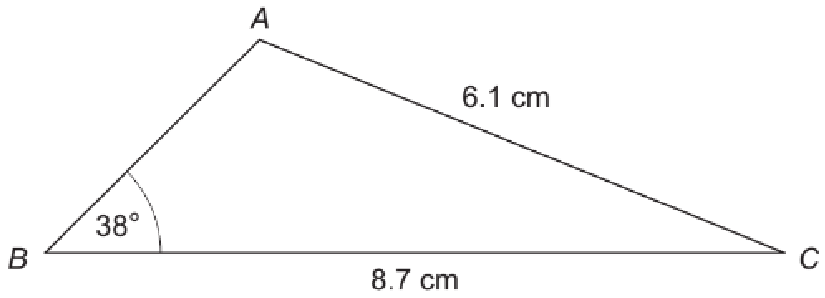
Pure Year 2 exam questions (A Level maths)-AQA

NOTE:
Please be aware that in the Year 12 collections, you will find questions from the Year 13 papers. However, these questions are intentionally included because they align with Year 12 content and topics. That's the reason why you are missing some questions here, please have a look at the Year 12 collection!
Happy studying!

Ch 6 Trigonometric functions

June 2022 Question 4 Paper 2

4



The diagram shows a triangle *ABC*.

AB is the shortest side. The lengths of *AC* and *BC* are 6.1 cm and 8.7 cm respectively.

The size of angle *ABC* is 38°

Find the size of the largest angle.

Give your answer to the nearest degree.

[3 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
4	Uses the sine rule Or Substitutes correctly into the cosine rule	1.1a	M1	$\frac{\sin\theta}{8.7}=\frac{\sin 38}{6.1}$ $\theta=61.4$ $A=180-61.4$ $=118.58...$ $=119^{\circ}$
	Obtains a value of 61 or 61.410964… rounded or truncated Condone answer (in radians) of 1.0718… or 0.4364… PI by correct obtuse angle or 81 Or Obtains correct length AB = 3.9367… Or $AB^2=15.4998...$	1.1b	A1	
	Deduces the largest angle is 119 AWRT CAO	2.2a	A1	
	Question 4 Total		**3**	

