Pure Year 1 exam questions Edexcel

Arancha Ruiz

Ch. 14: Exponentials and Logarithms

June 2023 Question 11

11. The height, h metres, of a plant, t years after it was first measured, is modelled by the equation

$$h = 2.3 - 1.7e^{-0.2t} \qquad t \in \mathbb{R} \quad t \geqslant 0$$

Using the model,

(a) find the height of the plant when it was first measured,

(2)

(b) show that, exactly 4 years after it was first measured, the plant was growing at approximately 15.3 cm per year.

(3)

According to the model, there is a limit to the height to which this plant can grow.

(c) Deduce the value of this limit.

(1)

ANSWER

Question	Scheme	Marks	AOs
11(a)	$h = 2.3 - 1.7e^{0}$	M1	3.4
	Either 0.6 {m} or 60 cm	A1	1.1b
		(2)	
(b)	$\left\{\frac{dh}{dt} =\right\} 0.34e^{-0.2t}$	M1	3.1b
	At $t = 4 \Rightarrow$ Rate of growth is $0.34e^{-0.2\times4} = 0.15277...\{\text{m / year}\}$	dM1	3.4
	0.153 {m per year} = 15.3 cm {per year} *	A1*	1.1b
		(3)	
(c)	2.3 (m)	B1	2.2a
		(1)	
		(6 marks)	

June 2023 Question 9

9. Using the laws of logarithms, solve the equation

$$2\log_5(3x - 2) - \log_5 x = 2$$

(5)

ANSWER

Question	Scheme	Marks	AOs
9	$2\log_5(3x-2)-\log_5 x=2$		
	Uses one correct law e.g. $2\log_5(3x-2)\to\log_5(3x-2)^2$ **or** $2\to\log_5 25$ **or** $\log_5 ...=2\to ...=5^2$	B1	1.1a
	Uses two correct log laws: either $2\log_5(3x-2)\to\log_5(3x-2)^2$ **and** $2\to\log_5 25$ or $2\log_5(3x-2)-\log_5 x\to\log_5\dfrac{(3x-2)^2}{x}$ leading to an equation without logs	M1	3.1a
	Correct equation without logs, usually $\dfrac{(3x-2)^2}{x}=25$	A1	1.1b
	$\dfrac{(3x-2)^2}{x}=25\Rightarrow 9x^2-37x+4=0\Rightarrow(9x-1)(x-4)=0\Rightarrow x=...$	dM1	1.1b
	$x=4$ only	A1 cso	3.2a
		(5)	
			(5 marks)

Arancha Ruiz

June 2023 Question 4 Paper 2 (A-Level)

4. Coffee is poured into a cup.

The temperature of the coffee, H °C, t minutes after being poured into the cup is modelled by the equation

$$H = Ae^{-Bt} + 30$$

where A and B are constants.

Initially, the temperature of the coffee was 85 °C.

(a) State the value of A.

(1)

Initially, the coffee was cooling at a rate of 7.5 °C per minute.

(b) Find a complete equation linking H and t, giving the value of B to 3 decimal places.

(3)

ANSWER

Question	Scheme	Marks	AOs
4(a)	$(A=)\,55$	**B1**	3.4
		(1)	
(b)	$\left\{\frac{dH}{dt}=\right\}-ABe^{-Bt}$ or $\left\{\frac{dH}{dt}=\right\}-\text{"55"}Be^{-Bt}$	**M1**	3.1b
	$-B\times\text{"55"}=-7.5 \Rightarrow B=\ldots\ \left(\frac{3}{22}=\text{awrt } 0.136\right)$	**M1**	1.1b
	$H=55e^{-0.136t}+30$	**A1cso**	3.3
		(3)	
			(4 marks)

June 2023 Question 3 Paper 2 (A-Level)

3. Given that

$$\log_2(x+3) + \log_2(x+10) = 2 + 2\log_2 x$$

(a) show that

$$3x^2 - 13x - 30 = 0$$

(3)

(b) (i) Write down the roots of the equation

$$3x^2 - 13x - 30 = 0$$

(ii) Hence state which of the roots in part (b)(i) is not a solution of

$$\log_2(x+3) + \log_2(x+10) = 2 + 2\log_2 x$$

giving a reason for your answer.

(2)

ANSWER

Question	Scheme	Marks	AOs
3(a)	Uses one correct log law e.g. $\log_2(x+3)+\log_2(x+10)=\log_2(x+3)(x+10)$ $2=\log_2 4,\quad 2\log_2 x=\log_2 x^2$	**M1**	1.1b
	$(x+3)(x+10)=4x^2$ oe	**dM1**	2.1
	$\Rightarrow 3x^2-13x-30=0$ *	**A1***	1.1b
		(3)	
(b)(i)	$(x=)\ 6,\ -\frac{5}{3}$	**B1**	1.1b
(ii)	$x\neq-\frac{5}{3}$ because $\log_{(2)}\left(-\frac{5}{3}\right)$ is not real	**B1**	2.4
		(2)	
			(5 marks)

June 2023 Question 11 Paper 1 (A-Level)

11.

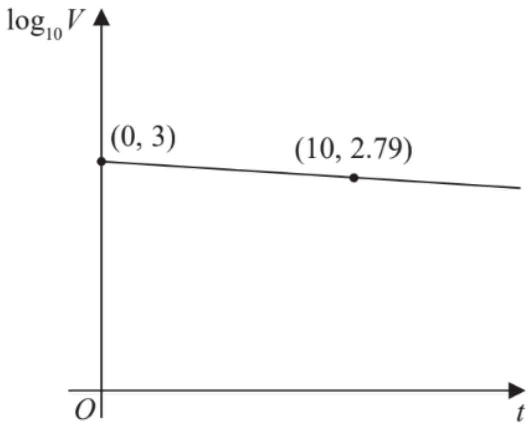


Figure 2

The value, V pounds, of a mobile phone, t months after it was bought, is modelled by

$$V = ab^t$$

where a and b are constants.

Figure 2 shows the linear relationship between $\log_{10} V$ and t.

The line passes through the points (0, 3) and (10, 2.79) as shown.

Using these points,

(a) find the initial value of the phone,

(2)

(b) find a complete equation for V in terms of t, giving the exact value of a and giving the value of b to 3 significant figures.

