Edexcel - Pure Year 2 exam questions

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Ch. 11: Integration

June 2023 Question 13 Paper 2

13. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$(3 + x)^{-2}$$

writing each term in simplest form.

(4)

(b) Using the answer to part (a) and using algebraic integration, estimate the value of

$$\int_{0.2}^{0.4} \frac{6x}{(3 + x)^2}\, dx$$

giving your answer to 4 significant figures.

(4)

(c) Find, using algebraic integration, the exact value of

$$\int_{0.2}^{0.4} \frac{6x}{(3 + x)^2}\, dx$$

giving your answer in the form $a \ln b + c$, where a, b and c are constants to be found.

(5)

ANSWER

Question	Scheme	Marks	AOs
13(a)	$3^{-2}\left(1+\frac{x}{3}\right)^{-2} = 3^{-2}(1+...x+...x^2)$	M1	1.1b
	$(-2)\left(\frac{x}{3}\right)$ or $\frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^2$	M1	1.1b
	$\left(1+\frac{x}{3}\right)^{-2} = 1+(-2)\left(\frac{x}{3}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^2$	A1	1.1b
	$3^{-2}\left(1+\frac{x}{3}\right)^{-2} = \frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$	A1	2.1
		(4)	
(b)	$\int 6x \times "\left(\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}\right)" \, dx = \int\left(\frac{2x}{3}-\frac{4x^2}{9}+\frac{2x^3}{9}\right)dx = ...$	M1	1.1b
	$\int\left(\frac{2x}{3}-\frac{4x^2}{9}+\frac{2x^3}{9}\right)dx = \frac{x^2}{3}-\frac{4x^3}{27}+\frac{x^4}{18}$ oe	A1	1.1b
	$\left["\frac{x^2}{3}-\frac{4x^3}{27}+\frac{x^4}{18}"\right]_{0.2}^{0.4} = \left(\frac{(0.4)^2}{3}-\frac{4(0.4)^3}{27}+\frac{(0.4)^4}{18}\right)-\left(\frac{(0.2)^2}{3}-\frac{4(0.2)^3}{27}+\frac{(0.2)^4}{18}\right)$	dM1	3.1a
	$=$ awrt 0.03304 or $\frac{223}{6750}$	A1	1.1b
		(4)	
(c)	Overall problem-solving mark (see notes)	M1	3.1a
	$u=3+x \Rightarrow \int_{3.2}^{3.4} f(u)\,du \Rightarrow \int_{3.2}^{3.4}\frac{6(u-3)}{u^2}\,du = \int_{3.2}^{3.4}\frac{6}{u}-\frac{18}{u^2}\,du \Rightarrow ...\ln u + ...u^{-1}$	M1	1.1b
	$\int_{3.2}^{3.4}\frac{6(u-3)}{u^2}\,du = \int_{3.2}^{3.4}\frac{6}{u}-\frac{18}{u^2}\,du \Rightarrow 6\ln u + 18u^{-1}$	A1	1.1b
	$\left[6\ln u+18u^{-1}\right]_{3.2}^{3.4} = \left(6\ln 3.4+\frac{18}{3.4}\right)-\left(6\ln 3.2+\frac{18}{3.2}\right) = ...$	ddM1	1.1b
	$6\ln\left(\frac{17}{16}\right)-\frac{45}{136}$ oe	A1	2.1
		(5)	

June 2023 Question 11 Paper 2

11.

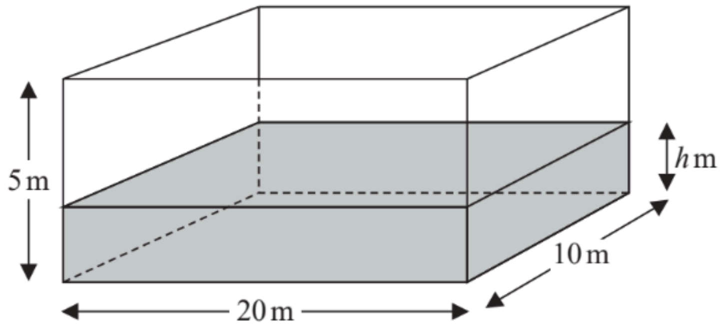

Figure 1

A tank in the shape of a cuboid is being filled with water.

The base of the tank measures 20 m by 10 m and the height of the tank is 5 m, as shown in Figure 1.

At time t minutes after water started flowing into the tank the height of the water was h m and the volume of water in the tank was V m^3

In a model of this situation

- the sides of the tank have negligible thickness
- the rate of change of V is inversely proportional to the square root of h

(a) Show that

$$\frac{dh}{dt} = \frac{\lambda}{\sqrt{h}}$$

where λ is a constant.

(3)

Given that

- initially the height of the water in the tank was 1.44 m
- exactly 8 minutes after water started flowing into the tank the height of the water was 3.24 m

(b) use the model to find an equation linking h with t, giving your answer in the form

$$h^{\frac{3}{2}} = At + B$$

where A and B are constants to be found.

(5)

(c) Hence find the time taken, from when water started flowing into the tank, for the tank to be completely full.

(2)

ANSWER

Question	Scheme	Marks	AOs
11(a)	$\frac{\mathrm{d}V}{\mathrm{d}h}=200$ oe e.g. $\frac{\mathrm{d}h}{\mathrm{d}V}=\frac{1}{200}$	B1	1.1b
	$\left(\frac{\mathrm{d}h}{\mathrm{d}t}=\right)\frac{\mathrm{d}h}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{1}{200}\times\frac{k}{\sqrt{h}}$	M1	3.1a
	$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\lambda}{\sqrt{h}}$ *	A1*	2.1
		(3)	
(b)	$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\lambda}{\sqrt{h}}\Rightarrow\int h^{\frac{1}{2}}\,dh=\int\lambda\,dt\Rightarrow\ldots h^{\frac{3}{2}}=\lambda t\{+c\}$	M1	1.1b
	$\frac{2}{3}h^{\frac{3}{2}}=\lambda t\{+c\}$ oe e.g. $\frac{h^{\frac{3}{2}}}{\frac{3}{2}}=\lambda t\{+c\}$	A1	1.1b
	$\frac{2}{3}(1.44)^{\frac{3}{2}}=\lambda\times 0+c\Rightarrow c=1.152\left(=\frac{144}{125}\right)$	dM1	3.4
	$\frac{2}{3}(3.24)^{\frac{3}{2}}=\lambda\times 8+"1.152"\Rightarrow\lambda=0.342\left(=\frac{171}{500}\right)$	ddM1	3.1b
	$h^{\frac{3}{2}}=0.513t+1.728$ oe e.g. $h^{\frac{3}{2}}=\frac{513}{1000}t+\frac{216}{125}$	A1	3.3
		(5)	

June 2023 Question 10 Paper 2

10. $$f(x) = \frac{3kx - 18}{(x+4)(x-2)}$$ where k is a positive constant

(a) Express $f(x)$ in partial fractions in terms of k.

(3)

(b) Hence find the exact value of k for which

$$\int_{-3}^{1} f(x)\,dx = 21$$

(4)

ANSWER

Question	Scheme	Marks	AOs
10(a)	e.g. $\frac{3kx-18}{(x+4)(x-2)} \equiv \frac{A}{x+4} + \frac{B}{x-2} \Rightarrow 3kx - 18 \equiv A(x-2) + B(x+4)$ **or** $\frac{3kx-18}{(x+4)(x-2)} \equiv \frac{A}{x-2} + \frac{B}{x+4} \Rightarrow 3kx - 18 \equiv A(x+4) + B(x-2)$	**B1**	1.1b
	$6k - 18 = 6B \Rightarrow B = \ldots$ or $-12k - 18 = -6A \Rightarrow A = \ldots$ **or** $3kx - 18 \equiv (A+B)x + 4B - 2A \Rightarrow A + B = 3k,\ -18 = 4B - 2A$ $\Rightarrow A = \ldots$ or $B = \ldots$	**M1**	1.1b
	$\frac{2k+3}{x+4} + \frac{k-3}{x-2}$	**A1**	1.1b
		(3)	
(b)	$\int \left(\frac{"2k+3"}{x+4} + \frac{"k-3"}{x-2} \right) dx = \ldots\ln(x+4) + \ldots\ln(x-2)$	**M1**	1.2
	$("2k+3")\ln(x+4) + ("k-3")\ln(x-2)$	**A1ft**	1.1b
	$("2k+3")\ln(5) - ("k-3")\ln(5) \Rightarrow ("k+6")\ln 5 = 21 \Rightarrow k = \ldots$	**dM1**	3.1a
	$(k =)\frac{21}{\ln 5} - 6$	**A1**	2.2a
		(4)	
		(7 marks)	