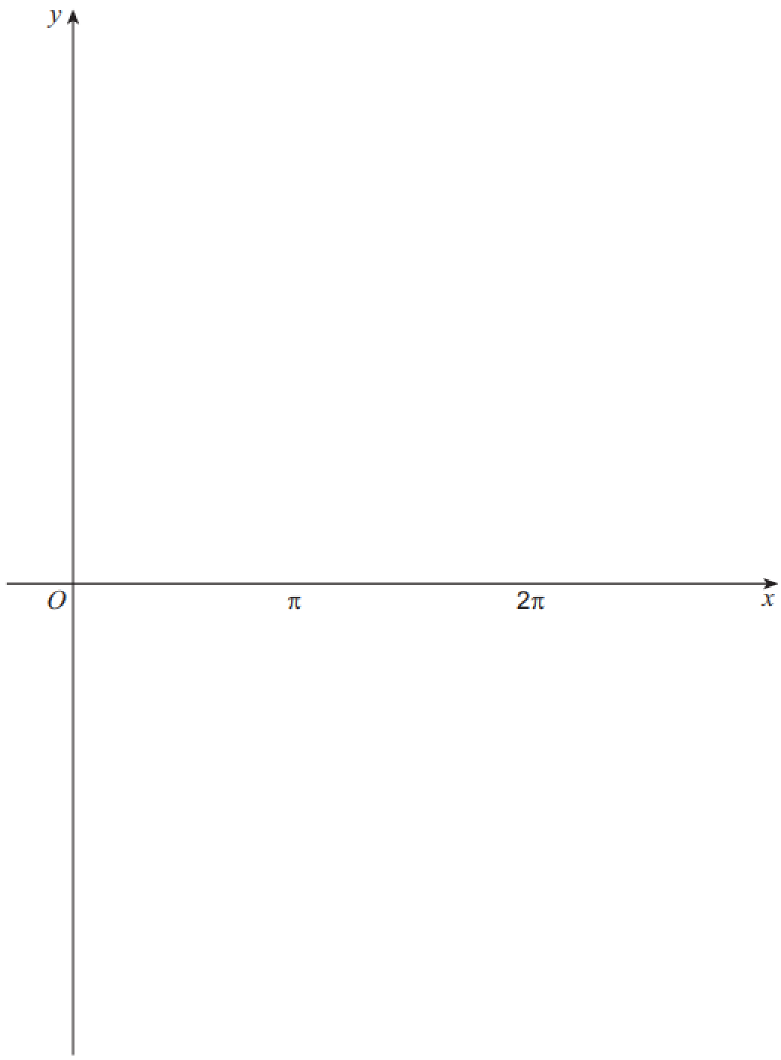
June 2022 Question 7 Paper 1

7 Sketch the graph of

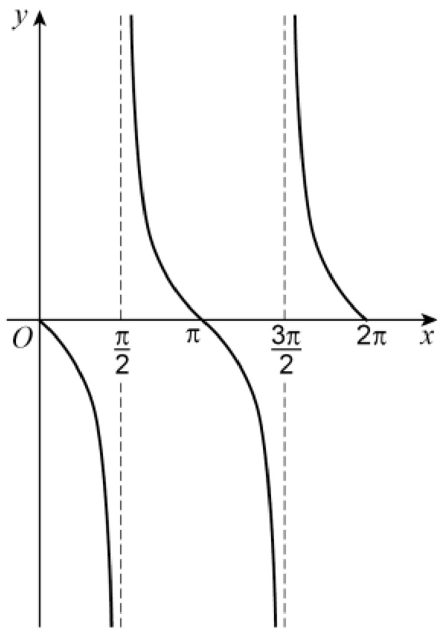
$$y = \cot\left(x - \frac{\pi}{2}\right)$$

for $0 \le x \le 2\pi$

[3 marks]



ANSWER

Q	Marking instructions	AO	Marks	Typical solution
7	Sketches one of the sections of the curve shown below Either  or  Condone translations Do not allow the end points of the curve turning to intersect the asymptote	1.2	B1	
	Sketches the three branches within the interval from 0 to 2π Condone overlapping branches Asymptotes need not be drawn	1.1a	M1	
	Completes fully correct sketch with asymptotes drawn at approximately the correct positions Labelling not required and can be ignored Ignore anything after 2π or to the left of O	1.1b	A1	
	Question 7 Total		**3**	

November 2021 Question 9 Paper 2

9 A robotic arm which is attached to a flat surface at the origin O, is used to draw a graphic design.

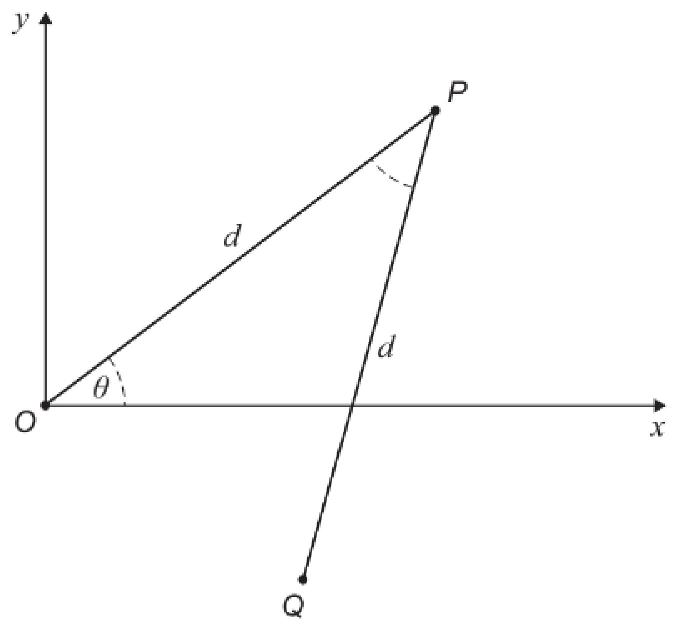
The arm is made from two rods OP and PQ, each of length d, which are joined at P.

A pen is attached to the arm at Q.

The coordinates of the pen are controlled by adjusting the angle OPQ and the angle θ between OP and the x-axis.

For this particular design the pen is made to move so that the two angles are always equal to each other with $0 \leq \theta \leq \frac{\pi}{2}$ as shown in **Figure 2**.

