Edexcel - Pure Year 2 exam questions

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Ch. 9: Differentiation

June 2023 Question 7 Paper 2

7. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$x^3 + 2xy + 3y^2 = 47$$

(a) Find $\frac{dy}{dx}$ in terms of x and y

(4)

The point $P(-2, 5)$ lies on the curve.

(b) Find the equation of the normal to the curve at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

ANSWER

Question	Scheme	Marks	AOs
7(a)	$x^3 \rightarrow \ldots x^2$ and $3y^2 \rightarrow \ldots y\frac{\mathrm{d}y}{\mathrm{d}x}$	**M1**	1.1b
	$2xy \rightarrow 2y + 2x\frac{\mathrm{d}y}{\mathrm{d}x}$	**B1**	1.1b
	$3x^2 + 2x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y + 6y\frac{\mathrm{d}y}{\mathrm{d}x} = \ldots \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \ldots.$	**M1**	2.1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2y+3x^2}{2x+6y}$	**A1**	1.1b
		(4)	
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2(5)+3(-2)^2}{2(-2)+6(5)}$ or e.g. $3(-2)^2 + 2(-2)\frac{\mathrm{d}y}{\mathrm{d}x} + 2\times5 + 6\times5\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \ldots \left(-\frac{11}{13}\right)$	**M1**	1.1b
	$y - 5 = \text{"}\frac{13}{11}\text{"}(x+2)$	**dM1**	1.1b
	$13x - 11y + 81 = 0$	**A1**	2.2a
		(3)	
			(7 marks)

June 2023 Question 1 Paper 2

1.

$$f(x) = x^3 + 2x^2 - 8x + 5$$

(a) Find $f''(x)$

(2)

(b) (i) Solve $f''(x) = 0$

(ii) Hence find the range of values of x for which $f(x)$ is concave.

(2)

ANSWER

Question	Scheme	Marks	AOs
1(a)	$\{f'(x) =\}\ldots x^2 + \ldots x + \ldots \Rightarrow \{f''(x) =\}\ldots x + \ldots$	**M1**	1.1b
	$\{f'(x) =\} 3x^2 + 4x - 8 \Rightarrow \{f''(x) =\} 6x + 4$	**A1cso**	1.1b
		(2)	
(b)(i)	$"6x+4" = 0 \Rightarrow x = "-\frac{2}{3}"$	**B1ft**	1.1b
(ii)	$x \leqslant "-\frac{2}{3}"$ or $x < "-\frac{2}{3}"$	**B1ft**	2.2a
		(2)	
			(4 marks)

June 2023 Question 12 Paper 1

12. $$y = \sin x$$

where x is measured in radians.

Use differentiation from first principles to show that

$$\frac{dy}{dx} = \cos x$$

You may

- use without proof the formula for $\sin(A \pm B)$
- assume that as $h \to 0$, $\dfrac{\sin h}{h} \to 1$ and $\dfrac{\cos h - 1}{h} \to 0$

(5)

ANSWER

Question	Scheme	Marks	AOs
12	$\dfrac{\sin(x+h)-\sin x}{h}$	B1	2.1
	$\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}$	M1 A1	1.1b 1.1b
	(As $h \to 0$), $\sin x\left(\dfrac{\cos h-1}{h}\right)+\cos x\left(\dfrac{\sin h}{h}\right) \to 0\times\sin x+1\times\cos x$	dM1	2.1
	so $\dfrac{dy}{dx}=\cos x$ *	A1*	2.5
			(5 marks)

June 2022 Question 16 Paper 2

16.

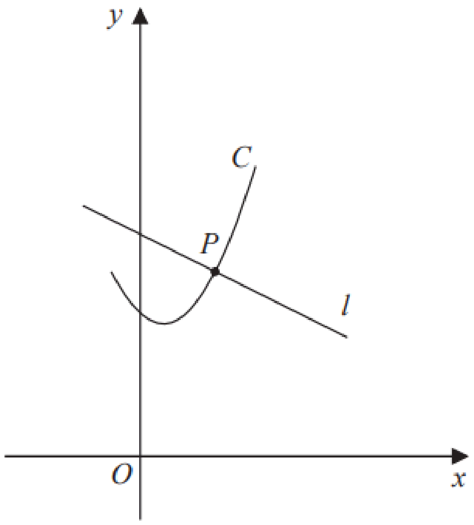

Figure 6

Figure 6 shows a sketch of the curve C with parametric equations

$$x = 2\tan t + 1 \qquad y = 2\sec^2 t + 3 \qquad -\frac{\pi}{4} \leqslant t \leqslant \frac{\pi}{3}$$

