Pure Year 2 exam questions (A Level maths)-AQA

NOTE:
Please be aware that in the Year 12 collections, you will find questions from the Year 13 papers. However, these questions are intentionally included because they align with Year 12 content and topics. That's the reason why you are missing some questions here, please have a look at the Year 12 collection!
Happy studying!

Arancha Ruiz
Copyright 2023 © Better your Maths

Ch 5 Differentiation

June 2022 Question 4 Paper 1

4 The graph of

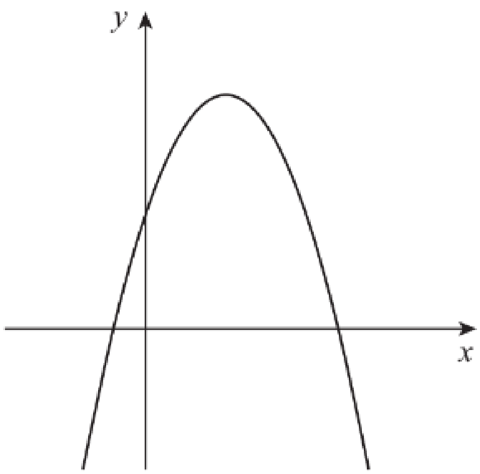
$$y = \mathrm{f}(x)$$

where

$$\mathrm{f}(x) = ax^2 + bx + c$$

is shown in **Figure 1**.

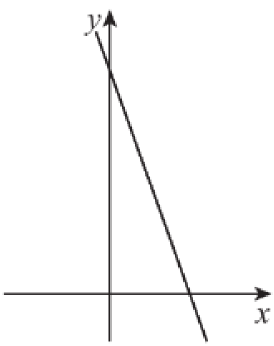
Figure 1



Which of the following shows the graph of $y = \mathrm{f}'(x)$?

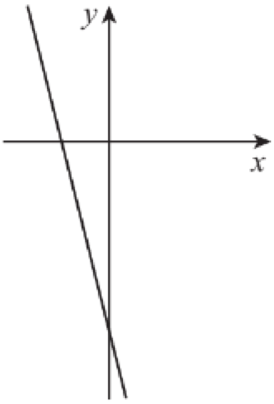
Tick (✓) **one** box.

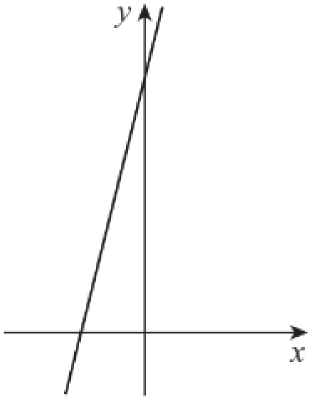
[1 mark]

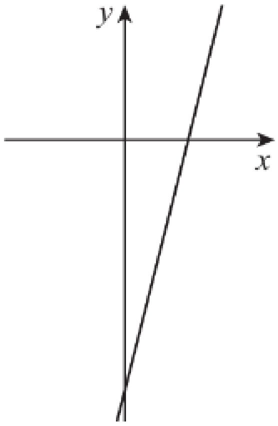


☐

Arancha Ruiz
Copyright 2023 © Better your Maths

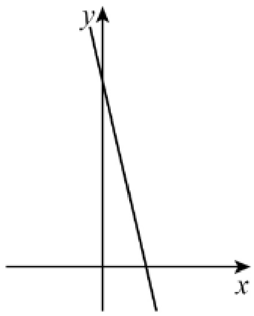






Arancha Ruiz
Copyright 2023 © Better your Maths

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
4	Ticks the correct box	2.2a	R1	
	Question 4 Total		**1**	

