Statistics Year 1 (AS) exam questions

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Ch. 5: Probability

June 2023 Question 3

3. In an after-school club, students can choose to take part in Art, Music, both or neither.

There are 45 students that attend the after-school club. Of these

- 25 students take part in Art
- 12 students take part in both Art and Music
- the number of students that take part in Music is x

(a) Find the range of possible values of x

(2)

One of the 45 students is selected at random.

Event A is the event that the student selected takes part in Art.

Event M is the event that the student selected takes part in Music.

(b) Determine whether or not it is possible for the events A and M to be independent.

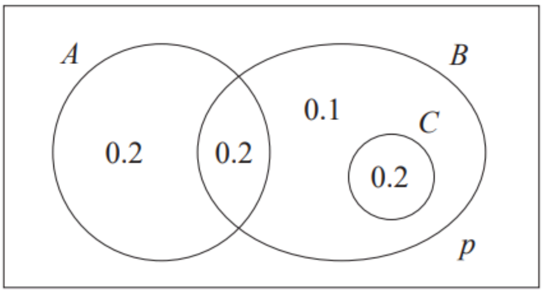
(4)

ANSWER

Que.	Scheme		Marks	AOs
3(a)	$45 - 25 = 20$ or e.g. '$25 \leqslant 13 + 12 + y \leqslant 45$'		M1	2.1
	$12 \leqslant x \leqslant 32$		A1	1.1b
			(2)	
(b)	To be independent $\mathrm{P}(A)\times\mathrm{P}(M) = \mathrm{P}(A \text{ and } M)$		M1	1.1a
	$\mathrm{P}(M) = \dfrac{\mathrm{P}(A \text{ and } M)}{\mathrm{P}(A)} = \dfrac{\frac{12}{45}}{\frac{25}{45}} = \dfrac{12}{25}$ <u>or</u> $\dfrac{25}{45}\times\mathrm{P}(M) = \dfrac{12}{45}$ <u>or</u> $\dfrac{25}{45}\times\dfrac{x}{45} = \dfrac{12}{45}$ <u>or</u> $\dfrac{25}{45}\times\dfrac{12+y}{45} = \dfrac{12}{45}$		A1	2.1
	The number of students taking part in music would be $\dfrac{12}{25}\times 45 = 21.6$	The number of students taking part in music but not art would be $y = 9.6$	A1	1.1b
	…so it is not possible for A and M to be independent (since it must be a whole number).		A1	2.2a
			(4)	
				(6 marks)

November 2021 question 1

1.



The Venn diagram, where p is a probability, shows the 3 events A, B and C with their associated probabilities.

(a) Find the value of p.

(1)

(b) Write down a pair of mutually exclusive events from A, B and C.

(1)

ANSWER

Qu	Scheme	Marks	AO
1 (a)	[$p = 1 - (0.2 + 0.2 + 0.1 + 0.2)$] = $\underline{\mathbf{0.3}}$	B1 **(1)**	1.1b
(b)	A and C are mutually exclusive. [NOT P(A) and P(C)]	B1 **(1)**	1.2
		(2 marks)	

November 2021 question 5

5. Two bags, **A** and **B**, each contain balls which are either red or yellow or green.

Bag **A** contains 4 red, 3 yellow and n green balls.
Bag **B** contains 5 red, 3 yellow and 1 green ball.

A ball is selected at random from bag **A** and placed into bag **B**.
A ball is then selected at random from bag **B** and placed into bag **A**.

The probability that bag **A** now contains an equal number of red, yellow and green balls is p.

Given that $p > 0$, find the possible values of n and p.

