# Edexcel - Pure Year 2 exam questions



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# Ch. 10: Numerical Methods

## June 2023 Question 15 Paper 1

**15.** A curve has equation $y = f(x)$, where

$$f(x) = \frac{7xe^x}{\sqrt{e^{3x} - 2}} \qquad x > \ln \sqrt[3]{2}$$

(a) Show that

$$f'(x) = \frac{7e^x(e^{3x}(2 - x) + Ax + B)}{2(e^{3x} - 2)^{\frac{3}{2}}}$$

where $A$ and $B$ are constants to be found.

**(5)**

(b) Hence show that the $x$ coordinates of the turning points of the curve are solutions of the equation

$$x = \frac{2e^{3x} - 4}{e^{3x} + 4}$$

**(2)**

The equation $x = \dfrac{2e^{3x} - 4}{e^{3x} + 4}$ has two positive roots $\alpha$ and $\beta$ where $\beta > \alpha$

A student uses the iteration formula

$$x_{n+1} = \frac{2e^{3x_n} - 4}{e^{3x_n} + 4}$$

in an attempt to find approximations for $\alpha$ and $\beta$

Diagram 1 shows a plot of part of the curve with equation $y = \dfrac{2e^{3x} - 4}{e^{3x} + 4}$ and part of the line with equation $y = x$

Using Diagram 1 on page 42

(c) draw a staircase diagram to show that the iteration formula starting with $x_1 = 1$ can be used to find an approximation for $\beta$

**(1)**

Arancha Ruiz

Use the iteration formula with $x_1 = 1$, to find, to 3 decimal places,

(d) (i) the value of $x_2$

(ii) the value of $\beta$

**(3)**

Using a suitable interval and a suitable function that should be stated

(e) show that $\alpha = 0.432$ to 3 decimal places.

**(2)**

Only use the copy of Diagram 1 if you need to redraw your answer to part (c).

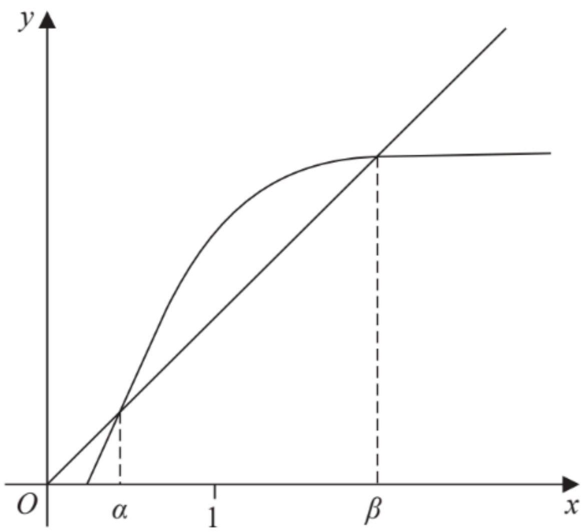
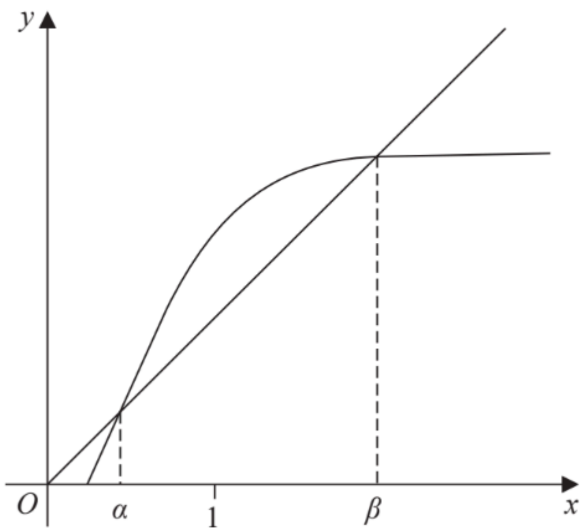



