Edexcel - Pure Year 2 exam questions

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Ch. 5: Radians

June 2023 Question 4 Paper 1

4. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation $y = \mathrm{f}(x)$ where $x \in \mathbb{R}$

Given that

- $\mathrm{f}'(x) = 2x + \dfrac{1}{2}\cos x$
- the curve has a stationary point with x coordinate α
- α is small

(a) use the small angle approximation for $\cos x$ to estimate the value of α to 3 decimal places.

(3)

The point $P(0, 3)$ lies on C

(b) Find the equation of the tangent to the curve at P, giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(2)

ANSWER

Question	Scheme	Marks	AOs
4a	$2\alpha+\frac{1}{2}\left(1-\frac{\alpha^2}{2}\right)$	M1	1.2
	$2\alpha+\frac{1}{2}\left(1-\frac{\alpha^2}{2}\right)=0 \Rightarrow 2\alpha+\frac{1}{2}-\frac{\alpha^2}{4}=0 \quad \Rightarrow \alpha=\dots$	dM1	1.1b
	$\alpha=-0.243$ (3dp) only	A1	2.3
		(3)	
b	$\mathrm{f}'(0)=\frac{1}{2}\cos 0 \Rightarrow \dots \Rightarrow y=\dots x+3$	M1	1.1b
	$y=\frac{1}{2}x+3$	A1	1.1b
		(2)	
			(5 marks)

June 2023 Question 8 Paper 1

8.

FRONT OF STAGE

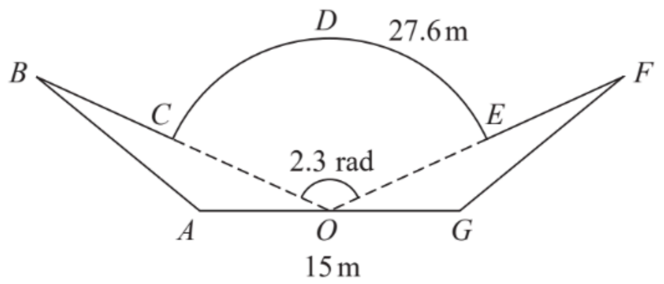


Diagram **NOT** accurately drawn

BACK OF STAGE

Figure 1

Figure 1 shows the plan view of a stage.

The plan view shows two congruent triangles ABO and GFO joined to a sector $OCDEO$ of a circle, centre O, where

- angle $COE = 2.3$ radians
- arc length $CDE = 27.6$ m
- AOG is a straight line of length 15 m

(a) Show that $OC = 12$ m.

(2)

(b) Show that the size of angle AOB is 0.421 radians correct to 3 decimal places.

(2)

Given that the total length of the front of the stage, $BCDEF$, is 35 m,

(c) find the total area of the stage, giving your answer to the nearest square metre.

(6)

ANSWER

Question	Scheme	Marks	AOs
8(a)	$OC \times 2.3 = 27.6$	M1	1.1b
	e.g. $OC = \dfrac{27.6}{2.3} = 12\text{ m}$ *	A1*	2.1
		(2)	
(b)	e.g. $(2AOB =)\ \pi - 2.3$	M1	1.1b
	$\dfrac{\pi - 2.3}{2} \Rightarrow 0.421$ rad *	A1*	2.1
		(2)	
(c)	Area $OCDE = \dfrac{1}{2} \times 12^2 \times 2.3$	M1	1.1b
	$= 165.6\ (\text{m}^2)$ (accept awrt 166)	A1	1.1b
	$(OB =)\ \dfrac{35 - 27.6}{2} + 12 = 15.7\text{ m}$	B1	2.1
	Area of OAB (or OFG) $= \dfrac{1}{2} \times "15.7" \times 7.5 \times \sin 0.421 \quad (= 24.0\ldots\text{m}^2)$	M1	1.1b
	Total area = $"165.6" + 2 \times "24.1"$	dM1	3.1a
	$=$ awrt $214\ (\text{m}^2)$	A1	1.1b
		(6)	
			(10 marks)

Video solution:

https://youtu.be/WWG2r1zfDMo

November 2021 question 4 Paper 2

4. Given that θ is small and measured in radians, use the small angle approximations to show that

$$4\sin\frac{\theta}{2} + 3\cos^2\theta \approx a + b\theta + c\theta^2$$

where a, b and c are integers to be found.