Figure 2



9 (a) Show that the x-coordinate of the pen can be modelled by the equation

$$x = d\left(\cos\theta + \sin\left(2\theta - \frac{\pi}{2}\right)\right)$$

[2 marks]

9 (b) Hence, show that

$$x = d(1 + \cos\theta - 2\cos^2\theta)$$

[2 marks]

9 (c) It can be shown that

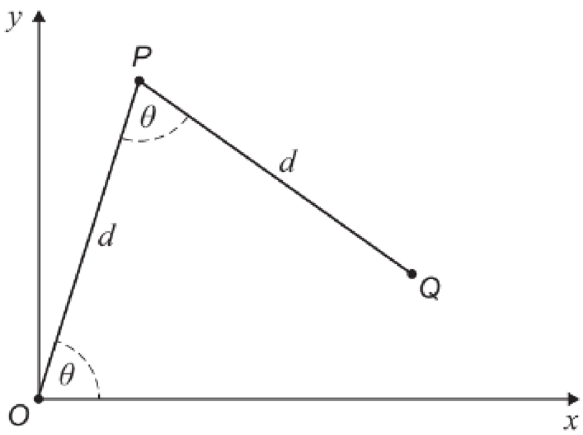
$$x = \frac{9d}{8} - d\left(\cos\theta - \frac{1}{4}\right)^2$$

State the greatest possible value of x and the corresponding value of $\cos\theta$

[2 marks]

9 (d) **Figure 3** below shows the arm when the x-coordinate is at its greatest possible value.

Figure 3



Find, in terms of d, the exact distance OQ.

[3 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Begins to find the required horizontal distance by considering an appropriate horizontal distance in a right-angled triangle For example: $d\cos\theta$, $dcos2\theta$ or $d\sin\left(\frac{\pi}{2}-2\theta\right)$ Award for correctly identifying at least one of the horizontal components required. This can be done on a sketch or by clearly stating which distance they are referring to	3.3	M1	QP makes angle of 2θ with horizontal $x = d\cos\theta - dcos2\theta$ $x = d\cos\theta - d\sin\left(\frac{\pi}{2}-2\theta\right)$ $x = d\left(\cos\theta + \sin\left(2\theta - \frac{\pi}{2}\right)\right)$
	Completes correct manipulation to show required result AG	2.1	R1	
	Subtotal		**2**	

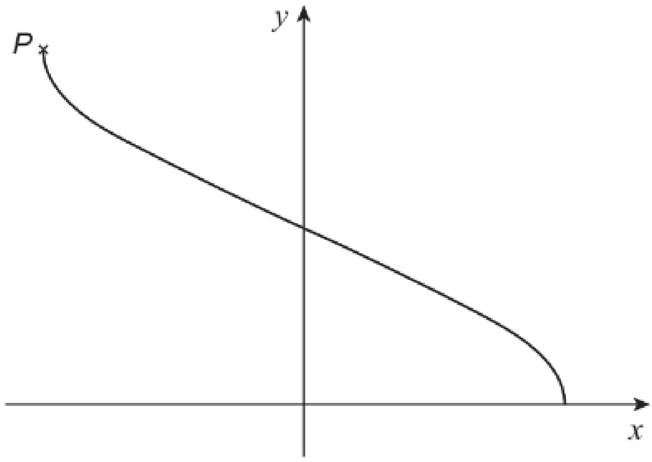
Q	Marking instructions	AO	Marks	Typical solution
9(b)	Uses a compound angle formula and expands $\sin\left(2\theta-\frac{\pi}{2}\right)$ or Uses complementary angles to obtain the formula without $\frac{\pi}{2}$ or States $x = d(\cos\theta - cos2\theta)$	3.1a	M1	$x = d\left(\cos\theta + \sin\left(2\theta - \frac{\pi}{2}\right)\right)$ $x = d(\cos\theta - cos2\theta)$ $x = d(\cos\theta - (2cos^2\theta - 1)$ $x = d(1 + \cos\theta - 2cos^2\theta)$
	Uses $cos2\theta = 2cos^2\theta - 1$ to show the required result AG	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(c)	States greatest value = $\frac{9d}{8}$	1.1b	B1	Greatest value = $\frac{9d}{8}$ $cos\,\theta = \frac{1}{4}$
	States $cos\,\theta = \frac{1}{4}$	1.1b	B1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(d)	Begins to find OQ by either using the cosine rule or the sine rule with d and θ Accept either $OQ^2 = d^2 + d^2 - 2d^2 cos\theta$ or $\frac{OQ}{sin\theta} = \frac{d}{sin\alpha}$ Note $\alpha = \frac{\pi - \theta}{2}$	3.1a	M1	$OQ^2 = d^2 + d^2 - 2d^2 cos\theta$ $= 2d^2 - 2d^2 \times \frac{1}{4}$ $= \frac{3d^2}{2}$ $OQ = \frac{\sqrt{6}}{2}d$
	Substitutes their exact value for $cos\theta$ into the cosine rule or Finds and substitutes their corresponding exact values for $sin\theta$ and $cos\frac{\theta}{2}$ into the sine rule Note When $cos\,\theta = \frac{1}{4}$, $sin\,\theta = \frac{\sqrt{15}}{4}$ and $cos\frac{\theta}{2} = \sqrt{\frac{5}{8}}$,	1.1a	M1	
	Obtains the correct exact value of OQ ACF	1.1b	A1	
	Subtotal		**3**	