**Diagram 1**




**copy of Diagram 1**

## ANSWER

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 15(a) | $...xe^x + ...e^x$ | M1 | 1.1b |
| | $k\left(xe^x + e^x\right)$ | A1 | 1.1b |
| | $\frac{d}{dx}\left(\sqrt{e^{3x}-2}\right) = \frac{1}{2} \times 3e^{3x}\left(e^{3x}-2\right)^{-\frac{1}{2}}$ | B1 | 1.1b |
| | $\left(f'(x) =\right) \dfrac{(e^{3x}-2)^{\frac{1}{2}}("7"xe^x + "7"e^x) - "\frac{3}{2}"e^{3x}(e^{3x}-2)^{-\frac{1}{2}} \times "7"xe^x}{e^{3x}-2}$ | dM1 | 2.1 |
| | $f'(x) = \dfrac{7e^x\left(e^{3x}(2-x)-4x-4\right)}{2(e^{3x}-2)^{\frac{3}{2}}}$ | A1 | 1.1b |
| | | **(5)** | |
| (b) | $e^{3x}(2-x)-4x-4=0 \Rightarrow x(...e^{3x} \pm ...) = ...e^{3x} \pm ...$ | M1 | 1.1b |
| | $\Rightarrow x = \dfrac{2e^{3x}-4}{e^{3x}+4}$ * | A1* | 2.1 |
| | | **(2)** | |
| (c) | Draws a vertical line $x=1$ up to the curve then across to the line $y=x$ then up to the curve finishing at the root (need to see a minimum of 2 vertical and horizontal lines tending to the root) | B1 | 2.1 |
| | | **(1)** | |
| (d)(i) | $x_2 = \dfrac{2e^3-4}{e^3+4} = 1.5017756...$ | M1 | 1.1b |
| | $x_2 =$ awrt 1.502 | A1 | 1.1b |
| (ii) | $\beta = 1.968$ | dB1 | 2.2b |
| | | **(3)** | |
| (e) | $h(x) = \dfrac{2e^{3x}-4}{e^{3x}+4} - x$<br>$h(0.4315) = -0.000297...$ $h(0.4325) = 0.000947...$ | M1 | 3.1a |
| | Both calculations correct and e.g. states:<br>• There is a change of sign<br>• e.g $f'(x)$ is continuous<br>• $\alpha = 0.432$ (to 3dp) | A1cao | 2.4 |
| | | **(2)** | |
| | | | **(13 marks)** |

## June 2023 Question 5 Paper 1

**5.** A continuous curve has equation $y = f(x)$.

The table shows corresponding values of $x$ and $y$ for this curve, where $a$ and $b$ are constants.

| $x$ | 3 | 3.2 | 3.4 | 3.6 | 3.8 | 4 |
|---|---|---|---|---|---|---|
| $y$ | $a$ | 16.8 | $b$ | 20.2 | 18.7 | 13.5 |

The trapezium rule is used, with all the $y$ values in the table, to find an approximate area under the curve between $x = 3$ and $x = 4$

Given that this area is 17.59

(a) show that $a + 2b = 51$

**(3)**

Given also that the sum of all the $y$ values in the table is 97.2

(b) find the value of $a$ and the value of $b$

**(3)**

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## ANSWER

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **5(a)** | $h = 0.2$ | B1 | 1.1b |
| | $\frac{1}{2} \times "0.2" \times \{a + 13.5 + 2(16.8 + b + 20.2 + 18.7)\} = 17.59$ | M1 | 1.1b |
| | e.g. $\Rightarrow a + 13.5 + 2b + 111.4 = 175.9 \Rightarrow a + 2b = 51*$ | A1* | 2.1 |
| | | **(3)** | |
| **(b)** | $a + 16.8 + b + 20.2 + 18.7 + 13.5 = 97.2 \Rightarrow a + b = 28 \Rightarrow a = \ldots$ ( or $b = \ldots$) | M1 | 3.1a |
| | $a = 5$ or $b = 23$ | A1 | 1.1b |
| | $a = 5$ and $b = 23$ | A1 | 1.1b |
| | | **(3)** | |
| | | | **(6 marks)** |

## June 2022 Question 6 Paper 2

**6.**

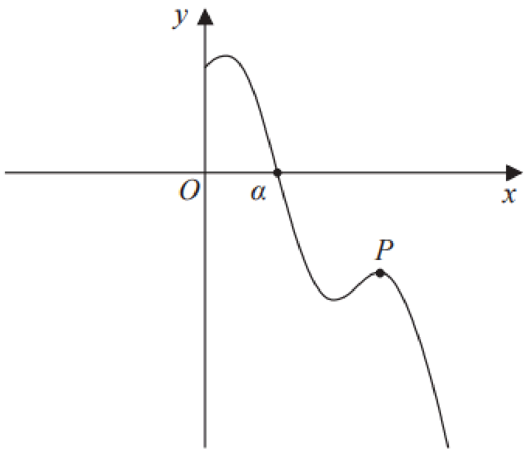


**Figure 2**

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$ where

$$f(x) = 8\sin\left(\frac{1}{2}x\right) - 3x + 9 \qquad x > 0$$

and $x$ is measured in radians.