(3)

Exactly 2 years after it was bought, the value of the phone was £320

(c) Use this information to evaluate the reliability of the model.

(2)

ANSWER

Question	Scheme	Marks	AOs
11(a)	$\log_{10} V = 3 \Rightarrow V = 10^3$	M1	1.1b
	$(V =)\ £1000$	A1	3.4
		(2)	
(b)	e.g. $(\log_{10} b =)\ \dfrac{2.79-3}{10-0} = -0.021$ **or** $\log_{10} V = 3 - 0.021t$ **or** $10^{2.79} = "1000"b^{10}$	M1	1.1b
	e.g. $b = 10^{-0.021}\ (= 0.952796....)$ **or** $V = 10^3 \times 10^{-0.021t}$ **or** $b = \sqrt[10]{"0.61659..."}$	M1	3.1b
	$V = 1000 \times 0.953^t$	A1ft	3.3
		(3)	
(c)	e.g. $V = 1000 \times "0.953"^{24}\ (= £315)$ or e.g. $\log_{10} V = 3 - "0.021" \times 24 \Rightarrow V = ...(= £313)$	M1	3.4
	which is close (to £320) so it is a suitable model	A1	3.2b
		(2)	
			(7 marks)

June 2023 Question 6 Paper 1 (A-Level)

6. $a = \log_2 x \qquad b = \log_2(x + 8)$

Express in terms of a and/or b

(a) $\log_2 \sqrt{x}$ **(1)**

(b) $\log_2(x^2 + 8x)$ **(2)**

(c) $\log_2\left(8 + \dfrac{64}{x}\right)$

Give your answer in simplest form. **(3)**

ANSWER

Question	Scheme	Marks	AOs
6(a)	$\frac{1}{2}a$	B1	2.2a
		(1)	
(b)	$\log_2 x(x+8) \Rightarrow \log_2 x + \log_2(x+8)$	M1	1.2
	$= a + b$	A1	2.2a
		(2)	
(c)	e.g. $8 + \frac{64}{x} = \frac{8x+64}{x}$	B1	1.1b
	$\log_2 \frac{8}{x}(x+8) = 3 - \log_2 x + \log_2(x+8)$	M1	1.1b
	$3 + b - a$	A1	2.2a
		(3)	
			(6 marks)

June 2022 Question 2 Paper 2 (A Level)

2. (a) Sketch the curve with equation

$$y = 4^x$$

stating any points of intersection with the coordinate axes.

(2)

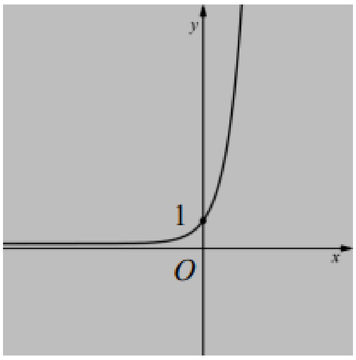
(b) Solve

$$4^x = 100$$

giving your answer to 2 decimal places.

(2)

ANSWER

<table>
<tr><th>Question</th><th colspan="2">Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="3">2(a)</td><td rowspan="2">
</td><td>Correct shape or correct intercept – see notes</td><td>B1</td><td>1.2</td></tr>
<tr><td>Fully correct – see notes</td><td>B1</td><td>1.1b</td></tr>
<tr><td colspan="2"></td><td>(2)</td><td></td></tr>
<tr><td rowspan="3">(b)</td><td colspan="2">$4^x = 100 \Rightarrow x = \log_4 100$
or
e.g. $x\log 4 = \log 100 \Rightarrow x = \dfrac{\log 100}{\log 4}$</td><td>M1</td><td>1.1b</td></tr>
<tr><td colspan="2">$\Rightarrow (x =)$ awrt 3.32</td><td>A1</td><td>1.1b</td></tr>
<tr><td colspan="2"></td><td>(2)</td><td></td></tr>
<tr><td colspan="5">(4 marks)</td></tr>
</table>

June 2022 Question 10 Paper 1 (A Level)

10. A scientist is studying the number of bees and the number of wasps on an island.

The number of bees, measured in thousands, N_b, is modelled by the equation

$$N_b = 45 + 220\,e^{0.05t}$$

where t is the number of years from the start of the study.

According to the model,

(a) find the number of bees at the start of the study,

(1)

(b) show that, exactly 10 years after the start of the study, the number of bees was increasing at a **rate** of approximately 18 thousand per year.

(3)

The number of wasps, measured in thousands, N_w, is modelled by the equation

$$N_w = 10 + 800\,e^{-0.05t}$$

where t is the number of years from the start of the study.

When $t = T$, according to the models, there are an equal number of bees and wasps.

(c) Find the value of T to 2 decimal places.

(4)

ANSWER

Question	Scheme	Marks	AOs
10 (a)	265 thousand	B1	3.4
		(1)	
(b)	Attempts $\frac{dN_b}{dt} = 11e^{0.05t}$	M1	1.1b
	Substitutes $t = 10$ into their $\frac{dN_b}{dt}$	M1	3.4
	$\frac{dN_b}{dt} =$ awrt 18.1 which is approximately 18 thousand per year *	A1*	2.1
		(3)	
(c)	Sets $45 + 220e^{0.05t} = 10 + 800e^{-0.05t} \Rightarrow 220e^{0.05t} + 35 - 800e^{-0.05t} = 0$	M1	3.1b
	Correct quadratic equation $\Rightarrow 220\left(e^{0.05t}\right)^2 + 35e^{0.05t} - 800 = 0$	A1	1.1b
	$e^{0.05t} = 1.829, (-1.988) \Rightarrow 0.05t = \ln(1.829)$	M1	2.1
	$T = 12.08$	A1	1.1b
		(4)	
			(8 marks)

Video solution:

https://youtu.be/Qc-RPb6Z1OU

June 2022 Question 9

9. (a) Given that $p = \log_3 x$, where $x > 0$, find in simplest form in terms of p,

(i) $\log_3\left(\frac{x}{9}\right)$

(ii) $\log_3\left(\sqrt{x}\right)$

(2)

(b) Hence, or otherwise, solve

$$2\log_3\left(\frac{x}{9}\right) + 3\log_3\left(\sqrt{x}\right) = -11$$

giving your answer as a simplified fraction.