June 2023 Question 5 Paper 2

5. The curve C has equation $y = \mathrm{f}(x)$

The curve

- passes through the point $P(3, -10)$
- has a turning point at P

Given that

$$\frac{dy}{dx} = 2x^3 - 9x^2 + 5x + k$$

where k is a constant,

(a) show that $k = 12$ **(2)**

(b) Hence find the coordinates of the point where C crosses the y-axis. **(3)**

ANSWER

Question	Scheme	Marks	AOs
5(a)	$2(3)^3 - 9(3)^2 + 5(3) + k = 0 \Rightarrow k = ...$	M1	1.1b
	$54 - 81 + 15 + k = 0 \Rightarrow k = 12*$ or $-12 + k = 0 \Rightarrow k = 12*$	A1*	1.1b
		(2)	
	(a) Alternative by verification:		
	$2(3)^3 - 9(3)^2 + 5(3) + 12 = 0$	M1	1.1b
	$54 - 81 + 15 + 12 = 0$ Hence $k = 12$ *	A1*	1.1b
		(2)	
(b)	$\int (2x^3 - 9x^2 + 5x + 12)\,dx ... x^4 \pm ... x^3 \pm ... x^2 \pm ... x \pm ...$	M1	3.1a
	$\frac{1}{2}(3)^4 - 3(3)^3 + \frac{5}{2}(3)^2 + 12(3) + c = -10 \Rightarrow c = ...$	dM1	1.1b
	$(0, -28)$	A1	2.2a
		(3)	
			(5 marks)

Arancha Ruiz

June 2023 Question 1 Paper 1

1. Find

$$\int \frac{x^{\frac{1}{2}}(2x-5)}{3}\,\mathrm{d}x$$

writing each term in simplest form.

(4)

ANSWER

Question	Scheme	Marks	AOs
1	$x^{\frac{1}{2}}(2x-5) = \ldots x^{\frac{3}{2}} + \ldots x^{\frac{1}{2}}$ or $\frac{x^{\frac{1}{2}}(2x-5)}{3} = \frac{\ldots x^{\frac{3}{2}} + \ldots x^{\frac{1}{2}}}{3}$	M1	1.1b
	$\frac{2x^{\frac{3}{2}}}{3} - \frac{5x^{\frac{1}{2}}}{3}$	A1	1.1b
	$\int \frac{2x^{\frac{3}{2}}}{3} - \frac{5x^{\frac{1}{2}}}{3}\,\mathrm{d}x = \ldots x^{\frac{5}{2}} \pm \ldots x^{\frac{3}{2}}\ \ (+c)$	dM1	1.1b
	$\frac{4}{15}x^{\frac{5}{2}} - \frac{10}{9}x^{\frac{3}{2}} + c$	A1	1.1b
		(4)	
			(4 marks)

June 2022 Question 14 Paper 2

14. (a) Express $\dfrac{3}{(2x-1)(x+1)}$ in partial fractions.

(3)

When chemical A and chemical B are mixed, oxygen is produced.

A scientist mixed these two chemicals and measured the total volume of oxygen produced over a period of time.

The total volume of oxygen produced, $V\,\text{m}^3$, t hours after the chemicals were mixed, is modelled by the differential equation

$$\frac{dV}{dt} = \frac{3V}{(2t-1)(t+1)} \qquad V \geqslant 0 \qquad t \geqslant k$$

where k is a constant.

Given that exactly 2 hours after the chemicals were mixed, a total volume of $3\,\text{m}^3$ of oxygen had been produced,

(b) solve the differential equation to show that

$$V = \frac{3(2t-1)}{(t+1)}$$

(5)

The scientist noticed that

- there was a **time delay** between the chemicals being mixed and oxygen being produced
- there was a **limit** to the total volume of oxygen produced

Deduce from the model

(c) (i) the **time delay** giving your answer in minutes,

(ii) the **limit** giving your answer in m^3

(2)

ANSWER

Question	Scheme	Marks	AOs
14(a)	$\frac{3}{(2x-1)(x+1)}=\frac{A}{2x-1}+\frac{B}{x+1}\Rightarrow A=...,\ B=...$	M1	1.1b
	Either $A=2$ or $B=-1$	A1	1.1b
	$\frac{3}{(2x-1)(x+1)}=\frac{2}{2x-1}-\frac{1}{x+1}$	A1	1.1b
		(3)	
(b)	$\int\frac{1}{V}\mathrm{d}V=\int\frac{3}{(2t-1)(t+1)}\mathrm{d}t$	B1	1.1a
	$\int\frac{2}{2t-1}-\frac{1}{t+1}\mathrm{d}t=...\ln(2t-1)-...\ln(t+1)(+c)$	M1	3.1a
	$\ln V=\ln(2t-1)-\ln(t+1)(+c)$	A1ft	1.1b
	Substitutes $t=2,V=3\Rightarrow c=(\ln 3)$	M1	3.4
	$\ln V=\ln(2t-1)-\ln(t+1)+\ln 3$ $V=\frac{3(2t-1)}{(t+1)}$ *	A1*	2.1
		(5)	

June 2022 Question 5 Paper 2

5. The table below shows corresponding values of x and y for $y = \log_3 2x$

The values of y are given to 2 decimal places as appropriate.

x	3	4.5	6	7.5	9
y	1.63	2	2.26	2.46	2.63

(a) Using the trapezium rule with all the values of y in the table, find an estimate for

$$\int_3^9 \log_3 2x \, dx$$

(3)

Using your answer to part (a) and making your method clear, estimate

(b) (i) $\displaystyle\int_3^9 \log_3 (2x)^{10} \, dx$

(ii) $\displaystyle\int_3^9 \log_3 18x \, dx$

(3)

ANSWER

Question	Scheme	Marks	AOs
5(a)	States or uses $h = 1.5$	B1	1.1a
	Full attempt at the trapezium rule $= \frac{\dots}{2}\{1.63 + 2.63 + 2\times(2 + 2.26 + 2.46)\}$	M1	1.1b
	$=$ awrt 13.3 or $\frac{531}{40}$	A1	1.1b
		(3)	
(b)(i)	$\int_3^9 \log_3(2x)^{10}\,dx = 10\times\text{"13.3"} = \text{awrt}\,133$ or e.g. $\frac{531}{4}$	B1ft	2.2a
(ii)	$\int_3^9 \log_3 18x\,dx = \int_3^9 \log_3(9\times 2x)\,dx = \int_3^9 2+\log_3 2x\,dx$ $= [2x]_3^9 + \int_3^9 \log_3 2x\,dx = 18 - 6 + \int_3^9 \log_3 2x\,dx = \dots$	M1	3.1a
	Awrt 25.3 or $\frac{1011}{40}$	A1ft	1.1b
		(3)	
			(6 marks)

June 2022 Question 16 Paper 1

16.

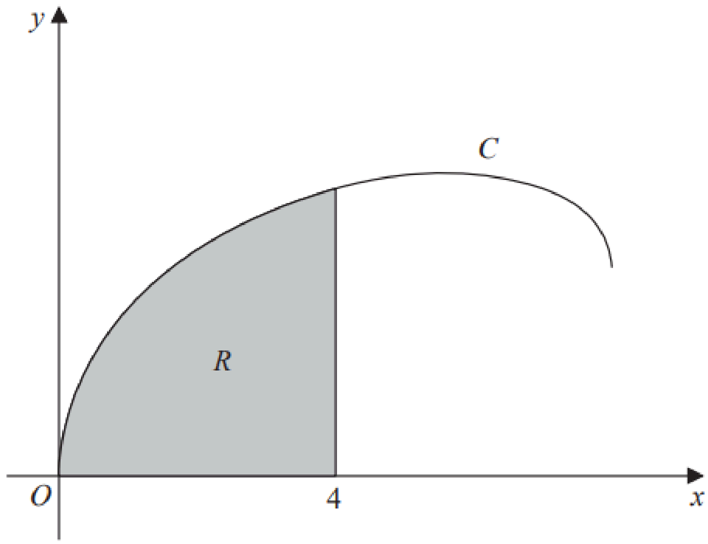

Figure 6

Figure 6 shows a sketch of the curve C with parametric equations

$$x = 8\sin^2 t \qquad y = 2\sin 2t + 3\sin t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 6, is bounded by C, the x-axis and the line with equation $x = 4$

(a) Show that the area of R is given by

$$\int_0^a \left(8 - 8\cos 4t + 48\sin^2 t\cos t\right) \mathrm{d}t$$

where a is a constant to be found.

