Statistics Year 1 (AS) exam questions

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Ch. 7: Hypothesis Testing

June 2023 Question 4

4. Past information shows that 25% of adults in a large population have a particular allergy.

Rylan believes that the proportion that has the allergy differs from 25%

He takes a random sample of 50 adults from the population.

Rylan carries out a test of the null hypothesis H_0: $p = 0.25$ using a 5% level of significance.

(a) Write down the alternative hypothesis for Rylan's test.

(1)

(b) Find the critical region for this test.
You should state the probability associated with each tail, which should be as close to 2.5% as possible.

(4)

(c) State the actual probability of incorrectly rejecting H_0 for this test.

(1)

Rylan finds that 10 of the adults in his sample have the allergy.

(d) State the conclusion of Rylan's hypothesis test.

(1)

ANSWER

Que.	Scheme	Marks	AOs
4(a)	[H_1 :] $p \neq 0.25$	B1	2.5
		(1)	
(b)	X~B(50, 0.25)	B1	3.3
	[P($X \leqslant 6$) =]0.0194 <u>or</u> [P($X \leqslant 18$) =]0.9713 <u>or</u> [P($X \geqslant 19$) =]0.0287 <u>or</u> $X \leqslant 6$ <u>or</u> $X \geqslant 19$	M1	3.4
	[P($X \leqslant 6$) =]awrt 0.0194 and [P($X \geqslant 19$) =]awrt 0.0287	A1	1.1b
	CR: $X \leqslant 6$ or $X \geqslant 19$	A1	1.1b
		(4)	
(c)	[0.0194 + 0.0287 =] awrt 0.048	B1ft	1.1b
		(1)	
(d)	(Do not reject H_0,) there is insufficient evidence to suggest that the **proportion** of those with the **allergy** differs from 25%/**Rylan's belief** not supported	B1	2.2b
		(1)	
			(7 marks)

June 2023 Question 2 (A-Level)

2. A machine fills packets with sweets and $\frac{1}{7}$ of the packets also contain a prize.

The packets of sweets are placed in boxes before being delivered to shops.
There are 40 packets of sweets in each box.

The random variable T represents the number of packets of sweets that contain a prize in each box.

(a) State a condition needed for T to be modelled by $\mathrm{B}(40, \frac{1}{7})$

(1)

A box is selected at random.

(b) Using $T \sim \mathrm{B}(40, \frac{1}{7})$ find

(i) the probability that the box has exactly 6 packets containing a prize,

(ii) the probability that the box has fewer than 3 packets containing a prize.

(2)

Kamil's sweet shop buys 5 boxes of these sweets.

(c) Find the probability that exactly 2 of these 5 boxes have fewer than 3 packets containing a prize.

(2)

Kamil claims that the proportion of packets containing a prize is less than $\frac{1}{7}$

A random sample of 110 packets is taken and 9 packets contain a prize.

(d) Use a suitable test to assess Kamil's claim.
You should

- state your hypotheses clearly
- use a 5% level of significance

(4)

ANSWER

Qu 2	Scheme	Marks	AO
(a)	Comment in context about either **independence** or **random** packing e.g. "prizes must be placed in packets at random/independently of each other" **or** about **constant probability** e.g. "the probability of a packet containing a prize is constant/ the same/fixed"	B1	3.5b
		(1)	
(b)(i)	[$\mathrm{P}(T=6)=$] 0.17273… awrt **0.173**	B1	1.1b
(ii)	[$\mathrm{P}(T<3)=\mathrm{P}(T \leqslant 2)=$] 0.061587… awrt **0.0616**	B1	1.1b
		(2)	
(c)	[K= no. of boxes with fewer than 3 packets containing a prize]		
	$K \sim \mathrm{B}(5, \text{"0.0616"})$	M1	1.1b
	$\mathrm{P}(K=2)=0.031344\ldots$ in the range **[0.0313~0.0314]**	A1	1.1b
		(2)	
(d)	$\mathrm{H}_0 : p=\frac{1}{7} \quad \mathrm{H}_1 : p<\frac{1}{7}$	B1	2.5
	[X = no of packets containing a prize] $X \sim \mathrm{B}(110, \frac{1}{7})$	M1	3.3
	[$\mathrm{P}(X \leqslant 9)$] = 0.038292…	A1	3.4
	[Significant result or reject H_0] E.g. there is evidence to support Kamil's claim	A1	2.2b
		(4)	
		(9 marks)	