Arancha Ruiz
Copyright 2023 © Better your Maths

June 2022 Question 5 Paper 1

5 Find an equation of the tangent to the curve

$$y = (x - 2)^4$$

at the point where $x = 0$

[3 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
5	Differentiates to obtain a correct derivative either $4(x-2)^3$ OE or $4x^3 - 24x^2 + 48x - 32$ PI by -32 obtained with no errors seen in evaluating $\frac{dy}{dx}$	1.1b	B1	
	Substitutes x = 0 into their $\frac{dy}{dx}$ to obtain a numerical value or PI by constant from their $\frac{dy}{dx}$ or PI by -32 obtained with no errors seen in evaluating $\frac{dy}{dx}$	1.1a	M1	$\frac{dy}{dx} = 4(x-2)^3$ When x = 0 $\frac{dy}{dx} = -32$ $y = 16$ $y = -32x + 16$
	Obtains $y = -32x + 16$ ACF Award the mark at the first opportunity and ISW any incorrect rearrangement No errors seen	1.1b	A1	
	Question 5 Total		**3**	

Arancha Ruiz
Copyright 2023 © Better your Maths

Copyright 2023 © Better your Maths

June 2022 Question 2 Paper 2

2 State the value of

$$\lim_{h \to 0} \frac{\sin(\pi + h) - \sin \pi}{h}$$

Circle your answer.

[1 mark]

$\cos h$ $\quad -1 \quad$ 0 $\quad 1$

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
2	Circles correct answer	2.2a	R1	-1
	Question 2 Total		**1**	

Arancha Ruiz
Copyright 2023 © Better your Maths

November 2021 Question 9 Paper 1

9 A function f is defined for all real values of x as

$$\mathrm{f}(x) = x^4 + 5x^3$$

The function has exactly two stationary points when $x = 0$ and $x = -\frac{15}{4}$

9 (a) (i) Find $\mathrm{f}''(x)$

[2 marks]

9 (a) (ii) Determine the nature of the stationary points.

Fully justify your answer.

[4 marks]

9 (b) State the range of values of x for which

$$\mathrm{f}(x) = x^4 + 5x^3$$

is an increasing function.

[1 mark]

9 (c) A second function g is defined for all real values of x as

$$\mathrm{g}(x) = x^4 - 5x^3$$

9 (c) (i) State the single transformation which maps f onto g.

[1 mark]

9 (c) (ii) State the range of values of x for which g is an increasing function.

[1 mark]

Arancha Ruiz
Copyright 2023 © Better your Maths

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
9(a)(i)	Differentiates $f(x)$ at least one correct term May be unsimplified	1.1a	M1	$f'(x) = 4x^3 + 15x^2$ $f''(x) = 12x^2 + 30x$
	Obtains $f''(x) = 12x^2 + 30x$	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)(ii)	Substitutes $x = -\frac{15}{4}$ into their $f''(x)$ or uses gradient test both sides of $x = -\frac{15}{4}$	1.1a	M1	$f''\left(-\frac{15}{4}\right) = 12\left(-\frac{15}{4}\right)^2 + 30\left(-\frac{15}{4}\right)$ $= \frac{225}{4} > 0$ Hence there is a minimum at $x = -\frac{15}{4}$ $f''(0) = 0$ $f''(1) = 12 + 30 > 0$ and f''(-1) = 12 – 30 < 0 hence point of inflection at $x = 0$
	Completes rigorous justification for minimum at $x = -\frac{15}{4}$ This must be correctly deduced using shape of graph or $f''\left(-\frac{15}{4}\right) = \frac{225}{4} > 0$	2.1	R1	
	Substitutes two values either side of $x = 0$ into their $f''(x)$ or uses gradient test both sides of $x = 0$ or argues using the shape of a quartic curve with two stationary points	1.1a	M1	
	Completes rigorous justification for point of inflection at $x = 0$ This must be correctly deduced using the shape of the graph or a completely correct test both sides of the point Other explanation eg quartic with two stationary points, one of the points must be a point of inflection	2.2a	R1	
	Subtotal		**4**	

Arancha Ruiz
Copyright 2023 © Better your Maths

Q	Marking instructions	AO	Marks	Typical solution
9(b)	Deduces $x > -\frac{15}{4}$ **OE** Condone use of '$\geq$'	2.2a	B1	$x > -\frac{15}{4}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
9(c)(i)	Deduces the transformation is a reflection in the y-axis **OE**	2.2a	B1	Reflection in the y-axis
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
9(c)(ii)	Deduces $x > \frac{15}{4}$ Condone use of '$\geq$' **FT** their answer in part (b) only if their value in (b) is negative	2.2a	B1F	$x > \frac{15}{4}$
	Subtotal		**1**	

Arancha Ruiz
Copyright 2023 © Better your Maths

June 2019 Question 13 Paper 1

13 A curve, C, has equation

$$y = \frac{e^{3x-5}}{x^2}$$

Show that C has exactly one stationary point.

