	
	Obtains correct coordinates of *D* Or shows that C to (-1,2) has required gradient of 0.5 Or shows that (-1, 2) lies on both lines	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
4(b)(i)	Calculates length of *AB* and *CD*. At least one correct.	1.1a	M1	$AB = \sqrt{(36 + 9)} = \sqrt{45} = 3\sqrt{5}$ $CD = \sqrt{(4 + 16)} = \sqrt{20} = 2\sqrt{5}$ $AB + CD = 5\sqrt{5}$
	Obtains correct simplified sum	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
4(b)(ii)	Calculates *AD* and applies trapezium area formula	1.1a	M1	$AD = \sqrt{(4 + 16)} = \sqrt{20} = 2\sqrt{5}$ $= \frac{1}{2}(5\sqrt{5} \times 2\sqrt{5})$ $= 25$
	Obtains correct area	1.1b	A1	
	Subtotal		**2**	

Video solution:

https://youtu.be/Ua72e1XNMSY

November 2021 Question 11 Paper 2

11 A circle C has centre (0, 10) and radius $\sqrt{20}$

A line L has equation $y = mx$

11 (a) (i) Show that the x-coordinate of any point of intersection of L and C satisfies the equation

$$(1 + m^2)x^2 - 20mx + 80 = 0$$

[3 marks]

11 (a) (ii) Find the values of m for which the equation in part **(a)(i)** has equal roots.

[3 marks]

11 (b) Two lines are drawn from the origin which are tangents to C.

Find the coordinates of the points of contact between the tangents and C.

[4 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
11(a)(i)	Writes down the equation of the circle in any correct form	1.1b	B1	Circle equation is $(x-0)^2 + (y-10)^2 = (\sqrt{20})^2$ $x^2 + y^2 - 20y + 100 = 20$ $x^2 + y^2 - 20y + 80 = 0$ Substitute $y = mx$ $x^2 + m^2x^2 - 20mx + 80 = 0$ $(1 + m^2)x^2 - 20mx + 80 = 0$
	Substitutes mx for y	1.1a	M1	
	Simplifies to the given quadratic AG	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
11(a)(ii)	Uses the discriminant of the given equation from (a)(i)	3.1a	M1	Using $b^2 = 4ac$ $400m^2 = 4 \times (1 + m^2) \times 80$ $5m^2 = 4 \times (1 + m^2)$ $m^2 = 4$ $m = \pm 2$
	Obtains a correct equation in m	1.1b	A1	
	Obtains $m = \pm 2$	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
11(b)	Uses one of their m values from **(a)(ii)**	3.1a	M1	Using $m = 2$ in equation from (a) $5x^2 - 40x + 80 = 0$ $x = 4$ giving $y = 8$ Using $m = -2$ in equation from (a) $5x^2 + 40x + 80 = 0$ $x = -4$ giving $y = 8$ So (4, 8) and (–4, 8)
	Obtains one correct x value	1.1b	A1	
	Uses line equation to calculate y value	1.1a	M1	
	Obtains two correct sets of coordinates	1.1b	A1	
	Subtotal		**4**	

November 2020 Question 6 Paper 2

6 A circle has equation

$$x^2 + y^2 + 10x - 4y - 71 = 0$$

6 (a) Find the centre of the circle.

[2 marks]

6 (b) Hence, find the equation of the tangent to the circle at the point (1, 10), giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

[4 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)	Completes a square correctly once OE (PI by one correct coordinate of the centre)	1.1a	M1	$x^2 + 10x + 25 + y^2 - 4y + 4 - 100 = 0$ $(x + 5)^2 + (y - 2)^2 = 100$ Centre = (–5, 2)
	Obtains correct centre	1.1b	A1	
	Subtotal		**2**	
6(b)	Finds correct gradient from 'their' centre from (a) to the point (1, 10)	1.1b	B1F	Gradient from (–5, 2) to (1, 10) is $\frac{8}{6}$ So perpendicular gradient is $-\frac{3}{4}$ $y - 10 = -\frac{3}{4}(x - 1)$ $4y - 40 = -3x + 3$ $3x + 4y - 43 = 0$
	Uses perpendicular gradient property for 'their' gradient	1.1a	M1	
	Finds 'their' equation of the line based on 'their' perpendicular gradient. Finding c = 10.75 (OE) is sufficient for M1	1.1a	M1	
	Rearranges "their" equation of the line into form $ax + by + c = 0$ (FT 'their' centre of the circle only)	1.1b	A1F	
	Subtotal		**4**	

November 2020 Question 6 Paper 2 (A-Level)

6 The line L has equation

$$5y + 12x = 298$$

A circle, C, has centre (7, 9)

L is a tangent to C.

