Statistics Year 1 (AS) exam questions

Ch. 6: Statistical Distributions **2**

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Ch. 6: Statistical Distributions

June 2023 Question 5

5. Julia selects 3 letters at random, one at a time without replacement, from the word

V A R I A N C E

The discrete random variable X represents the number of times she selects a letter A.

(a) Find the complete probability distribution of X.

(5)

Yuki selects 10 letters at random, one at a time **with** replacement, from the word

D E V I A T I O N

(b) Find the probability that he selects the letter E at least 4 times.

(3)

ANSWER

Que.	Scheme	Marks	AOs
5(a)	$X = 0, 1, 2$ only	B1	3.1b
	$[\mathrm{P}(X=0)=]\frac{6}{8}\times\frac{5}{7}\times\frac{4}{6}$	M1	1.1b
	$[\mathrm{P}(X=1)=]3\times\frac{2}{8}\times\frac{6}{7}\times\frac{5}{6}$ or $[\mathrm{P}(X=2)=]3\times\frac{2}{8}\times\frac{1}{7}\times\frac{6}{6}$	M1	2.1
	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td></tr><tr><td>$\mathrm{P}(X=x)$</td><td>$\frac{5}{14}$</td><td>$\frac{15}{28}$</td><td>$\frac{3}{28}$</td></tr></table>	A1 A1	1.1b 1.1b
		(5)	
(b)	$J \sim \mathrm{B}(10, \frac{1}{9})$	M1	3.1b
	$\mathrm{P}(J \geqslant 4) = 1 - \mathrm{P}(J \leqslant 3)$ or $\mathrm{P}(J \geqslant 4) = \mathrm{P}(J=4)+\mathrm{P}(J=5)+\ldots+\mathrm{P}(J=10)$ or $1-0.981(57\ldots)$	M1	3.4
	= awrt 0.0184	A1	1.1b
		(3)	
			(8 marks)

June 2022 Question 3 (A-level)

3. Dian uses the large data set to investigate the Daily Total Rainfall, r mm, for Camborne.

(a) Write down how a value of $0 < r \leqslant 0.05$ is recorded in the large data set.

(1)

Dian uses the data for the 31 days of August 2015 for Camborne and calculates the following statistics

$$n = 31 \qquad \sum r = 174.9 \qquad \sum r^2 = 3523.283$$

(b) Use these statistics to calculate

(i) the mean of the Daily Total Rainfall in Camborne for August 2015,

(ii) the standard deviation of the Daily Total Rainfall in Camborne for August 2015.

(3)

Dian believes that the mean Daily Total Rainfall in August is less in the South of the UK than in the North of the UK.
The mean Daily Total Rainfall in Leuchars for August 2015 is 1.72 mm to 2 decimal places.

(c) State, giving a reason, whether this provides evidence to support Dian's belief.

(2)

Dian uses the large data set to estimate the proportion of days with no rain in Camborne for 1987 to be 0.27 to 2 decimal places.

ANSWER

Question	Scheme	Marks	AOs
3(a)	tr	B1	1.2
		(1)	
(b)(i)	$\mu = \frac{174.9}{31} = 5.6419\ldots$ awrt 5.64	B1	1.1b
(ii)	$\sigma_r = \sqrt{\frac{3523.283}{31} - \mu^2}$	M1	1.1b
	$= 9.04559\ldots$ awrt 9.05	A1	1.1b
		(3)	
(c)	Leuchars is in the North and Camborne is in the South	M1	2.4
	The mean is smaller for Leuchars than Camborne therefore there is no evidence that Dian's belief is true	A1ft	2.2b
		(2)	
(d)	eg $p = 0.27$ is unlikely to be constant.	B1	2.4
		(1)	
			(7 marks)

June 2022 Question 5

5. Manon has two biased spinners, one red and one green.

The random variable R represents the score when the red spinner is spun.
The random variable G represents the score when the green spinner is spun.

