# Pure Year 2 exam questions (A Level maths)-AQA

**NOTE**:
*Please be aware that in the Year 12 collections, you will find questions from the Year 13 papers. However, these questions are intentionally included because they align with Year 12 content and topics. That's the reason why you are missing some questions here, please have a look at the Year 12 collection!*
*Happy studying!*

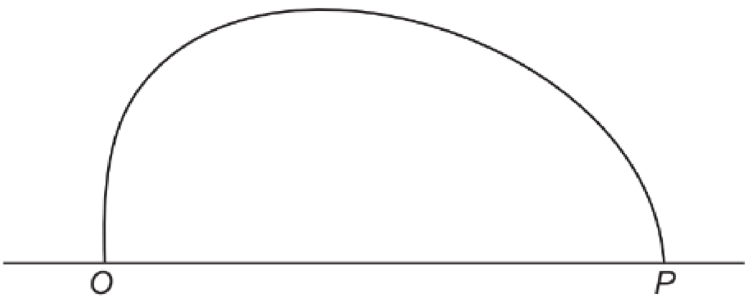
# Ch 9 Further differentiation

## June 2022 Question 13 Paper 1

**13** **Figure 2** shows the approximate shape of the vertical cross section of the entrance to a cave. The cave has a horizontal floor.

The entrance to the cave joins the floor at the points *O* and *P*.

**Figure 2**




Garry models the shape of the cross section of the entrance to the cave using the equation

$$x^2 + y^2 = a\sqrt{x} - y$$

where $a$ is a constant, and $x$ and $y$ are the horizontal and vertical distances respectively, in metres, measured from *O*.

**13 (a)** The distance *OP* is 16 metres.

Find the value of $a$ that Garry should use in the model.

**[2 marks]**

**13 (b)** Show that the maximum height of the cave above *OP* is approximately 10.5 metres.

**[6 marks]**

**13 (c)** Suggest one limitation of the model Garry has used.

**[1 mark]**

## ANSWER

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **13(a)** | Substitutes $y$ = 0 and $x$ = 16 correctly into $x^2+y^2=a\sqrt{x}-y$ | 3.4 | M1 | $x^2+y^2=a\sqrt{x}-y$<br>$16^2+0^2=a\sqrt{16}-0$ |
| | Obtains $a$ = 64 | 1.1b | A1 | $256=4a$<br>$a=64$ |
| | **Subtotal** | | **2** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **13(b)** | Differentiates implicitly with either $2y\frac{\mathrm{d}y}{\mathrm{d}x}$ or $-\frac{\mathrm{d}y}{\mathrm{d}x}$ seen | 3.1b | B1 | |
| | Differentiates any two of the four terms correctly.<br>Can be in terms of $a$ or with their $a$ value | 1.1a | M1 | |
| | Obtains a fully correct differentiated equation<br>Can be in terms of $a$<br>Follow through their $a$ value<br>$2x+2y\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{a}{2}x^{-\frac{1}{2}}-\frac{\mathrm{d}y}{\mathrm{d}x}$ | 1.1b | A1F | $2x+2y\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{64}{2}x^{-\frac{1}{2}}-\frac{\mathrm{d}y}{\mathrm{d}x}$<br>$\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow 2x=\frac{32}{\sqrt{x}}$<br>$x^{\frac{3}{2}}=16$<br>$x=6.3496...$ |
| | Uses $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ | 1.1a | M1 | $(6.3496...)^2+y^2=64\sqrt{6.3496...}-y$ |
| | Substitutes their numerical $x$ value where 0 < $x$ < 16, into the model with their $a$ value | 3.4 | M1 | $y=10.51$<br>Maximum height is approximately 10.5 metres |

| | | | | |
|---|---|---|---|---|
| | Obtains a value for $y$ AWRT 10.51 and concludes that the maximum height is approximately 10.5 metres AG<br>Condone equals<br>Must state units<br>CSO | 3.2a | R1 | |
| | **Subtotal** | | **6** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **13(c)** | States or infers that the entrance is unlikely to be a smooth curve<br>Accept:<br>• The cave has dents<br>• Entrance is not perfectly smooth<br><br>Ignore comments about the floor or the vertical cross section | 3.5b | E1 | The entrance to the cave is unlikely to be perfectly smooth |
| | **Subtotal** | | **1** | |

| | | | | |
|---|---|---|---|---|
| | **Question 13 Total** | | **9** | |

## November 2021 Question 2 Paper 1

**2** Given that $y = \ln(5x)$

find $\dfrac{dy}{dx}$

Circle your answer.

