Pure Year 1 exam questions (AS maths)-AQA

NOTE:
Please be aware that in the Year 12 collections, you will find questions from the A-Level (Year 13) papers. However, these questions are intentionally included because they align with Year 12 content and topics.
Happy studying!

Ch 6 Trigonometry

June 2022 Question 8 Paper 2

8 Triangle *ABC* has sides of length $(m-n)$, m and $(m+n)$ where $0 < 2n < m$

Angle *A* is the largest angle in the triangle.

8 (a) (i) Explain why angle *A* must be opposite the side of length $(m+n)$.

[1 mark]

8 (a) (ii) Using the cosine rule, show that $\cos A = \dfrac{m-4n}{2(m-n)}$

[3 marks]

8 (b) You are given that *BC* is the diameter of a circle, and *A* lies on the circumference of the circle. The value of m is 8

Calculate the value of n.

[3 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
8(a)(i)	Explains that $(m + n)$ is the largest side (which must be opposite the largest angle)	2.4	E1	The largest angle must be opposite the largest side which is $(m + n)$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)(ii)	Substitutes for a, b, c into cosine rule	1.1a	M1	$(m+n)^2 = m^2 + (m-n)^2 - 2m(m-n)\cos A$ $\cos A = \dfrac{m^2 + (m-n)^2 - (m+n)^2}{2m(m-n)}$ $\cos A = \dfrac{m^2 - 4mn}{2m(m-n)}$ $\cos A = \dfrac{m-4n}{2(m-n)}$
	Makes $\cos A$ the subject	1.1a	M1	
	Completes reasoned argument to obtain given result. AG	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)	Deduces that A is 90°, Accept marked on diagram PI by awarding of M1	2.2a	B1	A must be 90° So $\cos A = 0$ $\dfrac{m-4n}{2(m-n)} = 0$ $m = 4n$ $n = 2$
	Substitutes $\cos A = 0$ in **part (a)** equation PI by correct value for n Or uses Pythagoras	3.1a	M1	
	Obtains $n = 2$	1.1b	A1	
	Subtotal		**3**	

June 2022 Question 4 Paper 1

4 Find all the solutions of the equation

$$\cos^2\theta = 10\sin\theta + 4$$

for $0° < \theta < 360°$, giving your answers to the nearest degree.

Fully justify your answer.

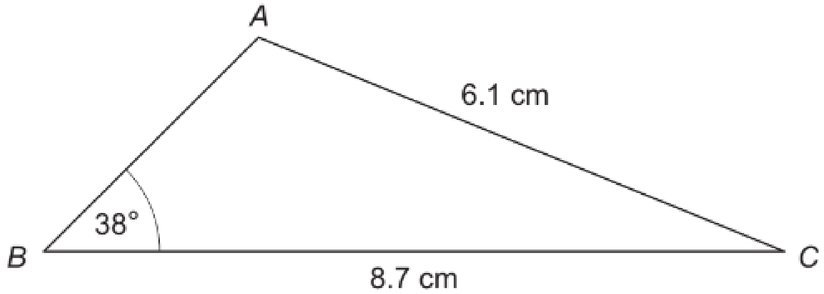
[5 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
4	Uses identity $\cos^2\theta = 1 - \sin^2\theta$	1.2	B1	$1 - \sin^2\theta = 10\sin\theta + 4$
	Solves their quadratic in $\sin\theta$ PI by at least one correct value of θ or - 18°	1.1a	M1	$0 = \sin^2\theta + 10\sin\theta + 3$ $\sin\theta = -0.3095$ or -9.6904 $\sin\theta = -9.6904$ (not valid)
	Explains that the second solution or both solutions is/are inappropriate. Accept N/A, out of range, no solutions, math error, reject OE Do not accept a '×' or –9.69 (OE) crossed out alone.	2.4	E1F	$\sin^{-1}(-0.3095) = -18.03°$ $\theta = 198°$ or $\theta = 342°$
	Obtains one correct value for θ AWRT 198° or 342°	1.1b	A1	
	Obtains two correct values for θ Condone - 18° included, but no other answers.	1.1b	A1	
	Question 4 Total		**5**	

June 2022 Question 4 Paper 2 (A-Level)

4



The diagram shows a triangle ABC.