(5)

(b) Hence, using algebraic integration, find the exact area of R.

(4)

ANSWER

Question	Scheme	Marks	AOs
16 (a)	Attempts $y.\frac{\mathrm{d}x}{\mathrm{d}t} = (2\sin 2t + 3\sin t) \times 16\sin t\cos t$ and uses $\sin 2t = 2\sin t\cos t$	M1	2.1
	Correct expanded integrand. **Usually** for one of $(R) = \int \underline{\underline{48\sin^2 t\cos t + 16\sin^2 2t\ \mathrm{d}t}}$ $(R) = \int \underline{\underline{48\sin^2 t\cos t + 64\sin^2 t\cos^2 t\ \mathrm{d}t}}$ $(R) = \int \underline{\underline{24\sin 2t\sin t + 16\sin^2 2t\ \mathrm{d}t}}$	A1	1.1b
	Attempts to use $\cos 4t = 1 - 2\sin^2 2t = \left(1 - 8\sin^2 t\cos^2 t\right)$	M1	1.1b
	$R = \int_0^a 8 - 8\cos 4t + 48\sin^2 t\cos t\ \mathrm{d}t$ *	A1*	2.1
	Deduces $a = \frac{\pi}{4}$	B1	2.2a
		(5)	
(b)	$\int 8 - 8\cos 4t + 48\sin^2 t\cos t\ \mathrm{d}t = 8t - 2\sin 4t + 16\sin^3 t$	M1 A1	2.1 1.1b
	$\left[8t - 2\sin 4t + 16\sin^3 t\right]_0^{\frac{\pi}{4}} = 2\pi + 4\sqrt{2}$	M1 A1	2.1 1.1b
		(4)	
			(9 marks)

June 2022 Question 12 Paper 1

12. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Show that

$$\int_1^{e^2} x^3 \ln x \, dx = ae^8 + b$$

where a and b are rational constants to be found.

(5)

ANSWER

Question	Scheme	Marks	AOs
12	$\int x^3 \ln x \, dx = \frac{x^4}{4}\ln x - \int \frac{x^4}{4} \times \frac{1}{x} \, dx$	M1	1.1b
	$= \frac{x^4}{4}\ln x - \frac{x^4}{16}(+c)$	M1 A1	1.1b 1.1b
	$\int_1^{e^2} x^3 \ln x \, dx = \left[\frac{x^4}{4}\ln x - \frac{x^4}{16}\right]_1^{e^2} = \left(\frac{e^8}{4}\ln e^2 - \frac{e^8}{16}\right) - \left(-\frac{1^4}{16}\right)$	M1	2.1
	$= \frac{7}{16}e^8 + \frac{1}{16}$	A1	1.1b
		(5)	
			(5 marks)

June 2022 Question 4 Paper 1

4. (a) Express $\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x}\delta x$ as an integral.

(1)

(b) Hence show that

$$\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x}\delta x = \ln k$$

where k is a constant to be found.

(2)

ANSWER

Question	Scheme	Marks	AOs
4 (a)	$\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x}\delta x = \int_{2.1}^{6.3} \frac{2}{x}\,dx$	B1	1.2
		(1)	
(b)	$= [2\ln x]_{2.1}^{6.3} = 2\ln 6.3 - 2\ln 2.1$	M1	1.1b
	$= \ln 9$ CSO	A1	1.1b
		(2)	
			(3 marks)

November 2021 Question 11 Paper 1

11.

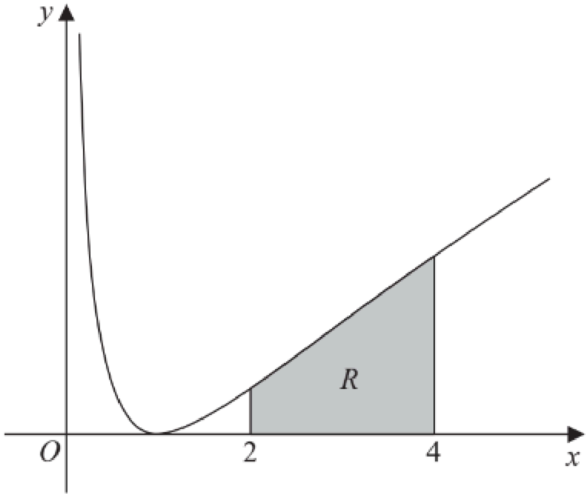

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = (\ln x)^2 \qquad x > 0$$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 4$

The table below shows corresponding values of x and y, with the values of y given to 4 decimal places.

x	2	2.5	3	3.5	4
y	0.4805	0.8396	1.2069	1.5694	1.9218

(a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of R, giving your answer to 3 significant figures.

(3)

(b) Use algebraic integration to find the exact area of R, giving your answer in the form

$$y = a(\ln 2)^2 + b\ln 2 + c$$

where a, b and c are integers to be found.

(5)

ANSWER

Question	Scheme	Marks	AOs
11(a)	$h = 0.5$	B1	1.1b
	$A \approx \frac{1}{2} \times \frac{1}{2}\left\{0.4805 + 1.9218 + 2\left(0.8396 + 1.2069 + 1.5694\right)\right\}$	M1	1.1b
	$= 2.41$	A1	1.1b
		(3)	
(b)	$\int (\ln x)^2 \, \mathrm{d}x = x(\ln x)^2 - \int x \times \frac{2\ln x}{x} \, \mathrm{d}x$	M1 A1	3.1a 1.1b
	$= x(\ln x)^2 - 2\int \ln x \, \mathrm{d}x = x(\ln x)^2 - 2\left(x\ln x - \int \mathrm{d}x\right)$ $= x(\ln x)^2 - 2\int \ln x \, \mathrm{d}x = x(\ln x)^2 - 2x\ln x + 2x$	dM1	2.1
	$\int_2^4 (\ln x)^2 \, \mathrm{d}x = \left[x(\ln x)^2 - 2x\ln x + 2x\right]_2^4$ $= 4(\ln 4)^2 - 2 \times 4\ln 4 + 2 \times 4 - \left(2(\ln 2)^2 - 2 \times 2\ln 2 + 2 \times 2\right)$ $= 4(2\ln 2)^2 - 16\ln 2 + 8 - 2(\ln 2)^2 + 4\ln 2 - 4$	ddM1	2.1
	$= 14(\ln 2)^2 - 12\ln 2 + 4$	A1	1.1b
		(5)	
			(8 marks)

November 2021 Question 12 Paper 2

12. (a) Use the substitution $u = 1 + \sqrt{x}$ to show that

$$\int_0^{16} \frac{x}{1+\sqrt{x}}\,dx = \int_p^q \frac{2(u-1)^3}{u}\,du$$

where p and q are constants to be found.

(3)

(b) Hence show that

$$\int_0^{16} \frac{x}{1+\sqrt{x}}\,dx = A - B\ln 5$$

where A and B are constants to be found.

