Mechanics Year 1 (AS) exam questions

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Ch. 11: Variable Acceleration

June 2023 Question 3

3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A fixed point O lies on a straight line.

A particle P moves along the straight line such that at time t seconds, $t \geqslant 0$, after passing through O, the velocity of P, $v\,\text{m}\,\text{s}^{-1}$, is modelled as

$$v = 15 - t^2 - 2t$$

(a) Verify that P comes to instantaneous rest when $t = 3$

(1)

(b) Find the magnitude of the acceleration of P when $t = 3$

(3)

(c) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 4$

(4)

ANSWER

Question	Scheme	Marks	AOs
3(a)	$15-3^2-2\times3=0$ *	B1*	1.1b
		(1)	
3(b)	Differentiate *v* wrt *t*	M1	2.1
	$-2t-2$	A1	1.1b
	8 $(\mathrm{m\,s^{-2}})$	A1	1.1b
		(3)	

3(b)	Differentiate *v* wrt *t*	M1	2.1
	$-2t-2$	A1	1.1b
	8 $(\mathrm{m\,s^{-2}})$	A1	1.1b
		(3)	
3(c)	Integrate *v* w.r.t. *t*	M1	1.1b
	$15t-\frac{1}{3}t^3-t^2$	A1	1.1b
	Total distance = $\left[15t-\frac{1}{3}t^3-t^2\right]_0^3-\left[15t-\frac{1}{3}t^3-t^2\right]_3^4$ **OR** $s_3+(s_3-s_4)$ where s_3 means the value of their integral when t = 3. **N.B.** Allow the negative of this.	M1	3.1a
	$\frac{94}{3}$ (m)	A1	1.1b
		(4)	
		(8 marks)	

June 2022 Question 3

3. A fixed point O lies on a straight line.

A particle P moves along the straight line.

At time t seconds, $t \geqslant 0$, the distance, s metres, of P from O is given by

$$s = \frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t$$

(a) Find the acceleration of P at each of the times when P is at instantaneous rest.

(6)

(b) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 4$

(3)

ANSWER

Question	Scheme	Marks	AOs
3(a)	Differentiate s wrt t	M1	3.1a
	$(v =)\ t^2 - 5t + 6$	A1	1.1b
	Equate their v to 0 and solve	M1	1.1b
	$t = 2$ or 3	A1	1.1b
	$(a =)\ 2t - 5$	**B1ft**	2.1
	$a =\ 1$ and -1 (m s^{-2}) isw (A0 if extras)	A1	1.1b
		(6)	
(b)	Attempt to find values of s for $t = 2, 3$ *and* 4 oe Correct values are $\left(s_2 = \frac{14}{3}, s_3 = \frac{9}{2}\ and\ s_4 = \frac{16}{3}\right)$ Could be implied by correct values for: s_2 , $(s_3 - s_2)$ and $(s_4 - s_3)$ which are $\frac{14}{3}$, $(-\frac{1}{6})$ and $\frac{5}{6}$	**DM1**	1.1b
	Total distance travelled $= s_2 + (s_2 - s_3) + s_4 - s_3$ **OR** $s_2 - (s_3 - s_2) + s_4 - s_3$ **OR** $\left[\frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t\right]_0^2 - \left[\frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t\right]_2^3 + \left[\frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t\right]_3^4$ **OR** $\frac{14}{3} - (-\frac{1}{6}) + \frac{5}{6}$ **OR** $s_2 + 2(s_2 - s_3) + s_4 - s_2$ $(= 2s_2 - 2s_3 + s_4)$ oe	M1	2.1
	$5\frac{2}{3}$ oe (m) Accept 5.7 or better	A1	1.1b
		(3)	
		(9 marks)	

Video solution:

https://youtu.be/RXP38AEYsw8

November 2021 question 2

2. A particle P moves along a straight line.

At time t seconds, the velocity $v\,\mathrm{m\,s^{-1}}$ of P is modelled as

$$v = 10t - t^2 - k \qquad t \geqslant 0$$

where k is a constant.

(a) Find the acceleration of P at time t seconds.

(2)

The particle P is instantaneously at rest when $t = 6$

(b) Find the other value of t when P is instantaneously at rest.

(4)

(c) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 6$

(4)

ANSWER

Question	Scheme	Marks	AOs
2(a)	Differentiate v w.r.t. t	M1	3.1a
	$a = \dfrac{\mathrm{d}v}{\mathrm{d}t} = 10 - 2t$ isw	A1	1.1b
		(2)	
2(b)	Solve problem using $v = 0$ when $t = 6$	M1	3.1a
	$0 = 10t - t^2 - 24$	A1	1.1b
	Solve quadratic oe to find other value of t	M1	1.1b
	$t = 4$	A1	1.1b
		(4)	
2(c)	Integrate v or $-v$ w.r.t. t	M1	3.1a
	$5t^2 - \dfrac{1}{3}t^3 - 24t$	A1	1.1b
	Total distance $= -\left[5t^2 - \dfrac{1}{3}t^3 - 24t\right]_0^4 + \left[5t^2 - \dfrac{1}{3}t^3 - 24t\right]_4^6$	M1	2.1
	$\dfrac{116}{3}$ (m)	A1	1.1b
		(4)	
		(10 marks)	

November 2020 question 3

3. A particle P moves along a straight line such that at time t seconds, $t \geqslant 0$, after leaving the point O on the line, the velocity, $v\,\mathrm{m\,s^{-1}}$, of P is modelled as

$$v = (7 - 2t)(t + 2)$$

(a) Find the value of t at the instant when P stops accelerating.