AB is the shortest side. The lengths of AC and BC are 6.1 cm and 8.7 cm respectively.

The size of angle ABC is 38°

Find the size of the largest angle.

Give your answer to the nearest degree.

[3 marks]

ANSWER

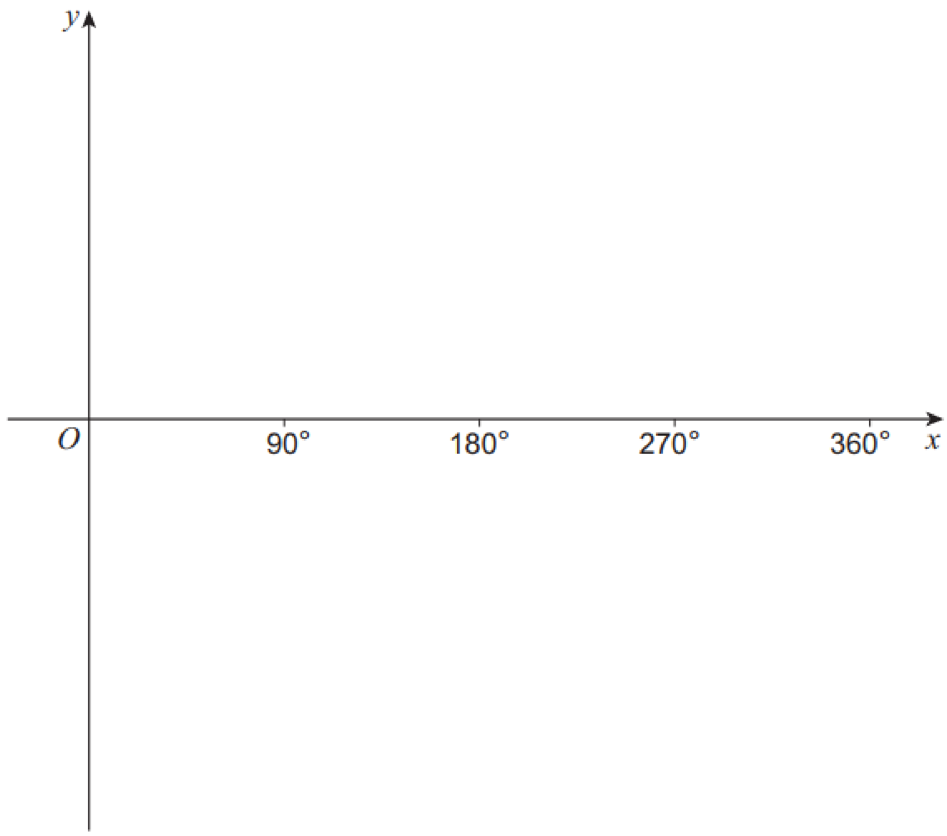
Q	Marking instructions	AO	Marks	Typical solution
4	Uses the sine rule Or Substitutes correctly into the cosine rule	1.1a	M1	$\frac{\sin\theta}{8.7}=\frac{\sin 38}{6.1}$ $\theta=61.4$ $A=180-61.4$ $=118.58...$ $=119^{\circ}$
	Obtains a value of 61 or 61.410964… rounded or truncated Condone answer (in radians) of 1.0718… or 0.4364… PI by correct obtuse angle or 81 Or Obtains correct length AB = 3.9367… Or $\mathrm{AB}^2=15.4998...$	1.1b	A1	
	Deduces the largest angle is 119 AWRT CAO	2.2a	A1	
	Question 4 Total		**3**	

