# Statistics Year 1 exam questions (AS maths) - AQA

**NOTE**:
*Please be aware that in the Year 12 collections, you will find questions from the Year 13 papers. However, these questions are intentionally included because they align with Year 12 content and topics.*
*Happy studying!*

# Ch 18 Hypothesis testing with binomial distribution

## June 2022 Question 19 Paper 3 (A-Level)

**19** A bank runs a campaign to promote Internet banking accounts to their customers.

Before the campaign, 42% of their customers had an Internet banking account.

One week after the campaign started, 35 customers were surveyed at random and 18 of them were found to have registered for an Internet banking account.

Using a binomial distribution, carry out a hypothesis test at the 10% significance level to investigate the claim that, since the campaign, there has been an increase in the proportion of customers registered for an Internet banking account.

**[6 marks]**

**ANSWER**

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **19** | States both hypotheses correctly for one-tailed test<br>0.42 OE | 2.5 | B1 | $X$ = number of customers with an internet banking account<br><br>$H_0$: $p = 0.42$<br>$H_1$: $p > 0.42$<br><br>Under null hypothesis<br>$X \sim B(35, 0.42)$<br><br>$P(X \geq 18) = 0.169$<br><br>$0.169 > 0.10$<br><br>Do not reject $H_0$<br><br>There is insufficient evidence to suggest an increase in the proportion of bank customers registered for an internet banking account. |
| | States or uses correct model<br>PI by calculation of one of the probabilities below<br>$P(X \leq 16) = [0.73, 0.733]$<br>$P(X \leq 17) = [0.83, 0.832]$<br>$P(X \leq 18) = [0.90, 0.903]$<br>$P(X \geq 18) = [0.168, 0.17]$<br>$P(X \geq 19) = [0.097, 0.10]$<br>or critical value of 19 or region $\geq 19$ | 3.3 | M1 | |
| | Obtains $P(X \geq 18)$ =[0.168, 0.17]<br>or<br>Obtains a critical value 19 or critical region $\geq 19$ | 1.1b | A1 | |
| | Evaluates binomial model by comparing their $P(X \geq 18)$ with 0.10<br>or<br>Evaluates binomial model by comparing their critical region with 18 | 3.5a | M1 | |
| | Infers $H_0$ or null hypothesis not rejected<br>Condone $H_0$ accepted<br>or $H_1$ or alternative hypothesis rejected<br>All figures must be correct | 2.2b | A1 | |
| | Concludes correctly in context that there is **insufficient evidence** to **suggest** an **increase** in the **proportion** of bank **customers** registered for an internet banking account<br><br>To be awarded R1, marks M1A1M1A1 must be scored as the minimum | 3.2a | R1 | |
| | **Question 19 Total** | | **6** | |

## November 2021 Question 18 Paper 2

**18** It is known from previous data that 14% of the visitors to a particular cookery website are under 30 years of age.

To encourage more visitors under 30 years of age a large advertising campaign took place to target this age group.

To test whether the campaign was effective, a sample of 60 visitors to the website was selected. It was found that 15 of the visitors were under 30 years of age.

**18 (a)** Explain why a one-tailed hypothesis test should be used to decide whether the sample provides evidence that the campaign was effective.

**[1 mark]**

**18 (b)** Carry out the hypothesis test at the 5% level of significance to investigate whether the sample provides evidence that the proportion of visitors under 30 years of age has increased.