The point $P$, shown in Figure 2, is a local maximum point on the curve.

Using calculus and the sketch in Figure 2,

(a) find the $x$ coordinate of $P$, giving your answer to 3 significant figures.

**(4)**

The curve crosses the $x$-axis at $x = \alpha$, as shown in Figure 2.

Given that, to 3 decimal places, $f(4) = 4.274$ and $f(5) = -1.212$

(b) explain why $\alpha$ must lie in the interval $[4, 5]$

**(1)**

(c) Taking $x_0 = 5$ as a first approximation to $\alpha$, apply the Newton-Raphson method once to $f(x)$ to obtain a second approximation to $\alpha$.

Show your method and give your answer to 3 significant figures.

**(2)**

**ANSWER**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 6(a) | $(\mathrm{f}'(x)=)4\cos\left(\frac{1}{2}x\right)-3$ | M1<br>A1 | 1.1b<br>1.1b |
| | Sets $\mathrm{f}'(x)=4\cos\left(\frac{1}{2}x\right)-3=0 \Rightarrow x=$ | dM1 | 3.1a |
| | $x=14.0$ Cao | A1 | 3.2a |
| | | **(4)** | |
| **(b)** | Explains that $\mathrm{f}(4)>0$, $\mathrm{f}(5)<0$<br>**and** the function is continuous | B1 | 2.4 |
| | | **(1)** | |
| **(c)** | Attempts $x_1=5-\frac{8\sin 2.5-15+9}{"4\cos 2.5-3"}$<br>(NB $\mathrm{f}(5)=-1.212\ldots$ and $\mathrm{f}'(5)=-6.204\ldots$) | M1 | 1.1b |
| | $x_1=$ awrt 4.80 | A1 | 1.1b |
| | | **(2)** | |
| | | | **(7 marks)** |

## November 2021 Question 4 Paper 1

**4.** The curve with equation $y = f(x)$ where

$$f(x) = x^2 + \ln(2x^2 - 4x + 5)$$

has a single turning point at $x = \alpha$

(a) Show that $\alpha$ is a solution of the equation

$$2x^3 - 4x^2 + 7x - 2 = 0$$

**(4)**

The iterative formula

$$x_{n+1} = \frac{1}{7}(2 + 4x_n^{\ 2} - 2x_n^{\ 3})$$

is used to find an approximate value for $\alpha$.

Starting with $x_1 = 0.3$

(b) calculate, giving each answer to 4 decimal places,

(i) the value of $x_2$

(ii) the value of $x_4$

**(3)**

Using a suitable interval and a suitable function that should be stated,

(c) show that $\alpha$ is 0.341 to 3 decimal places.

**(2)**

## ANSWER

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 4(a) | $\text{f}'(x) = 2x + \dfrac{4x-4}{2x^2-4x+5}$ | M1<br>A1 | 1.1b<br>1.1b |
| | $2x + \dfrac{4x-4}{2x^2-4x+5} = 0 \Rightarrow 2x(2x^2-4x+5)+4x-4=0$ | dM1 | 1.1b |
| | $2x^3-4x^2+7x-2=0$* | A1* | 2.1 |
| | | **(4)** | |
| (b) | (i) $x_2 = \frac{1}{7}\left(2+4(0.3)^2-2(0.3)^3\right)$ | M1 | 1.1b |
| | $x_2 = 0.3294$ | A1 | 1.1b |
| | (ii) $x_4 = 0.3398$ | A1 | 1.1b |
| | | **(3)** | |
| (c) | $\text{h}(x) = 2x^3-4x^2+7x-2$<br>$\text{h}(0.3415) = 0.00366...$ $\text{h}(0.3405) = -0.00130...$ | M1 | 3.1a |
| | States:<br>• there is a change of sign<br>• $\text{f}'(x)$ is continuous<br>• $\alpha = 0.341$ to 3dp | A1 | 2.4 |
| | | **(2)** | |
| | | | **(9 marks)** |