Solutions relying on calculator technology are not acceptable.

(4)

ANSWER

Question	Scheme	Marks	AOs
9(a)(i)	$\log_3\left(\frac{x}{9}\right) = \log_3 x - \log_3 9 = p - 2$	B1	1.2
(ii)	$\log_3\left(\sqrt{x}\right) = \frac{1}{2}p$	B1	1.1b
		(2)	
(b)	$2\log_3\left(\frac{x}{9}\right) + 3\log_3\left(\sqrt{x}\right) = -11 \Rightarrow 2p - 4 + \frac{3}{2}p = -11 \Rightarrow p = ...$	M1	1.1b
	$p = -2$	A1	1.1b
	$\log_3 x = -2 \Rightarrow x = 3^{-2}$	M1	1.1b
	$x = \frac{1}{9}$	A1	1.1b
		(4)	

Video solution:
https://youtu.be/-IE3ybfqRvg

June 2022 Question 8

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The air pressure, P kg/cm^2, inside a car tyre, t minutes from the instant when the tyre developed a puncture is given by the equation

$$P = k + 1.4e^{-0.5t} \qquad t \in \mathbb{R} \qquad t \geqslant 0$$

where k is a constant.

Given that the initial air pressure inside the tyre was 2.2 kg/cm^2

(a) state the value of k.

(1)

From the instant when the tyre developed the puncture,

(b) find the time taken for the air pressure to fall to 1 kg/cm^2
Give your answer in minutes to one decimal place.

(3)

(c) Find the rate at which the air pressure in the tyre is decreasing exactly 2 minutes from the instant when the tyre developed the puncture.
Give your answer in kg/cm^2 per minute to 3 significant figures.

(2)

ANSWER

Question	Scheme	Marks	AOs
8(a)	$(k =)\ 0.8$	B1	1.1b
		(1)	
(b)	$1 = 0.8 + 1.4\mathrm{e}^{-0.5t} \Rightarrow 1.4\mathrm{e}^{-0.5t} = 0.2$	M1	3.1b
	$-0.5t = \ln\left(\dfrac{0.2}{1.4}\right) \Rightarrow t = \ldots$	M1	1.1b
	awrt 3.9 minutes	A1	1.1b
		(3)	
(c)	$\left(\dfrac{\mathrm{d}P}{\mathrm{d}t} =\right) -0.7\mathrm{e}^{-0.5t}$ $\left(\dfrac{\mathrm{d}P}{\mathrm{d}t}\right)_{t=2} = -0.7\mathrm{e}^{-0.5\times 2}$	M1	3.1b
	$=$ awrt 0.258 (kg/cm^2 per minute)	A1	1.1b
		(2)	
			(6 marks)

Video solution:
https://youtu.be/Dnw1ESD5AhQ

June 2022 Question 5

5. The mass, A kg, of algae in a small pond, is modelled by the equation

$$A = pq^t$$

where p and q are constants and t is the number of weeks after the mass of algae was first recorded.

Data recorded indicates that there is a linear relationship between t and $\log_{10} A$ given by the equation

$$\log_{10} A = 0.03t + 0.5$$

(a) Use this relationship to find a complete equation for the model in the form

$$A = pq^t$$

giving the value of p and the value of q each to 4 significant figures.

(4)

(b) With reference to the model, interpret

(i) the value of the constant p,

(ii) the value of the constant q.

(2)

(c) Find, according to the model,

(i) the mass of algae in the pond when $t = 8$, giving your answer to the nearest 0.5 kg,

(ii) the number of weeks it takes for the mass of algae in the pond to reach 4 kg.

(3)

(d) State one reason why this may not be a realistic model in the long term.

(1)

ANSWER

Question	Scheme	Marks	AOs
5(a)	$p = 10^{0.5}$ (or $\log_{10} p = 0.5$) or $q = 10^{0.03}$ (or $\log_{10} q = 0.03$)	M1	1.1b
	$p =$ awrt 3.162 or $q =$ awrt 1.072	A1	1.1b
	$p = 10^{0.5}$ (or $\log_{10} p = 0.5$) and $q = 10^{0.03}$ (or $\log_{10} q = 0.03$)	dM1	3.1a
	$A = 3.162 \times 1.072^t$	A1	3.3
		(4)	
(b)(i)	The initial mass (in kg) of algae (in the pond).	B1	3.4
(b)(ii)	The ratio of algae from one week to the next.	B1	3.4
		(2)	
(c)(i)	5.5 kg	B1	2.2a
(c)(ii)	$4 = \text{"}3.162\text{"} \times \text{"}1.072\text{"}^t$ or $\log_{10} 4 = 0.03t + 0.5$	M1	3.4
	awrt 3.4 (weeks)	A1	1.1b
		(3)	
(d)	• The model predicts unlimited growth. • The weather may affect the rate of growth	B1	3.5b
		(1)	
		(10 marks)	

Video solution:
https://youtu.be/BxMnlf4OtqE

November 2021 Question 11

11. The owners of a nature reserve decided to increase the area of the reserve covered by trees.

Tree planting started on 1st January 2005.

The area of the nature reserve covered by trees, A km^2, is modelled by the equation

$$A = 80 - 45e^{ct}$$

where c is a constant and t is the number of years after 1st January 2005.

Using the model,

(a) find the area of the nature reserve that was covered by trees just before tree planting started.

(1)

On 1st January 2019 an area of $60\,\text{km}^2$ of the nature reserve was covered by trees.

(b) Use this information to find a complete equation for the model, giving your value of c to 3 significant figures.

(4)

On 1st January 2020, the owners of the nature reserve announced a long-term plan to have $100\,\text{km}^2$ of the nature reserve covered by trees.

(c) State a reason why the model is not appropriate for this plan.

(1)

ANSWER

Question	Scheme	Marks	AOs
11 (a)	$35\ (\text{km}^2)$	B1	3.4
		(1)	
(b)	Sets their $60 = 80 - 45e^{14c} \Rightarrow 45e^{14c} = 20$	M1 A1	1.1b 1.1b
	$\Rightarrow c = \frac{1}{14}\ln\left(\frac{20}{45}\right) = \cdots -0.0579$	dM1	3.1b
	$A = 80 - 45e^{-0.0579t}$	A1	3.3
		(4)	
(c)	Gives a suitable answer • The maximum area covered by trees is only 80km^2 • The "80" would need to be "100" • Substitutes 100 into the equation of the model and shows that the formula fails with a reason eg. you cannot take a log of a negative number	B1	3.5b
		(1)	
			(6 marks)

November 2021 Question 13

13.