6 (a) Find the coordinates of the point of intersection of L and C.

Fully justify your answer.

[5 marks]

6 (b) Find the equation of C.

[3 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Begins a valid method to find the coordinates Uses gradient of L to find gradient of perpendicular radius Or Forms equation of circle with unknown radius and solves simultaneously with equation of L Or differentiates equation of circle implicitly	3.1a	M1	$5y+12x=298$ $y=\frac{-12}{5}x+\frac{298}{5}$ $y-9=\frac{5}{12}(x-7)$ $12y-5x=73$ $x=19$ $y=14$ $(19,14)$
	Uses (7, 9) to find the equation of the radius Or Uses (7, 9) correctly in their equation of circle Or Uses $\frac{-12}{5}$ after their implicit differentiation	1.1a	M1	

	Obtains $12y-5x=73$ OE Or Correctly eliminates a variable to obtain a quadratic in x or y for example obtain a quadratic in x or y $(x-7)^2+\left(\frac{298-12x}{5}-9\right)^2=k$ $\Rightarrow (x-7)^2+\left(\frac{253-12x}{5}\right)^2=k$ $\left(\frac{298-5y}{12}-7\right)^2+(y-9)^2=k$ $\Rightarrow \left(\frac{214-5y}{12}\right)^2+(y-9)^2=k$	1.1b	A1	
	Equates discriminant to zero PI By correct answer or Solves their simultaneous equations of tangent and radius PI by correct answer	3.1a	M1	
	Obtains correct values for x and y $(19,14)$	1.1b	A1	
	Subtotal		**5**	

6(b)	Obtains $(x-7)^2+(y-9)^2=r^2$ **PI** if r^2 is replaced with correct value using their point from (a)	1.1a	M1	$(x-7)^2+(y-9)^2=r^2$ $(19-7)^2+(14-9)^2=r^2$ $12^2+5^2=169$ $(x-7)^2+(y-9)^2=169$
	Uses their point to find radius or radius squared. Must have obtained a point in part (a)	1.1a	M1	
	Obtains correct equation of circle CSO ACF	1.1b	A1	
	Subtotal		**3**	
	Question Total		**8**	

June 2018 Question 5 Paper 1

5 Point C has coordinates $(c, 2)$ and point D has coordinates $(6, d)$.

The line $y + 4x = 11$ is the perpendicular bisector of CD.

Find c and d.

[5 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
5	Forms an equation for gradient of CD = ¼ or –¼ of the form difference in y over difference in x (or vice versa = 4 or -4)	AO3.1a	M1	$\frac{d-2}{6-c} = \frac{1}{4}$ $4d - 8 = 6 - c$
	Obtains a correct equation for c & d	AO1.1b	A1	$c + 4d = 14$
	Forms an equation for the mid-point of CD lying on $y + 4x = 11$	AO3.1a	M1	$\frac{2+d}{2} + 4\left(\frac{c+6}{2}\right) = 11$
	Obtains correct equation for c & d (any correct form)	AO1.1b	A1	$4c + d = -4$
	Solves for c and d CAO	AO1.1b	A1	$c = -2 \quad d = 4$
	Total		**5**	

Arancha Ruiz

June 2019 Question 7 Paper 2

7 The points A $(a, 3)$ and B $(10, 6)$ lie on a circle.

AB is a diameter of the circle and passes through the point $(2, 4)$

The circle has equation

$$(x - c)^2 + (y - d)^2 = e$$

where c, d and e are rational numbers.

Find the values of a, c, d and e.