## November 2020 Question 7 Paper 2

**7.**

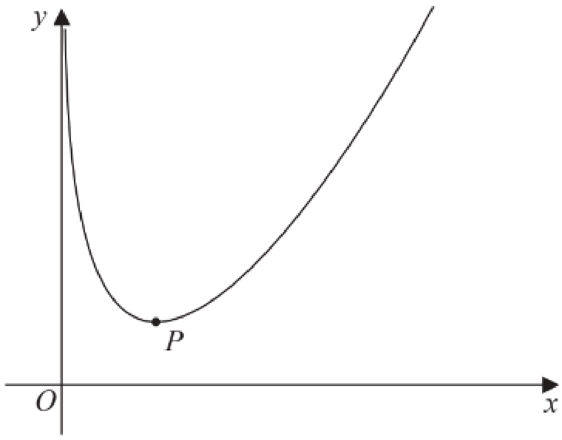


**Figure 1**

Figure 1 shows a sketch of the curve $C$ with equation

$$y = \frac{4x^2 + x}{2\sqrt{x}} - 4\ln x \qquad x > 0$$

(a) Show that

$$\frac{dy}{dx} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}}$$

**(4)**

The point $P$, shown in Figure 1, is the minimum turning point on $C$.

(b) Show that the $x$ coordinate of $P$ is a solution of

$$x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}$$

**(3)**

(c) Use the iteration formula

$$x_{n+1} = \left(\frac{4}{3} - \frac{\sqrt{x_n}}{12}\right)^{\frac{2}{3}} \qquad \text{with } x_1 = 2$$

to find (i) the value of $x_2$ to 5 decimal places,

(ii) the $x$ coordinate of $P$ to 5 decimal places.

**(3)**

**ANSWER**

| | | | |
|---|---|---|---|
| **7(a)** | $\ln x \to \frac{1}{x}$ | B1 | 1.1a |
| | Method to differentiate $\frac{4x^2+x}{2\sqrt{x}}$ – see notes | M1 | 1.1b |
| | E.g. $2\times\frac{3}{2}x^{\frac{1}{2}}+\frac{1}{2}\times\frac{1}{2}x^{-\frac{1}{2}}$ | A1 | 1.1b |
| | $\frac{dy}{dx}=3\sqrt{x}+\frac{1}{4\sqrt{x}}-\frac{4}{x}=\frac{12x^2+x-16\sqrt{x}}{4x\sqrt{x}}$ * | A1* | 2.1 |
| | | **(4)** | |
| **(b)** | $12x^2+x-16\sqrt{x}=0 \Rightarrow 12x^{\frac{3}{2}}+x^{\frac{1}{2}}-16=0$ | M1 | 1.1b |
| | E.g. $12x^{\frac{3}{2}}=16-\sqrt{x}$ | dM1 | 1.1b |
| | $x^{\frac{3}{2}}=\frac{4}{3}-\frac{\sqrt{x}}{12} \Rightarrow x=\left(\frac{4}{3}-\frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}$ * | A1* | 2.1 |
| | | **(3)** | |
| **(c)** | $x_2=\sqrt[3]{\left(\frac{4}{3}-\frac{\sqrt{2}}{12}\right)^2}$ | M1 | 1.1b |
| | $x_2=$ awrt 1.13894 | A1 | 1.1b |
| | $x=1.15650$ | A1 | 2.2a |
| | | **(3)** | |
| | | | **(10 marks)** |

## June 2019 Question 11 Paper 2

**11.**

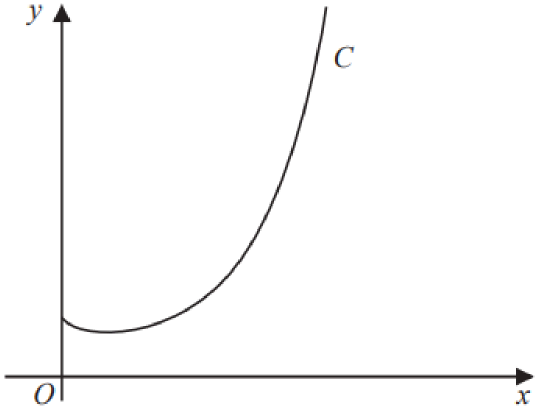


**Figure 8**

Figure 8 shows a sketch of the curve $C$ with equation $y = x^x$, $x > 0$

(a) Find, by firstly taking logarithms, the $x$ coordinate of the turning point of $C$.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

**(5)**

The point $P(\alpha, 2)$ lies on $C$.