(5)

ANSWER

Qu	Scheme	Marks	AO
5	Must end up with 3 of each colour or 4 of each colour	M1	3.1b
	$\underline{\boldsymbol{n = 2}}$ requires 1st red and 2nd green <u>or</u> red from **A** and green from **B**	M1	2.2a
	P(1st red and 2nd green) $= \frac{4}{9} \times \frac{1}{10} = \frac{4}{90}$ or $\frac{2}{45}$ $\quad \boldsymbol{p} = \underline{\underline{\frac{2}{45}}}$	A1	1.1b
	$\underline{\boldsymbol{n = 5}}$ requires 1st green and 2nd yellow <u>or</u> green from **A** and yellow from **B**	M1	2.2a
	P(1st green and 2nd yellow) $= \frac{5}{12} \times \frac{3}{10} = \frac{15}{120}$ or $\frac{1}{8}$ $\quad \boldsymbol{p} = \underline{\underline{\frac{1}{8}}}$	A1	1.1b
		(5)	
		(5 marks)	

October 2020 question 3

3. In a game, a player can score 0, 1, 2, 3 or 4 points each time the game is played.

The random variable S, representing the player's score, has the following probability distribution where a, b and c are constants.

s	0	1	2	3	4
$P(S = s)$	a	b	c	0.1	0.15

The probability of scoring less than 2 points is twice the probability of scoring at least 2 points.

Each game played is independent of previous games played.

John plays the game twice and adds the two scores together to get a total.

Calculate the probability that the total is 6 points.

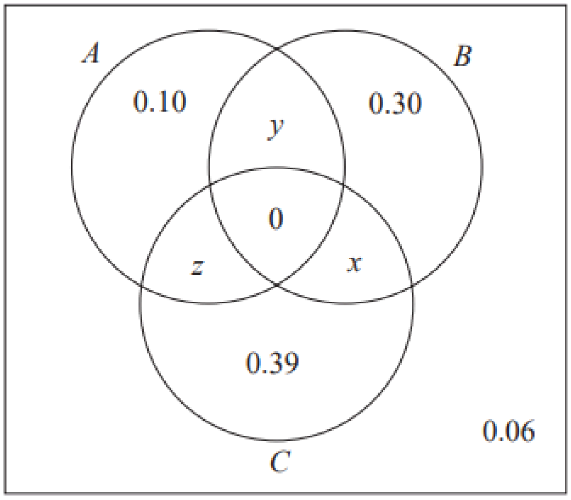
(6)

ANSWER

Question	Scheme	Marks	AOs
3	Overall method	M1	2.1
	$a+b=2c+0.5$ oe **or** $a+b=2(1-a-b)$	B1	2.2a
	$a+b+c=0.75$ oe	B1	1.1b
	$3c=0.25$ $\left[c=0.0833\ldots \text{ or } \frac{1}{12}\right]$	M1	1.1b
	P(scoring 2,4 or 4,2 or 3,3) = $2\times"\frac{1}{12}"\times0.15+0.1^2$	M1	3.1b
	$=0.035$ oe	A1cso	1.1b
		(6)	
			(6 marks)

June 2019 question 2

2. The Venn diagram shows three events, A, B and C, and their associated probabilities.



Events B and C are mutually exclusive.
Events A and C are independent.

Showing your working, find the value of x, the value of y and the value of z.

(5)

ANSWER

Question	Scheme	Marks	AOs
2	$x = 0$	B1	2.2a
	$\mathrm{P}(A) = 0.1 + z + y \quad \mathrm{P}(C) = 0.39 + z[+x] \quad \mathrm{P}(A \text{ and } C) = z$	M1	2.1
	$\mathrm{P}(A \text{ and } C) = \mathrm{P}(A) \times \mathrm{P}(C) \rightarrow z = (0.1 + z + y) \times (0.39 + z[+x])$	M1	1.1b
	$\left[\sum p = 1\right]$ $0.06 + 0.3 + 0.39 + 0.1 + z + y[+x] = 1 \rightarrow \quad [z + y[+x] = 0.15]$	M1	1.1b
	Solving (simultaneously) leading to $\underline{z = 0.13} \quad \underline{y = 0.02}$	A1	1.1b
			(5 marks)

June 2018 Question 2

2. A factory buys 10% of its components from supplier A, 30% from supplier B and the rest from supplier C. It is known that 6% of the components it buys are faulty.

