Mechanics Year 2 exam questions

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Ch. 7: Applications of Forces

June 2023 Question 6

6.

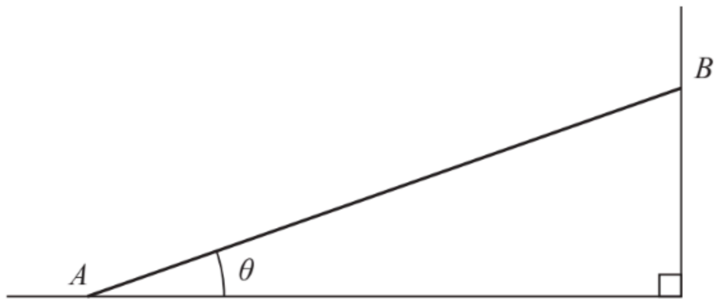


Figure 3

A rod AB has mass M and length $2a$.

The rod has its end A on rough horizontal ground and its end B against a smooth vertical wall.

The rod makes an angle θ with the ground, as shown in Figure 3.

The rod is at rest in limiting equilibrium.

(a) State the direction (left or right on Figure 3 above) of the frictional force acting on the rod at A. **Give a reason for your answer.**

(1)

The magnitude of the normal reaction of the wall on the rod at B is S.

In an initial model, the rod is modelled as being **uniform**.

Use this initial model to answer parts (b), (c) and (d).

(b) By taking moments about A, show that

$$S = \frac{1}{2}Mg\cot\theta$$

(3)

The coefficient of friction between the rod and the ground is μ

Given that $\tan\theta = \frac{3}{4}$

(c) find the value of μ

(5)

(d) find, in terms of M and g, the magnitude of the resultant force acting on the rod at A.

(3)

In a new model, the rod is modelled as being **non-uniform**, with its centre of mass closer to B than it is to A.

A new value for S is calculated using this new model, with $\tan\theta = \frac{3}{4}$

(e) State whether this new value for S is larger, smaller or equal to the value that S would take using the initial model. **Give a reason for your answer.**

(1)

ANSWER

Question	Scheme	Marks	AOs
6(a)	The normal reaction at *B* is acting to the left so it must act to the right, right as it needs to balance (oppose, counter) the force at *B*, right as it prevents the rod from sliding (slipping, falling), right as the weight (mass) of the rod will mean the rod tends to slip left, mass or weight will be pushing the rod to the left so friction will oppose that. **N.B.** You may see an arrow on the diagram at *A*, instead of 'right'. B0 if they say the rod is moving oe Accept towards the wall instead of to the right.	B1	2.4
		(1)	
6(b)	Take moments about *A*	M1	3.4
	$S\times 2a\sin\theta = Mga\cos\theta$	A1	1.1b
	$S=\frac{1}{2}Mg\cot\theta\,*$	A1*	2.2a
		(3)	
6(c)	Resolve vertically, *R* = *Mg*	B1	3.3
	Resolve horizontally, *F* = *S*	B1	3.3
	Other possible equations: Resolve along the rod, $F\cos\theta + R\sin\theta = S\cos\theta + Mg\sin\theta$ Resolve perp to the rod, $R\cos\theta + S\sin\theta = F\sin\theta + Mg\cos\theta$ M(*B*), $R\times 2a\cos\theta = F\times 2a\sin\theta + Mga\cos\theta$ M(*G*), $Ra\cos\theta = Fa\sin\theta + Sa\sin\theta$ **N.B.** When entering these two B marks on ePEN, **First** B1 is for a **vertical** resolution, **second** B1 is for a **horizontal** resolution, and if either is replaced by a different equation, enter appropriately. If both are replaced by other equations, enter in the order in which they appear in their working.		
	$F=\mu R$	B1	1.2

	$\frac{1}{2}Mg \times \frac{4}{3} = \mu Mg$	**dM**1	2.1
	$\mu = \frac{2}{3}$ oe Accept 0.67 or better	A1	2.2a
	S.C. For F „ μR, B0 $\frac{1}{2}Mg \times \frac{4}{3}$ „ μMg M1		
	$\frac{2}{3}$ „ μ A0 **N.B**. If $\mu = \frac{2}{3}$ follows this, they could score all the marks.		
		(5)	
6(d)	$\sqrt{F^2 + R^2}$	M1	3.1a
	$\sqrt{\left(\frac{2}{3}Mg\right)^2 + (Mg)^2}$	M1	1.1b
	$\frac{1}{3}Mg\sqrt{13}$ or 1.2Mg or better	A1	2.2a
		(3)	
6(e)	New value of *S* would be **larger** as the **moment** of the **weight** about ***A*** would be larger	B1	3.5a
		(1)	
			(13 marks)

June 2022 Question 4

4.