June 2022 Question 5 Paper 3 (A-Level)

5 (a) Sketch the graph of

$$y = \sin 2x$$

for $0° \leq x \leq 360°$



[2 marks]

5 (b) The equation

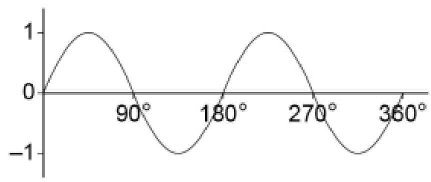
$$\sin 2x = A$$

has exactly two solutions for $0° \leq x \leq 360°$

State the possible values of A.

[1 mark]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Sketches sine wave with correct orientation through the origin to at least one period Ignore any numbers on y-axis	1.1a	M1	
	Sketches $y = \sin 2x$ through the correct axes intersections Condone only slight difference in amplitudes Ignore any numbers on y-axis Ignore any sections of the graph outside of $0^\circ \leq x \leq 360^\circ$	1.1b	A1	
	Subtotal		**2**	

November 2021 Question 8 Paper 1

8 (a) (i) Show that the equation

$$3\sin\theta\tan\theta = 5\cos\theta - 2$$

is equivalent to the equation

$$(4\cos\theta - 3)(2\cos\theta + 1) = 0$$

[3 marks]

8 (a) (ii) Solve the equation

$$3\sin\theta\tan\theta = 5\cos\theta - 2$$

for $-180° \leq \theta \leq 180°$

[2 marks]

8 (b) Hence, deduce all the solutions of the equation

$$3\sin\left(\frac{1}{2}\theta\right)\tan\left(\frac{1}{2}\theta\right) = 5\cos\left(\frac{1}{2}\theta\right) - 2$$

for $-180° \leq \theta \leq 180°$, giving your answers to the nearest degree.

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
8(a)(i)	Uses $\tan\theta = \frac{\sin\theta}{\cos\theta}$ identity	1.2	M1	$3\sin\theta\tan\theta = 5\cos\theta - 2$ $3\sin\theta\frac{\sin\theta}{\cos\theta} = 5\cos\theta - 2$ $3\sin^2\theta = 5\cos^2\theta - 2\cos\theta$ $3(1 - \cos^2\theta) = 5\cos^2\theta - 2\cos\theta$ $8\cos^2\theta - 2\cos\theta - 3 = 0$ $(4\cos\theta - 3)(2\cos\theta + 1) = 0$
	Uses $\sin^2\theta + \cos^2\theta = 1$ identity	1.2	M1	
	Manipulates to obtain the given equation	2.1	R1	
	Subtotal		**3**	
8(a)(ii)	Obtains any two solutions (AWRT)	1.1a	M1	$\theta = \pm 41°$ and $\pm 120°$
	Obtains all four solutions (AWRT)	1.1b	A1	
	Subtotal		**2**	
8(b)	Deduces that the required solutions are double their previous solutions PI by $\pm 83°$or $\pm 82°$ or $\pm 240°$	2.2a	M1	$\frac{1}{2}\theta = \pm 41.4$ and $\pm 120°$ $\theta = \pm 83°$
	Obtains $\pm 83°$ or AWRT $\pm 83°$ and no further solutions.	1.1b	A1	
	Subtotal		**2**	