[6 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
7	Uses gradient or equation of *AB* or vectors or proportionate division to find a PI by obtaining a = –2	3.1a	M1	Gradient (2, 4) to B = $\frac{6-4}{10-2} = \frac{1}{4}$ $\frac{6-3}{10-a} = \frac{1}{4}$ So a = –2
	Obtains a = –2	1.1b	A1	
	Finds midpoint of *AB* PI by either coordinate being correct NB Knowledge of value of a is not required for this mark	1.1a	M1	Midpoint = $(\frac{a+10}{2}, \frac{3+6}{2})$ = (4, 4.5) c = 4, d = 4.5
	Deduces c = 4 and d = 4.5	2.2a	A1	Radius^2 = 6^2 + 1.5^2 = 38.25 e = 38.25
	Uses an appropriate distance formula to find length of radius or radius squared NB Must be fully numerical PI by use of 'their' $(10 - c)^2 + (6 - d)^2$ or 38.25 seen anywhere or $\frac{1}{2}\sqrt{(10-a)^2 + 3^2}$ for 'their' a	1.1a	M1	
	Deduces correct value of e Accept 38.25 or $\frac{153}{4}$ or $38\frac{1}{4}$ OE CAO Do not ISW if e is square rooted or squared	2.2a	A1	
	Total		**6**	

June 2019 Question 4 Paper 1 (A-Level)

4 The point A has coordinates $(-1, a)$ and the point B has coordinates $(3, b)$

The line AB has equation $5x + 4y = 17$

Find the equation of the perpendicular bisector of the points A and B.

[4 marks]

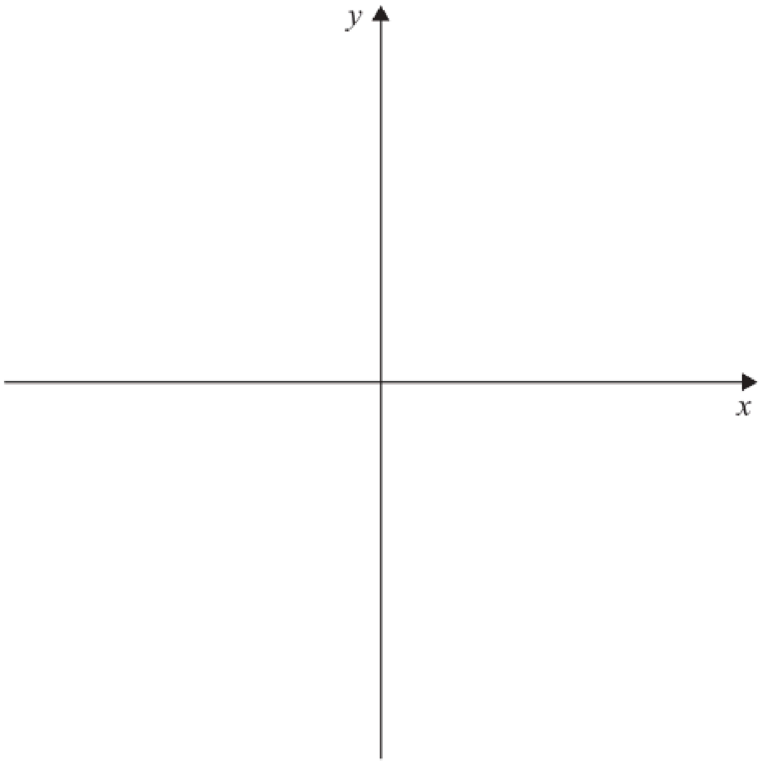
ANSWER

Q	Marking instructions	AO	Mark	Typical solution
4	Uses negative reciprocal to obtain equation with correct gradient	3.1a	M1	$-4x+5y=k$ $x=1$ $\Rightarrow 5+4y=17$ $\Rightarrow y=3$ $k=-4\times1+5\times3=11$ $5y-4x=11$ $y=\frac{4}{5}x+\frac{11}{5}$
	Obtains correct x coordinate of midpoint Or obtains correct equations of lines through A and B perpendicular to AB $5y-4x=31.5 \qquad 5y-4x=-9.5$ OE	1.1b	B1	
	Substitutes their mid-point value of x to obtain value of y coordinate of midpoint (not in terms of *a* or *b*) Or Finds a value for their $\frac{a+b}{2}$ Or Finds k by adding correct equations of lines through A and B perpendicular to AB Or equating intercepts.	1.1a	M1	
	Obtains correct equation **ACF** Eg $y=\frac{4}{5}x+c, \quad c=2.2$ **ISW** once correct answer seen.	1.1b	A1	
	Total		**4**	

June 2019 Question 7 Paper 2

7 (a) Sketch the graph of any cubic function that has **both** three distinct real roots **and** a positive coefficient of x^3

[2 marks]



7 (b) The function $f(x)$ is defined by

$$f(x) = x^3 + 3px^2 + q$$

where p and q are constants and $p > 0$

7 (b) (i) Show that there is a turning point where the curve crosses the y-axis.