**[1 mark]**

$\dfrac{dy}{dx} = \dfrac{1}{x}$ $\qquad$ $\dfrac{dy}{dx} = \dfrac{1}{5x}$ $\qquad$ $\dfrac{dy}{dx} = \dfrac{5}{x}$ $\qquad$ $\dfrac{dy}{dx} = \ln 5$

**ANSWER**

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **2** | Circles correct answer | 1.1b | B1 | $\dfrac{dy}{dx} = \dfrac{1}{x}$ |
| | **Total** | | **1** | |

## November 2021 Question 10 Paper 1

**10 (a)** Given that

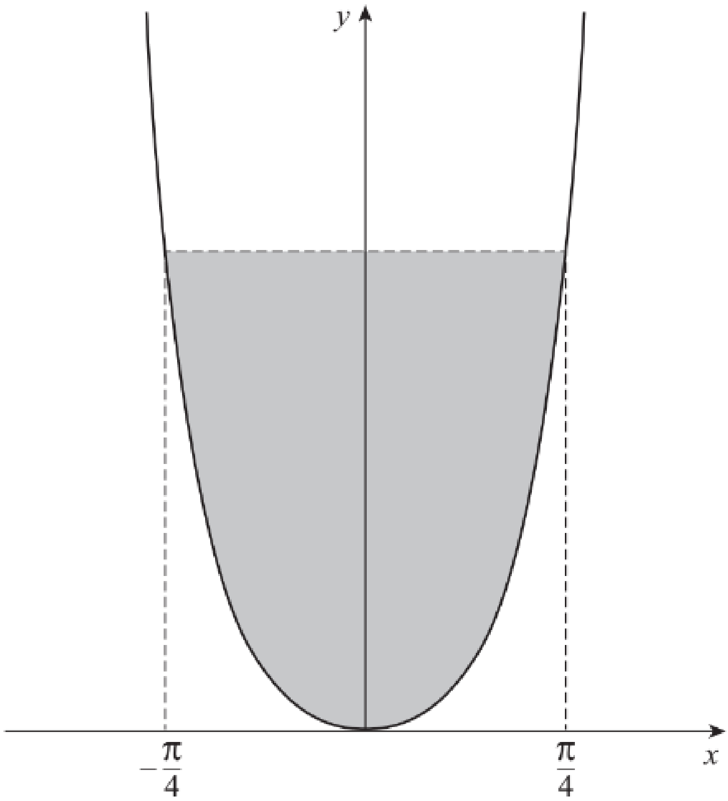
$$y = \tan x$$

use the quotient rule to show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \sec^2 x$$

**[3 marks]**

**10 (b)** The region enclosed by the curve $y = \tan^2 x$ and the horizontal line, which intersects the curve at $x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$, is shaded in the diagram below.




Show that the area of the shaded region is

$$\pi - 2$$

Fully justify your answer.

**[5 marks]**

## ANSWER

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **10(a)** | Recalls $\tan x = \dfrac{\sin x}{\cos x}$ | 1.2 | B1 | $\dfrac{d}{dx}(\tan x) = \dfrac{d}{dx}\left(\dfrac{\sin x}{\cos x}\right)$ $= \dfrac{\cos x \cos x - (-\sin x)\sin x}{\cos^2 x}$ $= \dfrac{\sin^2 x + \cos^2 x}{\cos^2 x}$ $= \dfrac{1}{\cos^2 x}$ $= \sec^2 x$ |
| | Uses the correct quotient rule. Condone sign error in differentiation of sin or cosine. | 1.1a | M1 | |
| | Completes rigorous argument to show the required result. Use of $\sin^2 x + \cos^2 x = 1$ or $\tan^2 x + 1 = \sec^2 x$ must be explicit. Must include $\dfrac{d}{dx}(\tan x) = \ldots$ or $\dfrac{dy}{dx} = \ldots$ | 2.1 | R1 | |
| | **Subtotal** | | **3** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **10(b)** | Writes down an integral of the form $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2 x\,dx$, $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (1-\tan^2 x)\,dx$ Condone missing or incorrect limits and missing dx | 3.1a | M1 | Area under curve $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2 x\,dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x - 1\,dx$ $= [\tan x - x]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$ $= \left(\tan\frac{\pi}{4} - \frac{\pi}{4}\right) - \left(\tan\left(-\frac{\pi}{4}\right) - -\frac{\pi}{4}\right)$ $= 1 - \frac{\pi}{4} + 1 - \frac{\pi}{4}$ $= 2 - \frac{\pi}{2}$ Area of shaded region $\frac{\pi}{2} - \left(2 - \frac{\pi}{2}\right)$ $= \pi - 2$ |
| | Uses $\tan^2 x + 1 = \sec^2 x$ to write integrand in a form which can be integrated, condone sign error. | 3.1a | M1 | |
| | Integrates their expression of the form $A\sec^2 x + B$ | 1.1b | A1F | |
| | Forms an expression for or evaluates the area of the relevant rectangle. $2\dfrac{\pi}{4}\tan^2\dfrac{\pi}{4}$ or $\dfrac{\pi}{4}\tan^2\dfrac{\pi}{4}$ Could be implicit within their integral | 1.1b | B1 | |
| | Completes rigorous argument to show the required result. Substitution of consistent limits should be explicit and no slips in algebra. Use of dx is required. AG | 2.1 | R1 | |
| | **Subtotal** | | **5** | |

