Pure Year 2 exam questions (A Level maths)-AQA

NOTE:
Please be aware that in the Year 12 collections, you will find questions from the Year 13 papers. However, these questions are intentionally included because they align with Year 12 content and topics. That's the reason why you are missing some questions here, please have a look at the Year 12 collection!
Happy studying!

Ch 4 Functions

June 2022 Question 3 Paper 1

3 The curve

$$y = \log_4 x$$

is transformed by a stretch, scale factor 2, parallel to the y-axis.

State the equation of the curve after it has been transformed.

Circle your answer.

[1 mark]

$y = \frac{1}{2}\log_4 x$ $\qquad$ $y = 2\log_4 x$ $\qquad$ $y = \log_4 2x$ $\qquad$ $y = \log_8 x$

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
3	Circles the correct answer	2.2a	R1	$y = 2\log_4 x$
	Question 3 Total		**1**	

June 2022 Question 3 Paper 3

3 The function f is defined by

$$f(x) = 2x + 1$$

Solve the equation

$$f(x) = f^{-1}(x)$$

Circle your answer.

[1 mark]

$x = -1$ $x = 0$ $x = 1$ $x = 2$

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
3	Circles correct answer	3.1a	B1	$x = -1$
	Question 3 Total		**1**	

Arancha Ruiz

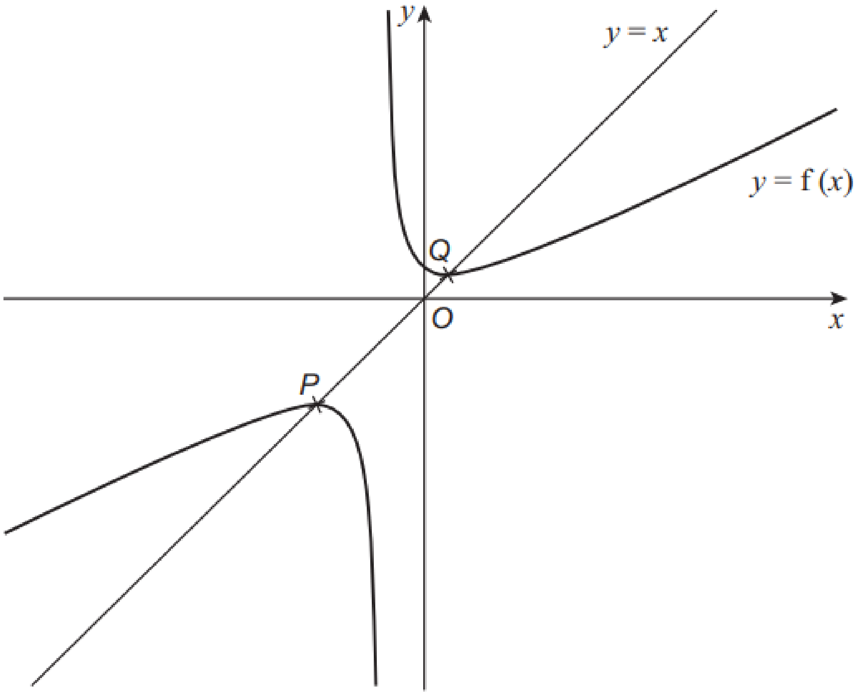
June 2022 Question 10 Paper 3

10 The function f is defined by

$$\mathrm{f}(x) = \frac{x^2 + 10}{2x + 5}$$

where f has its maximum possible domain.

The curve $y = \mathrm{f}(x)$ intersects the line $y = x$ at the points P and Q as shown below.



10 (b) Explain how you know that the function f is many-to-one.

[2 marks]

10 (c) (i) Show that the x-coordinates of P and Q satisfy the equation

$$x^2 + 5x - 10 = 0$$

[2 marks]

10 (c) (ii) Hence, find the exact x-coordinate of P and the exact x-coordinate of Q.

[1 mark]

10 (d) Show that *P* and *Q* are stationary points of the curve.

Fully justify your answer.

[5 marks]

10 (e) Using set notation, state the range of f.