Of the components bought from supplier A, 9% are faulty and of the components bought from supplier B, 3% are faulty.

(a) Find the percentage of components bought from supplier C that are faulty.

(3)

A component is selected at random.

(b) Explain why the event "the component was bought from supplier B" is not statistically independent from the event "the component is faulty".

(1)

ANSWER

Qu	Scheme	Marks	AO
2 (a)	[Let $p = \mathrm{P}(F \mid C)$] Tree diagram or some other method to find an equation for p $0.1\times0.09+0.3\times0.03+0.6\times p=0.06$ $p = 0.07$ i.e. <u>**7%**</u>	 M1 A1 A1 (3)	 2.1 1.1b 1.1b
(b)	e.g. $\mathrm{P}(B \text{ and } F) = 0.3\times0.03=0.009$ but $\mathrm{P}(B)\times\mathrm{P}(F)=0.3\times0.06=0.018$ These are not equal so not independent	 B1 (1)	 2.4
		(4 marks)	

June 2018 Question 3 Paper 3 (A-Level)

3. In an experiment a group of children each repeatedly throw a dart at a target.
For each child, the random variable H represents the number of times the dart hits the target in the first 10 throws.

Peta models H as B(10, 0.1)

(a) State two assumptions Peta needs to make to use her model.

(2)

(b) Using Peta's model, find $P(H \geqslant 4)$

(1)

For each child the random variable F represents the number of the throw on which the dart first hits the target.

Using Peta's assumptions about this experiment,

(c) find $P(F = 5)$

(2)

Thomas assumes that in this experiment no child will need more than 10 throws for the dart to hit the target for the first time. He models $P(F = n)$ as

$$P(F = n) = 0.01 + (n - 1) \times \alpha$$

where α is a constant.

(d) Find the value of α

(4)

(e) Using Thomas' model, find $P(F = 5)$

(1)

(f) Explain how Peta's and Thomas' models differ in describing the probability that a dart hits the target in this experiment.

(1)

ANSWER

Qu 3	Scheme	Marks	AO
(a)	The probability of a dart hitting the target is constant (from child to child and for each throw by each child) (o.e.)	B1	1.2
	The throws of each of the darts are independent (o.e.)	B1	1.2
		(2)	
(b)	$[\mathrm{P}(H \geqslant 4) = 1 - \mathrm{P}(H \leqslant 3) = 1 - 0.9872 = 0.012795.. =]$ awrt **0.0128**	B1	1.1b
		(1)	
(c)	$\mathrm{P}(F = 5) = 0.9^4 \times 0.1, = 0.06561$	M1,	3.4
	$=$ awrt **0.0656**	A1	1.1b
		(2)	
(d)	<table><tr><td>n</td><td>1</td><td>2</td><td>…</td><td>10</td></tr><tr><td>$\mathrm{P}(F = n)$</td><td>0.01</td><td>$0.01 + \alpha$</td><td>…</td><td>$0.01+9\alpha$</td></tr></table>	M1	3.1b
	Sum of probs = 1 $\Rightarrow \frac{10}{2}[2\times0.01+9\alpha]=1$	M1A1	3.1a 1.1b
	[i.e. $5(0.02 + 9\alpha) = 1$ or $0.1 + 45\alpha = 1$] so α = **0.02**	A1	1.1b
		(4)	
(e)	$\mathrm{P}(F = 5 \mid$ Thomas' model) = **0.09**	B1ft	3.4
		(1)	
(f)	Peta's model assumes the probability of hitting target is constant (o.e.) **and** Thomas' model assumes this probability increases with each attempt(o.e.)	B1	3.5a
		(1)	
		(11 marks)	