Pure Year 2 exam questions (A Level maths)-AQA

NOTE:
Please be aware that in the Year 12 collections, you will find questions from the Year 13 papers. However, these questions are intentionally included because they align with Year 12 content and topics. That's the reason why you are missing some questions here, please have a look at the Year 12 collection!
Happy studying!

Ch 8 Trigonometric identities

November 2021 Question 8 Paper 1

8 (a) Given that

$$9\sin^2\theta + \sin 2\theta = 8$$

show that

$$8\cot^2\theta - 2\cot\theta - 1 = 0$$

[4 marks]

8 (b) Hence, solve

$$9\sin^2\theta + \sin 2\theta = 8$$

in the interval $0 < \theta < 2\pi$

Give your answers to two decimal places.

[3 marks]

8 (c) Solve

$$9\sin^2\left(2x - \frac{\pi}{4}\right) + \sin\left(4x - \frac{\pi}{2}\right) = 8$$

in the interval $0 < x < \frac{\pi}{2}$

Give your answers to one decimal place.

[2 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
8(a)	Recalls and uses $\sin 2\theta = 2\sin\theta\cos\theta$	1.2	B1	$9\sin^2\theta + \sin 2\theta = 8$ $9\sin^2\theta + 2\sin\theta\cos\theta = 8$ $9 + 2\cot\theta = 8\operatorname{cosec}^2\theta$ $9 + 2\cot\theta = 8\left(\cot^2\theta + 1\right)$ $8\cot^2\theta - 2\cot\theta - 1 = 0$
	Uses $\cot^2\theta + 1 = \operatorname{cosec}^2\theta$ Or $\tan^2\theta + 1 = \sec^2\theta$ Condone a sign error	1.1a	M1	
	Divides through by $\cos^2\theta$ or $\sin^2\theta$	1.1a	M1	
	Completes rearrangement to achieve given result. **AG**	2.1	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Mark	Typical solution
8(b)	Solves to give values of cot θ or tan θ PI by sight of 2 and – 4 or $-\frac{1}{4}$ and $\frac{1}{2}$ or by two correct answers	1.1a	M1	$\cot\theta = -\frac{1}{4}$ or $\cot\theta = \frac{1}{2}$ tan θ = – 4 or tan θ = 2 θ = 1.82 θ = 1.82 + π = 4.96 θ = 1.11 θ = 1.11 + π = 4.25
	Obtains two correct values of θ. Condone AWRT correct answers.	1.1b	A1	θ = 1.11, 1.82, 4.25, 4.96
	Obtains all four solutions with no additional solutions or errors. Ignore additional solutions outside the interval. AWRT 1.11, 1.82, 4.25, 4.96 CAO	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Mark	Typical solution
8(c)	Sets $2x - \frac{\pi}{4}$ equal to at least one of their solutions. PI by a correct answer	3.1a	M1	$2x - \frac{\pi}{4} = 1.107..,1.815..$ x = 0.9, 1.3
	Obtains correct AWRT values. Correct values should be rounded from 0.94627… and 1.300058… ISW once correct answers seen. CSO Condone extra values outside of the interval.	1.1b	A1	
	Subtotal		**2**	

November 2020 Question 2 Paper 3

2 Given that

$$6\cos\theta + 8\sin\theta \equiv R\cos(\theta + \alpha)$$

find the value of R.

Circle your answer.

[1 mark]

6 8 10 14

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
2	Circles correct answer	1.1b	B1	10
	Total		1	

November 2020 Question 9 Paper 3

9 (a) For $\cos\theta \neq 0$, prove that

$$\operatorname{cosec} 2\theta + \cot 2\theta = \cot\theta$$

[4 marks]