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
10(a)	States -2.5 OE	2.2a	B1	-2.5
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)	Explains that many-to-one function is when distinct values of x give the same value for y	2.4	E1	Many-to-one function is when two or more x values give the same y value. This graph is many-to-one because you can draw a horizontal line and it will cross the graph twice.
	Uses the shape of the graph to justify their answer or gives an example of two x values eg $\mathrm{f}(0)=\mathrm{f}(4)$ or states turning or minimum or maximum points indicate many-to-one	2.4	E1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(c)(i)	Equates x and $\dfrac{x^2+10}{2x+5}$	3.1a	M1	$x=\dfrac{x^2+10}{2x+5}$ $x(2x+5)=x^2+10$ $2x^2+5x=x^2+10$ $x^2+5x-10=0$
	Rearranges with at least one intermediate step to obtain quadratic equation AG Condone $0 = x^2 + 5x - 10$	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(c)(ii)	Obtains $\dfrac{-5\pm\sqrt{65}}{2}$ Ignore any labels ISW	1.1b	B1	$x=\dfrac{-5\pm\sqrt{65}}{2}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
10(d)	Uses quotient rule to obtain an expression in the form of $\dfrac{Ax(2x+5)+B(x^2+10)}{(2x+5)^2}$ or uses product rule to obtain an expression in the form of $Cx(2x+5)^{-1}+D(x^2+10)(2x+5)^{-2}$ or uses implicit differentiation to obtain an equation of the form $Ax\dfrac{dy}{dx}+By+C\dfrac{dy}{dx}=Dx$ A, B, C and D can be any values but not 0 Condone missing brackets	3.1a	M1	$f'(x)=\dfrac{2x(2x+5)-2(x^2+10)}{(2x+5)^2}$ $=\dfrac{2x^2+10x-20}{(2x+5)^2}$ $f'(x)=0 \Leftrightarrow 2x^2+10x-20=0$ $x^2+5x-10=0$ This is the same equation solved in part c(i) so P and Q must be stationary points.
	Obtains fully correct $f'(x)$ or obtains $2x\frac{dy}{dx}+2y+5\frac{dy}{dx}=2x$ ACF May be unsimplified	1.1b	A1	

	Equates their $f'(x)$ or their numerator of $f'(x)$ to 0 or sets $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	1.1a	M1	
	Rearranges to obtain $x^2 + 5x - 10 = 0$ or $2x^2 + 10x - 20 = 0$ and links it to the equation in part **c(i)** or their answer to **c(ii)** or solves their quadratic $f'(x) = 0$ correctly or deduces $y = x$ and substitutes to get $x = \frac{x^2+10}{2x+5}$ then rearranges to get $x^2 + 5x - 10 = 0$	1.1a	M1	

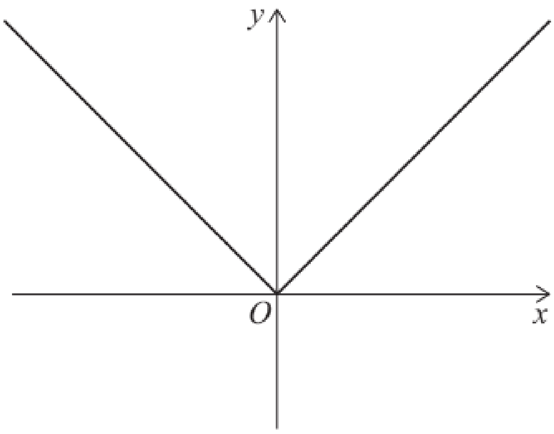
	Completes a reasoned argument by using $x = \frac{-5 \pm \sqrt{65}}{2}$ to conclude that P and Q are stationary points CSO Must have brackets correct throughout	2.1	R1	
	Subtotal		**5**	

Q	Marking instructions	AO	Marks	Typical solution
10(e)	Deduces critical regions from their answer to **c(ii)** condone strict inequalities or poor notation or decimal values	2.2a	M1	$x \le \frac{-5-\sqrt{65}}{2}$ and $x \ge \frac{-5+\sqrt{65}}{2}$ $\left\{x : x \le \frac{-5-\sqrt{65}}{2}\right\} \cup \left\{x : x \ge \frac{-5+\sqrt{65}}{2}\right\}$
	Writes correct range in correct set notation eg $\left(-\infty, \frac{-5-\sqrt{65}}{2}\right] \cup \left[\frac{-5+\sqrt{65}}{2}, \infty\right)$ Accept other letters for x or using $f(x)$ provided consistent throughout set Follow through their answer to **c(ii)**	2.5	A1F	
	Subtotal		**2**	