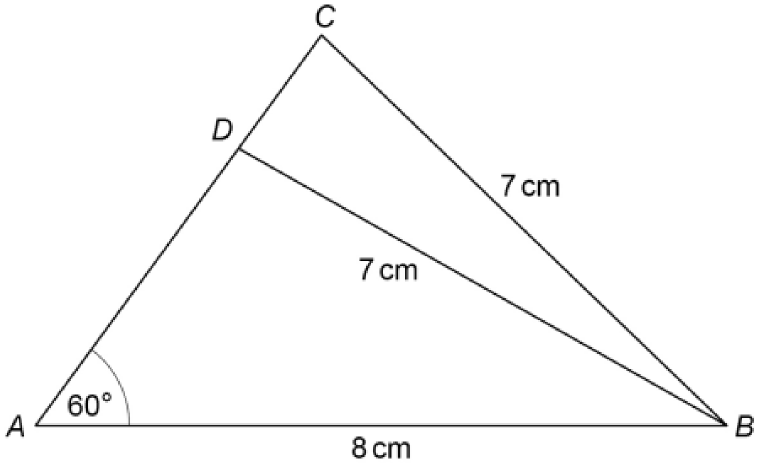
Video solution:

https://youtu.be/QNajms2RIVU

November 2021 Question 5 Paper 2

5 ABC is a triangle. The point D lies on AC.

$AB = 8$ cm, $BC = BD = 7$ cm and angle $A = 60°$ as shown in the diagram.



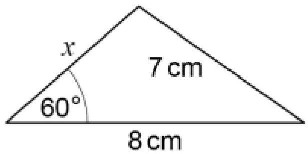
5 (a) Using the cosine rule, find the length of AC.

[3 marks]

5 (b) Hence, state the length of AD.

[1 mark]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Uses cosine rule or sine rule correctly to obtain any unknown angle or length	1.1b	B1	
	Uses the cosine rule to find an expression for, or the length of, AC	1.1a	M1	Using cosine rule $7^2 = 8^2 + x^2 - 2 \times 8 \times x \times \cos 60$ $x^2 - 8x + 15 = 0$
	Deduces that AC = 5cm (AWRT) Condone missing or incorrect units	2.2a	R1	$x = 3$ or 5 AC = 5cm
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)	States AD = 3cm Condone missing or incorrect units	1.1b	B1	AD = 3cm
	Subtotal		**1**	

November 2020 Question 3 Paper 1

3 Jia has to solve the equation

$$2 - 2\sin^2\theta = \cos\theta$$

where $-180° \leq \theta \leq 180°$

Jia's working is as follows:

$$2 - 2(1 - \cos^2\theta) = \cos\theta$$
$$2 - 2 + 2\cos^2\theta = \cos\theta$$
$$2\cos^2\theta = \cos\theta$$
$$2\cos\theta = 1$$
$$\cos\theta = 0.5$$
$$\theta = 60°$$

Jia's teacher tells her that her solution is incomplete.

3 (a) Explain the **two** errors that Jia has made.

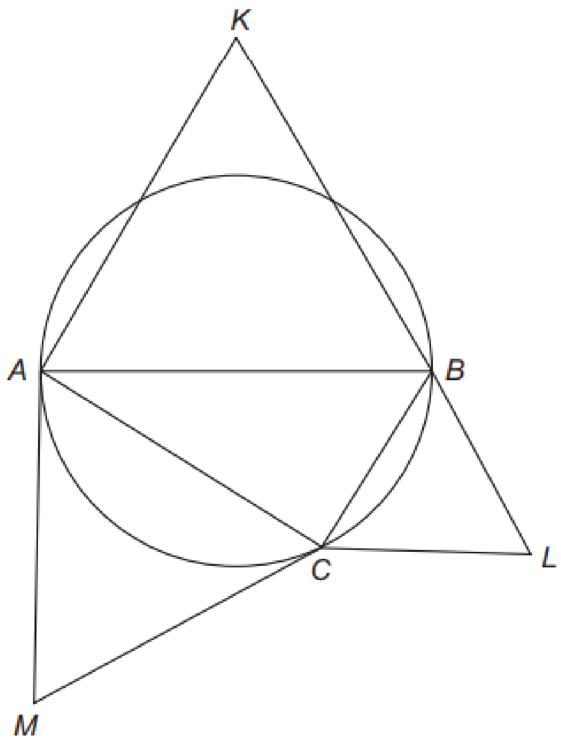
[2 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
3(a)	Explains that Jia should not have cancelled by $\cos\theta$ OE **Or** obtains solution $\cos\theta = 0$	2.3	E1	Jia should not have cancelled by $\cos\theta$ Jia has forgotten a second solution to $\cos\theta = 0.5$
	Explains that Jia has omitted the other solution to $\cos\theta = 0.5$ OE	2.3	E1	
	Subtotal		**2**	
3(b)	Obtains any two correct solutions. Accept answer written in part (a)	1.1a	M1	$\theta = \pm 60°$ and $\pm 90°$
	Obtains four correct solutions and no extra ones	1.1b	A1	
	Subtotal		**2**	
	Question Total		**4**	

