Pure Year 2 exam questions (A Level maths)-AQA

NOTE:
Please be aware that in the Year 12 collections, you will find questions from the Year 13 papers. However, these questions are intentionally included because they align with Year 12 content and topics. That's the reason why you are missing some questions here, please have a look at the Year 12 collection!
Happy studying!

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Ch 7 Further algebra

June 2022 Question 6 Paper 1

6 (a) Find the first two terms, in ascending powers of x, of the binomial expansion of

$$\left(1-\frac{x}{2}\right)^{\frac{1}{2}}$$

[2 marks]

6 (b) Hence, for small values of x, show that

$$\sin 4x + \sqrt{\cos x} \approx A + Bx + Cx^2$$

where A, B and C are constants to be found.

[4 marks]

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ANSWER

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Expands to obtain the first two terms Can be unsimplified Condone sign error	1.1a	M1	$\left(1-\frac{x}{2}\right)^{\frac{1}{2}} \approx 1+\left(\frac{1}{2}\right)\left(-\frac{x}{2}\right)$ $\approx 1-\frac{1}{4}x$
	Obtains $1-\frac{1}{4}x$ OE Accept if listed as two separate terms. Ignore any extra terms	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)	States or uses at least one small angle approximation correctly either $\sin kx \approx kx$ or $\sqrt{\cos x} \approx \sqrt{1-\frac{x^2}{2}}$	3.1a	M1	$\sin(4x)+\sqrt{\cos x} \approx 4x+\sqrt{1-\frac{x^2}{2}}$ $\approx 4x+\left(1-\frac{x^2}{4}\right)$ $\approx 1+4x-\frac{1}{4}x^2$
	Uses both small angle approximations correctly for sine and cosine $\sin kx \approx kx$ and $\sqrt{\cos x} \approx \sqrt{1-\frac{x^2}{2}}$ Must have eliminated all trig expressions Inconsistent variables for angles must eventually be consistent to be awarded A1	1.1b	A1	
	Uses their expansion from **(a)** Must have replaced x with x^2 or Applies binomial theorem correctly to $\left(1-\frac{x^2}{2}\right)^{\frac{1}{2}}$ ignore any extra terms	3.1a	M1	
	Completes argument to obtain $4x+\left(1-\frac{x^2}{4}\right)$ or $1+4x-\frac{1}{4}x^2$ Accept any order of terms Ignore higher powers of x Must be in terms of x Do not ISW	2.1	R1	
	Subtotal		**4**	

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November 2021 Question 5 Paper 2

5 Express

$$\frac{5(x-3)}{(2x-11)(4-3x)}$$

in the form

$$\frac{A}{(2x-11)}+\frac{B}{(4-3x)}$$

where A and B are integers.

[3 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
5	Forms the identity/equation $5(x-3) \equiv A(4-3x)+B(2x-11)$ and either Compares coefficients or Substitutes a value for x PI by correct A or B	1.1a	M1	$\frac{5(x-3)}{(2x-11)(4-3x)}=\frac{A}{(2x-11)}+\frac{B}{(4-3x)}$ $5(x-3)=A(4-3x)+B(2x-11)$ $x=\frac{4}{3} \Longrightarrow B=1$ $x=\frac{11}{2} \Rightarrow A=-1$
	Obtains $A=-1$	1.1b	A1	
	Obtains $B=1$	1.1b	A1	
	Total		**3**	

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November 2021 Question 2 Paper 3

2 Simplify fully

$$\frac{(x+3)(6-2x)}{(x-3)(3+x)} \quad \text{for } x \neq \pm 3$$

Circle your answer.

[1 mark]

-2 $\qquad$ 2 $\qquad$ $\frac{(6-2x)}{(x-3)}$ $\qquad$ $\frac{(2x-6)}{(x-3)}$

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
2	Circles correct answer	1.1b	B1	–2
	Total		**1**	

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November 2021 Question 6 Paper 3

6 Given that $x > 0$ and $x \neq 25$, fully simplify

$$\frac{10 + 5x - 2x^{\frac{1}{2}} - x^{\frac{3}{2}}}{5 - \sqrt{x}}$$

Fully justify your answer.