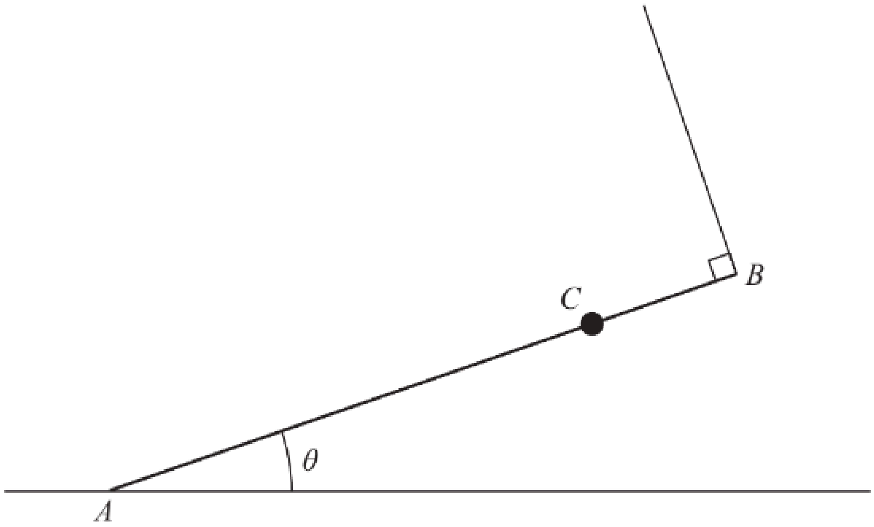


Figure 2

A uniform rod AB has mass M and length $2a$

A particle of mass $2M$ is attached to the rod at the point C, where $AC = 1.5a$

The rod rests with its end A on rough horizontal ground.

The rod is held in equilibrium at an angle θ to the ground by a light string that is attached to the end B of the rod.

The string is perpendicular to the rod, as shown in Figure 2.

(a) Explain why the frictional force acting on the rod at A acts horizontally to the right on the diagram.

(1)

The tension in the string is T

(b) Show that $T = 2Mg\cos\theta$

(3)

Given that $\cos\theta = \frac{3}{5}$

(c) show that the magnitude of the vertical force exerted by the ground on the rod at A is $\frac{57Mg}{25}$

(3)

The coefficient of friction between the rod and the ground is μ

Given that the rod is in limiting equilibrium,

(d) show that $\mu = \frac{8}{19}$

(4)

ANSWER

Question	Scheme	Marks	AOs
4(a)	The horizontal component of T acts to the left and since the **only** other horizontal force is friction, it must act to the right oe	B1	2.4
		(1)	
4(b)	Take moments about A or any other complete method to obtain **an equation in T, M and θ only.** (see possible equations below that they may use)	M1	3.1b
	$T.2a = Mga\cos\theta + 2Mg \times 1.5a\cos\theta$ (A0 if a's missing)	A1	1.1b
	Other possible equations but F and R would need to be eliminated. $(\nwarrow)$, $R\cos\theta + T = F\sin\theta + Mg\cos\theta + 2Mg\cos\theta$ $(\nearrow)$, $R\sin\theta + F\cos\theta = Mg\sin\theta + 2Mg\sin\theta$ $(\rightarrow)$, $F = T\sin\theta$ M(B), $R.2a\cos\theta = Mga\cos\theta + 2Mg \times 0.5a\cos\theta + F.2a\sin\theta$ M(G), $Fa\sin\theta + Ta = Ra\cos\theta + 2Mg \times 0.5a\cos\theta$ M(C), $R \times 1.5a\cos\theta = T \times 0.5a + Mg \times 0.5a\cos\theta + F \times 1.5a\sin\theta$		
	$T = 2Mg\cos\theta$ *	A1*	1.1b
		(3)	

	$(\nwarrow)$, $R\cos\theta + T = F\sin\theta + Mg\cos\theta + 2Mg\cos\theta$ $(\nearrow)$, $R\sin\theta + F\cos\theta = Mg\sin\theta + 2Mg\sin\theta$ $(\rightarrow)$, $F = T\sin\theta$ M(B), $R.2a\cos\theta = Mga\cos\theta + 2Mg\times 0.5a\cos\theta + F.2a\sin\theta$ M(G), $Fa\sin\theta + Ta = Ra\cos\theta + 2Mg\times 0.5a\cos\theta$ M(C), $R\times 1.5a\cos\theta = T\times 0.5a + Mg\times 0.5a\cos\theta + F\times 1.5a\sin\theta$		
	$F = \mu R$ used i.e. both F and R are substituted.	M1	3.1b
	$\mu = \frac{8}{19}$ *	A1*	2.2a
		(4)	
		(11 marks)	