The line l is the normal to C at the point P where $t = \frac{\pi}{4}$

(a) Using parametric differentiation, show that an equation for l is

$$y = -\frac{1}{2}x + \frac{17}{2}$$

(5)

(b) Show that all points on C satisfy the equation

$$y = \frac{1}{2}(x-1)^2 + 5$$

(2)

The straight line with equation

$$y = -\frac{1}{2}x + k \qquad \text{where } k \text{ is a constant}$$

intersects C at two distinct points.

(c) Find the range of possible values for k.

(5)

ANSWER

Question	Scheme	Marks	AOs
16(a)	Attempts $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4\sec^2 t\tan t}{2\sec^2 t} (= 2\tan t)$	M1 A1	1.1b 1.1b
	At $t = \frac{\pi}{4}$, $\frac{dy}{dx} = 2, x = 3, y = 7$	M1	2.1
	Attempts equation of normal $y - 7 = -\frac{1}{2}(x-3)$	M1	1.1b
	$y = -\frac{1}{2}x + \frac{17}{2}$ *	A1*	2.1
		(5)	
(b)	Attempts to use $\sec^2 t = 1 + \tan^2 t \Rightarrow \frac{y-3}{2} = 1 + \left(\frac{x-1}{2}\right)^2$	M1	3.1a
	$\Rightarrow y - 3 = 2 + \frac{(x-1)^2}{2} \Rightarrow y = \frac{1}{2}(x-1)^2 + 5$ *	A1*	2.1
		(2)	

(c)	**Attempts the lower limit for *k*:** $\frac{1}{2}(x-1)^2+5=-\frac{1}{2}x+k \Rightarrow x^2-x+(11-2k)=0$ $b^2-4ac=1-4(11-2k)=0 \Rightarrow k=\ldots$	M1	2.1
	$(k=)\frac{43}{8}$	A1	1.1b
	Attempts the upper limit for *k*: $(x,y)_{t=-\frac{\pi}{4}}: t=-\frac{\pi}{4} \Rightarrow x=2\tan\left(-\frac{\pi}{4}\right)+1=-1,\ y=2\sec^2\left(-\frac{\pi}{4}\right)+3=7$ $(-1,\ 7),\ y=-\frac{1}{2}x+k \Rightarrow 7=\frac{1}{2}+k \Rightarrow k=\ldots$	M1	2.1
	$(k=)\frac{13}{2}$	A1	1.1b
	$\frac{43}{8}<k\leqslant\frac{13}{2}$	A1	2.2a
		(5)	
			(12 marks)

June 2022 Question 12 Paper 2

12. The function f is defined by

$$f(x) = \frac{e^{3x}}{4x^2 + k}$$

where k is a positive constant.

(a) Show that

$$f'(x) = (12x^2 - 8x + 3k)\,g(x)$$

where $g(x)$ is a function to be found.

(3)

Given that the curve with equation $y = f(x)$ has at least one stationary point,

(b) find the range of possible values of k.

(3)

ANSWER

Question	Scheme	Marks	AOs
12(a)	$f(x) = \dfrac{e^{3x}}{4x^2+k} \Rightarrow f'(x) = \dfrac{\left(4x^2+k\right)3e^{3x} - 8xe^{3x}}{\left(4x^2+k\right)^2}$ or $f(x) = e^{3x}\left(4x^2+k\right)^{-1} \Rightarrow f'(x) = 3e^{3x}\left(4x^2+k\right)^{-1} - 8xe^{3x}\left(4x^2+k\right)^{-2}$	M1 A1	1.1b 1.1b
	$f'(x) = \dfrac{\left(12x^2 - 8x + 3k\right)e^{3x}}{\left(4x^2+k\right)^2}$	A1	2.1
		(3)	
(b)	If $y = f(x)$ has at least one stationary point then $12x^2 - 8x + 3k = 0$ has at least one root	B1	2.2a
	Applies $b^2 - 4ac\,(\geqslant)\,0$ with $a = 12, b = -8, c = 3k$	M1	2.1
	$0 < k \leqslant \dfrac{4}{9}$	A1	1.1b
		(3)	
			(6 marks)