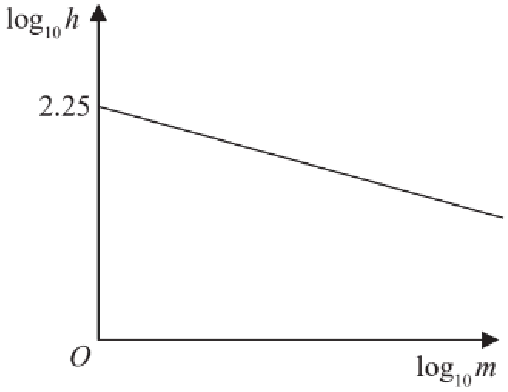


Figure 2

The resting heart rate, h, of a mammal, measured in beats per minute, is modelled by the equation

$$h = pm^q$$

where p and q are constants and m is the mass of the mammal measured in kg.

Figure 2 illustrates the linear relationship between $\log_{10} h$ and $\log_{10} m$

The line meets the vertical $\log_{10} h$ axis at 2.25 and has a gradient of -0.235

(a) Find, to 3 significant figures, the value of p and the value of q.

(3)

A particular mammal has a mass of 5 kg and a resting heart rate of 119 beats per minute.

(b) Comment on the suitability of the model for this mammal.

(3)

(c) With reference to the model, interpret the value of the constant p.

(1)

ANSWER

Question	Scheme		Marks	AOs
13 (a)	$\log_{10} h = 2.25 - 0.235\log_{10} m$ $\Rightarrow h = 10^{2.25-0.235\log_{10} m}$ $\Rightarrow h = 10^{2.25} \times m^{-0.235}$	$h = pm^q$ $\Rightarrow \log_{10} h = \log_{10} p + \log_{10} m^q$ $\Rightarrow \log_{10} h = \log_{10} p + q\log_{10} m$	M1	1.1b
	Either one of $p = 10^{2.25}$ $q = -0.235$	Or either one of $\log_{10} p = 2.25$ $q = -0.235$	A1	1.1b
	$\Rightarrow p = 178$ and $q = -0.235$		A1	2.2a
			(3)	
(b)	$h = "178" \times 5^{"-0.235"}$	$\log_{10} h = "2.25" - "0.235"\log_{10} 5$	M1	3.1b
	$h = 122$	$h = 122$	A1	1.1b
	Reasonably accurate (to 2 sf) so suitable		A1ft	3.2b
			(3)	
(c)	"p" would be the (resting) heart rate (in bpm) of a mammal with a mass of 1 kg		B1	3.4
			(1)	
				(7 marks)

November 2021 Question 8 Paper 1 (A-Level)

8. A scientist is studying the growth of two different populations of bacteria.

The number of bacteria, N, in the **first** population is modelled by the equation

$$N = Ae^{kt} \qquad t \geqslant 0$$

where A and k are positive constants and t is the time in hours from the start of the study.

Given that

- there were 1000 bacteria in this population at the start of the study
- it took exactly 5 hours from the start of the study for this population to double

(a) find a complete equation for the model.

(4)

(b) Hence find the rate of increase in the number of bacteria in this population exactly 8 hours from the start of the study. Give your answer to 2 significant figures.

(2)

The number of bacteria, M, in the **second** population is modelled by the equation

$$M = 500e^{1.4kt} \qquad t \geqslant 0$$

where k has the value found in part (a) and t is the time in hours from the start of the study.

Given that T hours after the start of the study, the number of bacteria in the two different populations was the same,

(c) find the value of T.

(3)

ANSWER

Question	Scheme	Marks	AOs
8(a)	$A = 1000$	B1	3.4
	$2000 = 1000e^{5k}$ or $e^{5k} = 2$	M1	1.1b
	$e^{5k} = 2 \Rightarrow 5k = \ln 2 \Rightarrow k = \ldots$	M1	2.1
	$N = 1000e^{\left(\frac{1}{5}\ln 2\right)t}$ or $N = 1000e^{0.139t}$	A1	3.3
		(4)	
(b)	$\frac{dN}{dt} = 1000 \times \left(\frac{1}{5}\ln 2\right)e^{\left(\frac{1}{5}\ln 2\right)t}$ or $\frac{dN}{dt} = 1000 \times 0.139e^{0.139t}$ $\left(\frac{dN}{dt}\right)_{t=8} = 1000 \times \left(\frac{1}{5}\ln 2\right)e^{8\times\frac{1}{5}\ln 2}$ or $\left(\frac{dN}{dt}\right)_{t=8} = 1000 \times 0.139e^{0.139\times 8}$	M1	3.1b
	$=$ awrt 420	A1	1.1b
		(2)	
(c)	$500e^{1.4\times\left(\frac{1}{5}\ln 2\right)T} = 1000e^{\left(\frac{1}{5}\ln 2\right)T}$ or $500e^{1.4\times"0.139"t} = 1000e^{"0.139"t}$	M1	3.4
	Correct method of getting a linear equation in T E.g. $0.08T\ln 2 = \ln 2$ or $1.4\times"0.339"T = \ln 2 + "0.339"t$	M1	2.1
	$T = 12.5$ hours	A1	1.1b
		(3)	
			(9 marks)

November 2021 Question 3 Paper 2 (A-Level)

3. Using the laws of logarithms, solve the equation

$$\log_3 (12y + 5) - \log_3 (1 - 3y) = 2$$

(3)

ANSWER

Question	Scheme	Marks	AOs
3	$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \log_3\dfrac{12y+5}{1-3y}=2$ or e.g. $2=\log_3 9$	B1 M1 on EPEN	1.1b
	$\log_3\dfrac{12y+5}{1-3y}=2\Rightarrow\dfrac{12y+5}{1-3y}=3^2\Rightarrow 9-27y=12y+5\Rightarrow y=...$ or e.g. $\log_3(12y+5)=\log_3(3^2(1-3y))\Rightarrow(12y+5)=3^2(1-3y)\Rightarrow y=...$	M1	2.1
	$y=\dfrac{4}{39}$	A1	1.1b
		(3)	
			(3 marks)

November 2021 Question 10 Paper 2 (A-Level)

10. The time, T seconds, that a pendulum takes to complete one swing is modelled by the formula

$$T = al^b$$

where l metres is the length of the pendulum and a and b are constants.