## November 2021 Question 12 Paper 1

**12** The equation of a curve is

$$(x+y)^2 = 4y + 2x + 8$$

The curve intersects the positive $x$-axis at the point $P$.

**12 (a)** Show that the gradient of the curve at $P$ is $-\dfrac{3}{2}$

**[6 marks]**

**12 (b)** Find the equation of the normal to the curve at $P$, giving your answer in the form $ax + by = c$, where $a$, $b$ and $c$ are integers.

**[2 marks]**

**ANSWER**

| Q | Marking instructions | AO | Mark | Typical solution |
|---|---|---|---|---|
| **12(a)** | Substitutes $y$ = 0 to form an equation for x<br>PI x = 4 | 3.1a | M1 | $(x+y)^2=4y+2x+8$<br>$y=0 \Rightarrow x^2=8+2x$<br>$\Rightarrow x=4\,or-2$<br>$x=4$ at P |
| | Obtains $x$ = 4 ignore any other value. | 1.1b | A1 | |
| | Expands and uses product rule to obtain the derivative of their $Axy$ term.<br><br>or<br><br>Uses chain rule to obtain $2(x+y)\left(1+\frac{\mathrm{d}y}{\mathrm{d}x}\right)$<br>Condone missing brackets. | 3.1a | M1 | $x^2+2xy+y^2=4y+2x+8$<br>$2x+2y+2x\frac{\mathrm{d}y}{\mathrm{d}x}+2y\frac{\mathrm{d}y}{\mathrm{d}x}=4\frac{\mathrm{d}y}{\mathrm{d}x}+2$<br><br>$x=4, y=0$<br>$\Rightarrow 8+8\frac{\mathrm{d}y}{\mathrm{d}x}=4\frac{\mathrm{d}y}{\mathrm{d}x}+2$ |
| | Uses implicit differentiation correctly to obtain the derivative of $4y$ or $y^2$. | 1.1b | B1 | $4\frac{\mathrm{d}y}{\mathrm{d}x}=-6$ |
| | Obtains correct equation from correct differentiation. | 1.1b | A1 | $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{3}{2}$ |
| | Substitutes $x$ = 4 and $y$ = 0 into $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2-2x-2y}{2x+2y-4}$ OE<br><br>and obtains $-\frac{3}{2}$<br>If substituting into an earlier equation must reach $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{3}{2}$<br>(AG) | 2.1 | R1 | |
| | **Subtotal** | | **6** | |

| Q | Marking instructions | AO | Mark | Typical solution |
|---|---|---|---|---|
| **12(b)** | Uses $\frac{2}{3}$ and $y$ = 0 and their $x$ = 4 from part (a) to form equation of line. | 1.1a | M1 | $y=\frac{2}{3}(x-4)$<br>$2x-3y=8$ |
| | Obtains their equation in correct form. | 1.1b | A1F | |
| | **Subtotal** | | **2** | |

## November 2020 Question 12 Paper 1

**12** A curve $C$ has equation

$$x^3 \sin y + \cos y = Ax$$

where $A$ is a constant.

$C$ passes through the point $P\left(\sqrt{3}, \frac{\pi}{6}\right)$

**12 (a)** Show that $A = 2$

**[2 marks]**

**12 (b) (i)** Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2 - 3x^2 \sin y}{x^3 \cos y - \sin y}$

**[5 marks]**

**12 (b) (ii)** Hence, find the gradient of the curve at $P$.

**[2 marks]**

**12 (b) (iii)** The tangent to $C$ at $P$ intersects the $x$-axis at $Q$.

Find the exact $x$-coordinate of $Q$.