(4)

ANSWER

Question	Scheme	Marks	AOs
12(a)	$u=1+\sqrt{x} \Rightarrow x=(u-1)^2 \Rightarrow \dfrac{dx}{du}=2(u-1)$ or $u=1+\sqrt{x} \Rightarrow \dfrac{du}{dx}=\dfrac{1}{2}x^{-\frac{1}{2}}$	B1	1.1b
	$\displaystyle\int \frac{x}{1+\sqrt{x}}dx=\int \frac{(u-1)^2}{u}2(u-1)du$ or $\displaystyle\int \frac{x}{1+\sqrt{x}}dx=\int \frac{x}{u}\times 2x^{\frac{1}{2}}du=\int \frac{2x^{\frac{3}{2}}}{u}du=\int \frac{2(u-1)^3}{u}du$	M1	2.1
	$\displaystyle\int_0^{16} \frac{x}{1+\sqrt{x}}dx=\int_1^5 \frac{2(u-1)^3}{u}du$	A1	1.1b
		(3)	
(b)	$\displaystyle 2\int \frac{u^3-3u^2+3u-1}{u}du=2\int\left(u^2-3u+3-\frac{1}{u}\right)du=\ldots$	M1	3.1a
	$=(2)\left[\dfrac{u^3}{3}-\dfrac{3u^2}{2}+3u-\ln u\right]$	A1	1.1b
	$=2\left[\dfrac{5^3}{3}-\dfrac{3(5)^2}{2}+3(5)-\ln 5-\left(\dfrac{1}{3}-\dfrac{3}{2}+3-\ln 1\right)\right]$	dM1	2.1
	$=\dfrac{104}{3}-2\ln 5$	A1	1.1b
		(4)	
		(7 marks)	

November 2021 Question 14 Paper 2

14.

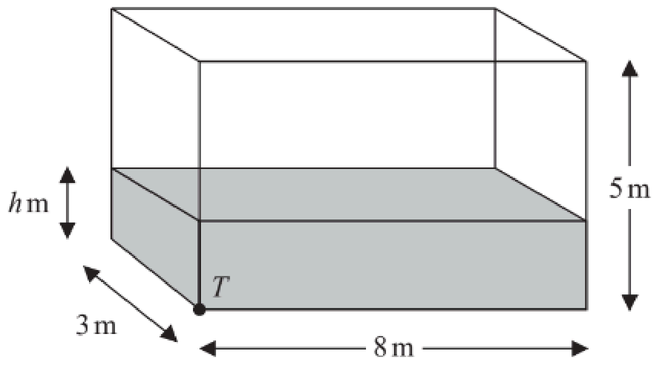

Figure 5

Water flows at a constant rate into a large tank.

The tank is a cuboid, with all sides of negligible thickness.

The base of the tank measures 8 m by 3 m and the height of the tank is 5 m.

There is a tap at a point T at the bottom of the tank, as shown in Figure 5.

At time t minutes after the tap has been opened

- the depth of water in the tank is h metres
- water is flowing into the tank at a constant rate of $0.48\,\text{m}^3$ per minute
- water is modelled as leaving the tank through the tap at a rate of $0.1h\,\text{m}^3$ per minute

(a) Show that, according to the model,

$$1200\frac{\mathrm{d}h}{\mathrm{d}t} = 24 - 5h$$

(4)

Given that when the tap was opened, the depth of water in the tank was 2 m,

(b) show that, according to the model,

$$h = A + B\mathrm{e}^{-kt}$$

where A, B and k are constants to be found.

(6)

Given that the tap remains open,

(c) determine, according to the model, whether the tank will ever become full, giving a reason for your answer.

(2)

ANSWER

Question	Scheme	Marks	AOs
14(a)	$\frac{\mathrm{d}V}{\mathrm{d}t} = 0.48 - 0.1h$	B1	3.1b
	$V = 24h \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}h} = 24$ or $\frac{\mathrm{d}h}{\mathrm{d}V} = \frac{1}{24}$	B1	3.1b
	$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \times \frac{\mathrm{d}h}{\mathrm{d}V} = \frac{0.48 - 0.1h}{24}$ or e.g. $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}h}\frac{\mathrm{d}h}{\mathrm{d}t} \Rightarrow 0.48 - 0.1h = 24\frac{\mathrm{d}h}{\mathrm{d}t}$	M1	2.1
	$1200\frac{\mathrm{d}h}{\mathrm{d}t} = 24 - 5h$ *	A1*	1.1b
		(4)	
(b)	$1200\frac{\mathrm{d}h}{\mathrm{d}t} = 24 - 5h \Rightarrow \int \frac{1200}{24-5h}\mathrm{d}h = \int \mathrm{d}t$ $\Rightarrow$ *e.g.* $\alpha \ln(24-5h) = t(+c)$ oe or $1200\frac{\mathrm{d}h}{\mathrm{d}t} = 24 - 5h \Rightarrow \frac{\mathrm{d}t}{\mathrm{d}h} = \frac{1200}{24-5h}$ $\Rightarrow$ *e.g.* $t(+c) = \alpha \ln(24-5h)$ oe	M1	3.1a
	$t = -240\ln(24-5h)(+c)$ oe	A1	1.1b
	$t = 0, h = 2 \Rightarrow 0 = -240\ln(24-10) + c \Rightarrow c = ...(240\ln 14)$	M1	3.4

	$t = 240\ln(14) - 240\ln(24-5h)$	A1	1.1b
	$t = 240\ln\frac{14}{24-5h} \Rightarrow \frac{t}{240} = \ln\frac{14}{24-5h} \Rightarrow e^{\frac{t}{240}} = \frac{14}{24-5h}$ $\Rightarrow 14e^{-\frac{t}{240}} = 24-5h \Rightarrow h = \ldots$	ddM1	2.1
	$h = 4.8 - 2.8e^{-\frac{t}{240}}$ oe e.g. $h = \frac{24}{5} - \frac{14}{5}e^{-\frac{t}{240}}$	A1	3.3
		(6)	
(c)	**Examples:** • As $t \to \infty, e^{-\frac{t}{240}} \to 0$ • When $h > 4.8$, $\frac{dV}{dt} < 0$ • Flow in = flow out at max h so $0.1h = 4.8 \rightarrow h = 4.8$ • As $e^{-\frac{t}{240}} > 0$, $h < 4.8$ • $h = 5 \Rightarrow \frac{dV}{dt} = -0.02$ or $\frac{dh}{dt} = -\frac{1}{1200}$ • $\frac{dh}{dt} = 0 \Rightarrow h = 4.8$ • $h = 5 \Rightarrow 4.8 - 2.8e^{-\frac{t}{240}} = 5 \Rightarrow e^{-\frac{t}{240}} < 0$	M1	3.1b
	• The limit for h (according to the model) is 4.8m and the tank is 5m high so the tank will never become full • If $h = 5$ the tank would be emptying so can never be full • The equation can't be solved when $h = 5$	A1	3.2a
		(2)	

Video solution:

https://youtu.be/TNLQCDmVqpM

November 2020 Question 8 Paper 1

8. A new smartphone was released by a company.

The company monitored the total number of phones sold, n, at time t days after the phone was released.

The company observed that, during this time,

the rate of increase of n was proportional to n

Use this information to write down a suitable equation for n in terms of t.

(*You do not need to evaluate any unknown constants in your equation.*)

(2)

ANSWER

Question	Scheme	Marks	AOs
8	Any equation involving an exponential of the correct form. See notes	M1	3.1b
	$n = Ae^{kt}$ (where A and k are positive constants)	A1	1.1b
		(2)	
			(2 marks)

November 2020 Question 10 Paper 1

10. (a) Use the substitution $x = u^2 + 1$ to show that

$$\int_5^{10} \frac{3\,\mathrm{d}x}{(x-1)\left(3+2\sqrt{x-1}\right)} = \int_p^q \frac{6\,\mathrm{d}u}{u(3+2u)}$$

where p and q are positive constants to be found.

(4)

(b) Hence, using algebraic integration, show that

$$\int_5^{10} \frac{3\,\mathrm{d}x}{(x-1)\left(3+2\sqrt{x-1}\right)} = \ln a$$

where a is a rational constant to be found.

(6)

ANSWER

Question	Scheme	Marks	AOs
10 (a)	$x = u^2 + 1 \Rightarrow \mathrm{d}x = 2u\mathrm{d}u$ oe	B1	1.1b
	Full substitution $\int \frac{3\mathrm{d}x}{(x-1)\left(3+2\sqrt{x-1}\right)} = \int \frac{3\times 2u\ \mathrm{d}u}{\left(u^2+1-1\right)(3+2u)}$	M1	1.1b
	Finds correct limits e.g. $p = 2, q = 3$	B1	1.1b
	$= \int \frac{3\times 2\cancel{u}\ \mathrm{d}u}{u^{\cancel{2}}(3+2u)} = \int \frac{6\ \mathrm{d}u}{u(3+2u)}$ *	A1*	2.1
		(4)	
(b)	$\frac{6}{u(3+2u)} = \frac{A}{u} + \frac{B}{3+2u} \Rightarrow A = ..., B = ...$	M1	1.1b
	Correct PF. $\frac{6}{u(3+2u)} = \frac{2}{u} - \frac{4}{3+2u}$	A1	1.1b
	$\int \frac{6\ \mathrm{d}u}{u(3+2u)} = 2\ln u - 2\ln(3+2u) \quad (+c)$	dM1 A1ft	3.1a 1.1b
	Uses limits $u = "3", u = "2"$ with some correct ln work leading to $k\ln b$ E.g. $(2\ln 3 - 2\ln 9) - (2\ln 2 - 2\ln 7) = 2\ln\frac{7}{6}$	M1	1.1b
	$\ln\frac{49}{36}$	A1	2.1
		(6)	
			(10 marks)

Video solution:

https://youtu.be/tsI94ty27S8

November 2020 Question 14 Paper 1

14. A large spherical balloon is deflating.

At time t seconds the balloon has radius r cm and volume $V\text{ cm}^3$

The volume of the balloon is modelled as decreasing at a constant rate.