(a) Show that this relationship can be written in the form

$$\log_{10} T = b\log_{10} l + \log_{10} a$$

(2)

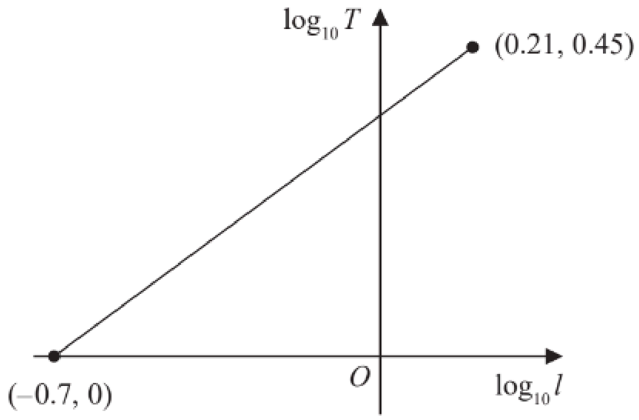


Figure 3

A student carried out an experiment to find the values of the constants a and b.

The student recorded the value of T for different values of l.

Figure 3 shows the linear relationship between $\log_{10} l$ and $\log_{10} T$ for the student's data. The straight line passes through the points (–0.7, 0) and (0.21, 0.45)

Using this information,

(b) find a complete equation for the model in the form

$$T = al^b$$

giving the value of a and the value of b, each to 3 significant figures.

(3)

(c) With reference to the model, interpret the value of the constant a.

(1)

ANSWER

Question	Scheme	Marks	AOs
10(a)	$T = al^b \Rightarrow \log_{10} T = \log_{10} a + \log_{10} l^b$	M1	2.1
	$\Rightarrow \log_{10} T = \log_{10} a + b\log_{10} l$ * or $\Rightarrow \log_{10} T = b\log_{10} l + \log_{10} a$ *	A1*	1.1b
		(2)	
(b)	$b = 0.495$ or $b = \dfrac{45}{91}$	B1	2.2a
	$0 = \text{"}0.495\text{"} \times -0.7 + \log_{10} a \Rightarrow a = 10^{0.346...}$ or $0.45 = \text{"}0.495\text{"} \times 0.21 + \log_{10} a \Rightarrow a = 10^{0.346...}$	M1	3.1a
	$T = 2.22l^{0.495}$	A1	3.3
		(3)	
(c)	The time taken for one swing of a pendulum of length 1 m	B1	3.2a
		(1)	
			(6 marks)

November 2020 Question 8

8. The temperature, θ°C, of a cup of tea t minutes after it was placed on a table in a room, is modelled by the equation

$$\theta = 18 + 65e^{-\frac{t}{8}} \qquad t \geqslant 0$$

Find, according to the model,

(a) the temperature of the cup of tea when it was placed on the table,

(1)

(b) the value of t, to one decimal place, when the temperature of the cup of tea was 35 °C.

(3)

(c) Explain why, according to this model, the temperature of the cup of tea could not fall to 15 °C.

(1)

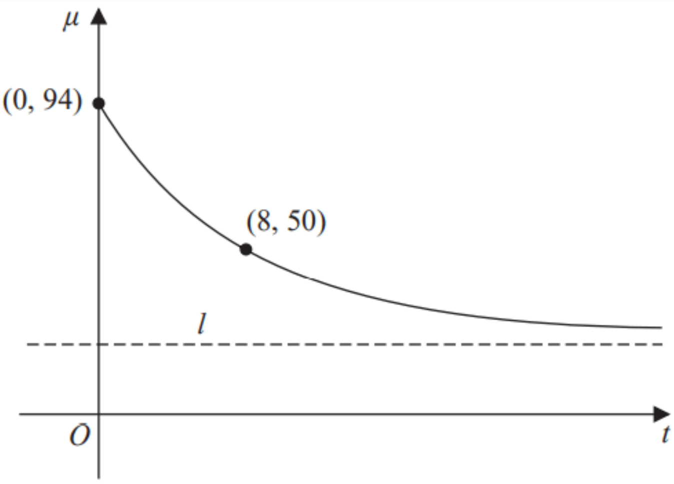


Figure 2

The temperature, μ°C, of a second cup of tea t minutes after it was placed on a table in a different room, is modelled by the equation

$$\mu = A + Be^{-\frac{t}{8}} \qquad t \geqslant 0$$

where A and B are constants.

Figure 2 shows a sketch of μ against t with two data points that lie on the curve.

The line l, also shown on Figure 2, is the asymptote to the curve.

Using the equation of this model and the information given in Figure 2

(d) find an equation for the asymptote l.

(4)

ANSWER

Question	Scheme		Marks	AOs
8 (a)	Temperature = 83°C		B1	3.4
			(1)	
(b)	$18+65e^{-\frac{t}{8}}=35 \Rightarrow 65e^{-\frac{t}{8}}=17$		M1	1.1b
	$t=-8\ln\left(\frac{17}{65}\right)$	$\ln 65-\frac{t}{8}=\ln 17 \Rightarrow t=...$	dM1	1.1b
	$t=10.7$		A1	1.1b
			(3)	
(c)	States a suitable reason • As $t \to \infty, \theta \to 18$ from above. • The minimum temperature is 18°C		B1	2.4
			(1)	
(d)	$A+B=94$ or $A+Be^{-1}=50$		M1	3.4
	$A+B=94$ and $A+Be^{-1}=50$		A1	1.1b
	Full method to find at least a value for A		dM1	2.1
	Deduces $\mu=\dfrac{50e-94}{e-1}$ or accept $\mu=$ awrt 24.4		A1	2.2a
			(4)	
			(9 marks)	

November 2020 Question 2 Paper 1 (A-Level)

2. By taking logarithms of both sides, solve the equation

$$4^{3p-1} = 5^{210}$$

giving the value of p to one decimal place.