9 (b) Explain why

$$\cot\theta \neq \operatorname{cosec} 2\theta + \cot 2\theta$$

when $\cos\theta = 0$

[1 mark]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
9(a)	Uses cosec2θ = $\frac{1}{\sin 2\theta}$ and cot2θ= $\frac{\cos 2\theta}{\sin 2\theta}$	1.2	B1	$\frac{1}{\sin 2\theta}+\frac{\cos 2\theta}{\sin 2\theta}$ $=\frac{1+\cos 2\theta}{\sin 2\theta}$ $=\frac{1+\cos^2\theta-\sin^2\theta}{2\sin\theta\cos\theta}$ $=\frac{2\cos^2\theta}{2\sin\theta\cos\theta}$ $=\frac{\cos\theta}{\sin\theta}=\cot\theta$
	Uses the identity for sin2θ = 2sinθ*cosθ* or an identity for cos2θ = cos$^2\theta$ – sin$^2\theta$ *or* 2cos$^2\theta$ – 1 or 1 – 2sin$^2\theta$ to commence proof	2.1	M1	
	Uses the identities for sin2θ and cos2θ in correct proof	1.1b	A1	
	Completes a reasoned argument leading to a single trigonometric fraction to prove given identity **AG**	2.1	R1	
9(b)	Deduces that when $\cos\theta=0$ then $\cot\theta$ is defined/zero/exists on LHS but cosec2θ or cot2θ or $\frac{1}{2\sin\theta\cos\theta}$ *or* $\frac{1}{\sin 2\theta}$ is undefined on RHS or deduces that LHS is defined but RHS is undefined Must compare both LHS and RHS	2.2a	E1	When $\cos\theta=0$ the value of $\cot\theta$ = 0 on LHS but because the value of sin2θ =0, cosec2θ and cot2θ are undefined on RHS.
	Total		**5**	

Arancha Ruiz

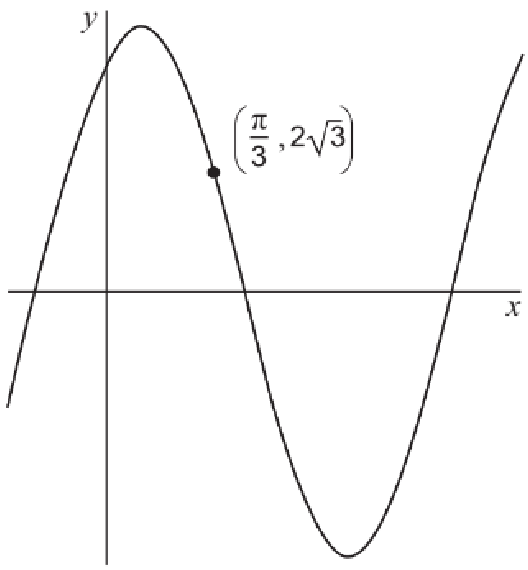
June 2019 Question 6 Paper 2

6 A curve has equation

$$y = a\sin x + b\cos x$$

where a and b are constants.

The maximum value of y is 4 and the curve passes through the point $\left(\frac{\pi}{3}, 2\sqrt{3}\right)$ as shown in the diagram.



Find the exact values of a and b.

[6 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
6	Compares with $R\cos(x\pm\alpha)$ or $R\sin(x\pm\alpha)$ by forming an identity e.g. $R\sin(x+\alpha)\equiv a\sin x+b\cos x$ **OE** or Differentiates correctly and equates to zero **CAO PI** by $a\cos x=b\sin x$ **PI** by *R* = 4 or $a^2+b^2=16$	3.1a	M1	$R\sin(x+\alpha)=a\sin x+b\cos x$ $R=4$ $4\sin\left(\dfrac{\pi}{3}+\alpha\right)=2\sqrt{3}$ $\alpha=\dfrac{\pi}{3}$ $a=4\cos\dfrac{\pi}{3}=2$ $b=4\sin\dfrac{\pi}{3}=2\sqrt{3}$
	Deduces *R* = 4 or $a^2+b^2=16$	2.2a	A1	
	Forms a correct equation for α **PI** by correct α or Forms the equation shown below $2\sqrt{3}=\dfrac{a\sqrt{3}}{2}+\dfrac{b}{2}$ **OE** **Must** substitute correct exact values for the trig functions	1.1b	B1	
	Solves their equation to obtain any correct value of α Correct values are shown below $\alpha=\dfrac{\pi}{3}$ or 0 for $R\sin(x\pm\alpha)$ $\alpha=\pm\dfrac{\pi}{6}$ for $R\cos(x\pm\alpha)$ or Eliminates a variable correctly from their two equations – **must** obtain a correct simplified equation	1.1a	M1	
	Deduces $a=2$	2.2a	R1	
	Deduces $b=2\sqrt{3}$	2.2a	R1	
	Total		**6**	