4(c)	e.g. Resolve vertically	M1	3.4
	$(\uparrow)$, $R + T\cos\theta = Mg + 2Mg$	A1	1.1b
	$R = \frac{57Mg}{25}$ *	A1*	1.1b
		(3)	
	Other possible equations but F would need to be eliminated. $(\nwarrow)$, $R\cos\theta + T = F\sin\theta + Mg\cos\theta + 2Mg\cos\theta$ $(\nearrow)$, $R\sin\theta + F\cos\theta = Mg\sin\theta + 2Mg\sin\theta$ $(\rightarrow)$, $F = T\sin\theta$ M(B), $R.2a\cos\theta = Mga\cos\theta + 2Mg\times 0.5a\cos\theta + F.2a\sin\theta$ M(G), $Fa\sin\theta + Ta = Ra\cos\theta + 2Mg\times 0.5a\cos\theta$ M(C), $R\times 1.5a\cos\theta = T\times 0.5a + Mg\times 0.5a\cos\theta + F\times 1.5a\sin\theta$		
4(d)	Find an equation containing F e.g. Resolve horizontally	M1	3.4
	$(\rightarrow)$, $F = T\sin\theta$	A1	1.1b
	Other possible equations		

Video solution:

https://youtu.be/tR8tNGHPYCI

June 2022 Question 2

2.

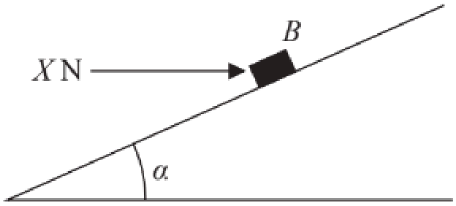


Figure 1

A rough plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

A small block B of mass 5 kg is held in equilibrium on the plane by a horizontal force of magnitude X newtons, as shown in Figure 1.

The force acts in a vertical plane which contains a line of greatest slope of the inclined plane.

The block B is modelled as a particle.

The magnitude of the normal reaction of the plane on B is 68.6 N.

Using the model,

(a) (i) find the magnitude of the frictional force acting on B,

(3)

(ii) state the direction of the frictional force acting on B.

(1)

The horizontal force of magnitude X newtons is now removed and B moves down the plane.

Given that the coefficient of friction between B and the plane is 0.5

(b) find the acceleration of B down the plane.

(6)

ANSWER

Question	Scheme	Marks	AOs
2(a)(i)	Resolve vertically	M1	3.1b
	F acting UP the plane: **OR** F acting DOWN the plane: $(\uparrow)\ F\sin\alpha + 68.6\cos\alpha = 5g$ $-F\sin\alpha + 68.6\cos\alpha = 5g$ Other possible equations from which X would need to be eliminated to give an equation in F **only** to earn the M mark are shown below. The equation in F **only** must then be correct to earn the A mark. Possible equations: $(\nwarrow)\ 68.6 = X\sin\alpha + 5g\cos\alpha$ (leads to $X = 49$ with $g = 9.8$) F acting UP the plane: **OR** F acting DOWN the plane: $(\nearrow)\ F + X\cos\alpha = 5g\sin\alpha$ $-F + X\cos\alpha = 5g\sin\alpha$ $(\rightarrow)\ F\cos\alpha + X = 68.6\sin\alpha$ $-F\cos\alpha + X = 68.6\sin\alpha$	A1	1.1b
	9.8 (N) (49/5 is A0) **N.B.** If sin and cos are interchanged in all equations, this leads to an answer of 9.8 in the wrong direction and can only score (a) (i)M1A0A0 (ii) A0	A1	1.1b
		(3)	

Video solution:

https://youtu.be/3l6D-YY5jKw

2(a)(ii)	Down the plane (Allow down or downwards or an arrow ↙, but must appear as the answer to (a) (ii) not just on the diagram.)	A1	2.2a
		(1)	
2(b)	**N.B.** If they use $R = 68.6$ in this part, the maximum they can score is M1A1M0A0M0A0 If they use $F = 9.8$ or their F from (a) in this part, the maximum they can score is M1A1M0A0M0A0		
	Equation of motion down the plane	M1	2.1
	$5g\sin\alpha - F = 5a$ Allow ($-a$) instead of a	A1	1.1b
	Resolve perpendicular to the plane	M1	3.1b
	$R = 5g\cos\alpha$	A1	1.1b
	$F = 0.5R$ seen	M1	3.4
	$a = 1.96$ or 2.0 or 2 (m s^{-2}) or $\frac{1}{5}g$	A1	1.1b
		(6)	

(10 marks)

October 2020 question 1

1. A rough plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

A brick P of mass m is placed on the plane.