(3)

ANSWER

Question	Scheme	Marks	AOs
4	**Examples:** $4\sin\frac{\theta}{2}\approx 4\left(\frac{\theta}{2}\right),\ 3\cos^2\theta\approx 3\left(1-\frac{\theta^2}{2}\right)^2$ $3\cos^2\theta = 3\left(1-\sin^2\theta\right)\approx 3\left(1-\theta^2\right)$ $3\cos^2\theta = 3\frac{(\cos 2\theta+1)}{2}\approx\frac{3}{2}\left(1-\frac{4\theta^2}{2}+1\right)$	M1	1.1a
	Examples: $4\sin\frac{\theta}{2}+3\cos^2\theta\approx 4\left(\frac{\theta}{2}\right)+3\left(1-\frac{\theta^2}{2}\right)^2$ $4\sin\frac{\theta}{2}+3\cos^2\theta = 4\left(\frac{\theta}{2}\right)+3\left(1-\sin^2\theta\right)\approx 2\theta+3\left(1-\theta^2\right)$ $4\sin\frac{\theta}{2}+3\cos^2\theta = 4\sin\frac{\theta}{2}+3\frac{(\cos 2\theta+1)}{2}\approx 4\left(\frac{\theta}{2}\right)+\frac{3}{2}\left(1-\frac{4\theta^2}{2}+1\right)$	dM1	1.1b
	$=2\theta+3\left(1-\theta^2+\ldots\right)=3+2\theta-3\theta^2$	A1	2.1
		(3)	
			(3 marks)

November 2021 Question 6 Paper 2

6.

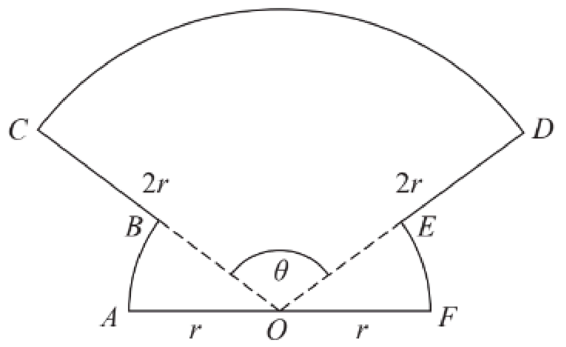


Figure 1

The shape $OABCDEFO$ shown in Figure 1 is a design for a logo.

In the design

- OAB is a sector of a circle centre O and radius r
- sector OFE is congruent to sector OAB
- ODC is a sector of a circle centre O and radius $2r$
- AOF is a straight line

Given that the size of angle COD is θ radians,

(a) write down, in terms of θ, the size of angle AOB

(1)

(b) Show that the area of the logo is

$$\frac{1}{2}r^2(3\theta + \pi)$$

(2)

(c) Find the perimeter of the logo, giving your answer in simplest form in terms of r, θ and π.

(2)

ANSWER

Question	Scheme	Marks	AOs
6(a)	Angle $AOB = \dfrac{\pi-\theta}{2}$	B1	2.2a
		(1)	
(b)	Area = $2\times\frac{1}{2}r^2\left(\frac{\pi-\theta}{2}\right)+\frac{1}{2}(2r)^2\theta$	M1	2.1
	$=\frac{1}{2}r^2\pi-\frac{1}{2}r^2\theta+2r^2\theta=\frac{3}{2}r^2\theta+\frac{1}{2}r^2\pi=\frac{1}{2}r^2(3\theta+\pi)$*	A1*	1.1b
		(2)	
(c)	Perimeter = $4r+2r\left(\frac{\pi-\theta}{2}\right)+2r\theta$	M1	3.1a
	$=4r+r\pi+r\theta$ or e.g. $r(4+\pi+\theta)$	A1	1.1b
		(2)	
			(5 marks)

November 2020 Question 11 Paper 1

11.

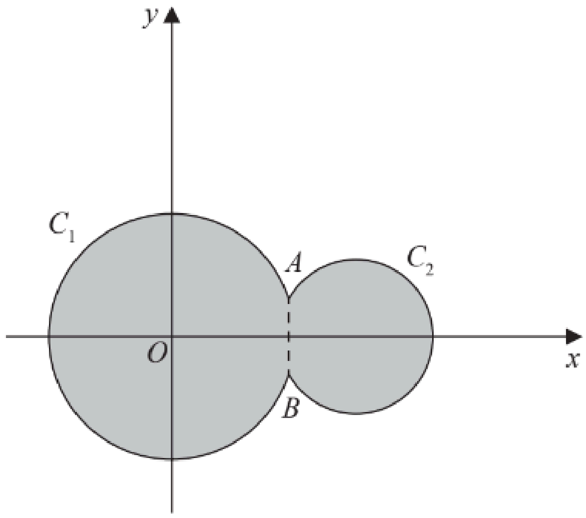


Figure 3

Circle C_1 has equation $x^2 + y^2 = 100$

Circle C_2 has equation $(x - 15)^2 + y^2 = 40$

The circles meet at points A and B as shown in Figure 3.