June 2019 Question 1 Paper 3

1 $f(x) = \arcsin x$

State the maximum possible domain of f

Tick (✓) **one** box.

[1 mark]

$\{x \in \mathbb{R} : -1 \leq x \leq 1\}$	☐
$\left\{x \in \mathbb{R} : -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}\right\}$	☐
$\{x \in \mathbb{R} : -\pi \leq x \leq \pi\}$	☐
$\{x \in \mathbb{R} : -90 \leq x \leq 90\}$	☐

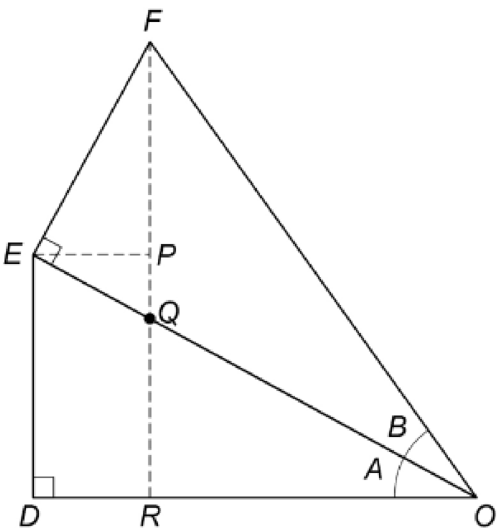
ANSWER

Q	Marking instructions	AO	Mark	Typical solution
1	Ticks the correct response	1.2	B1	$\{x \in \mathbb{R} : -1 \leq x \leq 1\}$
	Total		**1**	

June 2018 Question 14 Paper 1

14 Some students are trying to prove an identity for $\sin(A + B)$.

They start by drawing two right-angled triangles ODE and OEF, as shown.



The students' incomplete proof continues,

Let angle $DOE = A$ and angle $EOF = B$.

In triangle OFR,

Line 1 $\quad \sin(A + B) = \dfrac{RF}{OF}$

Line 2 $\quad = \dfrac{RP + PF}{OF}$

Line 3 $\quad = \dfrac{DE}{OF} + \dfrac{PF}{OF}$ since $DE = RP$

Line 4 $\quad = \dfrac{DE}{.....} \times \dfrac{.....}{OF} + \dfrac{PF}{EF} \times \dfrac{EF}{OF}$

Line 5 $\quad = ... + \cos A \sin B$

14 (a) Explain why $\dfrac{PF}{EF} \times \dfrac{EF}{OF}$ in Line 4 leads to $\cos A \sin B$ in Line 5

[2 marks]

14 (b) Complete Line 4 and Line 5 to prove the identity

Line 4 $$= \frac{DE}{\ldots\ldots} \times \frac{\ldots\ldots}{OF} + \frac{PF}{EF} \times \frac{EF}{OF}$$

Line 5 $$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + \cos A \sin B$$

[1 mark]

14 (c) Explain why the argument used in part **(a)** only proves the identity when A and B are acute angles.

[1 mark]

14 (d) Another student claims that by replacing B with $-B$ in the identity for $\sin(A + B)$ it is possible to find an identity for $\sin(A - B)$.

Assuming the identity for $\sin(A + B)$ is correct for all values of A and B, prove a similar result for $\sin(A - B)$.