The coefficient of friction between P and the plane is μ

Brick P is in equilibrium and on the point of sliding down the plane.

Brick P is modelled as a particle.

Using the model,

(a) find, in terms of m and g, the magnitude of the normal reaction of the plane on brick P

(2)

(b) show that $\mu = \frac{3}{4}$

(4)

For parts (c) and (d), you are not required to do any further calculations.

Brick P is now removed from the plane and a much heavier brick Q is placed on the plane.

The coefficient of friction between Q and the plane is also $\frac{3}{4}$

(c) Explain briefly why brick Q will remain at rest on the plane.

(1)

Brick Q is now projected with speed $0.5\,\text{m}\,\text{s}^{-1}$ down a line of greatest slope of the plane.

Brick Q is modelled as a particle.

Using the model,

(d) describe the motion of brick Q, giving a reason for your answer.

(2)

ANSWER

Question	Scheme	Marks	AOs
1.(a)	Resolve perpendicular to the plane	M1	3.4
	$R = mg\cos\alpha = \frac{4}{5}mg$	A1	1.1b
		(2)	
1(b)	Resolve parallel to the plane or horizontally or vertically	M1	3.4
	$F = mg\sin\alpha$ or $R\sin\alpha = F\cos\alpha$	A1	1.1b
	Use $F = \mu R$ and solve for μ	M1	2.1
	$\mu = \frac{3}{4}$ *	A1*	2.2a
		(4)	
1(c)	The forces acting on Q will still balance as the m's cancel oe Other possibilities: e.g. the friction will increase in the same proportion as the weight component or force down the plane. The force pulling the brick down the plane increases by the same amount as the friction oe This mark can be scored if they do the calculation.	B1	2.4
		(1)	
1(d)	Brick Q slides down the plane with **constant** speed.	B1	2.4
	No resultant force down the plane (so no acceleration) oe	B1	2.4
	These marks can be scored if they do the calculation.	**(2)**	
		(9 marks)	

October 2020 question 4

4.

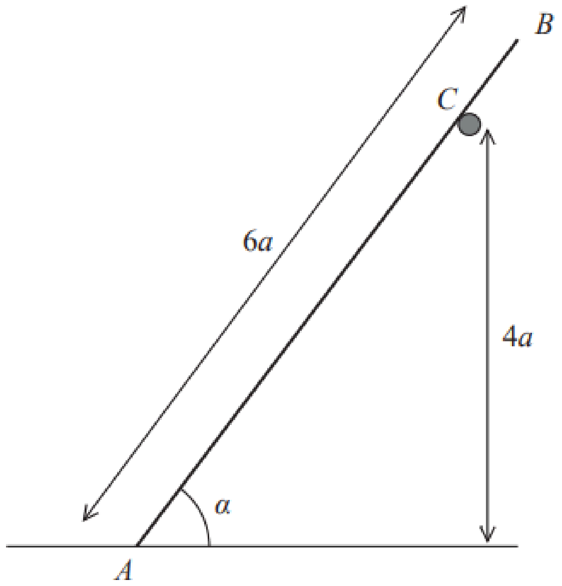


Figure 1

A ladder AB has mass M and length $6a$.

The end A of the ladder is on rough horizontal ground.

The ladder rests against a fixed smooth horizontal rail at the point C.

The point C is at a vertical height $4a$ above the ground.

The vertical plane containing AB is perpendicular to the rail.

The ladder is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{4}{5}$, as shown in Figure 1.

The coefficient of friction between the ladder and the ground is μ.

The ladder rests in limiting equilibrium.

The ladder is modelled as a uniform rod.

Using the model,

(a) show that the magnitude of the force exerted on the ladder by the rail at C is $\frac{9Mg}{25}$

(3)

(b) Hence, or otherwise, find the value of μ.