(3)

ANSWER

Question	Scheme	Marks	AOs
2	$4^{3p-1} = 5^{210} \Rightarrow (3p-1)\log 4 = 210\log 5$	M1	1.1b
	$\Rightarrow 3p = \dfrac{210\log 5}{\log 4} + 1 \Rightarrow p = ...$	dM1	2.1
	$p = \text{awrt } 81.6$	A1	1.1b
		(3)	
			(3 marks)

November 2020 Question 3 Paper 2 (A-Level)

3. (a) Given that

$$2\log(4-x) = \log(x+8)$$

show that

$$x^2 - 9x + 8 = 0$$

(3)

(b) (i) Write down the roots of the equation

$$x^2 - 9x + 8 = 0$$

(ii) State which of the roots in (b)(i) is **not** a solution of

$$2\log(4-x) = \log(x+8)$$

giving a reason for your answer.

(2)

ANSWER

3(a)	$2\log(4-x)=\log(4-x)^2$	B1	1.2
	$2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2=\log(x+8)$ $(4-x)^2=(x+8)$ **or** $2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2-\log(x+8)=0$ $\dfrac{(4-x)^2}{(x+8)}=1$	M1	1.1b
	$16-8x+x^2=x+8\Rightarrow x^2-9x+8=0\ *$	A1*	2.1
		(3)	
	(a) Alternative - working backwards:		
	$x^2-9x+8=0\Rightarrow(4-x)^2-x-8=0$	B1	1.2
	$\Rightarrow(4-x)^2=x+8$ $\Rightarrow\log(4-x)^2=\log(x+8)$	M1	1.1b
	$\Rightarrow 2\log(4-x)=\log(x+8)\ *$ Hence proved.	A1	2.1
(b)	(i) $(x=)\ 1,\ 8$	B1	1.1b
	(ii) 8 is **not** a solution as $\log(4-8)$ cannot be found	B1	2.3
		(2)	
			(5 marks)

November 2020 Question 5 Paper 2 (A-Level)

5. The curve with equation $y = 3 \times 2^x$ meets the curve with equation $y = 15 - 2^{x+1}$ at the point P.

Find, using algebra, the exact x coordinate of P.

(4)

ANSWER

Question	Scheme	Marks	AOs
5	$15 - 2^{x+1} = 3 \times 2^x$	B1	1.1b
	$\Rightarrow 15 - 2 \times 2^x = 3 \times 2^x \Rightarrow 2^x = 3$ or e.g. $\Rightarrow \dfrac{15}{2^x} - 2 = 3 \Rightarrow 2^x = 3$	M1	1.1b
	$2^x = 3 \Rightarrow x = \ldots$	dM1	1.1b
	$x = \log_2 3$	A1cso	1.1b
		(4)	
	Alternative		
	$y = 3 \times 2^x \Rightarrow 2^x = \dfrac{y}{3} \Rightarrow y = 15 - 2 \times \dfrac{y}{3}$	B1	1.1b
	$3y + 2y = 45 \Rightarrow y = 9 \Rightarrow 3 \times 2^x = 9 \Rightarrow 2^x = 3$	M1	1.1b
	$2^x = 3 \Rightarrow x = \ldots$	dM1	1.1b
	$x = \log_2 3$	A1cso	1.1b
			(4 marks)

November 2020 Question 9 Paper 2 (A-Level)

9. A quantity of ethanol was heated until it reached boiling point.

The temperature of the ethanol, θ °C, at time t seconds after heating began, is modelled by the equation

$$\theta = A - Be^{-0.07t}$$

where A and B are positive constants.

Given that

- the initial temperature of the ethanol was 18 °C
- after 10 seconds the temperature of the ethanol was 44 °C

(a) find a complete equation for the model, giving the values of A and B to 3 significant figures.

(4)

Ethanol has a boiling point of approximately 78 °C

(b) Use this information to evaluate the model.

(2)

ANSWER

Question	Scheme	Marks	AOs
9(a)	$t=0,\ \theta=18 \Rightarrow 18 = A - B$ **or** $t=10,\ \theta=44 \Rightarrow 44 = A - Be^{-0.7}$	M1	3.1b
	$t=0,\ \theta=18 \Rightarrow 18 = A - B$ **and** $t=10,\ \theta=44 \Rightarrow 44 = A - Be^{-0.7}$ **and** $\Rightarrow A = \ldots, B = \ldots$	M1	3.1a
	At least one of: $A = 69.6$, $B = 51.6$ but allow awrt 70/awrt 52	A1 **M1 on EPEN**	1.1b
	$\theta = 69.6 - 51.6e^{-0.07t}$	A1	3.3
		(4)	
(b)	The maximum temperature is "69.6"(°C) (according to the model) (The model has an) upper limit of "69.6"(°C) (The model suggests that) the boiling point is "69.6"(°C)	B1ft	3.4
	Model is not appropriate as 69.6(°C) is much lower than 78(°C)	B1ft	3.5a
		(2)	
			(6 marks)

June 2019 Question 7 Paper 1 (A-Level)

7. In a simple model, the value, £V, of a car depends on its age, t, in years.

The following information is available for car A

- its value when new is £20 000
- its value after one year is £16 000

(a) Use an exponential model to form, for car A, a possible equation linking V with t.

(4)

The value of car A is monitored over a 10-year period.
Its value after 10 years is £2 000

(b) Evaluate the reliability of your model in light of this information.

(2)

The following information is available for car B

- it has the same value, when new, as car A
- its value depreciates more slowly than that of car A

(c) Explain how you would adapt the equation found in (a) so that it could be used to model the value of car B.