November 2020 Question 9 Paper 1

9 The diagram below shows a circle and four triangles.



AB is a diameter of the circle. *C* is a point on the circumference of the circle.

Triangles *ABK*, *BCL* and *CAM* are equilateral.

Prove that the area of triangle *ABK* is equal to the sum of the areas of triangle *BCL* and triangle *CAM*.

[5 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
9	Uses 'angle in a semicircle' to justify ACB = 90°	2.4	E1	Angle ACB = 90° (Angle in a semicircle)
	Deduces that $AB^2 = AC^2 + BC^2$	2.2a	B1	$AB^2 = BC^2 + AC^2$ (Pythagoras)
	Applies area formula to one triangle	1.1a	M1	Area of ABK = $\frac{1}{2}AB^2 \sin 60°$ Area of BCL = $\frac{1}{2}BC^2 \sin 60°$ Area of CAM = $\frac{1}{2}AC^2 \sin 60°$
	Applies area formula to all three triangles	1.1b	A1	
	Forms a correct equation involving BCL + CAM and completes reasoned proof.	2.1	R1	BCL + CAM = $\frac{1}{2}\sin 60° (BC^2 + AC^2)$ $= \frac{1}{2}\sin 60° (AB^2)$ = ABK
	Total		**5**	

November 2020 Question 4 Paper 2

4 Find all the solutions of

$$9\sin^2 x - 6\sin x + \cos^2 x = 0$$

where $0° \leq x \leq 180°$

Give your solutions to the nearest degree.

Fully justify your answer.

[4 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
4	Uses substitution $\cos^2 x = 1 - \sin^2 x$ in any form	1.2	B1	$9\sin^2 x - 6\sin x + (1 - \sin^2 x) = 0$ $8\sin^2 x - 6\sin x + 1 = 0$ $(4\sin x - 1)(2\sin x - 1)$ $\sin x = \frac{1}{4}$ $\sin x = \frac{1}{2}$ 14°, 30°, 150°, 166°
	Solves 'their' quadratic to obtain two values for $\sin x$	1.1a	M1	
	Finds two correct solutions for x	1.1b	A1	
	Finds all four solutions for x and no extras (condone 14.5, 165.5 AWRT)	1.1b	A1	
	Total		**4**	

Arancha Ruiz

June 2019 Question 6 Paper 1

6 (a) (i) Show that $\cos\theta = \frac{1}{2}$ is one solution of the equation

$$6\sin^2\theta + 5\cos\theta = 7$$

[2 marks]

6 (a) (ii) Find all the values of θ that solve the equation

$$6\sin^2\theta + 5\cos\theta = 7$$

for $0° \le \theta \le 360°$

Give your answers to the nearest degree.

[2 marks]

6 (b) Hence, find all the solutions of the equation

$$6\sin^2 2\theta + 5\cos 2\theta = 7$$

for $0° \le \theta \le 360°$

Give your answers to the nearest degree.