June 2022 Question 4 (A-level)

4. A dentist knows from past records that 10% of customers arrive late for their appointment.

A new manager believes that there has been a change in the proportion of customers who arrive late for their appointment.

A random sample of 50 of the dentist's customers is taken.

(a) Write down

- a null hypothesis corresponding to no change in the proportion of customers who arrive late
- an alternative hypothesis corresponding to the manager's belief

(1)

(b) Using a 5% level of significance, find the critical region for a two-tailed test of the null hypothesis in (a)
You should state the probability of rejection in each tail, which should be less than 0.025

(3)

(c) Find the actual level of significance of the test based on your critical region from part (b)

(1)

The manager observes that 15 of the 50 customers arrived late for their appointment.

(d) With reference to part (b), comment on the manager's belief.

(1)

ANSWER

Question	Scheme	Marks	AOs
4(a)	$H_0: p = 0.1 \quad H_1: p \neq 0.1$	B1	2.5
		(1)	
(b)	Use of $X \sim B(50, 0.1)$ implied by sight of one of awrt 0.0052 or awrt 0.9755 or awrt 0.0245	M1	3.4
	Critical regions $X = 0$ or $X \geqslant 10$	A1	1.1b
	$X = 0$ and $X \geqslant 10$ plus $P(X = 0) =$ awrt 0.0052 and $P(X \geqslant 10) =$ awrt 0.0245	A1	1.1b
	SC: Both CR correct with no probabilities and no distribution seen scores M0A1A0		
		(3)	
(c)	0.0297	B1ft	1.1b
		(1)	
(d)	15 is in the critical region therefore there is evidence to support the **manager**'s belief	B1ft	2.2b
		(1)	
			(6 marks)

Video solution:

https://youtu.be/ImAuJLYpWdQ

June 2022 Question 2

2. A manufacturer of sweets knows that 8% of the bags of sugar delivered from supplier *A* will be damp.
A random sample of 35 bags of sugar is taken from supplier *A*.

(a) Using a suitable model, find the probability that the number of bags of sugar that are damp is

(i) exactly 2

(ii) more than 3

(3)

Supplier *B* claims that when it supplies bags of sugar, the proportion of bags that are damp is less than 8%

The manufacturer takes a random sample of 70 bags of sugar from supplier *B* and finds that only 2 of the bags are damp.

(b) Carry out a suitable test to assess supplier *B*'s claim.
You should state your hypotheses clearly and use a 10% level of significance.

(4)

ANSWER

Qu	Scheme	Mark	AO
2. (a)	[D = number of bags that are damp] $D \sim B(35, 0.08)$ NB $0.08 = \frac{2}{25}$	M1	3.3
(i)	$P(D = 2) = 0.2430497\ldots$ awrt **0.243**	A1	3.4
(ii)	$P(D > 3) = [1 - P(D \leqslant 3) = 1 - 0.69397...] = 0.30602\ldots$ awrt **0.306**	A1	1.1b
		(3)	
(b)	$H_0 : p = 0.08 \quad H_1 : p < 0.08$	B1	2.5
	$[X \sim]$ B(70, 0.08)	M1	2.1
	$[P(X \leqslant 2)] = 0.0739756\ldots$ awrt **0.074**	A1	1.1b
	[0.074 < 0.10 so significant, reject H_0 so…]		
	there is evidence to support supplier *B*'s claim (o.e.)	A1	2.2b
		(4)	
		(7 marks)	

Video solution:
https://youtu.be/0UCgR4qAd9k

November 2021 question 4

4. A nursery has a sack containing a large number of coloured beads of which 14% are coloured red.

Aliya takes a random sample of 18 beads from the sack to make a bracelet.