(b) Show that $1.5 < \alpha < 1.6$

**(2)**

A possible iteration formula that could be used in an attempt to find $\alpha$ is

$$x_{n+1} = 2x_n^{\,1-x_n}$$

Using this formula with $x_1 = 1.5$

(c) find $x_4$ to 3 decimal places,

**(2)**

(d) describe the long-term behaviour of $x_n$

**(2)**

## ANSWER

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **11 (a) Way 1** | $\{y = x^x \Rightarrow\}\ \ \ln y = x\ln x$ | B1 | 1.1a |
| | $\dfrac{1}{y}\dfrac{dy}{dx} = 1 + \ln x$ | M1<br>A1 | 1.1b<br>2.1 |
| | $\left\{\dfrac{dy}{dx} = 0 \Rightarrow\right\}\ \dfrac{x}{x} + \ln x = 0$ or $1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = \ldots$ | M1 | 1.1b |
| | $x = e^{-1}$ or awrt 0.368 | A1 | 1.1b |
| | **Note:** $k \neq 0$ | **(5)** | |
| **(a) Way 2** | $\{y = x^x \Rightarrow\}\ \ y = e^{x\ln x}$ | B1 | 1.1a |
| | $\dfrac{dy}{dx} = \left(\dfrac{x}{x} + \ln x\right)e^{x\ln x}$ | M1<br>A1 | 1.1b<br>2.1 |
| | $\left\{\dfrac{dy}{dx} = 0 \Rightarrow\right\}\ \dfrac{x}{x} + \ln x = 0$ or $1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = \ldots$ | M1 | 1.1b |
| | $x = e^{-1}$ or awrt 0.368 | A1 | 1.1b |
| | **Note:** $k \neq 0$ | **(5)** | |
| **(b) Way 1** | Attempts both $1.5^{1.5} = 1.8\ldots$ and $1.6^{1.6} = 2.1\ldots$ and at least one result is correct to awrt 1 dp | M1 | 1.1b |
| | $1.8\ldots < 2$ and $2.1\ldots > 2$ and as $C$ is continuous then $1.5 < \alpha < 1.6$ | A1 | 2.1 |
| | | **(2)** | |
| **(c)** | Attempts $x_{n+1} = 2x_n^{1-x_n}$ at least once with $x_1 = 1.5$<br>Can be implied by $2(1.5)^{1-1.5}$ or awrt 1.63 | M1 | 1.1b |
| | $\{x_4 = 1.67313\ldots \Rightarrow\}\ x_4 = 1.673$ (3 dp) **cao** | A1 | 1.1b |
| | | **(2)** | |

| Question | Scheme | | Marks | AOs |
|---|---|---|---|---|
| **(d)** | Give 1st B1 for any of<br>• oscillates<br>• periodic<br>• non-convergent<br>• divergent<br>• fluctuates<br>• goes up and down<br>• 1, 2, 1, 2, 1, 2<br>• alternates (condone) | Give B1 B1 for any of<br>• periodic {sequence} with period 2<br>• oscillates between 1 and 2 | B1 | 2.5 |
| | | Condone B1 B1 for any of<br>• fluctuates between 1 and 2<br>• keep getting 1, 2<br>• alternates between 1 and 2<br>• goes up and down between 1 and 2<br>• 1, 2, 1, 2, 1, 2, … | B1 | 2.5 |
| | | | **(2)** | |
| | | | | **(11 marks)** |