(a) Show that angle $AOB = 0.635$ radians to 3 significant figures, where O is the origin.

(4)

The region shown shaded in Figure 3 is bounded by C_1 and C_2

(b) Find the perimeter of the shaded region, giving your answer to one decimal place.

(4)

ANSWER

Question	Scheme	Marks	AOs
11 (a)	Solves $x^2+y^2=100$ and $(x-15)^2+y^2=40$ simultaneously to find x or y E.g. $(x-15)^2+100-x^2=40 \Rightarrow x=...$	M1	3.1a
	Either $\Rightarrow -30x+325=40 \Rightarrow x=9.5$ Or $y=\dfrac{\sqrt{39}}{2}=$ awrt ± 3.12	A1	1.1b
	Attempts to find the angle AOB in circle C_1 Eg Attempts $\cos\alpha=\dfrac{"9.5"}{10}$ to find α then $\times 2$	M1	3.1a
	Angle $AOB = 2\times\arccos\left(\dfrac{9.5}{10}\right)=0.635$ rads (3sf) *	A1*	2.1
		(4)	
(b)	Attempts $10\times(2\pi-0.635)$ $= 56.48$	M1	1.1b
	Attempts to find angle AXB or AXO in circle C_2 (see diagram) E.g. $\cos\beta=\dfrac{15-"9.5"}{\sqrt{40}} \Rightarrow \beta=...$ (Note AXB =1.03 rads)	M1	3.1a
	Attempts $10\times(2\pi-0.635)+\sqrt{40}\times(2\pi-2\beta)$	dM1	2.1
	$= 89.7$	A1	1.1b
		(4)	
			(8 marks)

June 2019 Question 2 Paper 1

2.

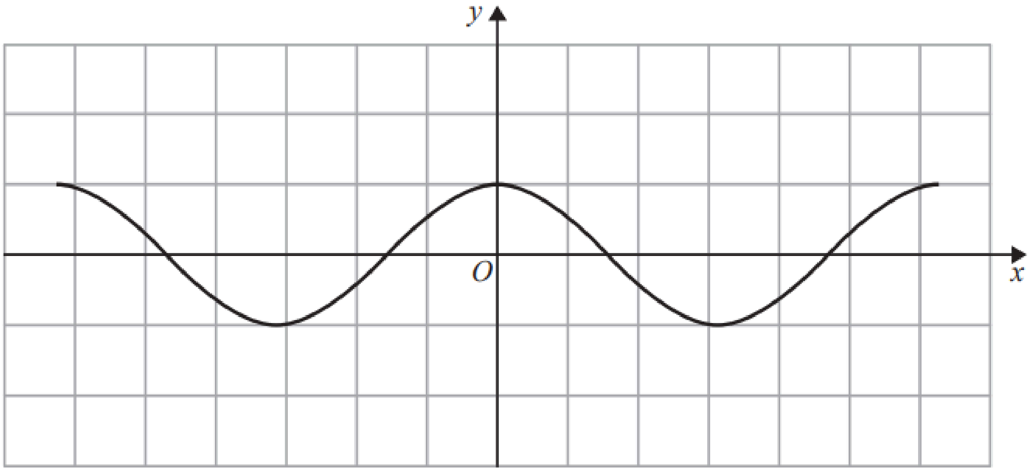


Figure 1

Figure 1 shows a plot of part of the curve with equation $y = \cos x$ where x is measured in radians. Diagram 1, on the opposite page, is a copy of Figure 1.

(a) Use Diagram 1 to show why the equation

$$\cos x - 2x - \frac{1}{2} = 0$$

has only one real root, giving a reason for your answer.

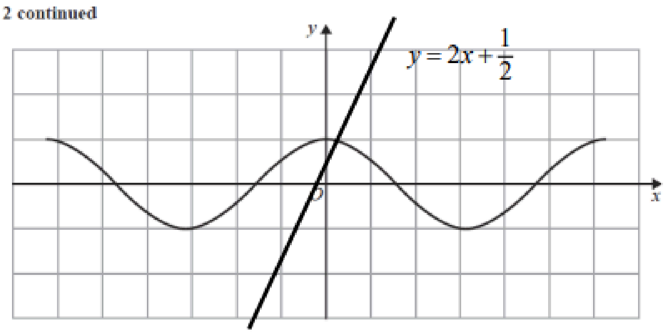
(2)

Given that the root of the equation is α, and that α is small,

(b) use the small angle approximation for $\cos x$ to estimate the value of α to 3 decimal places.