**[5 marks]**

**18 (c)** State one necessary assumption about the sample for the distribution used in part **(b)** to be valid.

**[1 mark]**

## ANSWER

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **18(a)** | Explains why this test is a one-tailed test. Must be in context. Condone explanation using 'number of' linked to increase | 3.2a | E1 | If the campaign is effective, then the proportion of under 30s visitors will be greater than 14%. So, a one-tailed test is required. |
| | **Subtotal** | | **1** | |
| **18(b)** | States both hypotheses correctly for a one-tailed test. Accept population proportion for p. Accept 14%, but not $x =$ or $\bar{x} =$ or $\mu =$ | 2.5 | B1 | $X$ is 'No of under 30's visitors to the website'<br><br>$H_0$: p = 0.14<br>$H_1$: p > 0.14 |
| | States model used (PI by 0.016(5), 0.0071(5), 0.035, 0.0029, 0.0093) (AWRT) | 1.1a | M1 | Under $H_0$: $X \sim B(60, 0.14)$<br><br>$P(X \geq 15) = 1 - P(X \leq 14)$<br>$= 1 - 0.98351 \ldots$<br>$= 0.01649$ |
| | Evaluates using calculator = 0.016(5) (AWRT) (condone 0.0071(5) for A1) | 1.1b | A1 | $= 0.0165$<br><br>As $0.0165 < 0.05$ |
| | Compares 0.016(5) to 0.05 and rejects $H_0$. (PI)(CSO)<br>No ft here. Must see clear comparison (inequality or diagram) | 3.5a | A1 | Reject $H_0$<br><br>There is sufficient evidence to suggest that the advertising campaign has been effective. |
| | Concludes correctly in context CSO 'sufficient evidence' OE required. Only award for full complete correct solution. | 3.2a | R1 | |
| | **Subtotal** | | **5** | |
| **18(c)** | Recalls that the sample would need to be Random. Accept 'not biased' OE | 1.2 | E1 | The sample would need to be a Random sample |
| | **Subtotal** | | **1** | |

## November 2021 Question 17 Paper 3 (A-Level)

**17** James is playing a mathematical game on his computer.

The probability that he wins is 0.6

As part of an online tournament, James plays the game 10 times.

Let $Y$ be the number of games that James wins.

**17 (a)** State two assumptions, in context, for $Y$ to be modelled as $B(10, 0.6)$

**[2 marks]**

**17 (b)** Find $\mathrm{P}(Y = 4)$

**[1 mark]**

**17 (c)** Find $\mathrm{P}(Y \geq 4)$

**[2 marks]**

**17 (d)** After practising the game, James claims that he has increased his probability of winning the game.

In a random sample of 15 subsequent games, he wins 12 of them.

Test a 5% significance level whether James's claim is correct.

**[6 marks]**

## ANSWER

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| **17(a)** | States one correct binomial assumption in context | 3.5b | E1 | The event of James winning one game is independent of him winning another game<br><br>The probability of James winning remains constant at 0.6 from game to game |
| | States a second correct binomial assumption in context eg each time he plays he can only win or not win or the number games is fixed or winning one game is independent of him winning another game<br><br>Condone omission of 0.6 from the statement | 3.5b | E1 | |
| | **Subtotal** | | **2** | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| **17(b)** | Obtains correct probability **AWRT** 0.11 | 3.1b | B1 | $0.111$ |
| | **Subtotal** | | **1** | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| 17(c) | Calculates either $P(Y \le 3) = 0.05476$ or $P(Y \le 4) = 0.16623$ using the Binomial distribution<br>or<br>states $P(Y \ge 4) = 1 - P(Y \le 3)$<br>or<br>subtracts their stated value of $P(Y \le 3)$ from 1 | 3.1b | M1 | $P(Y \le 3) = 0.05476$<br>$P(Y \ge 4) = 1 - P(Y \le 3)$<br>$= 1 - 0.05476$<br>$= 0.94524$ |
| | Obtains correct probability **AWFW** [0.94, 0.95] | 1.1b | A1 | |
| | **Subtotal** | | **2** | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| 17(d) | States both hypotheses correctly for a one-tailed test | 2.5 | B1 | $X$ = number of games won<br>$H_0$: $p = 0.6$<br>$H_1$: $p > 0.6$<br>X ~B(15,0.6)<br><br>P(X ≥ 12) = 1 – P($X \le 11$)<br>= 1 – 0.9094<br>= 0.0905<br><br>0.0905 > 0.05 so accept $H_0$<br><br>There is insufficient evidence to suggest that the probability of James winning the game has increased. |
| | Uses correct binomial model to obtain either P($X \le 11$) or $P(X \le 12)$ or $P(X \ge 13)$ or $P(X \ge 14)$<br>**PI** by critical region<br>$X \ge 13$ or $X \ge 14$ | 3.3 | M1 | |
| | Obtains the correct probability for $P(X \ge 12)$ or<br>obtains correct critical region $X \ge 13$ | 1.1b | A1 | |
| | Evaluates binomial model by comparing their $P(X \ge 12)$ with 0.05 or<br>Compares 12 with their critical region and makes their inference | 3.5a | M1 | |
| | Infer $H_0$ is not rejected<br>**CSO**<br>Allow reference to $H_1$ | 2.2b | A1 | |
| | Concludes correctly in context that there is **insufficient evidence** to suggest that the **probability** of winning the **game** has **increased.** | 3.2a | R1 | |
| | **Subtotal** | | **6** | |

## November 2020 Question 19 Paper 2

**19** It is known from historical data that 15% of the residents of a town buy the local weekly newspaper, 'Local News'.