(7)

ANSWER

Question	Scheme	Marks	AOs
4(a)	Take moments about A	M1	3.3
	$N \times \dfrac{4a}{\sin\alpha} = Mg \times 3a\cos\alpha$	A1	1.1b
	$\dfrac{9Mg}{25}$ *	A1*	1.1b
		(3)	
4(b)	Resolve horizontally	M1	3.4
	$(\rightarrow)\ F = \dfrac{9Mg}{25}\sin\alpha$	A1	1.1b
	Resolve vertically	M1	3.4
	$(\uparrow)\ R + \dfrac{9Mg}{25}\cos\alpha = Mg$	A1	1.1b
	Other possible equations: $(\nwarrow), R\cos\alpha + \dfrac{9Mg}{25} = Mg\cos\alpha + F\sin\alpha$ $(\nearrow), Mg\sin\alpha = F\cos\alpha + R\sin\alpha$ $\mathrm{M}(C), Mg.2a\cos\alpha + F.5a\sin\alpha = R.5a\cos\alpha$ $\mathrm{M}(G), \dfrac{9Mg}{25}.2a + F.3a\sin\alpha = R.3a\cos\alpha$ $\mathrm{M}(B), Mg.3a\cos\alpha + F.6a\sin\alpha = R.6a\cos\alpha + \dfrac{9Mg}{25}a$ $(F = \dfrac{36Mg}{125}\ , R = \dfrac{98Mg}{125})$		
	$F = \mu R$ used	M1	3.4
	Eliminate R and F and solve for μ	M1	3.1b
	$\mu = \dfrac{18}{49}$ (0.3673…..accept 0.37 or better)	A1	2.2a
		(7)	
		(10 marks)	

June 2019 question 3

3.

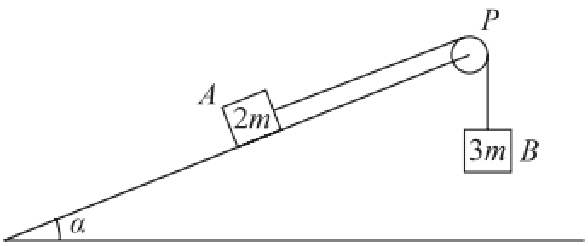


Figure 1

Two blocks, A and B, of masses $2m$ and $3m$ respectively, are attached to the ends of a light string.

Initially A is held at rest on a fixed rough plane.

The plane is inclined at angle α to the horizontal ground, where $\tan\alpha = \frac{5}{12}$

The string passes over a small smooth pulley, P, fixed at the top of the plane.

The part of the string from A to P is parallel to a line of greatest slope of the plane. Block B hangs freely below P, as shown in Figure 1.

The coefficient of friction between A and the plane is $\frac{2}{3}$

The blocks are released from rest with the string taut and A moves up the plane.

The tension in the string immediately after the blocks are released is T.

The blocks are modelled as particles and the string is modelled as being inextensible.

(a) Show that $T = \frac{12mg}{5}$

(8)

After B reaches the ground, A continues to move up the plane until it comes to rest before reaching P.

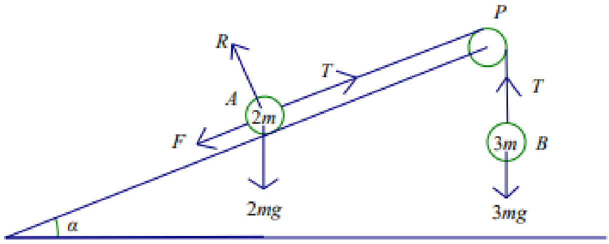
(b) Determine whether A will remain at rest, carefully justifying your answer.

(2)

(c) Suggest two refinements to the model that would make it more realistic.

(2)

ANSWER

Question	Scheme	Marks	AO
3(a)			
	$R = 2mg\cos\alpha$	B1	3.4
	$F = \frac{2}{3}R$	B1	1.2
	Equation of motion for A:	M1	3.3
	$T - F - 2mg\sin\alpha = 2ma$	A1	1.1b
	Equation of motion for B:	M1	3.3
	$3mg - T = 3ma$	A1	1.1b
	Complete strategy to find an equation in T, m and g only.	M1	3.1b
	$T = \frac{12mg}{5}$ *	A1*	2.2a
		(8)	
(b)	$(F_{\max} =)\frac{16mg}{13} > \frac{10mg}{13}$	M1	2.1
	…… so A will not move.	A1	2.2a
		(2)	
(c)	• Extensible string • Weight of string • Friction at pulley e.g. rough pulley • Allow for the dimensions of the blocks e.g. "Do not model blocks as particles"; "(include) air resistance";"include rotational effects of forces on blocks i.e. spin"	B1 B1	3.5c 3.5c
		(2)	
		(12)	

June 2019 question 4

4.