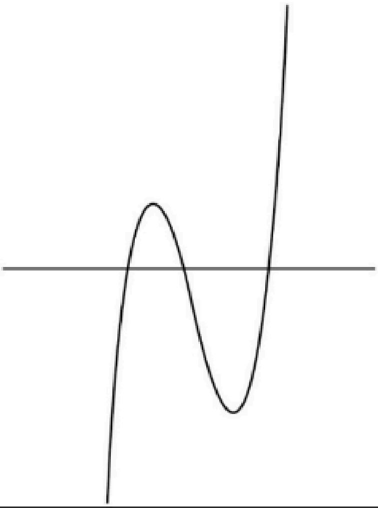
[3 marks]

7 (b) (ii) The equation $f(x) = 0$ has three distinct real roots.

By considering the positions of the turning points find, in terms of p, the range of possible values of q.

[5 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
7(a)	Sketches any cubic graph, crossing the x-axis in three places	1.2	B1	
	Sketches any cubic graph with a positive coefficient of x^3	1.2	B1	
7(b)(i)	Differentiates to obtain $f'(x)$ Two terms with at least one correct - either $3x^2$ or $6px$	1.1a	M1	For a turning point $f'(x) = 0$ $f(x) = x^3 + 3px^2 + q$ $f'(x) = 3x^2 + 6px$
	Solves $3x^2 + 6px = 0$ to obtain $x = 0$ or $x = -2p$ or Substitutes $x = 0$ in $f'(x) = 3x^2 + 6px$ and obtains 0	1.1b	A1	$3x^2 + 6px = 0$ $3x(x + 2p) = 0$ $x = 0$ $x = -2p$
	Obtains the correct two roots $x = 0$ and $x = -2p$ **OE** and states why there must be a turning point referring to root $x = 0$	2.4	R1	Since one of the roots is $x = 0$ there must be a turning point on the y axis

7(b)(ii)	Deduces that turning point at $x = -2p$ is a maximum or deduces that turning point $x = 0$ is a minimum May have been seen in part **(b)(i)** Accept a sketch showing correct relative positions of turning points	2.2a	B1	Since $p > 0$ $x = -2p$ is the maximum $x = 0$ is the minimum $\mathrm{f}(0) = q$ $\mathrm{f}(-2p) = (-2p)^3 + 3p(-2p)^2 + q$ $= 4p^3 + q$ $-4p^3 < q < 0$
	Substitutes their $x = -2p$ into $\mathrm{f}(x)$	1.1a	M1	
	Obtains correct $\mathrm{f}(0) = q$ and $\mathrm{f}(-2p) = 4p^3 + q$	1.1b	A1	
	Deduces either $q < 0$ or $-4p^3 < q$ Condone $\leq$	2.2a	R1	
	Deduces $-4p^3 < q < 0$ **CAO**	2.2a	R1	
	Total		**10**	

June 2018 Question 5 Paper 1

5 Point C has coordinates $(c, 2)$ and point D has coordinates $(6, d)$.

The line $y + 4x = 11$ is the perpendicular bisector of CD.

Find c and d.

[5 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
5	Forms an equation for gradient of CD = ¼ or –¼ of the form difference in y over difference in x (or vice versa = 4 or -4)	AO3.1a	M1	$\frac{d-2}{6-c} = \frac{1}{4}$ $4d - 8 = 6 - c$
	Obtains a correct equation for c & d	AO1.1b	A1	$c + 4d = 14$
	Forms an equation for the mid-point of CD lying on $y + 4x = 11$	AO3.1a	M1	$\frac{2+d}{2} + 4\left(\frac{c+6}{2}\right) = 11$
	Obtains correct equation for c & d (any correct form)	AO1.1b	A1	$4c + d = -4$
	Solves for c and d CAO	AO1.1b	A1	$c = -2 \quad d = 4$
	Total		**5**	

June 2018 Question 6 Paper 2

6 Points $A\ (-7, -7)$, $B\ (8, -1)$, $C\ (4, 9)$ and $D\ (-11, 3)$ are the vertices of a quadrilateral $ABCD$.

6 (a) Prove that $ABCD$ is a rectangle.

[4 marks]

6 (b) Find the area of $ABCD$.