[2 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
6 (a)(i)	Uses identity to replace $\sin^2\theta$ with $(1-\cos^2\theta)$ or uses $\sin\theta = \frac{\sqrt{3}}{2}$	1.2	M1	$6(1-\cos^2\theta) + 5\cos\theta = 7$ $6\cos^2\theta - 5\cos\theta + 1 = 0$ $(2\cos\theta - 1)(3\cos\theta - 1) = 0$ $\cos\theta = \frac{1}{2}$
	Solves quadratic equation to get one solution $\cos\theta = \frac{1}{2}$. Or verifying using $\cos\theta = \frac{1}{2}$	1.1a	A1	
6 (a)(ii)	States any two correct solutions	1.1b	B1	$\theta = 60°, 300°,$ Or $\cos\theta = \frac{1}{3}$ $\theta = 71°, 289°$
	States two additional correct solutions. Condone answers of 70.5 and 289.5 or greater accuracy. Ignore any additional answers outside the range but any additional answers inside range lose second B1	1.1b	B1	
6(b)	Writes down a set that is half the values given as their solutions in part (a) Accept 36° or 144°	2.2a	M1	$\theta = 30°, 150°, 35°, 145°$ $210°, 330°, 215°, 325°$
	Writes down an additional set that is 180° more than the first set. Condone AWRT integer values	1.1b	A1F	
	Total		**6**	

Video solution:

https://youtu.be/tZR3TkixZYY

June 2019 Question 10 Paper 1

10 On 18 March 2019 there were 12 hours of daylight in Inverness.

On 16 June 2019, 90 days later, there will be 18 hours of daylight in Inverness.

Jude decides to model the number of hours of daylight in Inverness, N, by the formula

$$N = A + B\sin t^\circ$$

where t is the number of days after 18 March 2019.

10 (a) (i) State the value that Jude should use for A.

[1 mark]

10 (a) (ii) State the value that Jude should use for B.

[1 mark]

10 (a) (iii) Using Jude's model, calculate the number of hours of daylight in Inverness on 15 May 2019, 58 days after 18 March 2019.

[1 mark]

10 (a) (iv) Using Jude's model, find how many days during 2019 will have at least 17.4 hours of daylight in Inverness.

[4 marks]

10 (a) (v) Explain why Jude's model will become inaccurate for 2020 and future years.

[1 mark]

10 (b) Anisa decides to model the number of hours of daylight in Inverness with the formula

$$N = A + B\sin\left(\frac{360}{365}t\right)^\circ$$

Explain why Anisa's model is better than Jude's model.

[1 mark]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
10 (a)(i)	States A = 12	3.3	B1	A = 12
10 (a)(ii)	States B = 6	3.3	B1	B = 6
10 (a)(iii)	Applies formula using their A and B 2 sf or better provided their answer >12 and <24	1.1b	B1F	12 + 6 sin 58 = 17.1
10 (a)(iv)	Substitutes 17.4 into formula with their A and B	1.1a	M1	17.4 = 12 + 6 sin t sin t = 0.9 t = 64 second value of t is 116 So answer is 53 days
	Evaluates first value of t. AWFW 64 to 65	1.1b	A1	
	Evaluates second value of t *as* 180 – their first value ±1 Or Subtracts their first value from 90 and doubles	3.4	M1	
	Obtains final answer AWRT 50	3.2a	A1	
10 (a)(v)	Explains 360 days is not the same as a year. Must mention 360	3.5b	E1	Jude's model will repeat after 360 days but a year has 365 days.
10(b)	Explains that Anisa's model adjusts the repeating pattern to match the number of days in a year. Mark may be supported by response seen in part (a)(v)	3.5c	E1	Anisa's model will repeat after 365 days because of the fraction
	Total		**9**	

June 2019 Question 3 Paper 2

3 It is given that $\sin\theta = -0.1$ and $180° < \theta < 270°$

Find the exact value of $\cos\theta$

[2 marks]