[4 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
6	Begins to solve the problem using an appropriate technique eg factorising or grouping terms in numerator or writing $y = \sqrt{x}$ **PI** if $2 + x$ or $25 - x$ or $5 - x^{1/2}$ seen or multiplies by $\frac{5+\sqrt{x}}{5+\sqrt{x}}$	3.1a	M1	$\frac{10+5x-2x^{\frac{1}{2}}-x^{\frac{3}{2}}}{5-\sqrt{x}} \times \frac{5+\sqrt{x}}{5+\sqrt{x}}$ $= \frac{50+25x-10x^{\frac{1}{2}}-5x^{\frac{3}{2}}+10\sqrt{x}+5x\sqrt{x}-2x-x^{\frac{3}{2}}\sqrt{x}}{25-x}$ $= \frac{50+23x-x^2}{25-x}$ $= \frac{(25-x)(2+x)}{25-x}$ $= 2+x$
	Obtains one correct common factor in numerator eg $2 + x$ or $25 - x$ or $5 - x^{1/2}$ or expands numerator condone one error may be unsimplified	1.1a	M1	
	Obtains second correct common factor in numerator or obtains correct simplified numerator and denominator **PI** in long division	1.1a	M1	
	Completes manipulation by cancelling common factor to obtain $2 + x$	1.1b	A1	
	Total		**4**	

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November 2020 Question 1 Paper 1

1 The first three terms, in ascending powers of x, of the binomial expansion of $(9+2x)^{\frac{1}{2}}$ are given by

$$(9+2x)^{\frac{1}{2}} \approx a + \frac{x}{3} - \frac{x^2}{54}$$

where a is a constant.

1 (a) State the range of values of x for which this expansion is valid.

Circle your answer.

[1 mark]

$|x| < \frac{2}{9}$ $\quad$ $|x| < \frac{2}{3}$ $\quad$ $|x| < 1$ $\quad$ $|x| < \frac{9}{2}$

1 (b) Find the value of a.

Circle your answer.

[1 mark]

1 $\quad$ 2 $\quad$ 3 $\quad$ 9

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
1a	Circles the correct answer	1.1b	B1	$\lvert x\rvert < \frac{9}{2}$
	Subtotal		**1**	
1b	Circles the correct answer	1.1b	B1	3
	Subtotal		**1**	
	Question Total		**2**	

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June 2019 Question 9 Paper 2

9 (a) Show that the first two terms of the binomial expansion of $\sqrt{4 - 2x^2}$ are

$$2 - \frac{x^2}{2}$$

[2 marks]

9 (b) State the range of values of x for which the expansion found in part **(a)** is valid.

[2 marks]

9 (c) Hence, find an approximation for

$$\int_0^{0.4} \sqrt{\cos x}\, \mathrm{d}x$$

giving your answer to five decimal places.

Fully justify your answer.

[4 marks]

9 (d) A student decides to use this method to find an approximation for

$$\int_0^{1.4} \sqrt{\cos x}\, \mathrm{d}x$$

Explain why this may not be a suitable method.

[1 mark]