(a) Using this model, show that

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{k}{r^2}$$

where k is a positive constant.

(3)

Given that

- the initial radius of the balloon is 40 cm
- after 5 seconds the radius of the balloon is 20 cm
- the volume of the balloon continues to decrease at a constant rate until the balloon is empty

(b) solve the differential equation to find a complete equation linking r and t.

(5)

(c) Find the limitation on the values of t for which the equation in part (b) is valid.

(2)

ANSWER

Question	Scheme	Marks	AOs
14 (a)	Uses the model to state $\frac{dV}{dt} = -c$ (for positive constant c)	B1	3.1b
	Uses $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ with their $\frac{dV}{dt} = -c$ and $\frac{dV}{dr} = 4\pi r^2$	M1	2.1
	$-c = 4\pi r^2 \times \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = -\frac{c}{4\pi r^2} = -\frac{k}{r^2}$ *	A1*	2.2a
		(3)	
(b)	$\frac{dr}{dt} = -\frac{k}{r^2} \Rightarrow \int r^2 dr = \int -k dt$ and integrates with one side "correct"	M1	2.1
	$\frac{r^3}{3} = -kt(+\alpha)$	A1	1.1b
	Uses $t = 0, r = 40 \Rightarrow \alpha = ...$ $\alpha = \frac{64000}{3}$	M1	1.1b
	Uses $t = 5, r = 20$ & $\alpha = ... \Rightarrow k = ...$	M1	3.4
	$r^3 = 64000 - 11200t$ or exact equivalent	A1	3.3
		(5)	
(c)	Uses the equation of their model and proceeds to a limiting value for t E.g. "$64000 - 11200t$" ... $0 \Rightarrow t$...	M1	3.4
	For times up to and including $\frac{40}{7}$ seconds	A1ft	3.5b
		(2)	
		(10 marks)	

Video solution:

https://youtu.be/lvzf21s7C4E

November 2020 Question 1 Paper 2

1 The table below shows corresponding values of x and y for $y = \sqrt{\frac{x}{1+x}}$

The values of y are given to 4 significant figures.

x	0.5	1	1.5	2	2.5
y	0.5774	0.7071	0.7746	0.8165	0.8452

(a) Use the trapezium rule, with all the values of y in the table, to find an estimate for

$$\int_{0.5}^{2.5} \sqrt{\frac{x}{1+x}}\, dx$$

giving your answer to 3 significant figures.

(3)

(b) Using your answer to part (a), deduce an estimate for $\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}}\, dx$

(1)

Given that

$$\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}}\, dx = 4.535 \text{ to 4 significant figures}$$

(c) comment on the accuracy of your answer to part (b).

(1)

ANSWER

Question	Scheme	Marks	AOs
1(a)	$h = 0.5$	B1	1.1a
	$A \approx \frac{0.5}{2}\{0.5774 + 0.8452 + 2(0.7071 + 0.7746 + 0.8165)\}$	M1	1.1b
	= awrt 1.50	A1	1.1b
	For reference: **The integration on a calculator gives 1.511549071** **The full accuracy for *y* values gives 1.504726147** **The accuracy from the table gives 1.50475**		
		(3)	

(b)	3 × their (a)
	If (a) is correct, allow awrt 4.50 or awrt 4.51 even with no working.
	Only allow 4.5 if (a) is correct and working is shown e.g. 3 × 1.5
	
	If (a) is incorrect allow 3 × their (a) given to at least 3sf but do not be too concerned about the accuracy (as they may use rounded or rounded value from (a))
		B1ft	2.2a

	For reference the integration on a calculator gives 4.534647213		
		(1)	

(c)	<u>**This mark depends on the B1 having been awarded in part (b) with awrt 4.5**</u> Look for a sensible comment. Some examples: • The answer is accurate to 2 sf or one decimal place • Answer to (b) is accurate as $4.535 \approx 4.50$ • Very accurate as 4.535 to 2 sf is 4.5 • $4.51425 < 4.535$ so my answer is underestimate but not too far off • It is an underestimate but quite close • It is a very good estimate • High accuracy • (Quite) accurate • It is less than 1% out • $4.535 - 4.5 = 0.035$ so not far out But **not** just "it is an underestimate" or Calculates percentage error correctly using awrt 4.50 or awrt 4.51 or 4.5 (No comment is necessary in these cases although one may be given) Examples: $\left\lvert\frac{4.535-4.50}{4.535}\right\rvert \times 100 = 0.77\%$ or $\left\lvert\frac{4.535-4.51}{4.535}\right\rvert \times 100 = 0.55\%$ or $\left\lvert\frac{4.535-4.51425}{4.535}\right\rvert \times 100 = 0.46\%$ or $\left\lvert\frac{4.50}{4.535}\right\rvert \times 100 = 99\%$ In these cases don't be too concerned about accuracy e.g. allow 1sf. **This mark should be withheld if there are any contradictory statements**	B1	3.2b
		(1)	
			(5 marks)

November 2020 Question

6. (a) Given that

$$\frac{x^2 + 8x - 3}{x + 2} \equiv Ax + B + \frac{C}{x + 2} \qquad x \in \mathbb{R} \;\; x \neq -2$$

find the values of the constants A, B and C

(3)

(b) Hence, using algebraic integration, find the exact value of

$$\int_0^6 \frac{x^2 + 8x - 3}{x + 2}\,\mathrm{d}x$$

giving your answer in the form $a + b\ln 2$ where a and b are integers to be found.

(4)

ANSWER

6(a)	$x^2+8x-3=(Ax+B)(x+2)+C$ or $Ax(x+2)+B(x+2)+C$ $\Rightarrow A=\ldots, B=\ldots, C=\ldots$ **or** $$\begin{array}{r l} & x+6 \\ x+2 & \overline{)x^2+8x-3} \\ & \underline{x^2+2x} \\ & \quad 6x-3 \\ & \quad \underline{6x+12} \\ & \quad\quad -15 \end{array}$$	M1	1.1b
	Two of $A=1, B=6, C=-15$	A1	1.1b
	All three of $A=1, B=6, C=-15$	A1	1.1b
		(3)	
6(b)	$\displaystyle\int \frac{x^2+8x-3}{x+2}\,\mathrm{d}x = \int x+6-\frac{15}{x+2}\,\mathrm{d}x = \ldots-15\ln(x+2)$	M1	1.1b
	$=\frac{1}{2}x^2+6x-15\ln(x+2) \quad (+c)$	A1ft	1.1b
	$\displaystyle\int_0^6 \frac{x^2+8x-3}{x+2}\,\mathrm{d}x = \left[\frac{1}{2}x^2+6x-15\ln(x+2)\right]_0^6$ $= (18+36-15\ln 8)-(0+0-15\ln 2)$ $= 18+36-(15-45)\ln 2$ or e.g. $18+36+15\ln\left(\frac{2}{8}\right)$	M1	2.1
	$= 54-30\ln 2$	A1	1.1b
		(4)	
			(7 marks)

November 2020 Question 12 Paper 2

12.