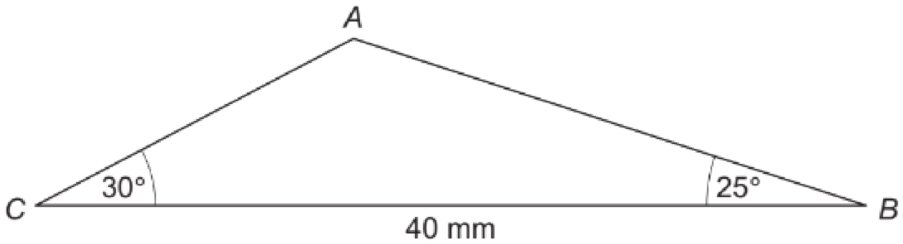
ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
3	Substitutes $\sin\theta$ value into the equation $\sin^2\theta + \cos^2\theta = 1$ ACF or Uses $\sin\theta = -0.1$ and right-angled triangle to get magnitude of $\cos\theta$ or Obtains $\cos^2\theta = 0.99$ CAO	1.1a	M1	$\sin^2\theta + \cos^2\theta = 1$ $0.01 + \cos^2\theta = 1$ $\cos^2\theta = 0.99$ $\cos\theta = -\frac{3}{10}\sqrt{11}$
	Solves and selects correct sign Accept $\cos\theta = -\sqrt{0.99}$ or exact equivalent $-\frac{3}{10}\sqrt{11}$ ISW if exact answer seen and then evaluated NB Any full numerical approach scores M0A0	1.1b	A1	
	Total		**2**	

Arancha Ruiz

June 2019 Question 5 Paper 2

5 A triangular prism has a cross section *ABC* as shown in the diagram below.



Angle $ABC = 25°$

Angle $ACB = 30°$

$BC = 40$ millimetres.

The length of the prism is 300 millimetres.

Calculate the volume of the prism, giving your answer to three significant figures.

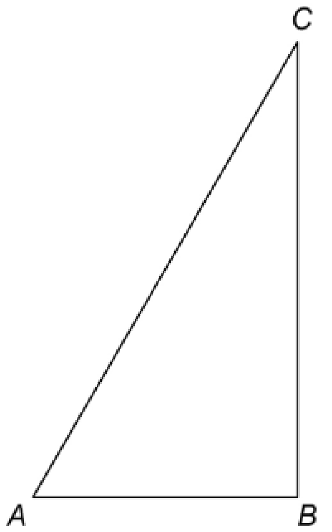
[4 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
5	Uses sine rule with 125° (or 55°)	1.1a	M1	$\frac{AB}{\sin 30} = \frac{40}{\sin 125}$
	Finds one of the sides as an expression or value given to at least 1 decimal place *AB* = 24.4 or *AC* = 20.6	1.1b	A1	$AB = \frac{40 \sin 30}{\sin 125}$ $AB = 24.415$
	Uses ½ ab sin C to find area for 'their' a, b and c OE NB Must be a valid set	1.1a	M1	$\text{Area} = \frac{1}{2} \times 40 \times \frac{40 \sin 30}{\sin 125} \times \sin 25$
	Obtains the correct volume of 61900 CAO Condone missing units	1.1b	A1	$= 206.4$ $\text{Volume} = 61900 \text{ mm}^3$
	Total		**4**	

June 2018 Question 6 Paper 1

6 ABC is a right-angled triangle.



D is the point on hypotenuse AC such that $AD = AB$.

The area of ΔABD is equal to half that of ΔABC.

6 (a) Show that $\tan A = 2 \sin A$

[4 marks]

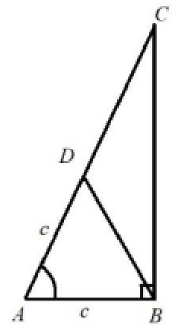
6 (b) (i) Show that the equation given in part **(a)** has two solutions for $0° \leq A \leq 90°$

[2 marks]

6 (b) (ii) State the solution which is appropriate in this context.