Video solution:

https://youtu.be/IKpbO3FbB1I

November 2021 Question 14 Paper 1

14. Given that

$$y = \frac{x-4}{2+\sqrt{x}} \qquad x > 0$$

show that

$$\frac{dy}{dx} = \frac{1}{A\sqrt{x}} \qquad x > 0$$

where A is a constant to be found.

(4)

ANSWER

Question	Scheme	Marks	AOs
14	$y=\frac{x-4}{2+\sqrt{x}} \Rightarrow \frac{dy}{dx}=\frac{2+\sqrt{x}-(x-4)\frac{1}{2}x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}$	M1 A1	2.1 1.1b
	$=\frac{2+\sqrt{x}-(x-4)\frac{1}{2}x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}=\frac{2+\sqrt{x}-\frac{1}{2}\sqrt{x}+2x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}=\frac{2\sqrt{x}+\frac{1}{2}x+2}{\sqrt{x}\left(2+\sqrt{x}\right)^2}$	M1	1.1b
	$=\frac{x+4\sqrt{x}+4}{2\sqrt{x}\left(2+\sqrt{x}\right)^2}=\frac{\left(2+\sqrt{x}\right)^2}{2\sqrt{x}\left(2+\sqrt{x}\right)^2}=\frac{1}{2\sqrt{x}}$	A1	2.1
		(4)	
			(4 marks)

November 2021 Question 8 Paper 2

8. The curve C has equation

$$px^3 + qxy + 3y^2 = 26$$

where p and q are constants.

(a) Show that

$$\frac{dy}{dx} = \frac{apx^2 + bqy}{qx + cy}$$

where a, b and c are integers to be found.

(4)

Given that

- the point $P(-1, -4)$ lies on C
- the normal to C at P has equation $19x + 26y + 123 = 0$

(b) find the value of p and the value of q.

(5)

ANSWER

Question	Scheme	Marks	AOs
8(a)	$\frac{\mathrm{d}}{\mathrm{d}x}(3y^2)=6y\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}}{\mathrm{d}x}(qxy)=qx\frac{\mathrm{d}y}{\mathrm{d}x}+qy$	M1	2.1
	$3px^2+qx\frac{\mathrm{d}y}{\mathrm{d}x}+qy+6y\frac{\mathrm{d}y}{\mathrm{d}x}=0$	A1	1.1b
	$(qx+6y)\frac{\mathrm{d}y}{\mathrm{d}x}=-3px^2-qy \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\ldots$	dM1	2.1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-3px^2-qy}{qx+6y}$	A1	1.1b
		(4)	
(b)	$p(-1)^3+q(-1)(-4)+3(-4)^2=26$	M1	1.1b
	$19x+26y+123=0 \Rightarrow m=-\frac{19}{26}$	B1	2.2a
	$\frac{-3p(-1)^2-q(-4)}{q(-1)+6(-4)}=\frac{26}{19}$ or $\frac{q(-1)+6(-4)}{3p(-1)^2+q(-4)}=-\frac{19}{26}$	M1	3.1a
	$p-4q=22,\ 57p-102q=624 \Rightarrow p=\ldots, q=\ldots$	dM1	1.1b
	$p=2,\ q=-5$	A1	1.1b
		(5)	
		(9 marks)	

November 2021 Question 13 Paper 2

13. The curve C has parametric equations

$$x = \sin 2\theta \qquad y = \operatorname{cosec}^3\theta \qquad 0 < \theta < \frac{\pi}{2}$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of θ

(3)

(b) Hence find the exact value of the gradient of the tangent to C at the point where $y = 8$

(3)