November 2021 Question 1 Paper 3

1 The graph of $y = \arccos x$ is shown.



State the coordinates of the end point P.

Circle your answer.

[1 mark]

$(-\pi,\ 1)$ $\qquad$ $(-1,\ \pi)$ $\qquad$ $\left(-\frac{\pi}{2},\ 1\right)$ $\qquad$ $\left(-1,\ \frac{\pi}{2}\right)$

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
1	Circles correct answer	1.2	B1	$(-1, \pi)$
	Total		**1**	

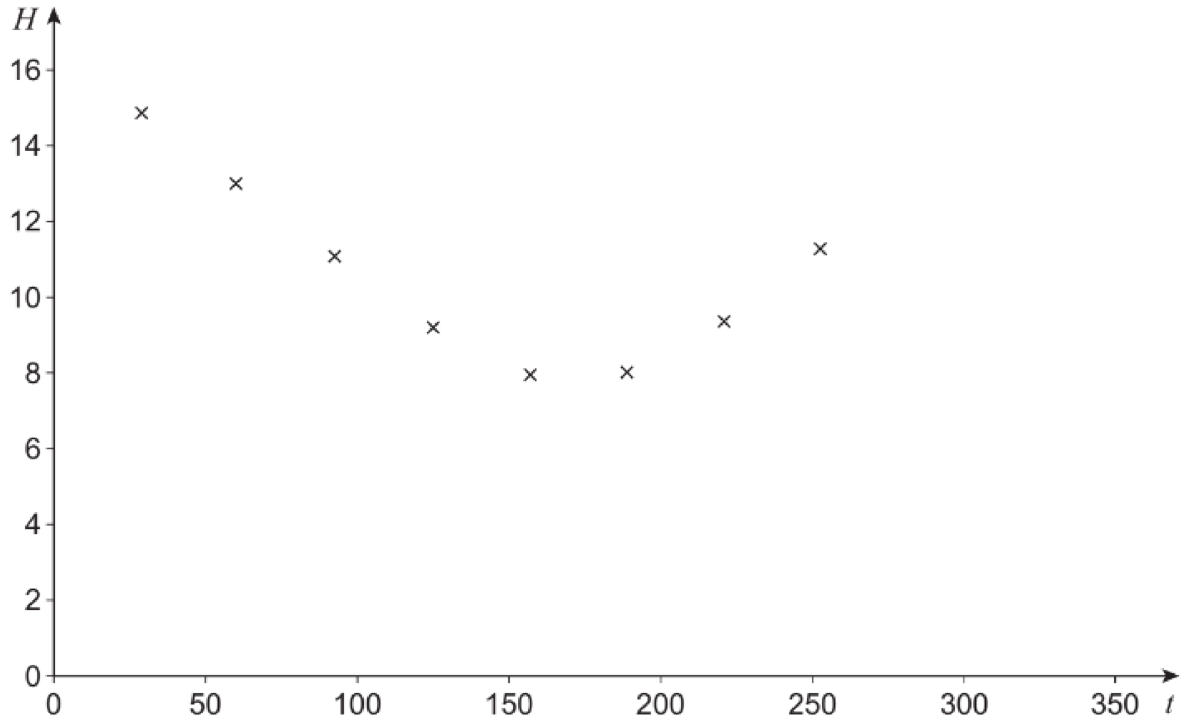
November 2020 Question 8 Paper 1

8 Mike, an amateur astronomer who lives in the South of England, wants to know how the number of hours of darkness changes through the year.