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ANSWER

Q	Marking instructions	AO	Mark	Typical solution
9(a)	Write in a form to which the binomial expansion can be applied Must be of form $a\left(1-\frac{x^2}{2}\right)^{\frac{1}{2}}$	3.1a	M1	$\sqrt{4-2x^2}=2\left(1-\frac{x^2}{2}\right)^{\frac{1}{2}}$ $\approx 2\left(1+\frac{1}{2}\left(-\frac{x^2}{2}\right)\right)$ $\approx 2-\frac{x^2}{2}$
	Completes rigorous argument to obtain correct expansion **AG**	2.1	R1	
9(b)	Compares their $\frac{x^2}{2}$ to 1 Condone incorrect inequality **PI** by $\left\|-2x^2\right\|<4$	1.1a	M1	$\left\|-\frac{x^2}{2}\right\|<1$ $\Rightarrow \|x\|<\sqrt{2}$
	Obtains correct range of values **ACF**	1.1b	A1	
9(c)	Explains that as 0.4 radians is small therefore $\cos x \approx 1-\frac{x^2}{2}$ **Must** refer to 0.4 and small angle approximation for cos x	2.4	E1	As 0.4 is small $\cos x \approx 1-\frac{x^2}{2}$ $\int_0^{0.4}\sqrt{\cos x}\,dx \approx \int_0^{0.4}\sqrt{1-\frac{x^2}{2}}\,dx$ $\approx \frac{1}{2}\int_0^{0.4} 2-\frac{x^2}{2}\,dx$ $\approx \int_0^{0.4} 1-\frac{x^2}{4}\,dx$ $\approx \left[x-\frac{x^3}{12}\right]_0^{0.4}$ $\approx 0.4-\frac{0.4^3}{12}$ ≈ 0.39467
	Uses half of their expansion from **9(a)** as the integrand	1.1a	M1	
	Integrates their expression with at least one term correct	1.1a	M1	
	Obtains correct value must be at least five decimal places Condone $\frac{148}{375}$ **CAO**	1.1b	A1	
9(d)	States that 1.4 radians is not a small angle so the approximation is not valid **Must** refer to small angle approximation **and** 1.4 or State invalid as 1.4 is bigger than 0.664 **NB** 0.664 is the limiting value for approximation to be valid	2.4	E1	Since 1.4 is not a small angle the approximation is not suitable
	Total		**9**	

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June 2018 Question 6 Paper 1

6 (a) Find the first three terms, in ascending powers of x, of the binomial expansion of $\dfrac{1}{\sqrt{4+x}}$

[3 marks]

6 (b) Hence, find the first three terms of the binomial expansion of $\dfrac{1}{\sqrt{4-x^3}}$

[2 marks]

6 (c) Using your answer to part **(b)**, find an approximation for $\displaystyle\int_0^1 \frac{1}{\sqrt{4-x^3}}\,\mathrm{d}x$, giving your answer to seven decimal places.

[3 marks]

6 (d) (i) Edward, a student, decides to use this method to find a more accurate value for the integral by increasing the number of terms of the binomial expansion used.

Explain clearly whether Edward's approximation will be an overestimate, an underestimate, or if it is impossible to tell.

[2 marks]

6 (d) (ii) Edward goes on to use the expansion from part **(b)** to find an approximation for $\displaystyle\int_{-2}^0 \frac{1}{\sqrt{4-x^3}}\,\mathrm{d}x$

Explain why Edward's approximation is invalid.

[2 marks]

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ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)	Writes in a form to which the binomial expansion can be applied Accept $A(1+\frac{x}{4})^{-\frac{1}{2}}$	AO3.1a	M1	$\frac{1}{\sqrt{4+x}}=\frac{1}{2}(1+\frac{x}{4})^{-\frac{1}{2}}$ $\approx\frac{1}{2}\left[1+\left(-\frac{1}{2}\right)\frac{x}{4}+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(\frac{x}{4}\right)^2}{2!}\right]$ $\approx\frac{1}{2}\left[1-\frac{x}{8}+\frac{3x^2}{128}\right]$ $\approx\frac{1}{2}-\frac{1}{16}x+\frac{3}{256}x^2$
	Uses binomial expansion for their $(1+kx)^{\pm\frac{1}{2}}$ with at least two terms correct (can be unsimplified)	AO1.1a	M1	
	Obtains correct simplified answer No need to expand brackets CAO	AO1.1b	A1	
(b)	Substitutes $-x^3$ in their three term expansion from part (a)	AO1.1a	M1	$\frac{1}{\sqrt{4-x^3}}\approx\frac{1}{2}-\frac{1}{16}\left(-x^3\right)+\frac{3}{256}\left(-x^3\right)^2$ $\approx\frac{1}{2}+\frac{x^3}{16}+\frac{3x^6}{256}$
	Obtains correct expansion. FT their (a)	AO1.1b	A1F	
(c)	Uses their three term expansion as the integrand ignore limits PI by next mark	AO1.1a	M1	$\int_0^1\frac{1}{\sqrt{4-x^3}}dx\approx\int_0^1\frac{1}{2}+\frac{x^3}{16}+\frac{3x^6}{256}dx$ $\approx\left[\frac{x}{2}+\frac{x^4}{64}+\frac{3x^7}{1792}\right]_0^1$ $\approx\frac{1}{2}+\frac{1}{64}+\frac{3}{1792}$ ≈ 0.5172991
	Integrates (at least two terms correct)	AO1.1a	M1	
	Obtains correct value CAO	AO1.1b	A1	
(d)(i)	Explains that each term in the expansion is positive	AO2.4	E1	Each term in the expansion is positive. So increasing the terms will increase the estimated value hence the value must be an underestimate.
	Deduces that increasing the number of terms will increase the estimated value and that the value must be an underestimate. (Condone inference if evidence given ie value calculated numerically and compared)	AO2.2a	R1	
(d)(ii)	States the validity of their binomial expansion for part (b) Provided their $k\neq\pm1$	AO3.1a	B1F	The binomial expansion is valid for $\lvert x\rvert<\sqrt[3]{4}$
	Compares integral lower limit with validity of correct expansion CAO	AO2.3	E1	$2>\sqrt[3]{4}$
	Total		**12**	