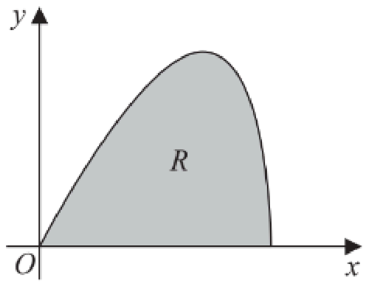

Figure 3

The curve shown in Figure 3 has parametric equations

$$x = 6\sin t \qquad y = 5\sin 2t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

(a) (i) Show that the area of R is given by $\displaystyle\int_0^{\frac{\pi}{2}} 60\sin t\cos^2 t\,\mathrm{d}t$

(3)

(ii) Hence show, by algebraic integration, that the area of R is exactly 20

(3)

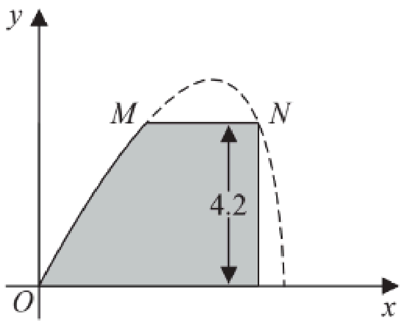

Figure 4

Part of the curve is used to model the profile of a small dam, shown shaded in Figure 4.
Using the model and given that

- x and y are in metres
- the vertical wall of the dam is 4.2 metres high
- there is a horizontal walkway of width MN along the top of the dam

(b) calculate the width of the walkway.

(5)

ANSWER

Question	Scheme	Marks	AOs
12(a)(i)	$y \times \dfrac{dx}{dt} = 5\sin 2t \times 6\cos t$ or $5 \times 2\sin t\cos t \times 6\cos t$	M1	1.2
	(Area =) $\displaystyle\int 5\sin 2t \times 6\cos t\,dt = \int 5 \times 2\sin t\cos t \times 6\cos t\,dt$ or $\displaystyle\int 5\sin 2t \times 6\cos t\,dt = \int 60\sin t\cos^2 t\,dt$	dM1	1.1b
	(Area =) $\displaystyle\int_0^{\frac{\pi}{2}} 60\sin t\cos^2 t\,dt$ *	A1*	2.1*
		(3)	
(a)(ii)	$\displaystyle\int 60\sin t\cos^2 t\,dt = -20\cos^3 t$	M1 A1	1.1b 1.1b
	Area $= \left[-20\cos^3 t\right]_0^{\frac{\pi}{2}} = 0 - (-20) = 20$ *	A1*	2.1
		(3)	
(b)	$5\sin 2t = 4.2 \Rightarrow \sin 2t = \dfrac{4.2}{5}$	M1	3.4
	$t = 0.4986..., 1.072...$	A1	1.1b
	Attempts to finds the x values at both t values	dM1	3.4
	$t = 0.4986... \Rightarrow x = 2.869...$ $t = 1.072 \Rightarrow x = 5.269...$	A1	1.1b
	Width of path = 2.40 metres	A1	3.2a
		(5)	
			(11 marks)

June 2019 Question 13 Paper 1

13. The curve C with equation

$$y = \frac{p - 3x}{(2x - q)(x + 3)} \qquad x \in \mathbb{R}, x \neq -3, x \neq 2$$

where p and q are constants, passes through the point $\left(3, \frac{1}{2}\right)$ and has two vertical asymptotes with equations $x = 2$ and $x = -3$

(a) (i) Explain why you can deduce that $q = 4$

(ii) Show that $p = 15$

(3)

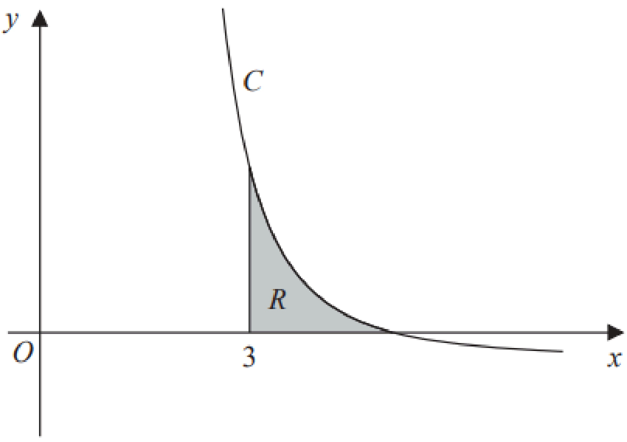

Figure 4

Figure 4 shows a sketch of part of the curve C. The region R, shown shaded in Figure 4, is bounded by the curve C, the x-axis and the line with equation $x = 3$

(b) Show that the exact value of the area of R is $a\ln 2 + b\ln 3$, where a and b are rational constants to be found.

(8)

ANSWER

Question	Scheme	Marks	AOs
13 (a)	(i) **Explains** $2x-q=0$ when $x=2$ oe Hence $q=4$ *	B1*	2.4
	(ii) Substitutes $\left(3,\frac{1}{2}\right)$ into $y=\frac{p-3x}{(2x-4)(x+3)}$ and solves	M1	1.1b
	$\frac{1}{2}=\frac{p-9}{(2)\times(6)}\Rightarrow p-9=6\Rightarrow p=15$*	A1*	2.1
		(3)	
(b)	Attempts to write $\frac{15-3x}{(2x-4)(x+3)}$ in PF's and integrates using lns between 3 and another value of x.	M1	3.1a
	$\frac{15-3x}{(2x-4)(x+3)}=\frac{A}{(2x-4)}+\frac{B}{(x+3)}$ leading to A and B	M1	1.1b
	$\frac{15-3x}{(2x-4)(x+3)}=\frac{1.8}{(2x-4)}-\frac{2.4}{(x+3)}$ or $\frac{0.9}{(x-2)}-\frac{2.4}{(x+3)}$ oe	A1	1.1b
	$\mathrm{I}=\int\frac{15-3x}{(2x-4)(x+3)}\mathrm{d}x=m\ln(2x-4)+n\ln(x+3)+(c)$	M1	1.1b
	$\mathrm{I}=\int\frac{15-3x}{(2x-4)(x+3)}\mathrm{d}x=0.9\ln(2x-4)-2.4\ln(x+3)$ oe	A1ft	1.1b
	Deduces that Area Either $\int_3^5\frac{15-3x}{(2x-4)(x+3)}\mathrm{d}x$ Or $[.............]_3^5$	B1	2.2a
	Uses correct ln work seen at least once for $\ln 6=\ln 2+\ln 3$ **or** $\ln 8=3\ln 2$ $[0.9\ln(6)-2.4\ln(8)]-[0.9\ln(2)-2.4\ln(6)]$ $=3.3\ln 6-7.2\ln 2-0.9\ln 2$	dM1	2.1
	$=3.3\ln 3-4.8\ln 2$	A1	1.1b
		(8)	
			(11marks)

June 2019 Question 5 Paper 2

5.

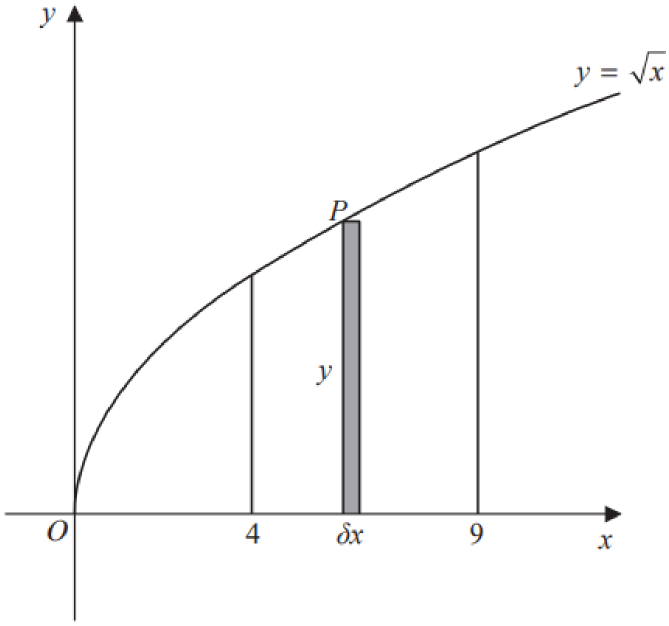

Figure 3

Figure 3 shows a sketch of the curve with equation $y = \sqrt{x}$

The point $P(x, y)$ lies on the curve.

The rectangle, shown shaded on Figure 3, has height y and width δx.