**[4 marks]**

## ANSWER

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 12(a) | Substitutes $x = \sqrt{3}$ and $y = \frac{\pi}{6}$ to obtain an equation or an expression for A | 1.1a | M1 | $(\sqrt{3})^3 \sin\frac{\pi}{6} + \cos\frac{\pi}{6} = A\sqrt{3}$<br>$\frac{3\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = A\sqrt{3}$<br>$\frac{3}{2} + \frac{1}{2} = A$<br>$A = 2$ |
| | Completes argument to show A = 2<br>Must clearly show use of $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ and $\sin\frac{\pi}{6} = \frac{1}{2}$<br>AG | 2.1 | R1 | |
| | **Subtotal** | | **2** | |
| 12(b)(i) | Uses implicit differentiation correctly at least once with sight of $\sin y \frac{dy}{dx}$ or $\cos y \frac{dy}{dx}$<br>Condone sign error | 3.1a | M1 | $3x^2 \sin y + x^3 \cos y \frac{dy}{dx} - \sin y \frac{dy}{dx} = 2$<br>$\frac{dy}{dx}(x^3 \cos y - \sin y) = 2 - 3x^2 \sin y$<br>$\frac{dy}{dx} = \frac{2 - 3x^2 \sin y}{x^3 \cos y - \sin y}$ |
| | Uses product rule with sight of $Px^2 \sin y \pm x^3 \cos y \frac{dy}{dx}$<br>Condone omission of $\frac{dy}{dx}$ | 3.1a | M1 | |
| | Obtains equation of the form $Px^2 \sin y \pm x^3 \cos y \frac{dy}{dx} \pm \sin y \frac{dy}{dx} = 2$ | 1.1b | A1 | |
| | Obtains completely correct equation | 1.1b | A1 | |
| | Isolates $\frac{dy}{dx}$ terms and factorises to complete rigorous argument with no slips to show the given result<br>AG | 2.1 | R1 | |
| | **Subtotal** | | **5** | |
| 12(b)(ii) | Substitutes $x = \sqrt{3}$ and $y = \frac{\pi}{6}$ to obtain an expression for the gradient | 1.1a | M1 | $\frac{dy}{dx} = \frac{2 - 3(\sqrt{3})^2 \sin\frac{\pi}{6}}{(\sqrt{3})^3 \cos\frac{\pi}{6} - \sin\frac{\pi}{6}}$<br>$= -\frac{5}{8}$ |
| | Obtains correct gradient of $-\frac{5}{8}$<br>OE | 1.1b | A1 | |
| | **Subtotal** | | **2** | |

| | | | | |
|---|---|---|---|---|
| **12(b)(iii)** | Forms equation for the tangent (condone normal) at P using '*their*' gradient and $\left(\sqrt{3}, \frac{\pi}{6}\right)$ ACF<br>or<br>Writes the equation as $y = mx + c$ using '*their*' gradient of tangent (condone normal) and substitutes $\left(\sqrt{3}, \frac{\pi}{6}\right)$ to obtain an equation in $c$<br>PI by correct exact value for $x$ | 3.1a | M1 | $y - \frac{\pi}{6} = -\frac{5}{8}(x - \sqrt{3})$<br><br>$0 - \frac{\pi}{6} = -\frac{5}{8}(x - \sqrt{3})$<br><br>$x = \sqrt{3} + \frac{4\pi}{15}$ |
| | Obtains fully correct equation for the '*their*' **tangent** at P ACF<br>Note $c = \frac{5\sqrt{3}}{8} + \frac{\pi}{6}$ or $c = 1.606..$<br>Follow through 'their' gradient of tangent from 12(b)(ii) must be to at least 3 dp | 1.1b | A1F | |
| | Substitutes $y = 0$ into '*their*' tangent (condone normal) equation and solves to find the $x$ coordinate of Q<br>Accept decimals | 3.1a | M1 | |
| | Obtains $x = \sqrt{3} + \frac{4\pi}{15}$<br>OE must be exact form<br>Eg $x = \frac{8}{5}(\frac{5\sqrt{3}}{8} + \frac{\pi}{6})$ | 1.1b | A1 | |
| | **Subtotal** | | **4** | |
| | **Question Total** | | **13** | |

## June 2019 Question 10 Paper 1

**10** The volume of a spherical bubble is increasing at a constant rate.

Show that the rate of increase of the radius, $r$, of the bubble is inversely proportional to $r^2$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