(4)

(b) Find the distance of P from O at the instant when P changes its direction of motion.

(5)

In this question, solutions relying on calculator technology are not acceptable.

ANSWER

Question	Scheme	Marks	AOs
3(a)	$v = 3t - 2t^2 + 14$ and differentiate	M1	3.1a
	$a = \frac{dv}{dt} = 3 - 4t$ or $(7 - 2t) - 2(t + 2)$ using product rule	A1	1.1b
	$3 - 4t = 0$ and solve for t	M1	1.1b
	$t = \frac{3}{4}$ oe	A1	1.1b
		(4)	
3(b)	Solve problem using $v = 0$ to find a value of t $\left(t = \frac{7}{2}\right)$	M1	3.1a
	$v = 3t - 2t^2 + 14$ and integrate	M1	1.1b
	$s = \frac{3t^2}{2} - \frac{2t^3}{3} + 14t$	A1	1.1b
	Substitute $t = \frac{7}{2}$ into their s expression (M0 if using *suvat*)	M1	1.1b
	$s = \frac{931}{24} = 38\frac{19}{24} = 38.79166..(\text{m})$ Accept 39 or better	A1	1.1b
		(5)	
			(9 marks)

Video solution:

https://youtu.be/BvgzBBXwnmU

June 2019 question 3

3. A particle, P, moves along a straight line such that at time t seconds, $t \geqslant 0$, the velocity of P, $v\,\mathrm{m\,s^{-1}}$, is modelled as

$$v = 12 + 4t - t^2$$

Find

(a) the magnitude of the acceleration of P when P is at instantaneous rest,

(5)

(b) the distance travelled by P in the interval $0 \leqslant t \leqslant 3$

(3)

ANSWER

Question	Scheme	Marks	AOs	Notes
3(a)	$v = 12 + 4t - t^2 = 0$ and solving	M1	3.1a	Equating v to 0 and solving the quadratic If no evidence of solving, and at least one answer wrong, M0
	$t = 6$ (or -2)	A1	1.1b	6 but allow -2 as well at this stage
	Differentiate v wrt t	M1	1.1a	For differentiation (both powers decreasing by 1)
	$(a = \frac{dv}{dt} =)\ 4 - 2t$	A1	1.1b	Cao; only need RHS
	When $t = 6$, $a = -8$; Magnitude is 8 ($\mathrm{m\,s^{-2}}$)	A1	1.1b	Substitute in $t = 6$ and get 8 ($\mathrm{m\,s^{-2}}$) as the answer . Must be **positive.** (A0 if two answers given)
		(5)		
(b)	Integrate v wrt t	M1	3.1a	For integration (at least two powers increasing by 1)
	$(s =)12t + 2t^2 - \frac{1}{3}t^3 (+C)$	A1	1.1b	Correct expression (ignore C) only need RHS Must be used in part (b)
	$t = 3$ => distance $= 45$ (m)	A1	1.1b	Correct distance. Ignore units
		(3)		
		(8 marks)		

June 2018 Question 8

8. A particle, P, moves along the x-axis. At time t seconds, $t \geqslant 0$, the displacement, x metres, of P from the origin O, is given by $x = \frac{1}{2}t^2(t^2 - 2t + 1)$

(a) Find the times when P is instantaneously at rest.

(5)

(b) Find the total distance travelled by P in the time interval $0 \leqslant t \leqslant 2$

(3)

(c) Show that P will never move along the negative x-axis.

(2)

ANSWER

Question	Scheme	Marks	AOs
8(a)	Multiply out and differentiate *wrt* to time (or use of product rule i.e. must have two terms with correct structure)	M1	1.1a
	$v = 2t^3 - 3t^2 + t$	A1	1.1b
	$2t^3 - 3t^2 + t = 0$ and solve: $t(2t-1)(t-1) = 0$	**DM**1	1.1b
	$t = 0$ or $t = \frac{1}{2}$ or $t = 1$; any two	A1	1.1b
	All three	A1	1.1b
		(5)	
(b)	Find x when $t = 0, \frac{1}{2}$, 1 and 2 : $(0, \frac{1}{32}, 0, 2)$	M1	2.1
	Distance $= \frac{1}{32} + \frac{1}{32} + 2$	M1	2.1
	$2\frac{1}{16}$ (m) oe or 2.06 or better	A1	1.1b
		(3)	
(c)	$x = \frac{1}{2}t^2(t-1)^2$	M1	3.1a
	$\frac{1}{2}$ perfect square so $x \geq 0$ i.e. never negative	A1 cso	2.4
		(2)	
		(10 marks)	