On various days between February and September he records the length of time, H hours, of darkness along with t, the number of days after 1 January.

His results are shown in **Figure 1** below.

Figure 1



Mike models this data using the equation

$$H = 3.87 \sin\left(\frac{2\pi(t + 101.75)}{365}\right) + 11.7$$

8 (a) Find the minimum number of hours of darkness predicted by Mike's model. Give your answer to the nearest minute.

[2 marks]

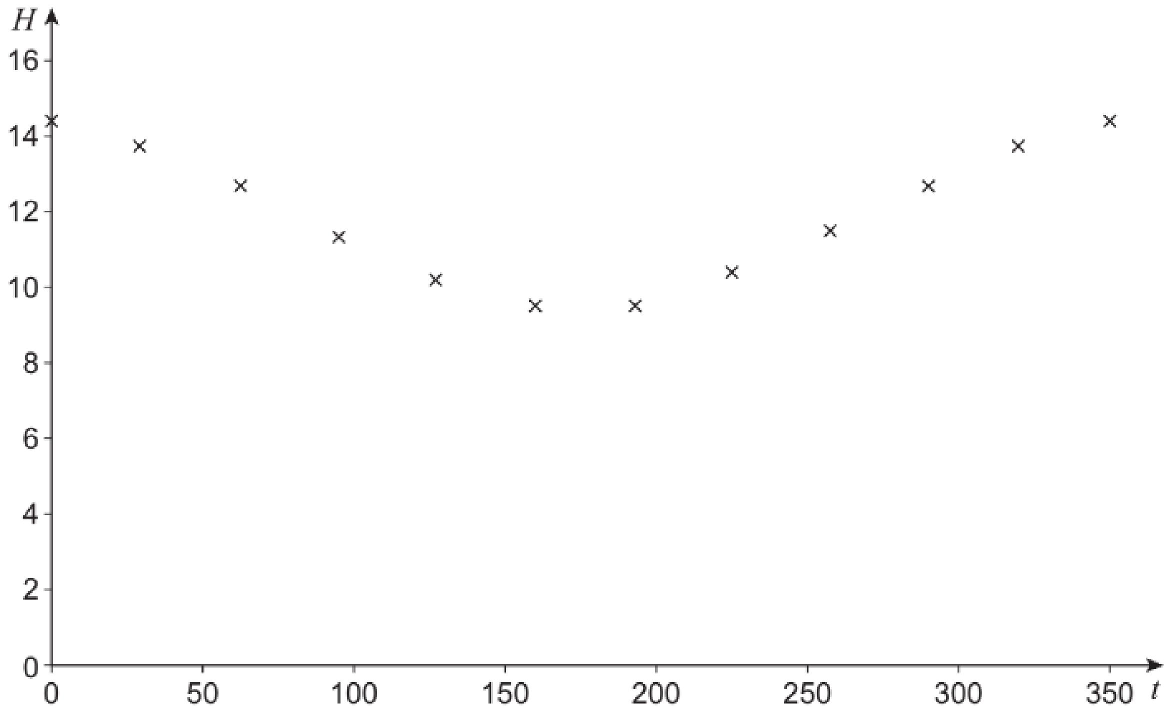
8 (b) Find the maximum number of consecutive days where the number of hours of darkness predicted by Mike's model exceeds 14

[3 marks]

8 (c) Mike's friend Sofia, who lives in Spain, also records the number of hours of darkness on various days throughout the year.

Her results are shown in **Figure 2** below.

Figure 2



Sofia attempts to model her data by refining Mike's model.

She decides to increase the 3.87 value, leaving everything else unchanged.

Explain whether Sofia's refinement is appropriate.