Calculate

$$\lim_{\delta x \to 0} \sum_{x=4}^{9} \sqrt{x}\, \delta x$$

(3)

ANSWER

Question	Scheme	Marks	AOs
5	States $\left\{ \lim_{\delta x \to 0} \sum_{x=4}^{9} \sqrt{x}\ \delta x \text{ is} \right\}$ $\int_4^9 \sqrt{x}\,\mathrm{d}x$	B1	1.2
	$= \left[\frac{2}{3} x^{\frac{3}{2}} \right]_4^9$	M1	1.1b
	$= \frac{2}{3} \times 9^{\frac{3}{2}} - \frac{2}{3} \times 4^{\frac{3}{2}} = \frac{54}{3} - \frac{16}{3}$		
	$= \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7	A1	1.1b
		(3)	
			(3 marks)

June 2019 Question 14 Paper 2

14. (a) Use the substitution $u = 4 - \sqrt{h}$ to show that

$$\int \frac{\mathrm{d}h}{4 - \sqrt{h}} = -8\ln\left|4 - \sqrt{h}\right| - 2\sqrt{h} + k$$

where k is a constant

(6)

A team of scientists is studying a species of slow growing tree.

The rate of change in height of a tree in this species is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{t^{0.25}\left(4 - \sqrt{h}\right)}{20}$$

where h is the height in metres and t is the time, measured in years, after the tree is planted.

(b) Find, according to the model, the range in heights of trees in this species.

(2)

One of these trees is one metre high when it is first planted.

According to the model,

(c) calculate the time this tree would take to reach a height of 12 metres, giving your answer to 3 significant figures.

(7)

ANSWER

Question	Scheme	Marks	AOs
14 (a)	$\{u = 4-\sqrt{h} \Rightarrow\}\ \dfrac{\mathrm{d}u}{\mathrm{d}h} = -\dfrac{1}{2}h^{-\frac{1}{2}}$ or $\dfrac{\mathrm{d}h}{\mathrm{d}u} = -2(4-u)$ or $\dfrac{\mathrm{d}h}{\mathrm{d}u} = -2\sqrt{h}$	B1	1.1b
	$\left\{\displaystyle\int \dfrac{\mathrm{d}h}{4-\sqrt{h}} = \right\} \displaystyle\int \dfrac{-2(4-u)}{u}\,\mathrm{d}u$	M1	2.1
	$= \displaystyle\int \left(-\dfrac{8}{u}+2\right)\mathrm{d}u$	M1	1.1b
	$= -8\ln u + 2u\ \{+c\}$	M1	1.1b
		A1	1.1b
	$= -8\ln\lvert 4-\sqrt{h}\rvert + 2(4-\sqrt{h}) + c\ =\ -8\ln\lvert 4-\sqrt{h}\rvert - 2\sqrt{h}\ + k$ *	A1*	2.1
		(6)	
(b)	$\left\{\dfrac{\mathrm{d}h}{\mathrm{d}t} = \dfrac{t^{0.25}(4-\sqrt{h})}{20} = 0 \Rightarrow\right\}\ 4-\sqrt{h} = 0$	M1	3.4
	Deduces any of $0 < h < 16,\ 0 \le h < 16,\ 0 < h \le 16,\ 0 \le h \le 16,$ $h < 16,\ h \le 16$ or all values up to 16	A1	2.2a
		(2)	
(c) Way 1	$\displaystyle\int \dfrac{1}{(4-\sqrt{h})}\,\mathrm{d}h = \int \dfrac{1}{20}t^{0.25}\,\mathrm{d}t$	B1	1.1b
	$-8\ln\lvert 4-\sqrt{h}\rvert - 2\sqrt{h} = \dfrac{1}{25}t^{1.25}\ \{+c\}$	M1	1.1b
		A1	1.1b
	$\{t=0,\ h=1 \Rightarrow\}\ -8\ln(4-1) - 2\sqrt{(1)} = \dfrac{1}{25}(0)^{1.25} + c$	M1	3.4
	$\Rightarrow c = -8\ln(3) - 2 \Rightarrow\ -8\ln\lvert 4-\sqrt{h}\rvert - 2\sqrt{h} = \dfrac{1}{25}t^{1.25} - 8\ln(3) - 2$ $\{h = 12 \Rightarrow\}\ -8\ln\lvert 4-\sqrt{12}\rvert - 2\sqrt{12} = \dfrac{1}{25}t^{1.25} - 8\ln(3) - 2$	dM1	3.1a
	$t^{1.25} = 221.2795202... \Rightarrow t = \sqrt[1.25]{221.2795...}$ or $t = (221.2795...)^{0.8}$	M1	1.1b
	$t = 75.154... \Rightarrow t =$ 75.2 (years) (3 sf) or awrt 75.2 (years)	A1	1.1b
	Note: You can recover work for part (c) in part (b)	**(7)**	
(c) Way 2	$\displaystyle\int_1^{12} \dfrac{20}{(4-\sqrt{h})}\,\mathrm{d}h = \int_0^T t^{0.25}\,\mathrm{d}t$	B1	1.1b
	$\left[20(-8\ln\lvert 4-\sqrt{h}\rvert - 2\sqrt{h})\right]_1^{12} = \left[\dfrac{4}{5}t^{1.25}\right]_0^T$	M1	1.1b
		A1	1.1b
	$20(-8\ln(4-\sqrt{12}) - 2\sqrt{12}) - 20(-8\ln(4-1) - 2\sqrt{1}) = \dfrac{4}{5}T^{1.25} - 0$	M1	3.4
		dM1	3.1a
	$T^{1.25} = 221.2795202... \Rightarrow T = \sqrt[1.25]{221.2795...}$ or $T = (221.2795...)^{0.8}$	M1	1.1b
	$T = 75.154... \Rightarrow T =$ 75.2 (years) (3 sf) or awrt 75.2 (years)	A1	1.1b
	Note: You can recover work for part (c) in part (b)	**(7)**	
			(15 marks)

June 2018 Question 7 Paper 1

7. Given that $k \in \mathbb{Z}^+$

(a) show that $\int_k^{3k} \frac{2}{(3x-k)}\,dx$ is independent of k, **(4)**

(b) show that $\int_k^{2k} \frac{2}{(2x-k)^2}\,dx$ is inversely proportional to k. **(3)**

ANSWER

Question	Scheme	Marks	AOs
7 (a)	$\int \frac{2}{(3x-k)}\,dx = \frac{2}{3}\ln(3x-k)$	M1 A1	1.1a 1.1b
	$\int_k^{3k} \frac{2}{(3x-k)}\,dx = \frac{2}{3}\ln(9k-k) - \frac{2}{3}\ln(3k-k)$	dM1	1.1b
	$= \frac{2}{3}\ln\left(\frac{8\cancel{k}}{2\cancel{k}}\right) = \frac{2}{3}\ln 4$ oe	A1	2.1
		(4)	
(b)	$\int \frac{2}{(2x-k)^2}\,dx = -\frac{1}{(2x-k)}$	M1	1.1b
	$\int_k^{2k} \frac{2}{(2x-k)^2}\,dx = -\frac{1}{(4k-k)} + \frac{1}{(2k-k)}$	dM1	1.1b
	$= \frac{2}{3k} \quad \left(\propto \frac{1}{k}\right)$	A1	2.1
		(3)	
			(7 marks)

June 2018 Question 10 Paper 1

10. The height above ground, H metres, of a passenger on a roller coaster can be modelled by the differential equation

$$\frac{dH}{dt} = \frac{H\cos(0.25t)}{40}$$

where t is the time, in seconds, from the start of the ride.

Given that the passenger is 5 m above the ground at the start of the ride,

(a) show that $H = 5e^{0.1\sin(0.25t)}$

(5)

(b) State the maximum height of the passenger above the ground.

(1)

The passenger reaches the maximum height, for the second time, T seconds after the start of the ride.

(c) Find the value of T.