(1)

ANSWER

Question	Scheme	Marks	AOs
7 (a)	Uses a model $V = Ae^{\pm kt}$ oe (See next page for other suitable models)	M1	3.3
	Eg. Substitutes $t = 0, V = 20\,000 \Rightarrow A = 20000$	M1	1.1b
	Eg. Substitutes $t = 1, V = 16000 \Rightarrow 16000 = 20000e^{-1k} \Rightarrow k = ..$	dM1	3.1b
	$V = 20000e^{-0.223t}$	A1	1.1b
		(4)	
(b)	Substitutes $t = 10$ in their $V = 20000e^{-0.223t} \Rightarrow V = (£\,2150)$	M1	3.4
	Eg. The model is reliable as £2150 ≈ £2000	A1	3.5a
		(2)	
(c)	Make the "-0.223" less negative. Alt: Adapt model to for example $V = 18000e^{-0.223t} + 2000$	B1ft	3.3
		(1)	
			(7 marks)

June 2019 Question 9 Paper 1 (A-Level)

9. Given that $a > b > 0$ and that a and b satisfy the equation

$$\log a - \log b = \log(a - b)$$

(a) show that

$$a = \frac{b^2}{b-1}$$

(3)

(b) Write down the full restriction on the value of b, explaining the reason for this restriction.

(2)

ANSWER

Question	Scheme	Marks	AOs
9 (a)	States $\log a - \log b = \log \frac{a}{b}$	B1	1.2
	Proceeds from $\frac{a}{b} = a - b \to \ldots\ldots \to ab - a = b^2$	M1	1.1b
	$ab - a = b^2 \to a(b-1) = b^2 \Rightarrow a = \frac{b^2}{b-1}$ *	A1*	2.1
		(3)	
(b)	States either $b > 1$ or $b \neq 1$ with reason $\frac{b^2}{b-1}$ is not defined at $b = 1$ oe	B1	2.2a
	States $b > 1$ and explains that as $a > 0 \Rightarrow \frac{b^2}{b-1} > 0 \Rightarrow b > 1$	B1	2.4
		(2)	
			(5 marks)

June 2019 Question 9 Paper 2 (A-Level)

9. A research engineer is testing the effectiveness of the braking system of a car when it is driven in wet conditions.

The engineer measures and records the braking distance, d metres, when the brakes are applied from a speed of $V \text{ km h}^{-1}$.

Graphs of d against V and $\log_{10} d$ against $\log_{10} V$ were plotted.

The results are shown below together with a data point from each graph.

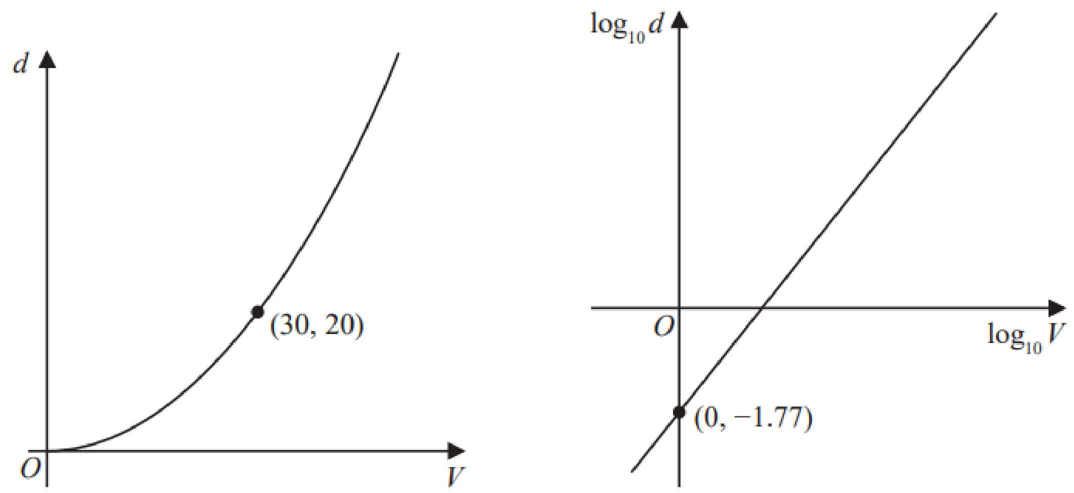


Figure 5 **Figure 6**

(a) Explain how Figure 6 would lead the engineer to believe that the braking distance should be modelled by the formula

$$d = kV^n \qquad \text{where } k \text{ and } n \text{ are constants}$$

with $k \approx 0.017$

(3)

Using the information given in Figure 5, with $k = 0.017$

(b) find a complete equation for the model giving the value of n to 3 significant figures.

(3)

Sean is driving this car at 60 km h^{-1} in wet conditions when he notices a large puddle in the road 100 m ahead. It takes him 0.8 seconds to react before applying the brakes.

(c) Use your formula to find out if Sean will be able to stop before reaching the puddle.

(3)

ANSWER

Question	Scheme	Marks	AOs
9 (a) Way 1	$\{d = kV^n \Rightarrow\}\ \log_{10} d = \log_{10} k + n\log_{10} V$ or $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$ seen or used as part of their argument	M1	2.1
	Alludes to $d = kV^n$ and gives a full explanation by comparing their result with a linear model e.g. $Y = MX + C$	A1	2.4
	$\{k =\}\ 10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
9 (a) Way 2	$\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$ or $\log_{10} d = \log_{10} k + n\log_{10} V$ seen or used as part of their argument	M1	2.1
	$\{d = kV^n \Rightarrow\}\ \log_{10} d = \log_{10}(kV^n)$ $\Rightarrow \log_{10} d = \log_{10} k + \log_{10} V^n \Rightarrow \log_{10} d = \log_{10} k + n\log_{10} V$	A1	2.4
	$\{k =\}\ 10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
(b)	$\{d = 20, V = 30 \Rightarrow\}\quad 20 = k(30)^n$ or $\log_{10} 20 = \log_{10} k + n\log_{10} 30$	M1	3.4
	$20 = k(30)^n \Rightarrow \log 20 = \log k + n\log 30 \Rightarrow n = \dfrac{\log 20 - \log k}{\log 30} \Rightarrow n = \ldots$ $\log_{10} 20 = \log_{10} k + n\log_{10} 30 \Rightarrow n = \dfrac{\log_{10} 20 - \log_{10} k}{\log_{10} 30} \Rightarrow n = \ldots$	M1	1.1b
	$\{n = \text{awrt } 2.08 \Rightarrow\}\ d = (0.017)V^{2.08}$ **or** $\log_{10} d = -1.77 + 2.08\log_{10} V$	A1	1.1b
	Note: You can recover the A1 mark for a correct model equation given in part (c)	**(3)**	
(c)	$d = (0.017)(60)^{2.08}$	M1	3.4
	• $13.333\ldots + 84.918\ldots = 98.251\ldots \Rightarrow$ Sean stops in time	M1	3.1b
	• $100 - 13.333\ldots = 86.666\ldots$ & $d = 84.918 \Rightarrow$ Sean stops in time	A1ft	3.2a
		(3)	
			(9 marks)

June 2018 Question 5

5. A student's attempt to solve the equation $2\log_2 x - \log_2 \sqrt{x} = 3$ is shown below.

$2\log_2 x - \log_2 \sqrt{x} = 3$	
$2\log_2 \left(\dfrac{x}{\sqrt{x}}\right) = 3$	using the subtraction law for logs
$2\log_2 \left(\sqrt{x}\right) = 3$	simplifying
$\log_2 x = 3$	using the power law for logs
$x = 3^2 = 9$	using the definition of a log

(a) Identify two errors made by this student, giving a brief explanation of each.