(a) State a suitable binomial distribution to model the number of red beads in Aliya's bracelet.

(1)

(b) Use this binomial distribution to find the probability that

(i) Aliya has just 1 red bead in her bracelet,

(ii) there are at least 4 red beads in Aliya's bracelet.

(3)

(c) Comment on the suitability of a binomial distribution to model this situation.

(1)

After several children have used beads from the sack, the nursery teacher decides to test whether or not the proportion of red beads in the sack has changed.
She takes a random sample of 75 beads and finds 4 red beads.

(d) Stating your hypotheses clearly, use a 5% significance level to carry out a suitable test for the teacher.

(4)

(e) Find the p-value in this case.

(1)

ANSWER

Qu	Scheme	Marks		AO
4. (a)	[R = no. of red beads in Aliya's bracelet] $R \sim B(18, 0.14)$	B1		3.3
		(1)		
(b)(i)	$P(R = 1) = 0.19403\ldots$ awrt **0.194**	B1		1.1b
(ii)	$P(R \geqslant 4) = 1 - P(R \leqslant 3) = 1 - [0.76184\ldots]$	M1		3.4
	$= 0.2381588\ldots$ awrt **<u>0.238</u>**	A1		1.1b
		(3)		
(c)	Requires $p = 0.14$ to be constant so need a large number of beads in the sack to ensure that removing 18 beads does not appreciably affect this probability, then it could be suitable.	B1		3.5b
		(1)		
(d)	$H_0 : p = 0.14 \quad H_1 : p \neq 0.14$	B1		2.5
	[X = number of red beads in the sample] $X \sim B(75, 0.14)$	M1		3.3
	$P(X \leqslant 4) = 0.01506\ldots$ or if $B(75, 0.14)$ seen awrt 0.02	A1		3.4
	$\{0.02 < 0.025$ so significant <u>or</u> reject $H_0\}$ There is evidence that the proportion of red beads has changed	A1		2.2b
		(4)		
(e)	p-value is $2 \times "0.01506..." = 0.030123... =$ awrt 0.03	B1ft		1.1b
		(1)		
		(10 marks)		

October 2020 question 5

5. Afrika works in a call centre.

She assumes that calls are independent and knows, from past experience, that on each sales call that she makes there is a probability of $\frac{1}{6}$ that it is successful.

Afrika makes 9 sales calls.

(a) Calculate the probability that at least 3 of these sales calls will be successful.

(2)

The probability of Afrika making a successful sales call is the same each day.

Afrika makes 9 sales calls on each of 5 different days.

(b) Calculate the probability that at least 3 of the sales calls will be successful on exactly 1 of these days.

(2)

Rowan works in the same call centre as Afrika and believes he is a more successful salesperson.

To check Rowan's belief, Afrika monitors the next 35 sales calls Rowan makes and finds that 11 of the sales calls are successful.

(c) Stating your hypotheses clearly test, at the 5% level of significance, whether or not there is evidence to support Rowan's belief.

(4)

ANSWER

Question	Scheme	Marks	AOs
5(a)	Let C = the number of successful calls. $C \sim B\left(9, \frac{1}{6}\right)$	M1	3.3
	$P(C \geq 3) = 1 - P(C \leq 2) = 0.1782\ldots$ **awrt 0.178**	A1	1.1b
		(2)	
(b)	Let X = the number of occasions when at least 3 calls are successful. $P(X = 1) = 5 \times (\text{"}0.1782...\text{"}) \times (\text{"}0.8217...\text{"})^4$	M1	1.1b
	$= 0.4061\ldots$ **awrt 0.406**	A1	1.1b
		(2)	
(c)	$H_0 : p = \frac{1}{6}$ $H_1 : p > \frac{1}{6}$	B1	2.5
	Let R = the number of successful calls $R \sim B\left(35, \frac{1}{6}\right)$	M1	3.3
	$P(R \geq 11) = 1 - P(R \leq 10) = 0.02...$	A1	3.4
	There is sufficient evidence to support that **Rowan** has more successful sales calls than Afrika.	A1	2.2b
		(4)	
			(8 marks)