ANSWER

Question	Scheme	Marks	AOs
13(a)	$y = \operatorname{cosec}^3\theta \Rightarrow \frac{dy}{d\theta} = -3\operatorname{cosec}^2\theta\operatorname{cosec}\theta\cot\theta$	B1	1.1b
	$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	M1	1.1b
	$\frac{dy}{dx} = \frac{-3\operatorname{cosec}^3\theta\cot\theta}{2\cos 2\theta}$	A1	1.1b
		(3)	
(b)	$y = 8 \Rightarrow \operatorname{cosec}^3\theta = 8 \Rightarrow \sin^3\theta = \frac{1}{8} \Rightarrow \sin\theta = \frac{1}{2}$	M1	3.1a
	$\theta = \frac{\pi}{6} \Rightarrow \frac{dy}{dx} = \frac{-3\operatorname{cosec}^3\left(\frac{\pi}{6}\right)\cot\left(\frac{\pi}{6}\right)}{2\cos\left(\frac{2\pi}{6}\right)} = \ldots$ **or** $\sin\theta = \frac{1}{2} \Rightarrow \frac{dy}{dx} = \frac{\frac{-3}{\sin^3\theta} \times \frac{\cos\theta}{\sin\theta}}{2\left(1-2\sin^2\theta\right)} = \frac{-3\times 8\times\frac{\sqrt{3}/2}{1/2}}{2\left(1-2\times\frac{1}{4}\right)}$	M1	2.1
	$= -24\sqrt{3}$	A1	2.2a
		(3)	
		(6 marks)	

November 2020 Question 9 Paper 1

9.

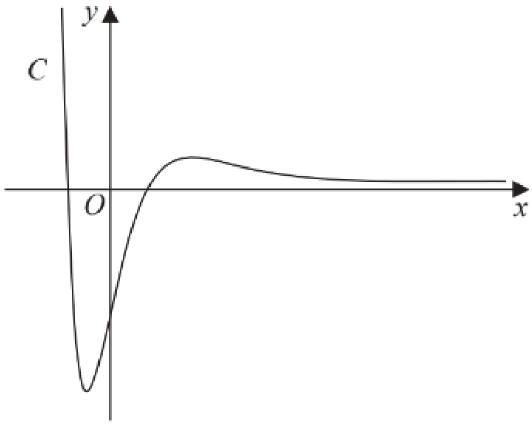

Figure 2

Figure 2 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 4(x^2 - 2)\mathrm{e}^{-2x} \qquad x \in \mathbb{R}$$

(a) Show that $\mathrm{f}'(x) = 8(2 + x - x^2)\mathrm{e}^{-2x}$

(3)

(b) Hence find, in simplest form, the exact coordinates of the stationary points of C.

(3)

The function g and the function h are defined by

$$\mathrm{g}(x) = 2\mathrm{f}(x) \qquad x \in \mathbb{R}$$

$$\mathrm{h}(x) = 2\mathrm{f}(x) - 3 \qquad x \geqslant 0$$

(c) Find (i) the range of g

(ii) the range of h

(3)

ANSWER

Question	Scheme	Marks	AOs
9(a)	$\mathrm{f}(x)=4\left(x^2-2\right)\mathrm{e}^{-2x}$		
	Differentiates to $\mathrm{e}^{-2x}\times 8x+4\left(x^2-2\right)\times -2\mathrm{e}^{-2x}$	M1 A1	1.1b 1.1b
	$\mathrm{f}'(x)=8\mathrm{e}^{-2x}\left\{x-\left(x^2-2\right)\right\}=8\left(2+x-x^2\right)\mathrm{e}^{-2x}$ *	A1*	2.1
		(3)	
(b)	States roots of $\mathrm{f}'(x)=0$ $x=-1,2$	B1	1.1b
	Substitutes one x value to find a y value	M1	1.1b
	Stationary points are $\left(-1,-4\mathrm{e}^2\right)$ and $\left(2,8\mathrm{e}^{-4}\right)$	A1	1.1b
		(3)	
(c)	(i) Range $\left[-8\mathrm{e}^2,\infty\right)$ o.e. such as $\mathrm{g}(x)\geqslant -8\mathrm{e}^2$	B1ft	2.5
	(ii) For • Either attempting to find $2\mathrm{f}(0)-3=2\times -8-3=(-19)$ and identifying this as the **lower bound** • Or attempting to find $2\times"8\mathrm{e}^{-4}"-3$ and identifying this as the **upper bound**	M1	3.1a
	Range $\left[-19,16\mathrm{e}^{-4}-3\right]$	A1	1.1b
		(3)	

