Mechanics Year 2 exam questions

Arancha Ruiz

Ch. 6: Projectiles

June 2023 Question 5

5.

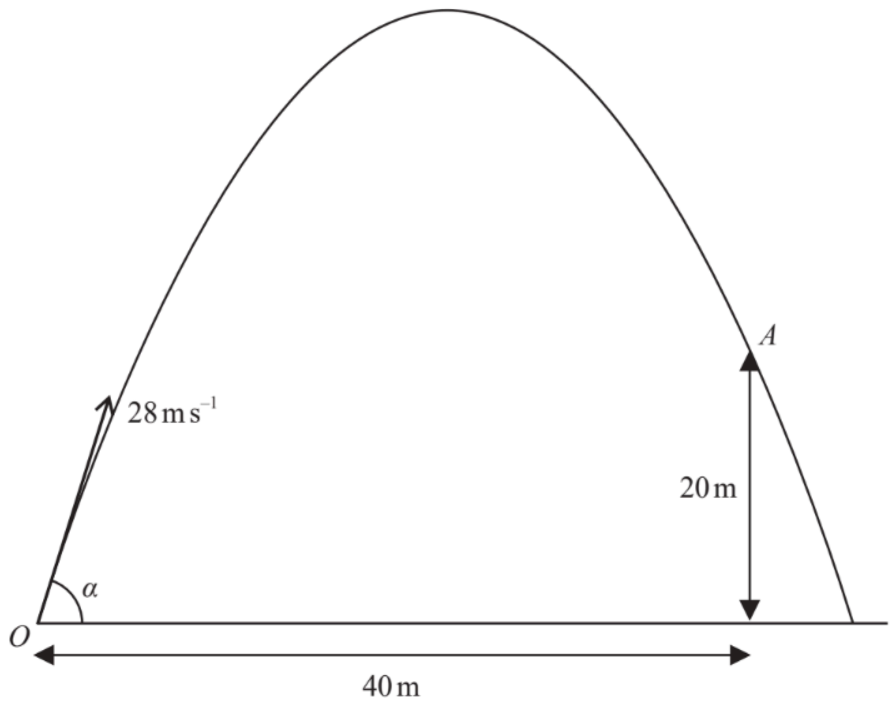

Figure 2

A small ball is projected with speed $28\,\text{m s}^{-1}$ from a point O on horizontal ground.

After moving for T seconds, the ball passes through the point A.

The point A is 40 m horizontally and 20 m vertically from the point O, as shown in Figure 2.

The motion of the ball from O to A is modelled as that of a particle moving freely under gravity.

Given that the ball is projected at an angle α to the ground, use the model to

(a) show that $T = \dfrac{10}{7\cos\alpha}$

(2)

(b) show that $\tan^2\alpha - 4\tan\alpha + 3 = 0$

(5)

(c) find the greatest possible height, in metres, of the ball above the ground as the ball moves from O to A.

(3)

The model does not include air resistance.

(d) State one other limitation of the model.

(1)

ANSWER

Question	Scheme	Marks	AOs
	N.B. In this question, allow misread of α for *a*.		
5(a)	Use horizontal motion to give an equation in *T* and α only: $28\cos\alpha \times T = 40$	M1	3.4
	$T = \dfrac{10}{7\cos\alpha}$ *	A1*	1.1b
		(2)	

5(b)	Use vertical motion to give an equation in *T* and α only	M1	3.3
	$20 = (28\sin\alpha)T - \dfrac{1}{2}gT^2$	A1	1.1b
	Eliminate *T* to give an unsimplified equation in α only: $20 = (28\sin\alpha)\times\dfrac{10}{7\cos\alpha} - \dfrac{1}{2}g\left(\dfrac{10}{7\cos\alpha}\right)^2$	M1	1.1b
	Use $\sec^2\alpha = 1+\tan^2\alpha$ oe to give an unsimplified equation in **tan** α **only** : $20 = 40\tan\alpha - \dfrac{1}{2}g\times\dfrac{100}{49}(1+\tan^2\alpha)$	M1	3.1b
	$\tan^2\alpha - 4\tan\alpha + 3 = 0$ * (allow $0 = \tan^2\alpha - 4\tan\alpha + 3$)	A1*	2.2a
		(5)	