Video solution:

https://youtu.be/D5PhpBhxxaQ

June 2019 question 5

5. Past records show that 15% of customers at a shop buy chocolate. The shopkeeper believes that moving the chocolate closer to the till will increase the proportion of customers buying chocolate.

After moving the chocolate closer to the till, a random sample of 30 customers is taken and 8 of them are found to have bought chocolate.

Julie carries out a hypothesis test, at the 5% level of significance, to test the shopkeeper's belief.

Julie's hypothesis test is shown below.

$H_0 : p = 0.15$

$H_1 : p \geqslant 0.15$

Let X = the number of customers who buy chocolate.

$X \sim B(30, 0.15)$

$P(X = 8) = 0.0420$

$0.0420 < 0.05$ so reject H_0

There is sufficient evidence to suggest that the proportion of customers buying chocolate has increased.

(a) Identify the first two errors that Julie has made in her hypothesis test.
(2)

(b) Explain whether or not these errors will affect the conclusion of her hypothesis test. Give a reason for your answer.
(1)

(c) Find, using a 5% level of significance, the critical region for a one-tailed test of the shopkeeper's belief. The probability in the tail should be less than 0.05
(2)

(d) Find the actual level of significance of this test.
(1)

ANSWER

Question	Scheme	Marks	AOs
5(a)	The alternative hypothesis should be $H_1: p > 0.15$	B1	2.5
	The calculation of the test statistic should be $P(X \geq 8)$ [= 0.0698]	B1	2.3
		(2)	
(b)	These will affect the conclusion (as the null hypothesis should not be rejected) since $P(X \geq 8)$ [= 0.0698] is greater than 0.05	B1	2.4
		(1)	
(c)	$P(X \leq 8) = 0.9722\ldots > 0.95$ or $P(X \geq 9) = 0.0277\ldots < 0.05$	M1	2.1
	CR: $\{X \geq 9\}$	A1	1.1b
		(2)	
(d)	awrt **<u>0.0278</u>**	B1ft	1.1b
		(1)	
			(6 marks)

June 2018 Question 3

3. Naasir is playing a game with two friends. The game is designed to be a game of chance so that the probability of Naasir winning each game is $\frac{1}{3}$

Naasir and his friends play the game 15 times.

(a) Find the probability that Naasir wins

(i) exactly 2 games,

(ii) more than 5 games.

(3)

Naasir claims he has a method to help him win more than $\frac{1}{3}$ of the games. To test this claim, the three of them played the game again 32 times and Naasir won 16 of these games.

(b) Stating your hypotheses clearly, test Naasir's claim at the 5% level of significance.

(4)

ANSWER

Qu	Scheme	Marks	AO
3 (a)	Let N = the number of games Naasir wins $N \sim B(15, \frac{1}{3})$	M1	3.3
(i)	$P(N = 2) = 0.059946\ldots$ **awrt 0.0599**	A1	1.1b
(ii)	$P(N > 5) = 1 - P(N \leqslant 5) = 0.38162\ldots$ **awrt 0.382**	A1	1.1b
		(3)	
(b)	$H_0 : p = \frac{1}{3}$ $H_1 : p > \frac{1}{3}$	B1	2.5
	Let X = the number of games Naasir wins $X \sim B(32, \frac{1}{3})$	M1	3.3
	$P(X \geqslant 16) = 1 - P(X \leqslant 15) = 0.03765$ (< 0.05)	A1	3.4
	[Significant result so reject H_0 (the null model) and conclude:] There is evidence to support Naasir's claim (o.e.)	A1	3.5a
		(4)	
		(7 marks)	