A new free weekly paper is introduced into the town.

The owners of 'Local News' are interested to know whether the introduction of the free newspaper has changed the proportion of residents who buy their paper.

In a random sample of 50 residents of the town taken after the free newspaper was introduced, it was found that 3 of them purchased 'Local News' regularly.

Investigate, at the 5% significance level, whether this sample provides evidence that the proportion of local residents who buy 'Local News' has changed.

**[6 marks]**

**ANSWER**

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| 19 | States both hypotheses correctly for a two-tailed test. Accept population proportion for $p$. Accept 15%, but not '$x$ =' or '$\overline{x}$ =' or 'μ =' | 2.5 | B1 | $X$ is 'Number of residents who buy Local News'<br><br>$H_0$: $p$ = 0.15<br>$H_1$: $p \neq$ 0.15 |
| | States model used (PI by 0.046, 0.032, 0.014, 0.112) (AWRT) | 1.1a | M1 | Under $H_0$: $X \sim$ B(50 , 0.15) |
| | Evaluates P($X \leq$ 3) = 0.046 (PI) (condone 0.014 for A1) | 1.1b | A1 | P($X \leq$ 3) = 0.04604657<br><br>As 0.046 > 0.025 |
| | Compares 0.046 or 0.014 with 0.025.<br><br>Must see clear comparison (inequality or diagram) | 3.5a | M1 | Accept $H_0$<br><br>There is insufficient evidence to suggest that the proportion of residents buying 'Local News' has changed. |
| | Infers $H_0$ is not rejected (PI)(CSO)(OE) | 2.2b | A1 | |
| | Concludes correctly in context CSO 'insufficient evidence' OE required. Only award for full complete correct solution. | 3.2a | R1 | |
| | **Total** | | **6** | |

## June 2020 Question 18 Paper 3 (A-Level)

**18** Tiana is a quality controller in a clothes factory. She checks for four possible types of defects in shirts.

Of the shirts with defects, the proportion of each type of defect is as shown in the table below.

| **Type of defect** | Colour | Fabric | Sewing | Sizing |
|---|---|---|---|---|
| **Probability** | 0.25 | 0.30 | 0.40 | 0.05 |

Shirts with defects are packed in boxes of 30 at random.

**18 (a)** Find the probability that:

**18 (a) (i)** a box contains exactly 5 shirts with a colour defect

**[2 marks]**

**18 (a) (ii)** a box contains fewer than 15 shirts with a sewing defect

**[2 marks]**

**18 (a) (iii)** a box contains at least 20 shirts which do **not** have a fabric defect.

**[3 marks]**

**18 (b)** Tiana wants to investigate the proportion, $p$, of defective shirts with a fabric defect.

She wishes to test the hypotheses

$$H_0 : p = 0.3$$

$$H_1 : p < 0.3$$

She takes a random sample of 60 shirts with a defect and finds that $x$ of them have a fabric defect.

**18 (b) (i)** Using a 5% level of significance, find the critical region for $x$.

**[5 marks]**

**18 (b) (ii)** In her sample she finds 13 shirts with a fabric defect.

Complete the test stating her conclusion in context.

**[2 marks]**

**ANSWER**

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| **18(a)(i)** | States the binomial distribution with $n$ = 30, $p$ = 0.25<br>or<br>states $P(X = 5) = P(X \leq 5) - P(X \leq 4)$<br>or<br>uses the binomial formula with $\binom{30}{5} 0.25^5 0.75^{25}$<br>**PI** by correct answer | 3.3 | M1 | $X$~B(30, 0.25)<br>$P(X = 5) = 0.1047$ |
| | Obtains correct probability<br>**AWRT** 0.105<br>**ISW** | 1.1b | A1 | |
| **18(a)(ii)** | States the binomial distribution with $n$ = 30, $p$ = 0.40<br>or<br>states $P(X < 15) = P(X \leq 14)$<br><br>**PI** by $P(X \leq 15)$= 0.9029 or by correct answer | 3.3 | M1 | $X$~B(30, 0.40)<br>$P(X < 15) = P(X \leq 14)$<br>=0.8246 |
| | Obtains correct probability<br>**AWRT** 0.825<br>**ISW** | 1.1b | A1 | |