Fully justify your answer.

[7 marks]

ANSWER

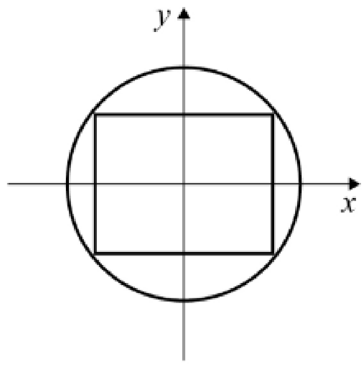
Q	Marking instructions	AO	Mark	Typical solution
13	Chooses an appropriate technique to differentiate accept any evidence of product rule or quotient rule	3.1a	M1	$x \neq 0$ as y is undefined $\frac{dy}{dx} = \frac{3e^{3x-5}x^2 - 2xe^{3x-5}}{x^4}$ At a turning point $\frac{dy}{dx} = 0$ $\Rightarrow \frac{3e^{3x-5}x^2 - 2xe^{3x-5}}{x^4} = 0$ $\Rightarrow 3e^{3x-5}x^2 - 2xe^{3x-5} = 0$ $\Rightarrow (3x-2)xe^{3x-5} = 0$ $\Rightarrow x = \frac{2}{3},\quad e^{3x-5} = 0$ $e^{3x-5} \neq 0$ $\therefore$ there is only one stationary point.
	Differentiates e^{3x-5} correctly	1.1b	B1	
	Obtains correct $\frac{dy}{dx}$ ACF	1.1b	A1	
	Explains that stationary points occur when $\frac{dy}{dx} = 0$	2.4	E1	
	Equates their $\frac{dy}{dx}$ to zero and solves their equation with at least one correct line of correct rearrangement. Resulting in a value for x.	1.1a	M1	
	Deduces their factor $e^{3x-5} \neq 0$	2.2a	B1F	
	Completes argument to show exactly one stationary point at $x = \frac{2}{3}$. Must include consideration of $x \neq 0$ somewhere.	2.1	R1	
	Total		**7**	

Arancha Ruiz
Copyright 2023 © Better your Maths

June 2018 Question 13 Paper 1

13 A company is designing a logo. The logo is a circle of radius 4 inches with an inscribed rectangle. The rectangle must be as large as possible.

The company models the logo on an x-y plane as shown in the diagram.



Use calculus to find the maximum area of the rectangle.

Fully justify your answer.

[10 marks]

Arancha Ruiz
Copyright 2023 © Better your Maths

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
13	Identifies and clearly defines consistent variables for length and width. Can be shown on diagram.	AO3.1b	B1	Width of rectangle = $2x$ Length of rectangle = $2y$
	Models the area of rectangle with an expression of the correct dimensions	AO3.3	M1	$A = 4xy$
	Eliminates either variable to form a model for the area in one variable.	AO1.1a	M1	$x^2 + y^2 = 16$
	Obtains a correct equation to model the area in one variable	AO1.1b	A1	$A = 4x\sqrt{16 - x^2}$
	Differentiates their expression for area. Condone one error	AO3.4	M1	$\frac{dA}{dx} = 4\sqrt{16 - x^2} - \frac{4x^2}{\sqrt{16-x^2}}$ $\frac{dA}{dx} = \frac{64-8x^2}{\sqrt{16-x^2}}$ For maximum point $\frac{\mathrm{d}A}{\mathrm{d}x} = 0$
	Explains that their derivative equals zero for a maximum or stationary point.	AO2.4	E1	$\frac{64-8x^2}{\sqrt{16-x^2}}=0$
	Equates area derivative to zero and obtains correct value for either variable. CAO	AO1.1b	A1	$x = 2\sqrt{2}$ When x = 2.8, $\frac{\mathrm{d}A}{\mathrm{d}x} = 0.448$
	Completes a gradient test or uses second derivative of their area function to determine nature of stationary point	AO1.1a	M1	When $x = 2.9$, $\frac{\mathrm{d}A}{\mathrm{d}x} = -1.191$ Therefore maximum
	Deduces that the area is a maximum at $x = 2\sqrt{2}$ or $\theta = \frac{\pi}{4}$ Values need not be exact	AO2.2a	R1	The maximum area is 32 sq in
	Obtains maximum area with correct units AWRT 32	AO3.2a	B1	
	Total		**10**	