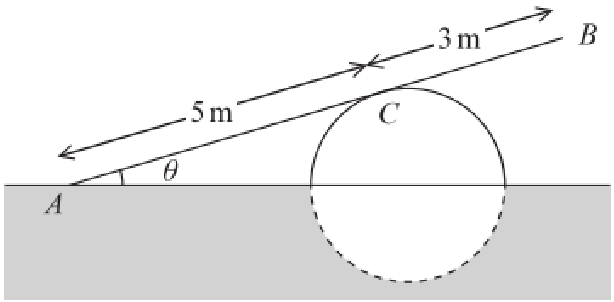


Figure 2

A ramp, AB, of length 8 m and mass 20 kg, rests in equilibrium with the end A on rough horizontal ground.

The ramp rests on a smooth solid cylindrical drum which is partly under the ground. The drum is fixed with its axis at the same horizontal level as A.

The point of contact between the ramp and the drum is C, where $AC = 5$ m, as shown in Figure 2.

The ramp is resting in a vertical plane which is perpendicular to the axis of the drum, at an angle θ to the horizontal, where $\tan\theta = \dfrac{7}{24}$

The ramp is modelled as a uniform rod.

(a) Explain why the reaction from the drum on the ramp at point C acts in a direction which is perpendicular to the ramp.

(1)

(b) Find the magnitude of the resultant force acting on the ramp at A.

(9)

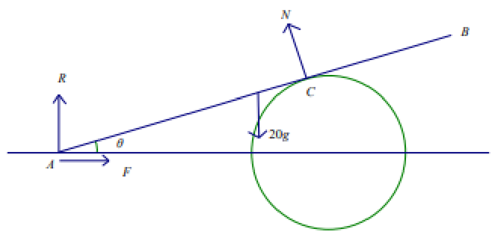
The ramp is still in equilibrium in the position shown in Figure 2 but the ramp is not now modelled as being uniform.

Given that the centre of mass of the ramp is assumed to be closer to A than to B,

(c) state how this would affect the magnitude of the normal reaction between the ramp and the drum at C.

(1)

ANSWER

Question	Scheme	Marks	AO
4(a)	Drum **smooth**, or no friction, (therefore reaction is perpendicular to the ramp)	B1	2.4
		(1)	
(b)	**N.B.** In (b), for a moments equation, if there is an extra $\sin\theta$ or $\cos\theta$ on a length, give M0 for the equation e.g. $\mathrm{M}(A)$: $20g \times 4\cos\theta = 5N\sin\theta$ would be given M0A0		
			
	Possible equns $(\nearrow): F\cos\theta + R\sin\theta = 20g\sin\theta$ $(\nwarrow): N + R\cos\theta = 20g\cos\theta + F\sin\theta$ $(\uparrow) R + N\cos\theta = 20g$ $(\rightarrow): F = N\sin\theta$ $\mathrm{M}(A)$: $20g \times 4\cos\theta = 5N$ $\mathrm{M}(B)$: $3N + R \times 8\cos\theta = F \times 8\sin\theta + 20g \times 4\cos\theta$ $\mathrm{M}(C)$: $R \times 5\cos\theta = F \times 5\sin\theta + 20g \times \cos\theta$ $\mathrm{M}(G)$: $R \times 4\cos\theta = F \times 4\sin\theta + N$	M1 A1 M1 A1 M1 A1	3.3 1.1b 3.4 1.1b 3.4 1.1b

(The values of the 3 unknowns are: $N = 150.528$; $F = 42.14784$; $R = 51.49312$)		

	Solve their 3 equations for F and R **OR** X and Y **OR** H and S	M1	1.1b
	$\lvert\text{Force}\rvert = \sqrt{R^2 + F^2}$ Main scheme **OR** $= \sqrt{X^2 + Y^2}$ Alternative 1 **OR** $= \sqrt{(H^2 + S^2 - 2HS\cos(90^o - \theta)}$ Alternative 2	M1	3.1b
	Magnitude = 67 or 66.5 (N)	A1	2.2a
		(9)	
(c)	Magnitude of the normal reaction (at C) will **decrease**.	B1	3.5a
		(1)	
		(11)	

June 2019 question 5

5.

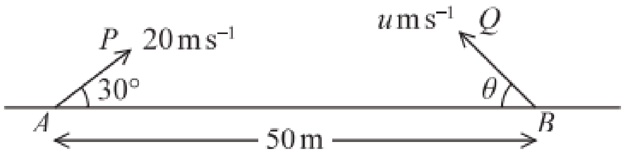


Figure 3

The points A and B lie 50 m apart on horizontal ground.