[2 marks]

ANSWER

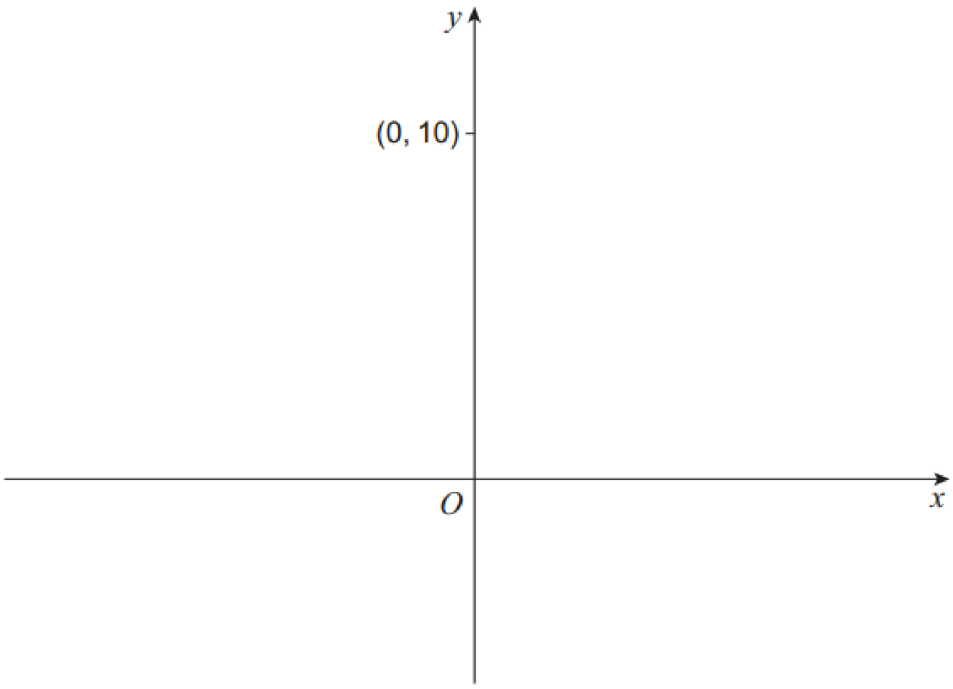
Q	Marking Instructions	AO	Marks	Typical Solution
6(a)	Selects a method leading to any calculation pertaining to one of the following methods seen (not necessarily correct); gradients of sides, lengths of sides or intersection or lengths of diagonals	AO3.1a	M1	Grad BC = -5/2 = Grad DA Grad AB = 2/5 = Grad DC Both pairs of opposite sides have equal gradient so parallel, so ABCD is a parallelogram Grad BC × grad AB = -1 ABC = 90° therefore all angles in ABCD are 90° so ABCD is a rectangle
	Finds gradients of all 4 sides or lengths of all 4 sides or midpoints of both diagonals correctly	AO1.1b	A1	
	Proves one angle is 90° by using gradients or Pythagoras	AO1.1a	M1	
	Completes proof that ABCD is a rectangle. There must be a clear statement that there are 2 pairs of parallel sides and all angles are 90°	AO2.1	R1	
	Note, there are various ways of proving that ABCD is a rectangle (1 – 5 below score M1 A1 M1 before final required statement for relevant R1 stating how their method used proves a rectangle) 1. As in the typical solution shown: show that both pairs of opposite sides are parallel, show that one angle is 90°. 2. Show that each pair of opposite sides is equal in length, show that one angle is 90°. 3. Show that one pair of opposite sides is parallel and equal in length, show that one angle is 90°. 4. Show that the diagonals bisect (the midpoint of one is also the midpoint of the other) and are equal in length. 5. Show that each pair of opposite sides are parallel and length of the two diagonals are the same NB May be expressed using vectors NB Diagonals AC and BD = $\sqrt{377}$			
(b)	Finds correct lengths of two adjacent sides (accept to at least 1dp accuracy)	AO1.1a	M1	AB (= DC) = $\sqrt{261} = 3\sqrt{29}$ BC (= DA) = $\sqrt{116} = 2\sqrt{29}$ Area = 174
	Obtains correct area (AWRT)	AO1.1b	A1	
	Total		**6**	

June 2018 Question 8 Paper 2

8 A circle of radius 6 passes through the points (0, 0) and (0, 10).

8 (a) Sketch the two possible positions of the circle.