(2)

(b) Write out the correct solution.

(3)

ANSWER

Question	Scheme		Marks	AOs
5 (a)	Identifies one of the two errors "You cannot use the subtraction law without dealing with the 2 first" " They undo the logs incorrectly. It should be $x = 2^3 = 8$ "		B1	2.3
	Identifies both errors. See above.		B1	2.3
			(2)	
(b)	$\log_2\left(\dfrac{x^2}{\sqrt{x}}\right) = 3$	$\dfrac{3}{2}\log_2(x) = 3$	M1	1.1b
	$x^{\frac{3}{2}} = 2^3$ or $\dfrac{x^2}{\sqrt{x}} = 2^3$	$x = 2^2$	M1	1.1b
	$x = \left(2^3\right)^{\frac{2}{3}} = 4$	$x = 4$	A1	1.1b
			(3)	
				(5 marks)

June 2018 Question 13

13.

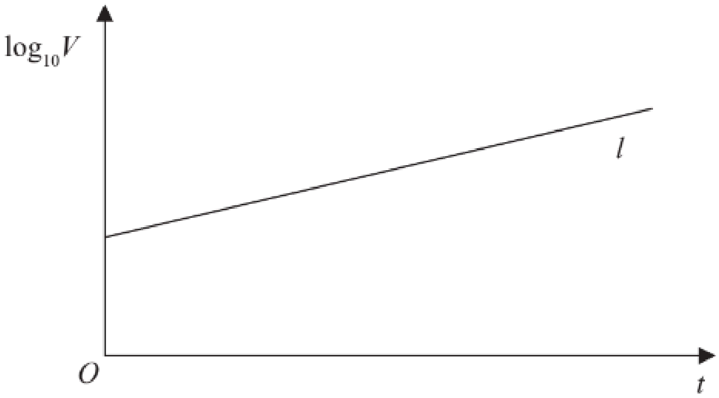


Figure 3

The value of a rare painting, £V, is modelled by the equation $V = pq^t$, where p and q are constants and t is the number of years since the value of the painting was first recorded on 1st January 1980.

The line l shown in Figure 3 illustrates the linear relationship between t and $\log_{10}V$ since 1st January 1980.

The equation of line l is $\log_{10}V = 0.05t + 4.8$

(a) Find, to 4 significant figures, the value of p and the value of q.

(4)

(b) With reference to the model interpret

(i) the value of the constant p,

(ii) the value of the constant q.

(2)

(c) Find the value of the painting, as predicted by the model, on 1st January 2010, giving your answer to the nearest hundred thousand pounds.

(2)

ANSWER

Question	Scheme	Marks	AOs
13(a)	For a correct equation in p or q $\quad p=10^{4.8}$ or $q=10^{0.05}$	M1	1.1b
	For $p=$ awrt 63100 or $q=$ awrt 1.122	A1	1.1b
	For correct equations in p and q $\quad p=10^{4.8}$ and $q=10^{0.05}$	dM1	3.1a
	For $p=$ awrt 63100 and $q=$ awrt 1.122	A1	1.1b
		(4)	
(b)	(i) The value of the painting on 1st January 1980	B1	3.4
	(ii) The proportional increase in value each year	B1	3.4
		(2)	
(c)	Uses $V=63100\times1.122^{30}$ or $\log V=0.05\times30+4.8$ leading to $V=$	M1	3.4
	$=$ awrt $(£)2000000$	A1	1.1b
		(2)	
			(8 marks)

June 2018 Question 12 Paper 1

12. The value, £V, of a vintage car t years after it was first valued on 1st January 2001, is modelled by the equation

$$V = Ap^t \qquad \text{where } A \text{ and } p \text{ are constants}$$

Given that the value of the car was £32 000 on 1st January 2005 and £50 000 on 1st January 2012

(a) (i) find p to 4 decimal places,

(ii) show that A is approximately 24 800

(4)

(b) With reference to the model, interpret

(i) the value of the constant A,

(ii) the value of the constant p.

(2)

Using the model,

(c) find the year during which the value of the car first exceeds £100 000

(4)

ANSWER

Question	Scheme	Marks	AOs
12 (a)	(i) Method to find p Eg. Divides $32000 = Ap^4$ by $50000 = Ap^{11}$ $$p^7 = \frac{50000}{32000} \Rightarrow p = \sqrt[7]{\frac{50000}{32000}} = \ldots$$	M1	3.1a
	$p = 1.0658$	A1	1.1b
	(ii) Substitutes their $p = 1.0658$ into either equation and finds A $$A = \frac{32000}{'1.0658'^4} \text{ or } A = \frac{50000}{'1.0658'^{11}}$$	M1	1.1b
	$A = 24795 \to 24805 \simeq 24\,800$ *	A1*	1.1b
		(4)	
(b)	A / $(£)24\,800$ is the value of the car on 1st January 2001	B1	3.4
	p /1.0658 is the factor by which the value rises each year. Accept that the value rises by 6.6 % a year (ft on their p)	B1	3.4
		(2)	
(c)	Attempts $100000 = '24800' \times '1.0658'^t$		
	$$'1.0658'^t = \frac{100000}{24800}$$	M1	3.4
	$$t = \log_{1.0658}\left(\frac{100000}{24800}\right)$$	dM1	1.1b
	$t = 21.8$ or 21.9	A1	1.1b
	cso 2022	A1	3.2a
		(4)	
			(10 marks)

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