At time $t = 0$ two small balls, P and Q, are projected in the vertical plane containing AB.

Ball P is projected from A with speed $20\,\text{m s}^{-1}$ at 30° to AB.

Ball Q is projected from B with speed $u\,\text{m s}^{-1}$ at angle θ to BA, as shown in Figure 3.

At time $t = 2$ seconds, P and Q collide.

Until they collide, the balls are modelled as particles moving freely under gravity.

(a) Find the velocity of P at the instant before it collides with Q.

(6)

(b) Find

(i) the size of angle θ,

(ii) the value of u.

(6)

(c) State one limitation of the model, other than air resistance, that could affect the accuracy of your answers.

(1)

ANSWER

Question	Scheme	Marks	AO
	In this question mark parts (a) and (b) together.		
5(a)	Horizontal speed $= 20\cos 30°$	B1	3.4
	Vertical velocity at $t = 2$	M1	3.4
	$= 20\sin 30° - 2g$	A1	1.1b
	$\theta = \tan^{-1}\left(\pm\dfrac{9.6}{10\sqrt{3}}\right)$	M1	1.1b
	Speed $= \sqrt{100\times 3 + 9.6^2}$ or e.g. speed $= \dfrac{9.6}{\sin\theta}$	M1	1.1b
	19.8 or 20 $\left(\text{m s}^{-1}\right)$ at 29.0° or 29° to the horizontal oe	A1	2.2a
		(6)	
(b)	Using sum of horizontal distances $= 50$ at $t = 2$	M1	3.3
	$(u\cos\theta)\times 2 + (20\cos 30°)\times 2 = 50$ $(u\cos\theta = 25 - 20\cos 30°)$	A1	1.1b
	Vertical distances equal	M1	3.4
	$\Rightarrow (20\sin 30°)\times 2 - \dfrac{g}{2}\times 4 = (u\sin\theta)\times 2 - \dfrac{g}{2}\times 4$ $(20\sin 30° = u\sin\theta)$	A1	1.1b
	Solving for both θ and u	M1	3.1b
	$\theta = 52°$ or better (52.47756849….°) $u = 13$ or better (12.6085128…)	A1	2.2a
		(6)	
(c)	It does not take account of the fact that they are not particles (moving freely under gravity) It does not take account of the size(s) of the balls It does not take account of the spin of the balls It does not take account of the wind g is not exactly 9.8 m s^{-2} **N.B.** If they refer to the mass or weight of the balls give B0	B1	3.5b
		(1)	
		(13)	

Arancha Ruiz

June 2018 Question 8

8. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given relative to the fixed point O.*]

A particle P moves with constant acceleration.
At time $t = 0$, the particle is at O and is moving with velocity $(2\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-1}$
At time $t = 2$ seconds, P is at the point A with position vector $(7\mathbf{i} - 10\mathbf{j})\,\text{m}$.

(a) Show that the magnitude of the acceleration of P is $2.5\,\text{m s}^{-2}$

(4)

At the instant when P leaves the point A, the acceleration of P changes so that P now moves with constant acceleration $(4\mathbf{i} + 8.8\mathbf{j})\,\text{m s}^{-2}$

At the instant when P reaches the point B, the direction of motion of P is north east.

(b) Find the time it takes for P to travel from A to B.

(4)

ANSWER

Question	Scheme	Marks	AOs
8(a)	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$: $(7\mathbf{i} - 10\mathbf{j}) = 2(2\mathbf{i} - 3\mathbf{j}) + \frac{1}{2}\mathbf{a}2^2$	M1	3.1b
	$\mathbf{a} = (1.5\mathbf{i} - 2\mathbf{j})$	A1	1.1b
	$\lvert\mathbf{a}\rvert = \sqrt{1.5^2 + (-2)^2}$	M1	1.1b
	$= 2.5\ \text{m s}^{-2}$ * GIVEN ANSWER	A1*	2.1
		(4)	
(b)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t = (2\mathbf{i} - 3\mathbf{j}) + 2(1.5\mathbf{i} - 2\mathbf{j})$	M1	3.1b
	$= (5\mathbf{i} - 7\mathbf{j})$	A1	1.1b
	$\mathbf{v} = (5\mathbf{i} - 7\mathbf{j}) + t(4\mathbf{i} + 8.8\mathbf{j}) = (5 + 4t)\mathbf{i} + (8.8t - 7)\mathbf{j}$ and $(5 + 4t) = (8.8t - 7)$	M1	3.1b
	$t = 2.5$ (s)	A1	1.1b
		(4)	
			(8 marks)

June 2018 Question 9

9.