5(c)	Solve and use of $\tan\alpha = 3$ or $\sin\alpha = \frac{3}{\sqrt{10}}$ or $\alpha = 71.565..^{o}$ to find an equation in *H* only.	M1	3.1b
	$0 = (28\sin\alpha)^2 - 2gH$ **where** $\tan\alpha = 3 \;\; (\alpha = 71.565..^{o})$	M1	3.4
	H = 36 or 36.0 (m)	A1	1.1b
		(3)	
5(d)	e.g. spin of the ball, the wind, the dimensions or shape of the ball, ball is modelled as a particle, uses an inaccurate value of *g*, motion takes place in 3D not in 2D, *g* could be variable. B0 if mass or weight are mentioned. B0 for ground may not be horizontal.	B1	3.5b
		(1)	

June 2022 Question 5

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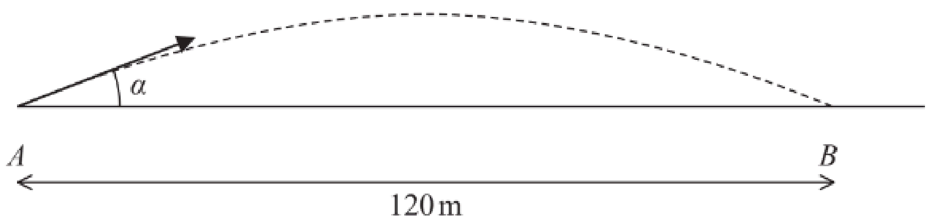

Figure 3

A golf ball is at rest at the point A on horizontal ground.

The ball is hit and initially moves at an angle α to the ground.

The ball first hits the ground at the point B, where $AB = 120$ m, as shown in Figure 3.

The motion of the ball is modelled as that of a particle, moving freely under gravity, whose initial speed is $U \text{ m s}^{-1}$

Using this model,

(a) show that $U^2 \sin\alpha \cos\alpha = 588$ **(6)**

The ball reaches a maximum height of 10 m above the ground.

(b) Show that $U^2 = 1960$ **(4)**

In a refinement to the model, the effect of air resistance is included.

The motion of the ball, from A to B, is now modelled as that of a particle whose initial speed is $V \text{ m s}^{-1}$

This refined model is used to calculate a value for V

(c) State which is greater, U or V, giving a reason for your answer. **(1)**

(d) State one further refinement to the model that would make the model more realistic. **(1)**

ANSWER

Question	Scheme	Marks	AOs
5(a)	Using horizontal motion	M1	3.3
	Whole Motion **Half way**		
	$U\cos\alpha \times t = 120$ $U\cos\alpha \times t = 60$	A1	1.1b
	Using vertical motion **OR**	M1	3.4
	$U\sin\alpha \times t - \frac{1}{2}gt^2 = 0$ $0 = U\sin\alpha - gt$	A1	1.1b
	Attempt to solve problem by eliminating t	DM1	3.1b
	$U^2\sin\alpha\cos\alpha = 588$ *	A1*	2.2a
		(6)	
	N.B. No credit given if they use the given answer from (b).		
5(b)	Using vertical motion **OR** conservation of energy	M1	3.4
	$0^2 = (U\sin\alpha)^2 - 2g\times 10$ $\frac{1}{2}mU^2 - \frac{1}{2}m(U\cos\alpha)^2 = mg\times 10$	A1	1.1b
	Attempt to solve problem by eliminating α : e.g. $U\sin\alpha = 14$ => $U\cos\alpha = 42$, from part (a) or from using $t = \frac{10}{7}$, then square and add to give result **OR:** $U^2\sin^2\alpha = 20g = 196$ and $U^2\sin\alpha\cos\alpha = 588$, divide to give $\tan\alpha = \frac{1}{3}$ then $\sin^2\alpha = \frac{1}{10}$, hence result **OR in ALTERNATIVE 2**: sub for U^2 using part (a), to give $\tan\alpha = \frac{1}{3}$ then $\sin^2\alpha = \frac{1}{10}$, hence result	DM1	3.1b

	N.B. Just stating that $\sin^2 \alpha = \frac{1}{10}$, with no working is DM0A0.		
	$U^2 = 1960$ *	A1*	2.2a
	N.B. Verification (i.e. starting with $U^2 = 1960$ and trying to work backwards) is not an acceptable method for this question.		
		(4)	
5(c)	V, since air resistance has to be overcome, or just 'because of <u>air resistance</u>' isw	B1	3.5a
		(1)	
5(d)	e.g. wind effects, more accurate value of *g*, spin of ball, size of ball, shape of ball, dimensions of ball, not a particle, variable acceleration, surface area of ball, humidity. Allow wind resistance and rotational resistance (Ignore any mention of air resistance or drag)	B1	3.5c
		(1)	
			(12 marks)

November 2021 question 4

4.