November 2020 Question 15 Paper 1

15. The curve C has equation

$$x^2 \tan y = 9 \qquad 0 < y < \frac{\pi}{2}$$

(a) Show that

$$\frac{dy}{dx} = \frac{-18x}{x^4 + 81}$$

(4)

(b) Prove that C has a point of inflection at $x = \sqrt[4]{27}$

(3)

ANSWER

Question	Scheme	Marks	AOs
15 (a)	$x^2 \tan y = 9 \Rightarrow 2x \tan y + x^2 \sec^2 y \dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$	M1 A1	3.1a 1.1b
	Full method to get $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x using $\sec^2 y = 1 + \tan^2 y = 1 + \mathrm{f}(x)$	M1	1.1b
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-2x \times \dfrac{9}{x^2}}{x^2\left(1+\dfrac{81}{x^4}\right)} = \dfrac{-18x}{x^4+81}$ *	A1*	2.1
		(4)	
(b)	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-18x}{x^4+81}$ $\Rightarrow \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \dfrac{-18\times\left(x^4+81\right)-(-18x)\left(4x^3\right)}{\left(x^4+81\right)^2} = \dfrac{54\left(x^4-27\right)}{\left(x^4+81\right)^2}$ o.e.	M1 A1	1.1b 1.1b
	States that when $x < \sqrt[4]{27} \Rightarrow \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} < 0$ when $x = \sqrt[4]{27} \Rightarrow \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$ **AND** when $x > \sqrt[4]{27} \Rightarrow \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} > 0$ giving a point of inflection when $x = \sqrt[4]{27}$	A1	2.4
		(3)	
			(7 marks)

November 2020 Question 13 Paper 2

13. The function g is defined by

$$g(x) = \frac{3\ln(x) - 7}{\ln(x) - 2} \qquad x > 0 \qquad x \neq k$$

where k is a constant.

(a) Deduce the value of k.

(1)

(b) Prove that

$$g'(x) > 0$$

for all values of x in the domain of g.

(3)

(c) Find the range of values of a for which

$$g(a) > 0$$

(2)

ANSWER

Question	Scheme	Marks	AOs
13(a)	$k=\mathrm{e}^2$ **or** $x \neq \mathrm{e}^2$	B1	2.2a
		(1)	
(b)	$$\mathrm{g}'(x)=\frac{(\ln x-2)\times\frac{3}{x}-(3\ln x-7)\times\frac{1}{x}}{(\ln x-2)^2}=\frac{1}{x(\ln x-2)^2}$$ or $$\mathrm{g}'(x)=\frac{\mathrm{d}}{\mathrm{d}x}\left(3-(\ln(x)-2)^{-1}\right)=(\ln x-2)^{-2}\times\frac{1}{x}=\frac{1}{x(\ln x-2)^2}$$ or $$\mathrm{g}'(x)=(\ln x-2)^{-1}\times\frac{3}{x}-(3\ln x-7)(\ln x-2)^{-2}\times\frac{1}{x}=\frac{1}{x(\ln x-2)^2}$$	M1 A1	1.1b 2.1
	As $x>0$ (or $1/x>0$) **AND** $\ln x-2$ is squared so $\mathrm{g}'(x)>0$	A1cso	2.4
		(3)	
(c)	Attempts to solve either $3\ln x-7\ \ldots\ 0$ or $\ln x-2\ \ldots\ 0$ or $3\ln a-7\ \ldots\ 0$ or $\ln a-2\ \ldots\ 0$ where … is "=" or ">" to reach a value for x or a but may be seen as an inequality e.g. $x>\ldots$ or $a>\ldots$	M1	3.1a
	$0<a<\mathrm{e}^2,\quad a>\mathrm{e}^{\frac{7}{3}}$	A1	2.2a
		(2)	
			(6 marks)

June 2019 Question 3 Paper 1

3. $$y = \frac{5x^2 + 10x}{(x+1)^2} \qquad x \neq -1$$

(a) Show that $\frac{dy}{dx} = \frac{A}{(x+1)^n}$ where A and n are constants to be found. **(4)**