[3 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
14(a)	Explains why $\angle EFQ = A$ Must be a fully correct explanation with reasons which may include: Vertically opposite angles and right angle implies similar triangles.	AO2.4	E1	$\angle OQR = \angle FQE$ vertically opposite angles $\angle ORQ = \angle FEQ = 90°$ So $\angle EFQ = A$
	Deduces $\frac{PF}{EF} = \cos(A)$ **AND** $\frac{EF}{OF} = \sin(B)$ Must have at least stated or implied that $\angle EFQ = A$ through similarity	AO2.2a	R1	Since $\angle EFQ = A$ $\frac{PF}{EF} = \cos(A)$ And $\frac{EF}{OF} = \sin(B)$ in triangle OEF
14(b)	Completes proof	AO2.2a	B1	$\frac{DE}{\boldsymbol{OE}} \times \frac{\boldsymbol{OE}}{OF} + \frac{PF}{EF} \times \frac{EF}{OF}$ $= \boldsymbol{\sin A \cos B} + \cos A \sin B$
14(c)	Explains that the proof is based on right angled triangles which limits A and B to acute angles	AO2.3	E1	Since the proof is based on the diagram which uses right-angled triangles it is assumed that A and B are acute. Therefore, the proof only holds for acute angles.
14(d)	Substitutes $-B$ into identity for $\sin(A+B)$ to give $\sin(A-B)$	AO2.1	R1	$\sin(A-B) = \sin A \cos(-B) + \cos A \sin(-B)$
	Recalls at least one of the identities $\sin(-B) = -\sin(B)$ $\cos(-B) = \cos(B)$ Must be explicitly stated	AO1.2	B1	$\sin(-B) = -\sin(B)$ $\cos(-B) = \cos(B)$
	Deduces correct identity with no errors. This must be clearly deduced from a correct argument and not simply stated.	AO2.2a	R1	Hence $\sin(A-B) = \sin A \cos B - \cos A \sin B$
	Total		**7**	

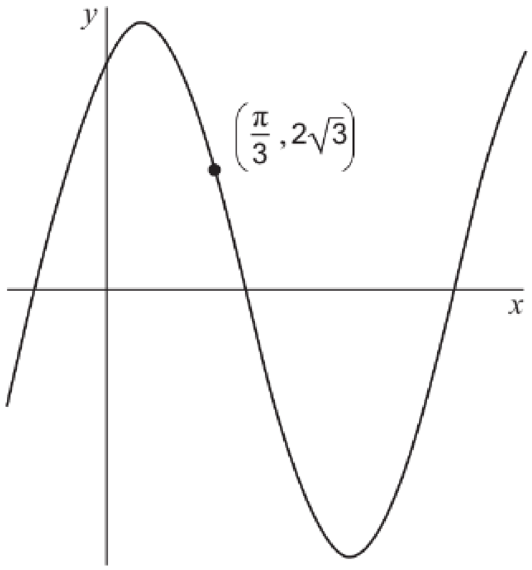
June 2018 Question 6 Paper 2

6 A curve has equation

$$y = a\sin x + b\cos x$$

where a and b are constants.

The maximum value of y is 4 and the curve passes through the point $\left(\frac{\pi}{3}, 2\sqrt{3}\right)$ as shown in the diagram.



Find the exact values of a and b.

[6 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
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	Deduces *R* = 4 or $a^2+b^2=16$	2.2a	A1	
	Forms a correct equation for α **PI** by correct α or Forms the equation shown below $2\sqrt{3} = \frac{a\sqrt{3}}{2} + \frac{b}{2}$ **OE** **Must** substitute correct exact values for the trig functions	1.1b	B1	
	Solves their equation to obtain any correct value of α Correct values are shown below $\alpha = \frac{\pi}{3}$ or 0 for $R\sin(x \pm \alpha)$ $\alpha = \pm\frac{\pi}{6}$ for $R\cos(x \pm \alpha)$ or Eliminates a variable correctly from their two equations – **must** obtain a correct simplified equation	1.1a	M1	
	Deduces $a = 2$	2.2a	R1	
	Deduces $b = 2\sqrt{3}$	2.2a	R1	
	Total		**6**	