## June 2018 Question 4 Paper 1

4. The curve with equation $y = 2\ln(8 - x)$ meets the line $y = x$ at a single point, $x = \alpha$.

(a) Show that $3 < \alpha < 4$

**(2)**

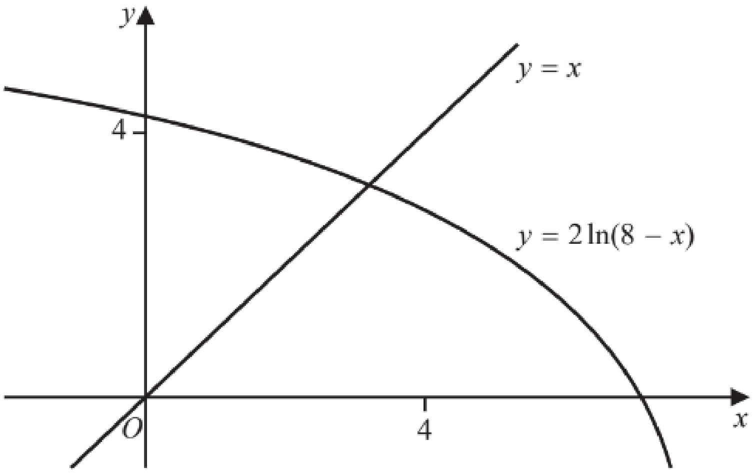


**Figure 2**

Figure 2 shows the graph of $y = 2\ln(8 - x)$ and the graph of $y = x$.

A student uses the iteration formula

$$x_{n+1} = 2\ln(8 - x_n), \quad n \in \mathbb{N}$$

in an attempt to find an approximation for $\alpha$.

Using the graph and starting with $x_1 = 4$

(b) determine whether or not this iteration formula can be used to find an approximation for $\alpha$, justifying your answer.

**(2)**

**ANSWER**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **4 (a)** | Attempts $\mathrm{f}(3)=$ and $\mathrm{f}(4)=$ where $\mathrm{f}(x)=\pm\left(2\ln(8-x)-x\right)$ | M1 | 2.1 |
| | $\mathrm{f}(3)=\left(2\ln(5)-x\right)=(+)0.22$ and $\mathrm{f}(4)=\left(2\ln(4)-4\right)=-1.23$<br>Change of sign and function continuous in interval $[3,4]\Rightarrow$ Root * | A1* | 2.4 |
| | | **(2)** | |
| **(b)** | For annotating the graph by drawing a cobweb diagram starting at $x_1=4$ It should have at least two spirals | M1 | 2.4 |
| | Deduces that the iteration formula **can be used** to find an approximation for $\alpha$ because **the cobweb spirals inwards** for the cobweb diagram | A1 | 2.2a |
| | | **(2)** | |
| | | | **(4 marks)** |

## June 2018 Question 5 Paper 2

**5.** The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root.

(a) Show that, for this equation, the Newton-Raphson formula can be written

$$x_{n+1} = \frac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n}$$

**(3)**

Using the formula given in part (a) with $x_1 = 1$

(b) find the values of $x_2$ and $x_3$

**(2)**

(c) Explain why, for this question, the Newton-Raphson method cannot be used with $x_1 = 0$

**(1)**

**ANSWER**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 5 | The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root | | |
| (a) | $\{f(x) = 2x^3 + x^2 - 1 \Rightarrow\}\ f'(x) = 6x^2 + 2x$ | B1 | 1.1b |
| | $\left\{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \Rightarrow\right\}\ \{x_{n+1}\} = x_n - \frac{2x_n^3 + x_n^2 - 1}{6x_n^2 + 2x_n}$ | M1 | 1.1b |
| | $= \frac{x_n(6x_n^2 + 2x_n) - (2x_n^3 + x_n^2 - 1)}{6x_n^2 + 2x_n} \Rightarrow x_{n+1} = \frac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n}$ * | A1* | 2.1 |
| | | **(3)** | |
| (b) | $\{x_1 = 1 \Rightarrow\}\ x_2 = \frac{4(1)^3 + (1)^2 + 1}{6(1)^2 + 2(1)}$ or $x_2 = 1 - \frac{2(1)^3 + (1)^2 - 1}{6(1)^2 + 2(1)}$ | M1 | 1.1b |
| | $\Rightarrow x_2 = \frac{3}{4},\ x_3 = \frac{2}{3}$ | A1 | 1.1b |
| | | **(2)** | |
| (c) | Accept any reasons why the Newton-Raphson **method** cannot be used with $x_1 = 0$ which refer or ***allude*** to either the stationary point or the tangent. E.g.<br>• There is a stationary point at $x = 0$<br>• Tangent to the curve (or $y = 2x^3 + x^2 - 1$) would not meet the $x$-axis<br>• Tangent to the curve (or $y = 2x^3 + x^2 - 1$) is horizontal | B1 | 2.3 |
| | | **(1)** | |
| | | | **(6 marks)** |

## June 2018 Question 14 Paper 2

**14.** A scientist is studying a population of mice on an island.

The number of mice, $N$, in the population, $t$ months after the start of the study, is modelled by the equation

$$N = \frac{900}{3 + 7e^{-0.25t}}, \quad t \in \mathbb{R}, \quad t \geqslant 0$$

(a) Find the number of mice in the population at the start of the study.