(2)

ANSWER

Question	Scheme	Marks	AOs
10(a)	$\frac{dH}{dt}=\frac{H\cos 0.25t}{40} \Rightarrow \int \frac{1}{H}\,dH = \int \frac{\cos 0.25t}{40}\,dt$	M1	3.1a
	$\ln H = \frac{1}{10}\sin 0.25t\,(+c)$	M1 A1	1.1b 1.1b
	Substitutes $t=0, H=5 \Rightarrow c=\ln(5)$	dM1	3.4
	$\ln\left(\frac{H}{5}\right)=\frac{1}{10}\sin 0.25t \Rightarrow H=5e^{0.1\sin 0.25t}$ *	A1*	2.1
		(5)	
(b)	Max height = $5e^{0.1}=5.53$ m (Condone lack of units)	B1	3.4
		(1)	
(c)	Sets $0.25t=\frac{5\pi}{2}$	M1	3.1b
	31.4	A1	1.1b
		(2)	
			(8 marks)

June 2018 Question 13 Paper 1

13. Show that

$$\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x = \frac{32}{15}\left(2+\sqrt{2}\right)$$

(7)

ANSWER

Question	Scheme for Substitution		Marks	AOs
13	Chooses a suitable method for $\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x$ Award for • Using a valid substitution $u=...$, changing the terms to u's • integrating and using appropriate limts .		M1	3.1a
	Substitution $u=\sqrt{x+2} \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u}=2u$ oe	Substitution $u=x+2 \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u}=1$ oe	B1	1.1b
	$\int 2x\sqrt{x+2}\,\mathrm{d}x$ $=\int A\left(u^2\pm 2\right)u^2\mathrm{d}u$	$\int 2x\sqrt{x+2}\,\mathrm{d}x$ $=\int A\left(u\pm 2\right)\sqrt{u}\mathrm{d}u$	M1	1.1b
	$=Pu^5 \pm Qu^3$	$=Su^{\frac{5}{2}} \pm Tu^{\frac{3}{2}}$	dM1	2.1
	$=\frac{4}{5}u^5-\frac{8}{3}u^3$	$=\frac{4}{5}u^{\frac{5}{2}}-\frac{8}{3}u^{\frac{3}{2}}$	A1	1.1b
	Uses limits 2 and $\sqrt{2}$ the correct way around	Uses limits 4 and 2 the correct way around	ddM1	1.1b
	$=\frac{32}{15}\left(2+\sqrt{2}\right)$ *		A1*	2.1
			(7)	
				(7 marks)

Arancha Ruiz

June 2018 Question 10 Paper 2

10. A spherical mint of radius 5 mm is placed in the mouth and sucked.
Four minutes later, the radius of the mint is 3 mm.

In a simple model, the rate of decrease of the radius of the mint is inversely proportional to the square of the radius.

Using this model and all the information given,

(a) find an equation linking the radius of the mint and the time.
(You should define the variables that you use.)

(5)

(b) Hence find the total time taken for the mint to completely dissolve. Give your answer in minutes and seconds to the nearest second.

(2)

(c) Suggest a limitation of the model.

(1)

ANSWER

Question	Scheme		Marks	AOs
10 (a)	$\frac{\mathrm{d}r}{\mathrm{d}t} \propto \pm\frac{1}{r^2}$ or $\frac{\mathrm{d}r}{\mathrm{d}t} = \pm\frac{k}{r^2}$ (for k or a numerical k)		M1	3.3
	$\int r^2\,\mathrm{d}r = \int \pm k\,\mathrm{d}t \Rightarrow ...$ (for k or a numerical k)		M1	2.1
	$\frac{1}{3}r^3 = \pm kt$ $\{+c\}$		A1	1.1b
	$t=0, r=5$ and $t=4, r=3$ gives $\frac{1}{3}r^3 = -\frac{49}{6}t + \frac{125}{3}$,	$t=0, r=5$ and $t=240, r=3$ gives $\frac{1}{3}r^3 = -\frac{49}{360}t + \frac{125}{3}$,	M1	3.1a
	where r, in mm, is the radius {of the mint} and t, in minutes, is the time from when it {the mint} was placed in the mouth	where r, in mm, is the radius {of the mint} and t, in seconds, is the time from when it {the mint} was placed in the mouth	A1	1.1b
			(5)	
(b)	$r=0 \Rightarrow 0 = -\frac{49}{6}t + \frac{125}{3} \Rightarrow 0 = -49t + 250 \Rightarrow t = ...$		M1	3.4
	time = 5 minutes 6 seconds		A1	1.1b
			(2)	
(c)	Suggests a suitable limitation of the model. E.g. • Model does not consider how the mint is sucked • Model does not consider whether the mint is bitten • Model is limited for times up to 5 minutes 6 seconds, o.e. • Not valid for times greater than 5 minutes 6 seconds, o.e. • Mint may not retain the shape of a sphere (or have uniform radius) as it is being sucked • The model indicates that the radius of the mint is negative after it dissolves • Model does not consider the temperature in the mouth • Model does not consider rate of saliva production • Mint could be swallowed before it dissolves in the mouth		B1	3.5b
			(1)	
			(8 marks)	

June 2018 Question 13 Paper 2

13.

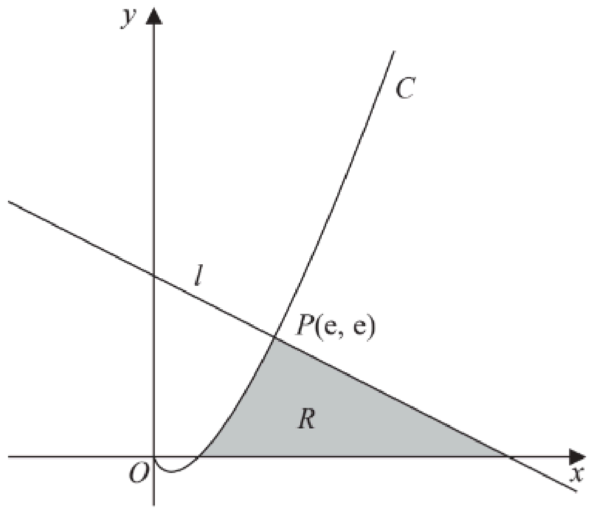

Figure 2

Figure 2 shows a sketch of part of the curve C with equation $y = x\ln x, \quad x > 0$

The line l is the normal to C at the point $P(\mathrm{e}, \mathrm{e})$

The region R, shown shaded in Figure 2, is bounded by the curve C, the line l and the x-axis.

Show that the exact area of R is $A\mathrm{e}^2 + B$ where A and B are rational numbers to be found.

(10)

ANSWER

Question	Scheme	Marks	AOs
13	$C: y = x\ln x$; l is a normal to C at $P(\mathrm{e}, \mathrm{e})$ Let x_A be the x-coordinate of where l cuts the x-axis		
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \ln x + x\left(\dfrac{1}{x}\right) \quad \{= 1 + \ln x\}$	M1 A1	2.1 1.1b
	$x = \mathrm{e},\ m_T = 2 \Rightarrow m_N = -\dfrac{1}{2} \Rightarrow y - \mathrm{e} = -\dfrac{1}{2}(x - \mathrm{e})$ $y = 0 \Rightarrow -\mathrm{e} = -\dfrac{1}{2}(x - \mathrm{e}) \Rightarrow x = \ldots$	M1	3.1a
	l meets x-axis at $x = 3\mathrm{e}$ (allow $x = 2\mathrm{e} + \mathrm{e}\ln \mathrm{e}$)	A1	1.1b
	{Areas:} **either** $\displaystyle\int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ \ldots\ ]_1^{\mathrm{e}} = \ldots$ **or** $\dfrac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$	M1	2.1
	$\left\{\displaystyle\int x\ln x\,\mathrm{d}x = \right\}\ \dfrac{1}{2}x^2\ln x - \displaystyle\int \dfrac{1}{x}\cdot\left(\dfrac{x^2}{2}\right)\{\mathrm{d}x\}$	M1	2.1
	$\left\{= \dfrac{1}{2}x^2\ln x - \displaystyle\int \dfrac{1}{2}x\,\{\mathrm{d}x\}\right\} = \dfrac{1}{2}x^2\ln x - \dfrac{1}{4}x^2$	dM1 A1	1.1b 1.1b
	$\text{Area}(R_1) = \displaystyle\int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ \ldots\ ]_1^{\mathrm{e}} = \ldots$; $\text{Area}(R_2) = \dfrac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$ and so, $\text{Area}(R) = \text{Area}(R_1) + \text{Area}(R_2)$ $\quad \{= \frac{1}{4}\mathrm{e}^2 + \frac{1}{4} + \mathrm{e}^2\}$	M1	3.1a
	$\text{Area}(R) = \frac{5}{4}\mathrm{e}^2 + \frac{1}{4}$	A1	1.1b
		(10)	