The probability distributions for R and G are given below.

r	2	3
$P(R = r)$	$\frac{1}{4}$	$\frac{3}{4}$

g	1	4
$P(G = g)$	$\frac{2}{3}$	$\frac{1}{3}$

Manon spins each spinner once and adds the two scores.

(a) Find the probability that

(i) the sum of the two scores is 7

(ii) the sum of the two scores is less than 4

(3)

The random variable $X = mR + nG$ where m and n are integers.

$$P(X = 20) = \frac{1}{6} \qquad \text{and} \qquad P(X = 50) = \frac{1}{4}$$

(b) Find the value of m and the value of n

(5)

ANSWER

Qu	Scheme	Mark	AO
5. (a)(i)	Require $R = 3$ and $G = 4$ so probability is $\frac{3}{4} \times \frac{1}{3}$	M1	2.1
	$= \underline{\frac{1}{4}}$ or $\underline{\mathbf{0.25}}$	A1	1.1b
(ii)	[R must be 2 and $G = 1$ so $\frac{1}{4} \times \frac{2}{3}$] $= \underline{\frac{1}{6}}$	A1	1.1b
		(3)	
(b)	$P(X = 50) = 0.25$ must mean $R = 3$ and $G = 4$	M1	3.1a
	so $3m + 4n = 50$	A1	1.1b
	$P(X = 20) = \frac{1}{6} \Rightarrow R = 2, G = 1$ so $2m + n = 20$	A1	2.1
	Solving: $3m + 4(20 - 2m) = 50$ (o.e.)	M1	1.1b
	$\underline{\boldsymbol{m = 6}}$ and $\underline{\boldsymbol{n = 8}}$	A1	3.2a
		(5)	
		(8 marks)	

November 2021 question 4

Included in chapter 7 as it has hypothesis testing.

October 2020 question 5

Same with this question, so you will find it in chapter 7.

June 2019 question 3

3. A fair 5-sided spinner has sides numbered 1, 2, 3, 4 and 5

The spinner is spun once and the score of the side it lands on is recorded.

(a) Write down the name of the distribution that can be used to model the score of the side it lands on.

(1)

The spinner is spun 28 times.

The random variable X represents the number of times the spinner lands on 2

(b) (i) Find the probability that the spinner lands on 2 at least 7 times.

(ii) Find $P(4 \leqslant X < 8)$

(5)

ANSWER

Question	Scheme	Marks	AOs
3(a)	(Discrete) uniform (distribution)	B1	1.2
		(1)	
(b)	B(28, 0.2)	B1	3.3
(i)	$P(X \geq 7) = 1 - P(X \leq 6)$ $[= 1 - 0.6784...]$	M1	3.4
	awrt **<u>0.322</u>**	A1	1.1b
(ii)	$P(4 \leq X < 8) = P(X \leq 7) - P(X \leq 3)$ $[= 0.818... - 0.160...]$	M1	3.1b
	awrt **<u>0.658</u>**	A1	1.1b
		(5)	
			(6 marks)

June 2019 question 4 (A level, not AS)

4. Magali is studying the mean total cloud cover, in oktas, for Leuchars in 1987 using data from the large data set. The daily mean total cloud cover for all 184 days from the large data set is summarised in the table below.

Daily mean total cloud cover (oktas)	0	1	2	3	4	5	6	7	8
Frequency (number of days)	0	1	4	7	10	30	52	52	28

One of the 184 days is selected at random.

(a) Find the probability that it has a daily mean total cloud cover of 6 or greater.

(1)

Magali is investigating whether the daily mean total cloud cover can be modelled using a binomial distribution.

She uses the random variable X to denote the daily mean total cloud cover and believes that $X \sim B(8, 0.76)$

Using Magali's model,

(b) (i) find $P(X \geqslant 6)$

(2)

(ii) find, to 1 decimal place, the expected number of days in a sample of 184 days with a daily mean total cloud cover of 7

(2)

(c) Explain whether or not your answers to part (b) support the use of Magali's model.