November 2021 Question 4 Paper 2

4 **Figure 1** shows the graph of $y = |2x|$

Figure 1



4 (a) On **Figure 1** add a sketch of the graph of

$$y = |3x - 6|$$

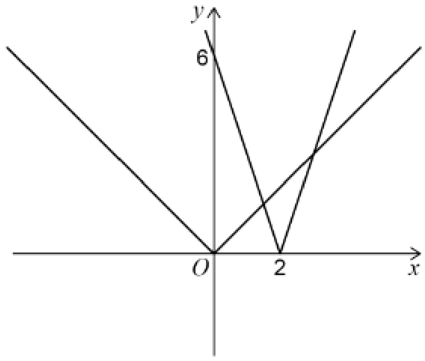
[2 marks]

4 (b) Find the coordinates of the points of intersection of the two graphs.

Fully justify your answer.

[4 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
4(a)	Sketches any V shaped graph with the apex on the positive x axis	1.1a	M1	
	Sketches a roughly symmetrical v-shaped graph touching the positive x-axis and intersecting $y = \|2x\|$ twice in the first quadrant Condone missing or incorrect labels on the axes	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
4(b)	Forms the equation $\|3x - 6\| = \|2x\|$ and selects an appropriate method to begin removing modulus signs For example Squares both sides to obtain $9x^2 - 36x + 36 = 4x^2$ or Considers $3x - 6 = 2x$ or $3x - 6 = -2x$	3.1a	M1	$\|3x - 6\| = \|2x\|$ $3x - 6 = 2x$ $x = 6$ $-3x + 6 = 2x$ $x = 1.2$ when $x = 6$ $y = 12$ when $x = 1.2$ $y = 2.4$
	Obtains $x = 6$	1.1b	A1	
	Obtains $x = 1.2$ OE	1.1b	A1	
	Obtains $y = 12$ and $y = 2.4$	1.1b	A1	
	Subtotal		**4**	

November 2021 Question 10 Paper 2

10 The function h is defined by

$$h(x) = \frac{\sqrt{x}}{x - 3}$$

where h has its maximum possible domain.

10 (a) Find the domain of h.

Give your answer using set notation.

[3 marks]

10 (b) Alice correctly calculates

$$h(1) = -0.5 \quad \text{and} \quad h(4) = 2$$

She then argues that since there is a change of sign there must be a value of x in the interval $1 < x < 4$ that gives $h(x) = 0$

Explain the error in Alice's argument.

[2 marks]

10 (c) By considering any turning points of h, determine whether h has an inverse function.

Fully justify your answer.

[6 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
10(a)	Obtains a domain excluding negatives or excluding 3 Condone $x > 0$	1.1a	M1	$\{x: x \geq 0, x \neq 3\}$
	Deduces both $x \geq 0$ and $x \neq 3$ with no extras Condone $x > 0$	2.2a	A1	
	Obtains correct domain correctly stated in set notation	2.5	R1	
	Subtotal		**3**	