**[4 marks]**

**ANSWER**

| Q | Marking instructions | AO | Mark | Typical solution |
|---|---|---|---|---|
| 10 | Models the rate of change of volume with a differential equation of the correct form. With respect to time, not contradicted. | 3.3 | B1 | $\frac{dv}{dt} = k$ |
| | Obtains $4\pi r^2$ by differentiation. | 1.1b | B1 | $\frac{dv}{dr} = 4\pi r^2$ |
| | Uses the chain rule to connect rates of change substituting their expressions for dv/dt and dv/dr.<br>Or<br>Integrates to obtain expression for v=kt+c<br>Then differentiates wrt r<br>dv/dr=kdt/dr<br>Substitutes their expression for dv/dr | 3.1b | M1 | $\frac{dv}{dt} = \frac{dv}{dr} \times \frac{dr}{dt}$ |
| | Completes argument, obtaining a correct expression for $\frac{dr}{dt}$ and concluding that $\frac{dr}{dt} \propto \frac{1}{r^2}$<br>OE statement | 2.1 | R1 | $k = 4\pi r^2 \times \frac{dr}{dt}$<br>$\frac{dr}{dt} = \frac{k}{4\pi r^2}$<br>$\therefore \frac{dr}{dt} \propto \frac{1}{r^2}$ |
| | | | | |
| | **Total** | | **4** | |

## June 2019 Question 9 Paper 3

**9** A curve has equation

$$x^2y^2 + xy^4 = 12$$

**9 (a)** Prove that the curve does not intersect the coordinate axes.

**[2 marks]**

**9 (b) (i)** Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{2xy + y^3}{2x^2 + 4xy^2}$

**[5 marks]**

**9 (b) (ii)** Prove that the curve has no stationary points.

**[4 marks]**

**9 (b) (iii)** In the case when $x > 0$, find the equation of the tangent to the curve when $y = 1$

**[4 marks]**

ANSWER

| Q | Marking instructions | AO | Mark | Typical solution |
|---|---|---|---|---|
| **9(a)** | Demonstrates by substitution that $x=0$ or $y=0$ leads to value on the LHS = 0 | 2.4 | E1 | When $x$ = 0 $0^2y^2 + 0y^4 = 0$ When $y$ = 0 $x^2 0^2 + x0^4 = 0$ |
| | Completes rigorous argument to show required result | 2.1 | R1 | This is a contradiction because $x^2y^2 + xy^4 = 12$ so the curve does not intersect either axis |
| **9 (b)(i)** | Uses implicit differentiation | 3.1a | M1 | $2xy^2+2x^2y\frac{dy}{dx}+y^4+4xy^3\frac{dy}{dx}=0$ |
| | Product rule used LHS (at least one pair of terms correct) | 1.1a | M1 | $\frac{dy}{dx}=-\frac{2xy^2+y^4}{2x^2y+4xy^3}$ |
| | Differentiates equation of curve fully correctly | 1.1b | A1 | |
| | Collects their $\frac{dy}{dx}$ terms in an equation and factorises | 3.1a | M1 | $= -\frac{y\,(2xy+y^3)}{y(2x^2+4xy^2)}$ |
| | Completes convincing argument to obtain required result by **factorising** then simplifying $y$ **AG** | 2.1 | R1 | $=-\frac{2xy+y^3}{2x^2+4xy^2}$, |

| | | | | |
|---|---|---|---|---|
| **9 (b)(ii)** | Begins argument by setting $\frac{dy}{dx}=0$ to form an equation for $x$ and $y$<br>**PI** by $2xy+y^3=0$ | 2.1 | M1 | For stationary points<br>$\frac{dy}{dx}=0$<br>$\Rightarrow 2xy+y^3=0$<br>$\Rightarrow y^2=-2x$<br>$\Rightarrow x^2y^2+x(-2x)y^2=12$<br>$\Rightarrow -x^2y^2=12$<br>Since $-x^2y^2<0$ there can be no stationary points. |
| | Obtains $y^2=-2x$ or $y=\sqrt{-2x}$ or $x=\frac{-y^2}{2}$ | 1.1b | A1 | |
| | Substitutes $y^2=-2x$ or $x=\frac{-y^2}{2}$ into equation for curve | 1.1a | M1 | |
| | Completes convincing argument to deduce the required result | 2.2a | R1 | |
| **9 (b)(iii)** | Substitutes $y = 1$ into equation of curve to obtain correct quadratic<br>**ACF** | 3.1a | M1 | $y=1\Rightarrow x^2+x-12=0$<br>$\Rightarrow x=3 \qquad (x>0)$<br>$\Rightarrow \frac{dy}{dx}=-\frac{7}{30}$<br>$y-1=-\frac{7}{30}(x-3)$ |
| | Deduces $x = 3$<br>**PI** by substituting their $x$ in their dy/dx | 2.2a | R1 | |
| | Substitutes their $x$ and $y = 1$ in their dy/dx | 1.1a | M1 | |
| | Obtains correct equation of tangent<br>**ACF**<br>**ISW** | 1.1b | A1 | |
| | **Total** | | **15** | |