Edexcel - Pure Year 2 exam questions

Arancha Ruiz

Ch. 8: Parametric Equations

June 2023 Question 9 Paper 2

9. The curve C has parametric equations

$$x = t^2 + 6t - 16 \qquad y = 6\ln(t+3) \qquad t > -3$$

(a) Show that a Cartesian equation for C is

$$y = A\ln(x+B) \qquad x > -B$$

where A and B are integers to be found.

(3)

The curve C cuts the y-axis at the point P

(b) Show that the equation of the tangent to C at P can be written in the form

$$ax + by = c\ln 5$$

where a, b and c are integers to be found.

(4)

ANSWER

Question	Scheme	Marks	AOs
9(a) **Way 1**	$x=(t+3)^2-25$	**M1**	1.1b
	$\Rightarrow x+25=(t+3)^2 \Rightarrow (x+25)^{\frac{1}{2}}=(t+3) \Rightarrow y=\ldots$	**M1**	2.1
	$y=6\ln(x+25)^{\frac{1}{2}} \Rightarrow y=3\ln(x+25)$	**A1cso**	1.1b
		(3)	
(b)	$x=0,\ y=3\ln 25$ oe e.g. $6\ln 5$	**B1ft**	2.2a
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{x+"25"} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{0+"25"}\ \left(=\frac{3}{25}\right)$ **or** $\frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}=\frac{\left(\frac{6}{t+3}\right)}{2t+6} \Rightarrow \frac{\frac{6}{2+3}}{2\times 2+6}\ \left(=\frac{6}{50}=\frac{3}{25}\right)$	**M1**	2.1
	$y-"3\ln 25"="\frac{3}{25}"(x\{-0\})$	**dM1**	3.1a
	$25y-3x=150\ln 5$	**A1**	2.2a
		(4)	
		(7 marks)	

Notes

Choose the mark scheme that best matches their chosen method.

November 2021 Question 13 Paper 1

13. A curve C has parametric equations

$$x = \frac{t^2+5}{t^2+1} \qquad y = \frac{4t}{t^2+1} \qquad t \in \mathbb{R}$$

Show that all points on C satisfy

$$(x-3)^2 + y^2 = 4$$

(3)

ANSWER

Question	Scheme	Marks	AOs
13	$(x-3)^2 + y^2 = \left(\frac{t^2+5}{t^2+1} - 3\right)^2 + \left(\frac{4t}{t^2+1}\right)^2$	M1	3.1a
	$= \frac{(2-2t^2)^2 + 16t^2}{(t^2+1)^2} = \frac{4+8t^2+4t^4}{(t^2+1)^2}$	dM1	1.1b
	$\frac{4(t^4+2t^2+1)}{(t^2+1)^2} = \frac{4(t^2+1)^2}{(t^2+1)^2} = 4^*$	A1*	2.1
		(3)	

June 2019 Question 4 Paper 2

4.

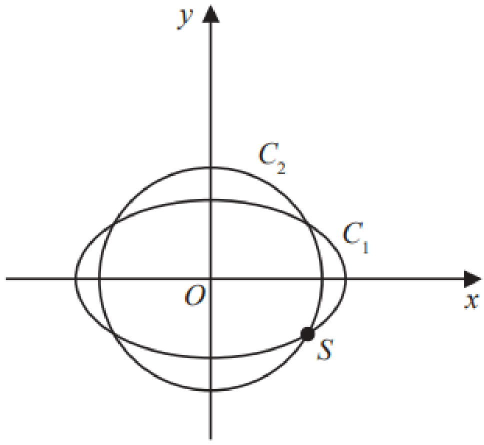

Figure 2

The curve C_1 with parametric equations

$$x = 10\cos t, \quad y = 4\sqrt{2}\sin t, \quad 0 \leqslant t < 2\pi$$

meets the circle C_2 with equation

$$x^2 + y^2 = 66$$

at four distinct points as shown in Figure 2.

Given that one of these points, S, lies in the 4th quadrant, find the Cartesian coordinates of S.

(6)