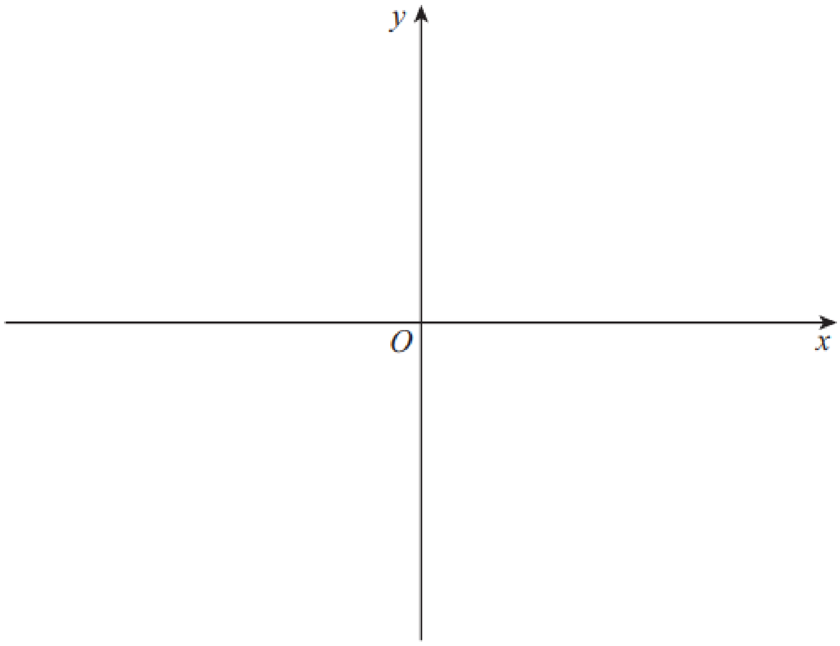
Q	Marking instructions	AO	Marks	Typical solution
10(b)	States that $h(x)$ has a discontinuity/asymptote at x = 3 or in the interval (1 , 4) OE	2.4	M1	$h(x)$ is not continuous at x = 3
	Explains that the discontinuity is at x = 3 and this is in the interval $(1, 4)$	2.3	A1	This means that a change of sign between x = 1 and 4 does not imply a root
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(c)	Selects an appropriate method to differentiate and reaches $h'(x)$ of the form $ax^{-\frac{1}{2}}(x-3)^{-1} + bx^{\frac{1}{2}}(x-3)^{-2}$ OE	3.1a	M1	$h(x) = \dfrac{\sqrt{x}}{x-3}$ $h'(x) = \dfrac{\frac{1}{2}x^{-\frac{1}{2}}(x-3) - x^{\frac{1}{2}}}{(x-3)^2}$
	Obtains correct $h'(x)$ ACF	1.1b	A1	
	Equates their $h'(x)$ to 0	1.1a	M1	$h'(x) = 0 \Rightarrow \dfrac{\frac{1}{2}x^{-\frac{1}{2}}(x-3) - x^{\frac{1}{2}}}{(x-3)^2} = 0$
	Obtains $x = -3$	1.1b	A1	$\frac{1}{2}x^{-\frac{1}{2}}(x-3) - x^{\frac{1}{2}} = 0$
	Explains that a continuous function with no turning points is one to one and therefore the inverse exists. Condone omission of 'continuous'	2.4	E1	$\dfrac{(x-3)}{2\sqrt{x}} - \sqrt{x} = 0$ $x - 3 - 2x = 0$
	Completes a reasoned argument to correctly show that the function is one to one and deduces that $h(x)$ has an inverse Must explain that $x = -3$ is not in the domain and therefore there are no turning points and considers the sign of $h(x)$ either side of x = 3	2.1	R1	$x = -3$ $x = -3$ is not in the domain of h hence the function has no turning points $h(x) > 0$ for $x > 3$ $h(x) < 0$ for $x < 3$ Hence function is one to one and has an inverse
	Subtotal		**6**	

November 2020 Question 4 Paper 1

4 (a) Sketch the graph of

$$y = 4 - |2x - 6|$$



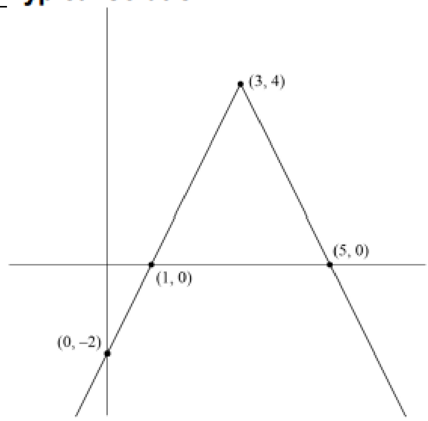
[3 marks]