[1 mark]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)	Uses area of ADB = area of CDB or area of ADB = ½ area of ABC Possibly by use of "½ab sinC" twice	AO3.1a	M1	 Area of ADB = Area of CDB
	Deduces that AC = 2 x AD = 2 x AB or equivalent	AO1.1b	A1	CD = AD = c AC = 2c
	Uses trigonometry involving sin and tan based on triangle with AC = 2 x AB	AO1.1a	M1	cos A = $\frac{c}{2c} = \frac{1}{2}$ so A = 60° tan A = √3
	Obtains correct conclusion (AG) Sets out a well-constructed mathematical argument. Use of 60° or equivalent must be justified	AO2.1	R1	sin A = $\frac{\sqrt{3}}{2}$ 2sin A = √3 = tan A
(b)(i)	Uses tan A = $\frac{\sin A}{\cos A}$ and multiplies Or Uses sketch of two graphs to show two intersections	AO1.1a	M1	$\frac{\sin A}{\cos A}$ = 2 sin A sin A = 2 sin A cos A
	Solves the equation to give A = 0° and 60° Or interprets intersections of graphs of the correct shape between 0° and 90° to be the solutions Special case 0° and 60° stated but not justified award B1. Stated and verified award B2	AO1.1b	A1	sin A(1 – 2cos A) = 0 sin A = 0 or cos A = ½ A = 0° and A = 60°
(b)(ii)	Selects A = 60°. (Can be earned with no other working shown)	AO3.2a	B1	We need A = 60°
	Total		**7**	

June 2018 Question 4 Paper 2

4 Solve the equation $\tan^2 2\theta - 3 = 0$ giving all the solutions for $0° \leq \theta \leq 360°$

[4 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
4	Solves to find $\tan 2\theta$ (allow M1 even if only $+\sqrt{3}$ given PI by any correct final value of θ)	AO1.1a	M1	$\tan 2\theta = \pm\sqrt{3}$ $\tan 2\theta = \sqrt{3} \rightarrow 2\theta = 60°, 240°, 420°, 600°$ $\tan 2\theta = -\sqrt{3} \rightarrow 2\theta = 120°, 300°, 480°, 660°$ $\theta = 30°, 60°, 120°, 150°, 210°, 240°, 300, 330°$
	Obtains at least 3 correct final values for θ ignore extra incorrect terms or terms outside range	AO1.1b	A1	
	Obtains at least 3 correct final values of θ from $\tan 2\theta = \sqrt{3}$ **and** at least 3 correct values from $\tan 2\theta = -\sqrt{3}$ ignore extra incorrect terms or terms outside range	AO1.1a	M1	
	Obtains complete set of exactly 8 correct values for θ	AO1.1b	A1	
	Total		**4**	

June 2018 Question 9 Paper 2

9 It is given that $\cos 15° = \frac{1}{2}\sqrt{2+\sqrt{3}}$ and $\sin 15° = \frac{1}{2}\sqrt{2-\sqrt{3}}$

Show that $\tan^2 15°$ can be written in the form $a + b\sqrt{3}$, where a and b are integers.

Fully justify your answer.

[3 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
9	Uses $\tan\theta = \frac{\sin\theta}{\cos\theta}$ correctly (PI) (OE)	AO1.1a	M1	$\tan 15° = \frac{\sqrt{(2-\sqrt{3})}}{\sqrt{(2+\sqrt{3})}}$
	Squares and multiplies by appropriate $\frac{\text{conjugate}}{\text{conjugate}}$ or vice versa	AO1.1b	A1	$\tan^2 15° = \frac{2-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$ $= (2-\sqrt{3})^2$
	Obtains correct values of a and b $(a = 7, b = -4)$ If no explicit evidence of the use of $\frac{\text{conjugate}}{\text{conjugate}}$ seen award max 1/3 (M1 A0 R0)	AO2.1	R1	$\tan^2 15° = 7 - 4\sqrt{3}$
	Total		**3**	

Arancha Ruiz