(b) Hence deduce the range of values for x for which $\frac{dy}{dx} < 0$ **(1)**

ANSWER

Question	Scheme	Marks	AOs
3 (a)	Correct method used in attempting to differentiate $y = \frac{5x^2+10x}{(x+1)^2}$	M1	3.1a
	$\frac{dy}{dx} = \frac{(x+1)^2 \times (10x+10) - (5x^2+10x) \times 2(x+1)}{(x+1)^4}$ oe	A1	1.1b
	Factorises/Cancels term in $(x+1)$ and attempts to simplify $\frac{dy}{dx} = \frac{(x+1)\times(10x+10)-(5x^2+10x)\times 2}{(x+1)^3} = \frac{A}{(x+1)^3}$	M1	2.1
	$\frac{dy}{dx} = \frac{10}{(x+1)^3}$	A1	1.1b
		(4)	
(b)	For $x < -1$ Follow through on their $\frac{dy}{dx} = \frac{A}{(x+1)^n}, n = 1, 3$	B1ft	2.2a
		(1)	
		(5 marks)	

June 2019 Question 14 Paper 1

14. The curve C, in the standard Cartesian plane, is defined by the equation

$$x = 4\sin 2y \qquad \frac{-\pi}{4} < y < \frac{\pi}{4}$$

The curve C passes through the origin O

(a) Find the value of $\frac{dy}{dx}$ at the origin.

(2)

(b) (i) Use the small angle approximation for $\sin 2y$ to find an equation linking x and y for points close to the origin.

(ii) Explain the relationship between the answers to (a) and (b)(i).

(2)

(c) Show that, for all points (x, y) lying on C,

$$\frac{dy}{dx} = \frac{1}{a\sqrt{b - x^2}}$$

where a and b are constants to be found.

(3)

ANSWER

Question	Scheme	Marks	AOs
14 (a)	Attempts to differentiate $x = 4\sin 2y$ and inverts $\frac{dx}{dy} = 8\cos 2y \Rightarrow \frac{dy}{dx} = \frac{1}{8\cos 2y}$	M1	1.1b
	At (0,0) $\frac{dy}{dx} = \frac{1}{8}$	A1	1.1b
		(2)	
(b)	(i) Uses $\sin 2y \approx 2y$ when y is small to obtain $x \approx 8y$	B1	1.1b
	(ii) The value found in (a) is the gradient of the line found in (b)(i)	B1	2.4
		(2)	
(c)	Uses their $\frac{dy}{dx}$ as a function of y and, using both $\sin^2 2y + \cos^2 2y = 1$ and $x = 4\sin 2y$ in an attempt to write $\frac{dy}{dx}$ or $\frac{dx}{dy}$ as a function of x Allow for $\frac{dy}{dx} = k\frac{1}{\cos 2y} = ..\frac{1}{\sqrt{1-(..x)^2}}$	M1	2.1
	A correct answer $\frac{dy}{dx} = \frac{1}{8\sqrt{1-\left(\frac{x}{4}\right)^2}}$ or $\frac{dx}{dy} = 8\sqrt{1-\left(\frac{x}{4}\right)^2}$	A1	1.1b
	and in the correct form $\frac{dy}{dx} = \frac{1}{2\sqrt{16-x^2}}$	A1	1.1b
		(3)	
			(7 marks)

June 2018 Question 5 Paper 1

5. Given that

$$y = \frac{3\sin\theta}{2\sin\theta + 2\cos\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

show that

$$\frac{dy}{d\theta} = \frac{A}{1 + \sin 2\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

where A is a rational constant to be found.

(5)

ANSWER

Question	Scheme	Marks	AOs
5	$\frac{dy}{d\theta} = \frac{(2\sin\theta + 2\cos\theta)3\cos\theta - 3\sin\theta(2\cos\theta - 2\sin\theta)}{(2\sin\theta + 2\cos\theta)^2}$	M1 A1	1.1b 1.1b
	Expands and uses $\sin^2\theta + \cos^2\theta = 1$ at least once in the numerator or the denominator or uses $2\sin\theta\cos\theta = \sin 2\theta$ in $\Rightarrow \frac{dy}{d\theta} = \frac{\ldots}{\ldots\ldots\ldots C\sin\theta\cos\theta}$	M1	3.1a
	Expands and uses $\sin^2\theta + \cos^2\theta = 1$ the numerator and the denominator AND uses $2\sin\theta\cos\theta = \sin 2\theta$ in $\Rightarrow \frac{dy}{d\theta} = \frac{P}{Q + R\sin 2\theta}$	M1	2.1
	$\Rightarrow \frac{dy}{d\theta} = \frac{3}{2 + 2\sin 2\theta} = \frac{3/2}{1 + \sin 2\theta}$	A1	1.1b
			(5 marks)

June 2018 Question 9 Paper 1

9.