| | | | | |
|---|---|---|---|---|
| **18(a)(iii)** | States the binomial distribution with $n$ = 30, $p$ = 0.70<br>**PI** by $P(X \leq 19)$= 0.2696 or 0.27 of $P(X \geq 21)$= 0.5888 or by correct answer<br>or<br>states the binomial distribution with $n$ = 30, $p$ = 0.30<br>**PI** by $P(Y \geq 10)$ = 0.4112 or 0.41 or $P(Y \geq 11)$ = 0.2696 or 0.27 or by correct answer | 1.1b | B1 | $X \sim$B(30, 0.70)<br>$P(X \geq 20) = 1 - P(X \leq 19)$<br>= 1 – 0.2696<br>= 0.7304 |
| | Uses B(30, 0.7) to calculate $P(X \leq 19)$= 0.2696 or 0.27<br>or<br>states $P(X \geq 20) = 1 - P(X \leq 19)$<br>or<br>uses B(30, 0.3) to calculate $P(Y \geq 10)$ = 0.4111 or 0.41 or $P(Y \geq 11)$ = 0.2696 or 0.27 | 3.4 | M1 | |
| | Obtains correct probability<br>**AWFW** [0.73, 0.7304] | 1.1b | A1 | |

| | | | | |
|---|---|---|---|---|
| **18(b)(i)** | States binomial distribution with $n$ = 60, $p$ = 0.30<br>**PI** by finding values of P($X \le x$) where $x$ = [0, 15] correct to at least 3dp<br>P($X \le x$) = [0, 0.244], $x$ = [0, 15]<br>**OE** | 3.3 | B1 | $X$~B(60, 0.30)<br>P($X \le 11$) = 0.0295 < 0.05<br>P($X \le 12$) = 0.0568 > 0.05<br><br>Critical region is $x \le 11$ |
| | Finds of P($X \le x$) where $x$ = [0, 15] correct to at least 3dp<br>Do not accept P($X = x$)<br>P($X \le x$) = [0, 0.244], $x$ = [0, 15]<br>**OE** | 1.1a | M1 | |
| | Obtains P($X \le 12$) = 0.0568<br>**AWFW** [0.05677, 0.0568]<br>Condone wrong inequality sign if [0.05677, 0.0568] seen | 1.1b | A1 | |
| | Obtains P($X \le 11$) = 0.0295<br>**AWFW** [0.02947, 0.0295]<br>Condone wrong inequality sign if [0.02947, 0.0295] seen | 1.1b | A1 | |
| | Obtains correct critical region $x$<br>Accept critical region of 0≤ $x$ ≤11 or $x$ <12 or<br>$\{0,1,2,3,4,5,6,7,8,9,10,11\}$ | 3.2a | R1 | |

| | | | | |
|---|---|---|---|---|
| **18(b)(ii)** | Compares 13 with their critical region from **b(i)** and makes their correct inference<br>or<br>compares $P(X \le 13)$= 0.09996 (accept **AWRT** 0.1) with 0.05 and infers $H_0$ is not rejected<br>Allow reference to $H_1$ | 2.2b | M1 | 13 > 11 so accept $H_0$<br><br>There is insufficient evidence to suggest that the proportion of shirts with a fabric defect has decreased. |
| | Concludes correctly in context that there is **insufficient evidence** to suggest that the **proportion** of **shirts** with a fabric defect has **decreased.**<br><br>Follow though correct conclusion for their critical region | 3.2a | R1F | |
| | **Total** | | **14** | |

## June 2019 Question 16  Paper 2

**16** Andrea is the manager of a company which makes mobile phone chargers.

In the past, she had found that 12% of all chargers are faulty.

**16 (a)** Andrea decides to move the manufacture of chargers to a different factory.

Andrea tests 60 of the new chargers and finds that 4 chargers are faulty.

Investigate, at the 10% level of significance, whether the proportion of faulty chargers has reduced.

**[7 marks]**

**16 (b)** State, **in context**, two assumptions that are necessary for the distribution that you have used in part **(a)** to be valid.