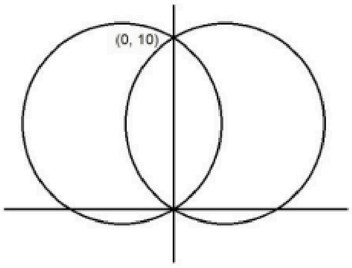
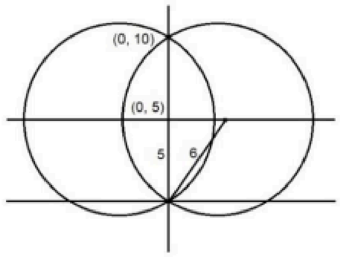
[1 mark]



8 (b) Find the equations of the two circles.

[3 marks]

ANSWER

Q 8	Marking Instructions	AO	Marks	Typical Solution
(a)	Produces a combined diagram showing circles intersecting at origin and (0, 10) or two separate diagrams. Allow reasonable 'hand drawn' circles which illustrate symmetry. Circles must cut the x axis again. Do not accept circles that go off the page.	AO2.2a	B1	
(b)	Deduces that y coordinate of centre is 5. (PI by any use of $(y - 5)$ in any circle equation or marked on diagram or seen as a y coordinate or used in Pythagoras)	AO2.2a	B1	
	Forms correct equation for x coordinate of centres using Pythagoras (PI)	AO1.1a	M1	$6^2 = 5^2 + a^2$ $a = +\sqrt{11}$ or $-\sqrt{11}$
	Obtains two correct circle equations (either form) Condone 3.3 or better provided $a = \sqrt{11}$ seen earlier	AO1.1b	A1	$(x - \sqrt{11})^2 + (y - 5)^2 = 6^2 = 36$ $(x + \sqrt{11})^2 + (y - 5)^2 = 6^2 = 36$ Or $x^2 + 2\sqrt{11}x + y^2 - 10y = 0$ $x^2 - 2\sqrt{11}x + y^2 - 10y = 0$
	Total		**4**	

June 2018 Question 7 Paper 1 (A-Level)

7 Three points A, B and C have coordinates A (8, 17), B (15, 10) and C (−2, −7)

7 (a) Show that angle ABC is a right angle.

[3 marks]

7 (b) A, B and C lie on a circle.

7 (b) (i) Explain why AC is a diameter of the circle.

[1 mark]

7 (b) (ii) Determine whether the point D (−8, −2) lies inside the circle, on the circle or outside the circle.

Fully justify your answer.

[4 marks]

Arancha Ruiz

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
7(a)	Uses a technique which could lead to showing two lines are perpendicular. Obtains at least one correct distance (or distance^2) or gradient.	AO3.1a	M1	$AB^2=(8-15)^2+(17-10)^2$ $=98$ $AC^2=(8--2)^2+(17--7)^2$ $=676$
	Obtains three correct distances (or distance^2) or two gradients. Lengths: $7\sqrt{2}, 17\sqrt{2}, 26$ Gradients: $AB=-\frac{7}{7}, BC=\frac{17}{17}$	AO1.1b	A1	$CB^2=(15--2)^2+(10--7)^2$ $=578$ $AB^2+BC^2=98+578$ $=676$ $=AC^2$
	Completes correct rigorous argument to show required result Uses Pythagoras OR Multiplies gradients to show product is -1 AND Writes a concluding statement.	AO2.1	R1	Angle ABC is a right angle.
(b)(i)	Explains why AC is a diameter Must reference angle subtended by diameter (condone "angle in a semi-circle") or give full explanation.	AO2.4	E1	The angle subtended by a diameter is 90° $\therefore AC$ must be a diameter of the circle
(b)(ii)	Deduces correct radius (or radius^2)	AO2.2a	B1	Radius $\frac{\sqrt{676}}{2}=13$
	Obtains mid-point of diameter	AO1.1b	B1	Centre $\left(\frac{8-2}{2}, \frac{17-7}{2}\right)=(3,5)$
	Uses $D(-8,-2)$ to find the distance or (distance^2) from *their* centre OE	AO1.1a	M1	
	Completes rigorous argument by comparing $\sqrt{170}>13$ (or $170>169$) to show that D lies outside the circle	AO2.1	R1	Distance from centre to D $(3--8)^2+(5--2)^2=11^2+7^2$ $=170>169$ So D lies outside the circle.
	Total		**8**	