(3)

ANSWER

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="3">2(a)</td><td rowspan="2">
Diagram 1
For an allowable linear graph and explaining that there is only one intersection</td><td>B1</td><td>3.1a</td></tr>
<tr><td>B1</td><td>2.4</td></tr>
<tr><td></td><td>(2)</td><td></td></tr>
<tr><td rowspan="4">(b)</td><td>$\cos x - 2x - \frac{1}{2} = 0 \Rightarrow 1 - \frac{x^2}{2} - 2x - \frac{1}{2} = 0$</td><td>M1</td><td>1.1b</td></tr>
<tr><td>Solves their $x^2 + 4x - 1 = 0$</td><td>dM1</td><td>1.1b</td></tr>
<tr><td>Allow awrt 0.236 but accept $-2 + \sqrt{5}$</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td>(3)</td><td></td></tr>
<tr><td colspan="4" align="right">(5 marks)</td></tr>
</table>

June 2019 Question 3 Paper 2

3.

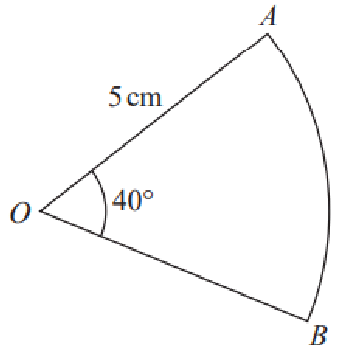


Figure 1

Figure 1 shows a sector AOB of a circle with centre O, radius 5 cm and angle $AOB = 40°$

The attempt of a student to find the area of the sector is shown below.

$$\text{Area of sector} = \frac{1}{2}r^2\theta$$
$$= \frac{1}{2} \times 5^2 \times 40$$
$$= 500\,\text{cm}^2$$

(a) Explain the error made by this student.

(1)

(b) Write out a correct solution.

(2)

ANSWER

Question	Scheme	Marks	AOs
3 (a)	Allow explanations such as • student should have worked in radians • they did not convert degrees to radians • 40 should be in radians • θ should be in radians • angle (or θ) should be $\frac{40\pi}{180}$ or $\frac{2\pi}{9}$ • correct formula is $\pi r^2\left(\frac{\theta}{360}\right)$ {where θ is in degrees} • correct formula is $\pi r^2\left(\frac{40}{360}\right)$	B1	2.3
		(1)	
(b) Way 1	{Area of sector = } $\frac{1}{2}(5^2)\left(\frac{2\pi}{9}\right)$	M1	1.1b
	$= \frac{25}{9}\pi$ {cm^2} or awrt 8.73 {cm^2}	A1	1.1b
		(2)	
(b) Way 2	{Area of sector = } $\pi(5^2)\left(\frac{40}{360}\right)$	M1	1.1b
	$= \frac{25}{9}\pi$ {cm^2} or awrt 8.73 {cm^2}	A1	1.1b
		(2)	
			(3 marks)

June 2018 Question 1 Paper 1

1. Given that θ is small and is measured in radians, use the small angle approximations to find an approximate value of

$$\frac{1 - \cos 4\theta}{2\theta \sin 3\theta}$$

(3)

ANSWER

Question	Scheme	Marks	AOs
1	Attempts either $\sin 3\theta \approx 3\theta$ or $\cos 4\theta \approx 1 - \frac{(4\theta)^2}{2}$ in $\frac{1-\cos 4\theta}{2\theta \sin 3\theta}$	M1	1.1b
	Attempts both $\sin 3\theta \approx 3\theta$ and $\cos 4\theta \approx 1 - \frac{(4\theta)^2}{2} \rightarrow \frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta}$ and attempts to simplify	M1	2.1
	$= \frac{4}{3}$ oe	A1	1.1b
		(3)	
		(3 marks)	

June 2018 Question 3 Paper 1

3.

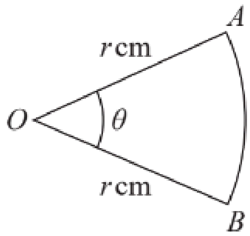


Figure 1

Figure 1 shows a sector AOB of a circle with centre O and radius r cm.

The angle AOB is θ radians.
The area of the sector AOB is 11 cm^2

Given that the perimeter of the sector is 4 times the length of the arc AB, find the exact value of r.

(4)

ANSWER

Question	Scheme	Marks	AOs
3	States or uses $\frac{1}{2}r^2\theta = 11$	B1	1.1b
	States or uses $2r + r\theta = 4r\theta$	B1	1.1b
	Attempts to solve, full method $r = \ldots$	M1	3.1a
	$r = \sqrt{33}$	A1	1.1b
			[4]
			(4 marks)