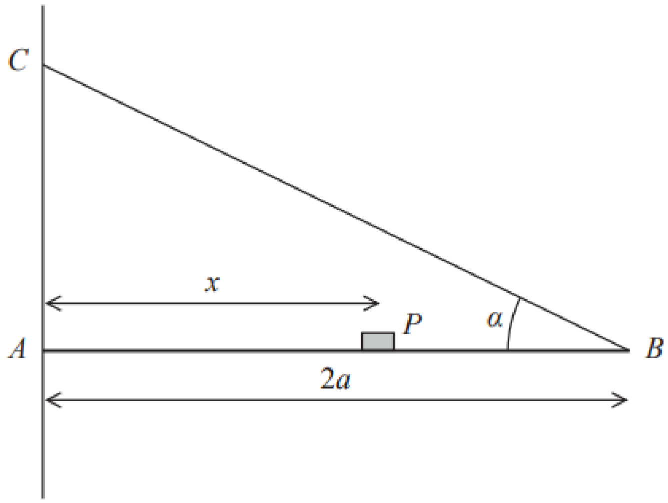


Figure 3

A plank, AB, of mass M and length $2a$, rests with its end A against a rough vertical wall. The plank is held in a horizontal position by a rope. One end of the rope is attached to the plank at B and the other end is attached to the wall at the point C, which is vertically above A.

A small block of mass $3M$ is placed on the plank at the point P, where $AP = x$. The plank is in equilibrium in a vertical plane which is perpendicular to the wall.

The angle between the rope and the plank is α, where $\tan\alpha = \frac{3}{4}$, as shown in Figure 3.

The plank is modelled as a uniform rod, the block is modelled as a particle and the rope is modelled as a light inextensible string.

(a) Using the model, show that the tension in the rope is $\dfrac{5Mg(3x + a)}{6a}$

(3)

The magnitude of the horizontal component of the force exerted on the plank at A by the wall is $2Mg$.

(b) Find x in terms of a.

(2)

The force exerted on the plank at A by the wall acts in a direction which makes an angle β with the horizontal.

(c) Find the value of $\tan\beta$

(5)

The rope will break if the tension in it exceeds $5Mg$.

(d) Explain how this will restrict the possible positions of P. You must justify your answer carefully.

(3)

ANSWER

Question	Scheme	Marks	AOs
9(a)	Moments about A (or any other complete method)	M1	3.3
	$T2a\sin\alpha = Mga + 3Mgx$	A1	1.1b
	$T = \dfrac{Mg(a+3x)}{2a \times \frac{3}{5}} = \dfrac{5Mg(3x+a)}{6a}$ * GIVEN ANSWER	A1*	2.1
		(3)	
(b)	$\dfrac{5Mg(3x+a)}{6a}\cos\alpha = 2Mg$ **OR** $2Mg.2a\tan\alpha = Mga + 3Mgx$	M1	3.1b
	$x = \dfrac{2a}{3}$	A1	2.2a
		(2)	

(c)	Resolve vertically **OR** Moments about B	M1	3.1b
	$Y = 3Mg + Mg - \dfrac{5Mg(3.\frac{2a}{3}+a)}{6a}\sin\alpha$ $\quad 2aY = Mga + 3Mg(2a - \frac{2a}{3})$ **Or**: $Y = 3Mg + Mg - \left(\dfrac{2Mg}{\cos\alpha}\right)\sin\alpha$	A1ft	1.1b
	$Y = \dfrac{5Mg}{2}$ **N.B.** May use $R\sin\beta$ for Y and/or $R\cos\beta$ for X throughout	A1	1.1b
	$\tan\beta = \dfrac{Y}{X}$ or $\dfrac{R\sin\beta}{R\cos\beta} = \dfrac{\frac{5Mg}{2}}{2Mg}$	M1	3.4
	$= \dfrac{5}{4}$	A1	2.2a
		(5)	
(d)	$\dfrac{5Mg(3x+a)}{6a} \le 5Mg$ and solve for x	M1	2.4
	$x \le \dfrac{5a}{3}$	A1	2.4
	For rope not to break, block can't be more than $\dfrac{5a}{3}$ from A oe Or just: $x \le \dfrac{5a}{3}$, if no incorrect statement seen. **N.B. If the correct inequality is not found,** their comment must mention 'distance from A'.	B1 **A1**	2.4
		(3)	
		(13 marks)	

Video solution:

https://youtu.be/NWhzZxPcpFw