Arancha Ruiz
Copyright 2023 © Better your Maths

June 2018 Question 9 Paper 3

9 A curve has equation

$$x^2y^2 + xy^4 = 12$$

9 (a) Prove that the curve does not intersect the coordinate axes.

[2 marks]

9 (b) (i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2xy + y^3}{2x^2 + 4xy^2}$

[5 marks]

9 (b) (ii) Prove that the curve has no stationary points.

[4 marks]

9 (b) (iii) In the case when $x > 0$, find the equation of the tangent to the curve when $y = 1$

[4 marks]

Arancha Ruiz
Copyright 2023 © Better your Maths

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
9(a)	Demonstrates by substitution that $x=0$ or $y=0$ leads to value on the LHS = 0	2.4	E1	When x = 0 $0^2y^2 + 0y^4 = 0$ When y = 0 $x^2 0^2 + x0^4 = 0$ This is a contradiction because $x^2y^2 + xy^4 = 12$ so the curve does not intersect either axis
	Completes rigorous argument to show required result	2.1	R1	
9 (b)(i)	Uses implicit differentiation	3.1a	M1	$2xy^2 + 2x^2y\frac{dy}{dx} + y^4 + 4xy^3\frac{dy}{dx} = 0$ $\frac{dy}{dx} = -\frac{2xy^2 + y^4}{2x^2y + 4xy^3}$ $= -\frac{y\,(2xy+y^3)}{y(2x^2+4xy^2)}$ $= -\frac{2xy + y^3}{2x^2 + 4xy^2}$,
	Product rule used LHS (at least one pair of terms correct)	1.1a	M1	
	Differentiates equation of curve fully correctly	1.1b	A1	
	Collects their $\frac{dy}{dx}$ terms in an equation and factorises	3.1a	M1	
	Completes convincing argument to obtain required result by **factorising** then simplifying y **AG**	2.1	R1	

Copyright 2023 © Better your Maths

9 (b)(ii)	Begins argument by setting $\frac{dy}{dx}=0$ to form an equation for x and y **PI** by $2xy+y^3=0$	2.1	M1	For stationary points $\frac{dy}{dx}=0$ $\Rightarrow 2xy+y^3=0$ $\Rightarrow y^2=-2x$ $\Rightarrow x^2y^2+x(-2x)y^2=12$ $\Rightarrow -x^2y^2=12$ Since $-x^2y^2<0$ there can be no stationary points.
	Obtains $y^2=-2x$ or $y=\sqrt{-2x}$ or $x=\frac{-y^2}{2}$	1.1b	A1	
	Substitutes $y^2=-2x$ or $x=\frac{-y^2}{2}$ into equation for curve	1.1a	M1	
	Completes convincing argument to deduce the required result	2.2a	R1	
9 (b)(iii)	Substitutes y = 1 into equation of curve to obtain correct quadratic **ACF**	3.1a	M1	$y=1\Rightarrow x^2+x-12=0$ $\Rightarrow x=3 \qquad (x>0)$ $\Rightarrow \frac{dy}{dx}=-\frac{7}{30}$ $y-1=-\frac{7}{30}(x-3)$
	Deduces x = 3 **PI** by substituting their x in their dy/dx	2.2a	R1	
	Substitutes their x and y = 1 in their dy/dx	1.1a	M1	
	Obtains correct equation of tangent **ACF** **ISW**	1.1b	A1	

Arancha Ruiz
Copyright 2023 © Better your Maths