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ANSWER

<table>
<tr><th>Question</th><th colspan="2">Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>4</td><td colspan="2">$C_1: x = 10\cos t,\ y = 4\sqrt{2}\sin t,\ \ 0 \le t < 2\pi;\ \ C_2: x^2 + y^2 = 66$</td><td></td><td></td></tr>
<tr><td rowspan="7">Way 1</td><td colspan="2">$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$</td><td>M1</td><td>3.1a</td></tr>
<tr><td rowspan="2">$100(1-\sin^2 t) + 32\sin^2 t = 66$</td><td rowspan="2">$100\cos^2 t + 32(1-\cos^2 t) = 66$</td><td>M1</td><td>2.1</td></tr>
<tr><td>A1</td><td>1.1b</td></tr>
<tr><td>$100 - 68\sin^2 t = 66 \Rightarrow \sin^2 t = \frac{1}{2}$
$\Rightarrow \sin t = \ldots$</td><td>$68\cos^2 t + 32 = 66 \Rightarrow \cos^2 t = \frac{1}{2}$
$\Rightarrow \cos t = \ldots$</td><td>dM1</td><td>1.1b</td></tr>
<tr><td colspan="2">Substitutes their solution back into the relevant original equation(s) to get the value of the x-coordinate and value of the corresponding y-coordinate.
Note: These may not be in the correct quadrant</td><td>M1</td><td>1.1b</td></tr>
<tr><td colspan="2">$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$</td><td>A1</td><td>3.2a</td></tr>
<tr><td colspan="2"></td><td>(6)</td><td></td></tr>
<tr><td rowspan="7">Way 2</td><td colspan="2">$\{\cos^2 t + \sin^2 t = 1 \Rightarrow\}\ \left(\frac{x}{10}\right)^2 + \left(\frac{y}{4\sqrt{2}}\right)^2 = 1\ \{\Rightarrow 32x^2 + 100y^2 = 3200\}$</td><td>M1</td><td>3.1a</td></tr>
<tr><td rowspan="2">$\frac{x^2}{100} + \frac{66 - x^2}{32} = 1$</td><td rowspan="2">$\frac{66 - y^2}{100} + \frac{y^2}{32} = 1$</td><td>M1</td><td>2.1</td></tr>
<tr><td>A1</td><td>1.1b</td></tr>
<tr><td>$32x^2 + 6600 - 100x^2 = 3200$
$x^2 = 50 \Rightarrow x = \ldots$</td><td>$2112 - 32y^2 + 100y^2 = 3200$
$y^2 = 16 \Rightarrow y = \ldots$</td><td>dM1</td><td>1.1b</td></tr>
<tr><td colspan="2">Substitutes their solution back into the relevant original equation(s) to get the value of the corresponding x-coordinate or y-coordinate.
Note: These may not be in the correct quadrant</td><td>M1</td><td>1.1b</td></tr>
<tr><td colspan="2">$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$</td><td>A1</td><td>3.2a</td></tr>
<tr><td colspan="2"></td><td>(6)</td><td></td></tr>
</table>

June 2018 Question 14 Paper 1

14. A curve C has parametric equations

$$x = 3 + 2\sin t, \qquad y = 4 + 2\cos 2t, \qquad 0 \leqslant t < 2\pi$$

(a) Show that all points on C satisfy $y = 6 - (x-3)^2$

(2)

(b) (i) Sketch the curve C.

(ii) Explain briefly why C does not include all points of $y = 6 - (x-3)^2, \quad x \in \mathbb{R}$

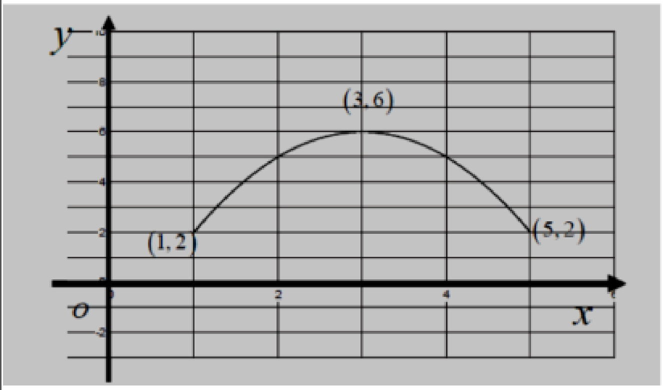
(3)

The line with equation $x + y = k$, where k is a constant, intersects C at two distinct points.

(c) State the range of values of k, writing your answer in set notation.

(5)

ANSWER

Question	Scheme	Marks	AOs
14(a)	Attempts to use $\cos 2t = 1 - 2\sin^2 t \Rightarrow \dfrac{y-4}{2} = 1 - 2\left(\dfrac{x-3}{2}\right)^2$	M1	2.1
	$\Rightarrow y - 4 = 2 - 4 \times \dfrac{(x-3)^2}{4} \Rightarrow y = 6 - (x-3)^2$ *	A1*	1.1b
		(2)	
(b)	 $\cap$ shaped parabola	M1	1.1b
	Fully correct with 'ends' at (1,2) & (5,2)	A1	1.1b
	Suitable reason : Eg states as $x = 3 + 2\sin t$, $1 \leqslant x \leqslant 5$	B1	2.4
		(3)	
(c)	Either finds the lower value for $k = 7$ or deduces that $k < \dfrac{37}{4}$	B1	2.2a

	Finds where $x+y=k$ meets $y=6-(x-3)^2$ $\Rightarrow k-x=6-(x-3)^2$ and proceeds to 3TQ in x or y	M1	3.1a
	Correct 3TQ in x $\quad x^2-7x+(k+3)=0$ Or y $\quad y^2+(7-2k)y+(k^2-6k+3)=0$	A1	1.1b
	Uses $b^2-4ac=0 \Rightarrow \quad 49-4\times1\times(k+3)=0 \Rightarrow k=\left(\frac{37}{4}\right)$ or $(7-2k)^2-4\times1\times(k^2-6k+3)=0 \Rightarrow k=\left(\frac{37}{4}\right)$	M1	2.1
	Range of values for $k=\left\{k:7\leqslant k<\frac{37}{4}\right\}$	A1	2.5
		(5)	
			(10 marks)

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