(1)

There were 28 days that had a daily mean total cloud cover of 8
For these 28 days the daily mean total cloud cover for the **following** day is shown in the table below.

Daily mean total cloud cover (oktas)	0	1	2	3	4	5	6	7	8
Frequency (number of days)	0	0	1	1	2	1	5	9	9

(d) Find the proportion of these days when the daily mean total cloud cover was 6 or greater.

(1)

(e) Comment on Magali's model in light of your answer to part (d).

(2)

ANSWER

Question	Scheme	Marks	AOs
4 (a)	$\frac{132}{184} = 0.71739...$ awrt **<u>0.717</u>**	B1	1.1b
		(1)	
(b)(i)	$P(X \geq 6) = 1 - P(X \leq 5)$ or $P([X=]6) + P([X=]7) + P([X=]8)$	M1	3.4
	$= 1 - 0.296722...$ awrt **<u>0.703</u>**	A1	1.1b
		(2)	
(b)(ii)	$184 \times P(X = 7)$ $[= 184 \times 0.2811\ldots]$	M1	1.1b
	$= 51.7385...$ awrt **<u>51.7</u>**	A1	1.1b
		(2)	
(c)	Part (a) and part (b)(i) are similar **and** the expected number of 7s (51.7 or 0.281) matches with the number of 7s found in the data set (52 or 0.283) so Magali's model is supported.	B1ft	3.5a
		(1)	
(d)	$\frac{23}{28} = 0.82142...$ awrt **<u>0.821</u>**	B1	1.1b
		(1)	
(e)	Any one of… • Part (d)/'0.821' differs from part (a)/(b)(i)/(0.7…) • there is a **greater/different probability of high cloud cover/more likely to have high cloud cover** if the previous day had high cloud cover • **independence(o.e.)** does not hold	B1	2.4
	…therefore Magali's (binomial) model may not be suitable.	dB1	3.5a
		(2)	
			(9 marks)

June 2018 Question 5

5. A biased spinner can only land on one of the numbers 1, 2, 3 or 4. The random variable X represents the number that the spinner lands on after a single spin and $P(X = r) = P(X = r + 2)$ for $r = 1, 2$

Given that $P(X = 2) = 0.35$

(a) find the complete probability distribution of X.

(2)

Ambroh spins the spinner 60 times.

(b) Find the probability that more than half of the spins land on the number 4
Give your answer to 3 significant figures.

(3)

The random variable $Y = \dfrac{12}{X}$

(c) Find $P(Y - X \leqslant 4)$

(3)

ANSWER

Qu	Scheme	Marks	AO
5(a)	$P(X=4)=P(X=2)$ so $P(X=4)=0.35$	M1	2.1
	$P(X=1)=P(X=3)$ and $P(X=1)+P(X=3)=1-0.7$		
	So $\begin{array}{\|c\|c\|c\|c\|c\|}\hline x & 1 & 2 & 3 & 4 \\ \hline P(X=x) & 0.15 & 0.35 & 0.15 & [0.35] \\ \hline\end{array}$	A1	1.1b
		(2)	
(b)	Let A = number of spins that land on 4 $A \sim B(60, \text{“}0.35\text{”})$	B1ft	3.3
	$[P(A>30)=]$ $1-P(A \leqslant 30)$	M1	3.4
	$= 1 - 0.99411\ldots$ = **awrt 0.00589**	A1	1.1b
		(3)	
(c)	$Y - X \leqslant 4 \Rightarrow \dfrac{12}{X} - X \leqslant 4$ <u>or</u> $12 - X^2 \leqslant 4X$ (since $X > 0$) o.e.	M1	3.1a
	i.e. $0 \leqslant X^2 + 4X - 12 \Rightarrow 0 \leqslant (X+6)(X-2)$ so $X \geqslant 2$	M1	1.1b
	$P(Y - X \leqslant 4) = P(X \geqslant 2) = 0.35 + 0.15 + 0.35 = \underline{\mathbf{0.85}}$	A1	3.2a
		(3)	
		(8 marks)	