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
8(a)	Uses sin = -1 in the model to obtain -3.87 + 11.7 If a *t* value is used then the sine must evaluate to -1 or Differentiates, sets the derivative equal to 0 and obtains a value for *t* which they substitute back into the formula	3.4	M1	$sin\left(\frac{2\pi(t+101.75)}{365}\right) = -1$ $-3.87 + 11.7 = 7.83$ $7\text{hours } 50\text{mins}$
	Obtains correct answer Accept 470 minutes, $\frac{47}{6}$ or $7\frac{5}{6}$ hours	3.2a	A1	
	Subtotal		**2**	
8(b)	Uses model to form equation or inequality with H =14 Condone incorrect inequality	3.4	M1	$3.87\,sin\left(\frac{2\pi(t+101.75)}{365}\right) + 11.7 = 14$
	Solves equation to obtain at least two correct values of t Can be rounded or truncated Eg -64.77, 43.779, 300.22, 408.77	1.1b	A1	$t = 300.22$ or $t = 408.77$ $408 - 300 = 108$
	Subtracts an appropriate pair of t values to obtain number of consecutive days Condone any rounding to the nearest whole number or truncation of their pair of values Accept 109 or 107 Alternative method = 43 + (365 – 300) = 108	3.2a	A1	
	Subtotal		**3**	
8(c)	Explains that Sofia's refinement would increase the amplitude of the graph Accept The range of the graph would increase It would increase the fluctuation of the graph	3.3	M1	Sofia's refinement would increase the range of the graph Sofia's graph suggests this is not the case, so the refinement is not appropriate
	Explains that Sofia's refinement is not appropriate as her data/graph suggests a lower amplitude OE	3.5c	A1	
	Subtotal		**2**	
	Question Total		**7**	

November 2020 Question 2 Paper 2

2 Which one of the following equations has no real solutions?

Tick (✓) **one** box.

[1 mark]

$\cot x = 0$ ☐

$\ln x = 0$ ☐

$|x + 1| = 0$ ☐

$\sec x = 0$ ☐

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
2	Ticks correct box	2.2a	B1	$\sec x = 0$
	Total		**1**	

June 2019 Question 12 Paper 1

12 (a) Show that the equation

$$2\cot^2 x + 2\operatorname{cosec}^2 x = 1 + 4\operatorname{cosec} x$$

can be written in the form

$$a\operatorname{cosec}^2 x + b\operatorname{cosec} x + c = 0$$

[2 marks]

12 (b) Hence, given x is obtuse and

$$2\cot^2 x + 2\operatorname{cosec}^2 x = 1 + 4\operatorname{cosec} x$$

find the exact value of $\tan x$

Fully justify your answer.

[5 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
12(a)	Uses appropriate trig identity to form quadratic equation **in single trigonometrical** term. Condone $2(\pm 1 \pm \operatorname{cosec}^2 x) + 2\operatorname{cosec}^2 x = 1 + 4\operatorname{cosec} x$	1.1a	M1	$2\cot^2 x + 2\operatorname{cosec}^2 x = 1 + 4\operatorname{cosec} x$ $2(\operatorname{cosec}^2 x - 1) + 2\operatorname{cosec}^2 x = 1 + 4\operatorname{cosec} x$ $4\operatorname{cosec}^2 x - 4\operatorname{cosec} x - 3 = 0$
	Completes rigorous argument to show the required result	2.1	R1	
12(b)	Solves quadratic and Obtains one of $\operatorname{cosec} x = \frac{3}{2}$ or $\operatorname{cosec} x = -\frac{1}{2}$ OE	1.1b	B1	$\operatorname{cosec} x = \frac{3}{2}$ or $\operatorname{cosec} x = -\frac{1}{2}$ reject since $\lvert\operatorname{cosec} x\rvert \geq 1$ $\cot^2 x = \left(\frac{3}{2}\right)^2 - 1 = \frac{5}{4}$ $\tan x = -\frac{2\sqrt{5}}{5}$ Since *x* is obtuse
	Explains why their spurious solution(s) is rejected referring to the range of cosec or sine with explicit comparison to ± 1 May accept later rejection for valid reason ie sq root of negative OE	2.4	E1F	
	Uses trig identity or right-angled triangle/Pythagoras or given equation with their exact value of cosec *x or sin x to obtain an exact value of tan x* value used must satisfy $\lvert\operatorname{cosec} x\rvert \geq 1$ OE	1.1a	M1	
	Completes rigorous argument to find correct exact magnitude of tan *x* *ACF*	2.1	R1	
	Deduces tan x is negative. May be seen anywhere without contradiction by a positive final answer.	2.2a	B1	
	Total		**7**	