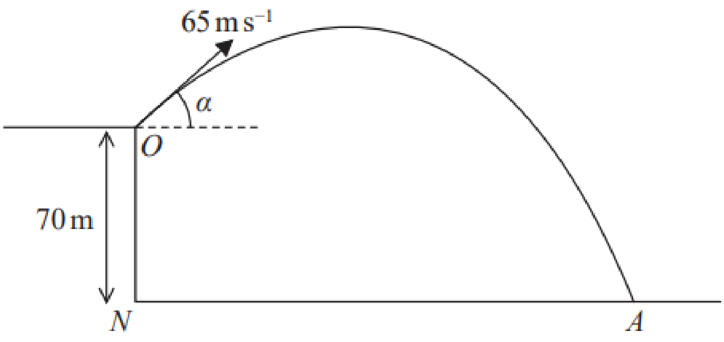

Figure 3

A small stone is projected with speed $65\,\text{m}\,\text{s}^{-1}$ from a point O at the top of a vertical cliff.

Point O is 70 m vertically above the point N.

Point N is on horizontal ground.

The stone is projected at an angle α above the horizontal, where $\tan\alpha = \frac{5}{12}$

The stone hits the ground at the point A, as shown in Figure 3.

The stone is modelled as a particle moving freely under gravity.

The acceleration due to gravity is modelled as having magnitude $10\,\text{m}\,\text{s}^{-2}$

Using the model,

(a) find the time taken for the stone to travel from O to A,

(4)

(b) find the speed of the stone at the instant just before it hits the ground at A.

(5)

One limitation of the model is that it ignores air resistance.

(c) State one other limitation of the model that could affect the reliability of your answers.

(1)

ANSWER

Question	Scheme	Marks	AOs
	Note that $g = 10$; penalise once for whole question if $g = 9.8$		
4(a)	Use $s = ut + \frac{1}{2}at^2$ vertically or any complete method to give an equation in t only	M1	3.4
	$-70 = 65\sin\alpha \times t - \frac{1}{2} \times g \times t^2$	A1	1.1b
		M (A)1	1.1b
	$t = 7$ (s)	A1	1.1b
		(4)	
4(b)	Horizontal velocity component at A = $65\cos\alpha$ (60)	B1	3.4
	Complete method to find vertical velocity component at A	M1	3.4
	$65\sin\alpha - g \times 7$ **OR** $\sqrt{(-25)^2 + 2g \times 70}$ (45)	A1**ft**	1.1b
	Sub for trig and square, add and square root : $\sqrt{60^2 + (-45)^2}$	M1	3.1b
	75 Accept 80 ($\mathrm{m\ s^{-1}}$)	A1	1.1b
		(5)	
4(c)	e.g. an approximate value of g has been used, the dimensions of the stone could affect its motion, spin of the stone, $g = 10$ instead of 9.8 has been used, g has been assumed to be constant, wind effect, shape of the stone	B1	3.5b
		(1)	
			(10 marks)

October 2020 question 5

5.

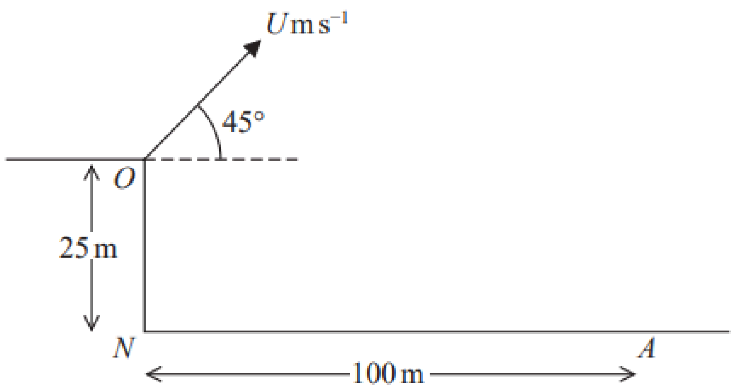

Figure 2

A small ball is projected with speed U m s^{-1} from a point O at the top of a vertical cliff.

The point O is 25 m vertically above the point N which is on horizontal ground.

The ball is projected at an angle of 45° above the horizontal.

The ball hits the ground at a point A, where $AN = 100$ m, as shown in Figure 2.

The motion of the ball is modelled as that of a particle moving freely under gravity.

Using this initial model,

(a) show that $U = 28$

(6)

(b) find the greatest height of the ball above the horizontal ground NA.

(3)

In a refinement to the model of the motion of the ball from O to A, the effect of air resistance is included.

This refined model is used to find a new value of U.

(c) How would this new value of U compare with 28, the value given in part (a)?

(1)

(d) State one further refinement to the model that would make the model more realistic.