**(1)**

(b) Show that the rate of growth $\frac{dN}{dt}$ is given by $\frac{dN}{dt} = \frac{N(300 - N)}{1200}$

**(4)**

The rate of growth is a maximum after $T$ months.

(c) Find, according to the model, the value of $T$.

**(4)**

According to the model, the maximum number of mice on the island is $P$.

(d) State the value of $P$.

**(1)**

**ANSWER**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **14** | $N=\dfrac{900}{3+7\mathrm{e}^{-0.25t}}=900(3+7\mathrm{e}^{-0.25t})^{-1},\ t\in\mathbb{R},\ t\geq 0\ ;\quad \dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{N(300-N)}{1200}$ | | |
| **(a)** | 90 | B1 | 3.4 |
| | | **(1)** | |
| **(b) Way 1** | $\dfrac{\mathrm{d}N}{\mathrm{d}t}=-900(3+7\mathrm{e}^{-0.25t})^{-2}\left(7(-0.25)\mathrm{e}^{-0.25t}\right)\ \left\{=\dfrac{900(0.25)(7)\mathrm{e}^{-0.25t}}{(3+7\mathrm{e}^{-0.25t})^{2}}\right\}$ | M1 | 2.1 |
| | | A1 | 1.1b |
| | $\Rightarrow \dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{900(0.25)\left(\left(\dfrac{900}{N}-3\right)\right)}{\left(\dfrac{900}{N}\right)^{2}}$ | dM1 | 2.1 |
| | correct algebra leading to $\dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{N(300-N)}{1200}$ * | A1* | 1.1b |
| | | **(4)** | |
| **(b) Way 2** | $\dfrac{\mathrm{d}N}{\mathrm{d}t}=-900(3+7\mathrm{e}^{-0.25t})^{-2}\left(7(-0.25)\mathrm{e}^{-0.25t}\right)\ \left\{=\dfrac{900(0.25)(7)\mathrm{e}^{-0.25t}}{(3+7\mathrm{e}^{-0.25t})^{2}}\right\}$ | M1 | 2.1 |
| | | A1 | 1.1b |
| | $\dfrac{N(300-N)}{1200}=\dfrac{\left(\dfrac{900}{3+7\mathrm{e}^{-0.25t}}\right)\left(300-\dfrac{900}{3+7\mathrm{e}^{-0.25t}}\right)}{1200}$ | dM1 | 2.1 |
| | LHS $=\dfrac{1575\mathrm{e}^{-0..25t}}{(3+7\mathrm{e}^{-0.25t})^{2}}$ o.e., <br> RHS $=\dfrac{900(300(3+7\mathrm{e}^{-0.25t})-900)}{1200(3+7\mathrm{e}^{-0.25t})^{2}}=\dfrac{1575\mathrm{e}^{-0..25t}}{(3+7\mathrm{e}^{-0.25t})^{2}}$ o.e. <br> and states hence $\dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{N(300-N)}{1200}$ (or LHS = RHS) * | A1* | 1.1b |
| | | **(4)** | |
| **(c)** | Deduces $N=150$ (can be implied) | B1 | 2.2a |
| | so $150=\dfrac{900}{3+7\mathrm{e}^{-0.25T}}\ \Rightarrow\ \mathrm{e}^{-0.25T}=\dfrac{3}{7}$ | M1 | 3.4 |
| | $T=-4\ln\left(\dfrac{3}{7}\right)$ or $T=$ awrt 3.4 (months) | dM1 | 1.1b |
| | | A1 | 1.1b |
| | | **(4)** | |
| **(d)** | either one of 299 or 300 | B1 | 3.4 |
| | | **(1)** | |
| | | **(10 marks)** | |