4 (b) Solve the inequality

$$4 - |2x - 6| > 2$$

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
4(a)	Sketches an inverted V shape graph Condone lack of symmetry	1.1a	M1	
	Sketches an inverted V shape in the correct quadrants Condone lack of symmetry or absence of curve to the left of (0 , - 2)	1.1b	A1	
	Correctly labels all three intersections with coordinate axis. Accept the coordinates of each point or x values on x axis and y value on y axis Ignore any other values	1.1b	A1	
	Total		**3**	
4(b)	Obtains at least one correct critical value using a correct method. Can be read off graph or calculator Condone use of equals or incorrect inequality sign	1.1a	M1	$2 < x < 4$
	Writes correct solution in a correct form Accept $x > 2$, $x < 4$ or (2 , 4)	1.1b	A1	
	Subtotal		**2**	
	Question Total		**5**	

November 2020 Question 13 Paper 1

13 The function f is defined by

$$\mathrm{f}(x) = \frac{2x+3}{x-2} \qquad x \in \mathbb{R},\ x \neq 2$$

13 (a) (i) Find f^{-1}

[3 marks]

13 (a) (ii) Write down an expression for $\mathrm{ff}(x)$.

[1 mark]

13 (b) The function g is defined by

$$\mathrm{g}(x) = \frac{2x^2 - 5x}{2} \qquad x \in \mathbb{R},\ 0 \leq x \leq 4$$

13 (b) (i) Find the range of g.

[3 marks]

13 (b) (ii) Determine whether g has an inverse.

Fully justify your answer.

[2 marks]

13 (c) Show that

$$\mathrm{gf}(x) = \frac{48 + 29x - 2x^2}{2x^2 - 8x + 8}$$

[4 marks]

13 (d) It can be shown that fg is given by

$$\mathrm{fg}(x) = \frac{4x^2 - 10x + 6}{2x^2 - 5x - 4}$$

with domain $\{x \in \mathbb{R} : 0 \leq x \leq 4,\ x \neq a\}$

Find the value of a.

Fully justify your answer.

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
13(a)(i)	Rearranges to make x the subject by isolating x terms or Swaps x and y and isolates y terms	1.1a	M1	$y = \frac{2x+3}{x-2}$ $xy - 2y = 2x + 3$ $xy - 2x = 2y + 3$ $x(y-2) = 2y + 3$ $x = \frac{2y+3}{y-2}$ $f^{-1}(x) = \frac{2x+3}{x-2}\ x \neq 2$
	Obtains correct rearrangement and factorises ACF PI by final correct answer	1.1b	A1	
	Obtains $f^{-1}(x)$ and states domain Must use fully correct notation	2.5	R1	
	Subtotal		**3**	
13(a)(ii)	Obtains any valid expression in x for $ff(x)$ Can be left unsimplified ISW	1.1b	B1	$ff(x) = x$
	Subtotal		**1**	
13(b)(i)	Deduces the greatest value of g by evaluating g (4)	2.2a	B1	$g(4)$ = 6 Vertex at (1.25 , -1.5625) $\{y : -1.5625 \le y \le 6\}$
	Obtains the minimum value of g	3.1a	B1	
	States the range using their finite greatest value and finite minimum value using set notation or interval notation Accept $[-1.5625, 6]$ in interval notation For set notation - use of none curly brackets or commas scores R0	2.5	R1F	
	Subtotal		**3**	
13(b)(ii)	Demonstrates that g is a many to one function by using an appropriate method eg Sketches the function Or Evaluates $g(x)$ at two points that give the same answer.	2.4	E1	$g(0) = 0 = g(2.5)$ g is many to one so it does not have an inverse.
	Deduces that g is many to one and states that g has no inverse Or Explains that g is not one to one and states that g has no inverse	2.2a	E1	

13(c)	Substitutes $f(x)$ into $g(x)$ correctly	1.1a	M1	$gf(x) = \dfrac{2\left(\frac{2x+3}{x-2}\right)^2 - 5\left(\frac{2x+3}{x-2}\right)}{2}$ $= \dfrac{2(2x+3)^2 - 5(2x+3)(x-2)}{2(x-2)^2}$ $= \dfrac{2(4x^2+12x+9) - 5(2x^2-x-6)}{2(x^2-4x+4)}$ $= \dfrac{48 + 29x - 2x^2}{2x^2 - 8x + 8}$
	Obtains common denominator of $2(x-2)^2$ or $(x-2)^2$ correctly The fraction(s) must have the fully correct structure	1.1b	A1	
	Expands at least two quadratics correctly	1.1a	M1	
	Completes rigorous argument to show the required result Must have expanded all three quadratics correctly Terms in the numerator and denominator can be in any order AG	2.1	R1	
	Subtotal		**4**	