Video solution:

https://youtu.be/_geLVXvTYs0

June 2018 Question 12 Paper 1

12 $\mathrm{p}(x) = 30x^3 - 7x^2 - 7x + 2$

12 (a) Prove that $(2x+1)$ is a factor of $\mathrm{p}(x)$

[2 marks]

12 (b) Factorise $\mathrm{p}(x)$ completely.

[3 marks]

12 (c) Prove that there are no real solutions to the equation

$$\frac{30\sec^2 x + 2\cos x}{7} = \sec x + 1$$

[5 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
12(a)	Begins a proof using a valid method Eg. Factor theorem, algebraic division, multiplication of correct factors	AO1.1a	M1	$p\left(-\frac{1}{2}\right)=30\times\left(-\frac{1}{2}\right)^3-7\left(-\frac{1}{2}\right)^2-7\left(-\frac{1}{2}\right)+2$ $=0$ $\therefore 2x+1$ is a factor of $\mathrm{p}(x)$
	Constructs rigorous mathematical proof. To achieve this mark: Factor theorem the student must clearly substitute and state that p(-1/2)=0 **and** clearly state that this implies that $2x$ + 1 is a factor Algebraic division OR Multiplication of correct factors The method must be completely correct with a concluding statement	AO2.1	R1	
(b)	Obtains quadratic factor PI	AO1.1a	M1	$\mathrm{p}(x)=(2x+1)(15x^2-11x+2)$ $=(2x+1)(5x-2)(3x-1)$
	Obtains second linear factor	AO1.1b	A1	
	Writes $p(x)$ as the product of the correct three linear factors. NMS correct answer 3/3	AO1.1b	A1	
(c)	Rearranges to achieve a cubic equation in $\sec x$ (or $\cos x$)	AO3.1a	M1	$\frac{30\sec^2 x+2\cos x}{7}=\sec x+1$ $\Rightarrow 30\sec^2 x+2\cos x=7\sec x+7$ $\Rightarrow 30\sec^3 x+2=7\sec^2 x+7\sec x$ $30\sec^3 x-7\sec^2 x-7\sec x+2=0$ $\Rightarrow(2\sec x+1)(5\sec x-2)(3\sec x-1)=0$ $\Rightarrow \sec x=-\frac{1}{2},\frac{1}{3},\frac{2}{5}$ These values do not fall within the range of $\sec x$ as they are between -1 and 1 $\therefore \frac{30\sec^2 x+2\cos x}{7}=\sec x+1$ has no real solutions.
	Equates to zero and uses result from (b) or factorises	AO1.1a	M1	
	Deduces that if solutions exist they must be of the form $\sec x$ = -½, $\sec x$ =1/3 or $\sec x$ = 2/5 OE	AO2.2a	A1	
	Explains that the range of $\sec x$ is $(-\infty,-1]\cup[1,\infty)$ OE OE for $\cos x$	AO2.4	E1	
	Completes argument explaining that there cannot be any real solutions as values are outside of the function's range.	AO2.1	R1	
	Total		**10**	

June 2018 Question 1 Paper 3

1 $\mathrm{f}(x) = \arcsin x$

State the maximum possible domain of f

Tick (✓) **one** box.

[1 mark]

$\{x \in \mathbb{R} : -1 \le x \le 1\}$ ☐

$\left\{x \in \mathbb{R} : -\frac{\pi}{2} \le x \le \frac{\pi}{2}\right\}$ ☐

$\{x \in \mathbb{R} : -\pi \le x \le \pi\}$ ☐

$\{x \in \mathbb{R} : -90 \le x \le 90\}$ ☐

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
1	Ticks the correct response	1.2	B1	$\{x \in \mathbb{R} : -1 \le x \le 1\}$
	Total		**1**	