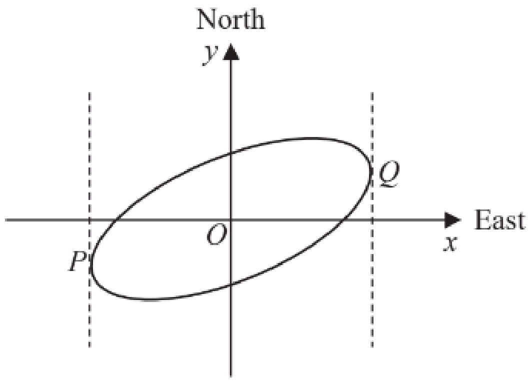

Figure 4

Figure 4 shows a sketch of the curve with equation $x^2 - 2xy + 3y^2 = 50$

(a) Show that $\dfrac{dy}{dx} = \dfrac{y - x}{3y - x}$

(4)

The curve is used to model the shape of a cycle track with both x and y measured in km.

The points P and Q represent points that are furthest west and furthest east of the origin O, as shown in Figure 4.

Using part (a),

(b) find the exact coordinates of the point P.

(5)

(c) Explain briefly how to find the coordinates of the point that is furthest north of the origin O. (You **do not** need to carry out this calculation).

(1)

ANSWER

Question	Scheme	Marks	AOs
9(a)	Either $3y^2 \to Ay\dfrac{dy}{dx}$ or $2xy \to 2x\dfrac{dy}{dx} + 2y$	M1	2.1
	$2x - 2x\dfrac{dy}{dx} - 2y + 6y\dfrac{dy}{dx} = 0$	A1	1.1b
	$(6y - 2x)\dfrac{dy}{dx} = 2y - 2x$	M1	2.1
	$\dfrac{dy}{dx} = \dfrac{2y-2x}{6y-2x} = \dfrac{y-x}{3y-x}$ *	A1*	1.1b
		(4)	
(b)	$\left(\text{At } P \text{ and } Q \dfrac{dy}{dx} \to \infty \Rightarrow\right)$ Deduces that $3y - x = 0$	M1	2.2a
	Solves $y = \dfrac{1}{3}x$ and $x^2 - 2xy + 3y^2 = 50$ simultaneously	M1	3.1a
	$\Rightarrow x = (\pm)5\sqrt{3}$ OR $\Rightarrow y = (\pm)\dfrac{5}{3}\sqrt{3}$	A1	1.1b
	Using $y = \dfrac{1}{3}x \Rightarrow x = ..$ AND $y = ..$	dM1	1.1b
	$P = \left(-5\sqrt{3}, -\dfrac{5}{3}\sqrt{3}\right)$	A1	2.2a
		(5)	
(c)	Explains that you need to solve $y = x$ and $x^2 - 2xy + 3y^2 = 50$ simultaneously and choose the positive solution	B1ft	2.4
		(1)	
			(10 marks)

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9. Given that θ is measured in radians, prove, from first principles, that

$$\frac{d}{d\theta}(\cos\theta) = -\sin\theta$$

You may assume the formula for $\cos(A \pm B)$ and that as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

(5)

ANSWER

Question	Scheme	Marks	AOs
9	$\frac{d}{d\theta}(\cos\theta) = -\sin\theta$; as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$		
	$\frac{\cos(\theta + h) - \cos\theta}{h}$	B1	2.1
	$= \frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}$	M1 A1	1.1b 1.1b
	$= -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta$		
	As $h \to 0$, $-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta \to -1\sin\theta + 0\cos\theta$	dM1	2.1
	so $\frac{d}{d\theta}(\cos\theta) = -\sin\theta$ *	A1*	2.5
		(5)	
			(5 marks)