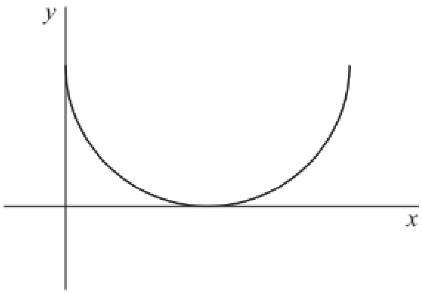
13(d)	States $g(x) = 2$ or States $2x^2 - 5x - 4 = 0$ PI by solving correct quadratic PI by sight of $\frac{5+\sqrt{57}}{4}$ or $\frac{5-\sqrt{57}}{4}$	3.1a	M1	$2x^2 - 5x - 4 = 0$ $x = \dfrac{5 \pm \sqrt{57}}{4}$ $a > 0$ since $0 \leq x \leq 4$ $a = \dfrac{5+\sqrt{57}}{4}$
	Determines the exact value of a giving a clear reason for the rejection of the negative root	2.4	R1	
	Subtotal		**2**	
	Question Total		**15**	

November 2020 Question 3 Paper 3

3 Determine which one of these graphs does **not** represent y as a function of x.

Tick (✓) **one** box.

[1 mark]



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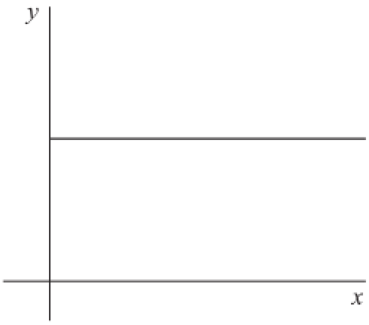

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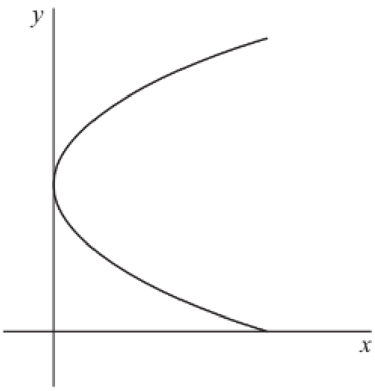

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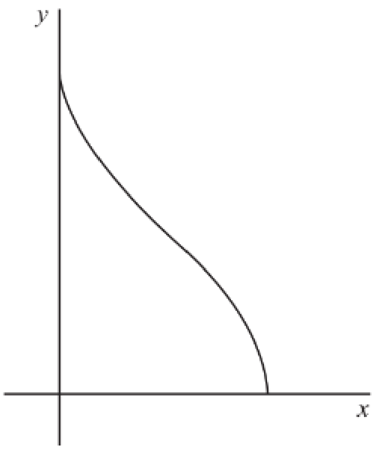

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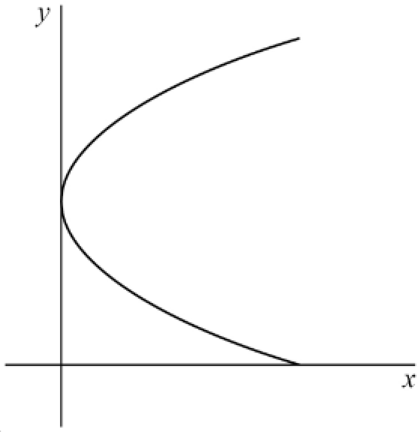
ANSWER

Q	Marking instructions	AO	Marks	Typical solution
3	Ticks correct box	1.2	B1	
	Total		1	

June 2019 Question 6 Paper 1

6 The function f is defined by

$$\mathrm{f}(x) = \frac{1}{2}(x^2 + 1),\ x \geq 0$$

6 (a) Find the range of f.