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June 2018 Question 9 Paper 2

9 (a) Show that the first two terms of the binomial expansion of $\sqrt{4 - 2x^2}$ are

$$2 - \frac{x^2}{2}$$

[2 marks]

9 (b) State the range of values of x for which the expansion found in part **(a)** is valid.

[2 marks]

9 (c) Hence, find an approximation for

$$\int_0^{0.4} \sqrt{\cos x}\, \mathrm{d}x$$

giving your answer to five decimal places.

Fully justify your answer.

[4 marks]

9 (d) A student decides to use this method to find an approximation for

$$\int_0^{1.4} \sqrt{\cos x}\, \mathrm{d}x$$

Explain why this may not be a suitable method.

[1 mark]

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ANSWER

Q	Marking instructions	AO	Mark	Typical solution
9(a)	Write in a form to which the binomial expansion can be applied Must be of form $a\left(1-\frac{x^2}{2}\right)^{\frac{1}{2}}$	3.1a	M1	$\sqrt{4-2x^2}=2\left(1-\frac{x^2}{2}\right)^{\frac{1}{2}}$ $\approx 2\left(1+\frac{1}{2}\left(-\frac{x^2}{2}\right)\right)$ $\approx 2-\frac{x^2}{2}$
	Completes rigorous argument to obtain correct expansion **AG**	2.1	R1	
9(b)	Compares their $\frac{x^2}{2}$ to 1 Condone incorrect inequality **PI** by $\left\|-2x^2\right\|<4$	1.1a	M1	$\left\|-\frac{x^2}{2}\right\|<1$ $\Rightarrow \|x\|<\sqrt{2}$
	Obtains correct range of values **ACF**	1.1b	A1	
9(c)	Explains that as 0.4 radians is small therefore $\cos x \approx 1-\frac{x^2}{2}$ **Must** refer to 0.4 and small angle approximation for cos x	2.4	E1	As 0.4 is small $\cos x \approx 1-\frac{x^2}{2}$ $\int_0^{0.4}\sqrt{\cos x}\,\mathrm{d}x \approx \int_0^{0.4}\sqrt{1-\frac{x^2}{2}}\,\mathrm{d}x$ $\approx \frac{1}{2}\int_0^{0.4} 2-\frac{x^2}{2}\,\mathrm{d}x$ $\approx \int_0^{0.4} 1-\frac{x^2}{4}\,\mathrm{d}x$ $\approx \left[x-\frac{x^3}{12}\right]_0^{0.4}$ $\approx 0.4-\frac{0.4^3}{12}$ ≈ 0.39467
	Uses half of their expansion from **9(a)** as the integrand	1.1a	M1	
	Integrates their expression with at least one term correct	1.1a	M1	
	Obtains correct value must be at least five decimal places Condone $\frac{148}{375}$ **CAO**	1.1b	A1	
9(d)	States that 1.4 radians is not a small angle so the approximation is not valid **Must** refer to small angle approximation **and** 1.4 or State invalid as 1.4 is bigger than 0.664 **NB** 0.664 is the limiting value for approximation to be valid	2.4	E1	Since 1.4 is not a small angle the approximation is not suitable
	Total		**9**	

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