(1)

ANSWER

Question	Scheme	Marks	AOs
5(a)	Using horizontal motion	M1	3.3
	$U\cos 45^\circ t = 100$	A1	1.1b
	Using vertical motion	M1	3.4
	$U\sin 45^\circ t - \frac{1}{2}gt^2 = -25$	A1	1.1b
	Solve problem by eliminating t and solving for U	M1	3.1b
	$U = 28$*	A1*	1.1b
		(6)	
5(b)	Using vertical motion	M1	3.4
	$0^2 = (28\sin 45^\circ)^2 - 2gh$	A1	1.1b
	Greatest height = 45 m	A1	1.1b
		(3)	
5(c)	New value > 28	B1	3.5a
		(1)	
5(d)	e.g. wind effects, more accurate value of g, spin of ball, include size of the ball, not model as a particle, shape of ball	B1	3.5c
		(1)	
			(11 marks)

June 2018 Question 10

10.

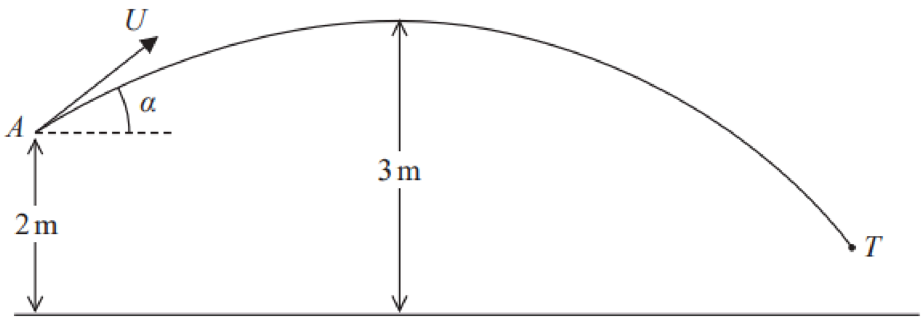

Figure 4

A boy throws a ball at a target. At the instant when the ball leaves the boy's hand at the point A, the ball is 2 m above horizontal ground and is moving with speed U at an angle α above the horizontal.

In the subsequent motion, the highest point reached by the ball is 3 m above the ground. The target is modelled as being the point T, as shown in Figure 4.
The ball is modelled as a particle moving freely under gravity.

Using the model,

(a) show that $U^2 = \dfrac{2g}{\sin^2 \alpha}$.

(2)

The point T is at a horizontal distance of 20 m from A and is at a height of 0.75 m above the ground. The ball reaches T without hitting the ground.

(b) Find the size of the angle α

(9)

(c) State one limitation of the model that could affect your answer to part (b).

(1)

(d) Find the time taken for the ball to travel from A to T.

(3)

ANSWER

Question	Scheme	Marks	AOs
10(a)	Using the model and vertical motion: $0^2=(U\sin\alpha)^2-2g(3-2)$	M1	3.3
	$U^2=\dfrac{2g}{\sin^2\alpha}$ * GIVEN ANSWER	A1*	2.2a
		(2)	
(b)	Using the model and horizontal motion: $s=ut$	M1	3.4
	$20=Ut\cos\alpha$	A1	1.1b
	Using the model and vertical motion: $s=ut+\dfrac{1}{2}at^2$	M1	3.4
	$-\dfrac{5}{4}=Ut\sin\alpha-\dfrac{1}{2}gt^2$	A1	1.1b
	sub for *t*: $-\dfrac{5}{4}=U\sin\alpha\left(\dfrac{20}{U\cos\alpha}\right)-\dfrac{1}{2}g\left(\dfrac{20}{U\cos\alpha}\right)^2$	M1 (I)	3.1b
	sub for U^2	M1(II)	3.1b
	$-\dfrac{5}{4}=20\tan\alpha-100\tan^2\alpha$	A1(I)	1.1b
	$(4\tan\alpha-1)(100\tan\alpha+5)=0$	M1(III)	1.1b
	$\tan\alpha=\dfrac{1}{4}\Rightarrow\ \alpha=14^0$ or better	A1(II)	2.2a
		(9)	

(c)	The target will have dimensions so in practice there would be a range of possible values of α **Or** There will be air resistance **Or** The ball will have dimensions **Or** Wind effects **Or** Spin of the ball	B1	3.5b
		(1)	
(d)	Find U using their α e.g. $U=\sqrt{\dfrac{2g}{\sin^2\alpha}}$	M1	3.1b
	Use $20=Ut\cos\alpha$ (or use vertical motion equation)	A1 **M1**	1.1b
	$t=\dfrac{5}{\sqrt{2g}}$ or 1.1 or 1.13	B1 **A1**	1.1b
		(3)	