[1 mark]

6 (b) (i) Find $\mathrm{f}^{-1}(x)$

[3 marks]

6 (b) (ii) State the range of $\mathrm{f}^{-1}(x)$

[1 mark]

6 (c) State the transformation which maps the graph of $y = \mathrm{f}(x)$ onto the graph of $y = \mathrm{f}^{-1}(x)$

[1 mark]

6 (d) Find the coordinates of the point of intersection of the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$

[2 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)	Deduces the range of f Accept $\mathrm{f}(x) \geq \frac{1}{2}$, $y \geq \frac{1}{2}$ *or* $[0.5, \infty)$ OE	AO2.2a	**B1**	$\{y : y \geq \frac{1}{2}\}$
6(b)(i)	Rearranges formula, isolating squared term with at least one correct step seen.	1.1a	**M1**	$y = \frac{1}{2}(x^2 + 1)$ $2y = x^2 + 1$ $2y - 1 = x^2$ $x = \sqrt{2y - 1}$ $\mathrm{f}^{-1}(x) = \sqrt{2x - 1}$ $x \geq \frac{1}{2}$
	Obtains inverse function in any correct form.	1.1b	**A1**	
	Obtains correct inverse function using $\mathrm{f}^{-1}(x) = \ldots$ and states correct domain	2.5	**A1**	
6 (b)(ii)	States correct range Accept $\mathrm{f}^{-1}(x) \geq 0$ OE	1.1b	**B1**	$\{y : y \geq 0\}$
6(c)	Recalls correct transformation	1.2	**B1**	Reflection in $y = x$
6(d)	Forms equation using two of the three expressions $x = \frac{x^2+1}{2} = \sqrt{2x-1}$ allow their $\sqrt{2x-1}$ PI by correct answer	3.1a	**M1**	$x = \frac{x^2+1}{2}$ (1, 1)
	Obtains x=1 and y=1 CSO	1.1b	**A1**	
	Total		**8**	

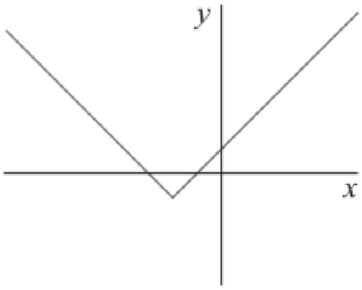
June 2019 Question 1 Paper 2

1 Identify the graph of $y = 1 - |x + 2|$ from the options below.

Tick (✓) **one** box.

[1 mark]

A



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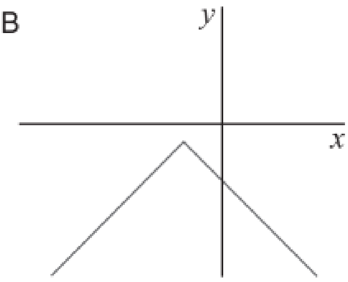

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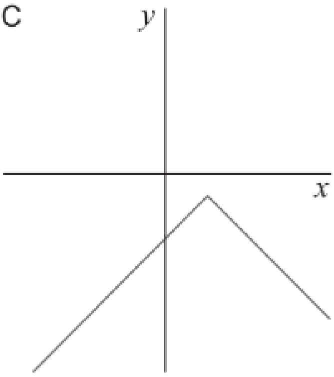

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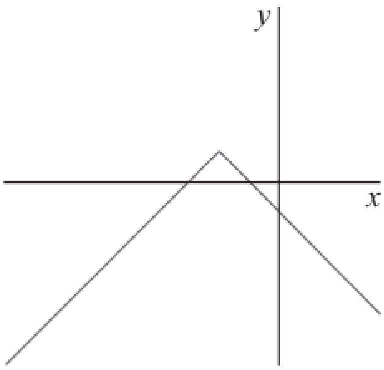

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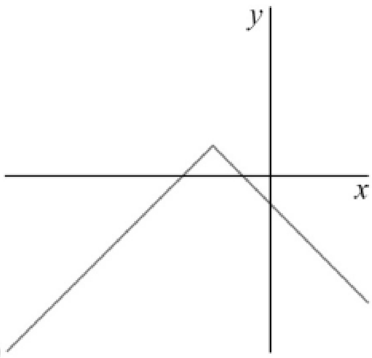
ANSWER

Q	Marking instructions	AO	Mark	Typical solution
1	Ticks the correct response	2.2a	R1	D 
	Total		**1**	

June 2019 Question 3 Paper 2

3 Each of these functions has domain $x \in \mathbb{R}$

Which function does **not** have an inverse?