**[2 marks]**

## ANSWER

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| **16(a)** | States both hypotheses correctly for one-tailed test<br><br>Accept:<br>• Population proportion = 0.12<br>• $p$ = 12%<br>• $H_1: p \leq 0.12$ or 12%<br><br>Do not accept:<br>$x$ = 0.12, $\mu$ = 0.12 or $\bar{x}$ = 0.12 | 2.5 | B1 | $H_0: p = 0.12$<br><br>$H_1: p < 0.12$<br><br>Under null hypothesis,<br>$X \sim B(60\,,0.12)$ |
| | States model used PI<br><br>Can be implied by AWRT 0.06, 0.14, 0.040, 0.079 | 3.3 | M1 | $P(X \leq 4) = 0.139$<br><br>$0.139 > 0.10$ |
| | Calculates $P(X \leq 4)$ or $P(X \leq 3)$<br>$P(X \leq 3) = 0.060$<br><br>Do **not** accept $P(X = 4)$ or $P(X = 3)$ | 1.1a | M1 | Accept $H_0$ |
| | Obtains correct value for $P(X \leq 4)$<br>Accept 0.14 AWRT | 1.1b | A1 | There is insufficient evidence to suggest that the proportion of faulty chargers has reduced |
| | Evaluates Binomial model by comparing 0.139 (accept 0.14) or 0.060 with 0.10<br><br>Do **not** accept use of $P(X = 4)$ or $P(X = 3)$<br><br>Must be a clear comparison in words or inequality or diagram | 3.5a | M1 | |
| | Infers $H_0$ accepted or $H_1$ rejected<br><br>Condone 'do not reject'<br><br>If no hypothesis after comparison assume $H_0$ | 2.2b | A1 | |
| | Concludes correctly in context.<br>'Insufficient evidence' required OE<br><br>Only award for full complete solution | 3.2a | R1 | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| **16(b)** | States a first assumption in context<br><br>Must include 'faulty' if assumption refers to probability or independence | 3.5b | E1 | The probability of a faulty charger is fixed |
| | States a second assumption in context<br><br>Must include 'faulty' if assumption refers to probability or independence<br><br>Also accept:<br>The sample of chargers was randomly selected<br><br>Do **not** accept the number of trials is fixed at 60<br>Do **not** accept the charger is either faulty or not faulty | 3.5b | E1 | A charger being faulty is independent of any other charger being faulty |
| | **Total** | | **9** | |

## June 2018 Question 19 Paper 2

**19** Martin grows cucumbers from seed.

In the past, he has found that 70% of all seeds successfully germinate and grow into cucumber plants.

He decides to try out a new brand of seed.

The producer of this brand claims that these seeds are more likely to successfully germinate than other brands of seeds.

Martin sows 20 of this new brand of seed and 18 successfully germinate.

Carry out a hypothesis test at the 5% level of significance to investigate the producer's claim.

**[7 marks]**

### ANSWER

**Accept percentages throughout this question**

| Q | Marking Instructions | AO | Marks | Typical Solution |
|---|---|---|---|---|
| 19 | States both hypotheses correctly for one-tailed test accept 'population proportion' = 0.7 or 70%, but NOT $x$ = or $\overline{x}$ = or $\mu$ = | AO2.5 | B1 | $X$ = number of seeds which germinate<br>$H_0$:$p$=0.7<br>$H_1$:$p$>0.7 |
| | States model used **PI** can also be implied by 0.0278 or 0.00684 | AO3.3 | M1 | Under null hypothesis $X$~B(20,0.7) |
| | Calculates $P(X \geq 18)$ *or* $P(X \geq 19)$, (PI by 0.035(5) or 0.0076) but not $P(X = 18)$ or $P(X = 19)$ | AO1.1a | M1 | $P(X \geq 18) = 1 - P(X \leq 17)$<br>$= 1 - 0.965$ |
| | Obtains correct probability for $P(X \geq 18)$ | AO1.1b | A1 | $= 0.035$ |
| | Evaluates binomial model by comparing 0.035(5) or 0.0076 with 0.05 but **not** $P(X = 18)$ or $P(X = 19)$ | AO3.5a | M1 | 0.035 < 0.05 |
| | Infers $H_0$ rejected (FT 0.0076), condone 'accept $H_1$' | AO2.2b | A1F | Reject $H_0$ |
| | Concludes correctly in context. 'Sufficient evidence' or equivalent required. Only award for full complete correct solution. | AO3.2a | R1 | There is sufficient evidence to conclude that the new seeds are more likely to germinate. |
| | **Total** | | **7** | |