Circle your answer.

[1 mark]

$\mathrm{f}(x) = x^3$ $\quad\quad$ $\mathrm{f}(x) = 2x + 1$ $\quad\quad$ $\mathrm{f}(x) = x^2$ $\quad\quad$ $\mathrm{f}(x) = \mathrm{e}^x$

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
3	Circles the correct response	1.2	B1	$f(x) = x^2$
	Total		**1**	

June 2018 Question 4 Paper 1

4 The function f is defined by $\mathrm{f}(x) = \mathrm{e}^{x-4}$, $x \in \mathbb{R}$

Find $\mathrm{f}^{-1}(x)$ and state its domain.

[3 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
4	Takes logs of an equation. Must be correct use of logs.	AO1.1a	M1	$y = e^{x-4}$ $\ln y = x - 4$ $4 + \ln y = x$ $\mathrm{f}^{-1}(x) = 4 + \ln x, x > 0$
	Obtains correct inverse function in any correct form	AO1.1b	A1	
	Deduces correct domain	AO2.2a	B1	
	Total		**3**	

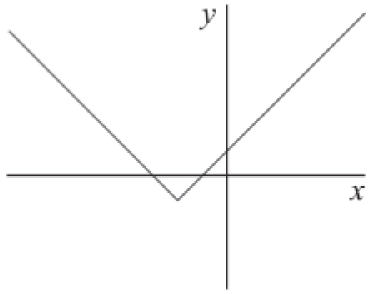
June 2018 Question 1 Paper 2

1 Identify the graph of $y = 1 - |x + 2|$ from the options below.

Tick (✓) **one** box.

[1 mark]

A



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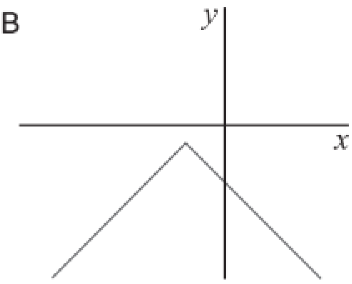
B



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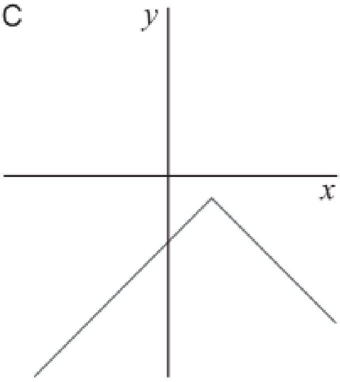
C



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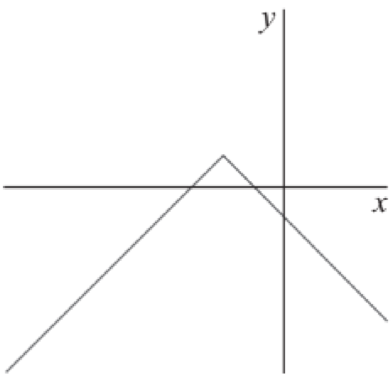
D



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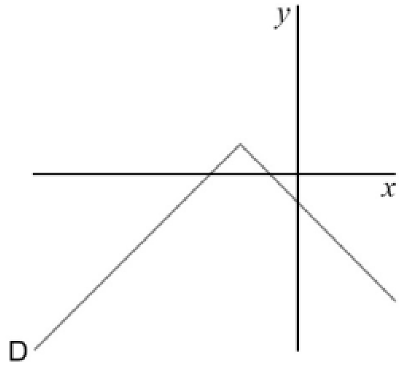
ANSWER

Q	Marking instructions	AO	Mark	Typical solution
1	Ticks the correct response	2.2a	R1	
	Total		**1**	

June 2018 Question 3 Paper 2

3 Each of these functions has domain $x \in \mathbb{R}$

Which function does **not** have an inverse?

Circle your answer.

[1 mark]

$f(x) = x^3$ $\quad$ $f(x) = 2x + 1$ $\quad$ $f(x) = x^2$ $\quad$ $f(x) = e^x$

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
3	Circles the correct response	1.2	B1	$f(x) = x^2$
	Total		**1**	