Statistics Year 2 exam questions - AQA

NOTE:
Please be aware that in the Year 12 collections, you will find questions from the Year 13 papers. However, these questions are intentionally included because they align with Year 12 content and topics. That's the reason why you are missing some questions here, please have a look at the Year 12 collection!
Happy studying!

Ch 17 Statistical hypothesis testing

June 2022 Question 17 Paper 3

17 The number of working hours per week of employees in a company is modelled by a normal distribution with mean of 34 hours and a standard deviation of 4.5 hours.

The manager claims that the mean working hours per week of the company's employees has increased.

A random sample of 30 employees in the company was found to have mean working hours per week of 36.2 hours.

Carry out a hypothesis test at the 2.5% significance level to investigate the manager's claim.

[6 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
17	States both hypotheses correctly for one-tailed test	2.5	B1	X = working hours per week $H_0: \mu = 34$ $H_1: \mu > 34$
	States or uses correct model PI by mean 34 and variance $\frac{4.5^2}{30}$ or standard deviation $\frac{4.5}{\sqrt{30}}$ OE or by correct probability AWFW [0.0037, 0.004] or test statistic $\pm\frac{36.2-34}{4.5\div\sqrt{30}}$ or test statistic value AWFW ±[2.67, 2.68] or critical region > AWFW [35.6, 35.632] condone ≥ or critical value $\pm$[35.6, 35.632]	1.1a	M1	$\bar{X} \sim N(34, \frac{4.5^2}{30})$ $P(\bar{X} > 36.2) = 0.004$ $0.004 < 0.025$ Reject H_0 There is sufficient evidence to suggest that the mean working hours have increased.
	Obtains AWFW [0.0037, 0.004] or obtains the correct value of the test statistic AWFW [2.67, 2.68] or obtains the correct critical region > AWFW [35.6, 35.632] condone ≥ condone incorrect missing or incorrect labels	1.1b	A1	
	Compares their [0.0037, 0.004] with 0.025 or compares their [2.67, 2.68] with 1.96 allow positive values up to 4 for 1.96 or compares their region > [35.6, 35.632] with 36.2 condone ≥	3.5a	M1	

	Infers H_0 or null hypothesis rejected Condone H_1 or alternative hypothesis accepted All figures must be correct	2.2b	A1	
	Concludes correctly in context that there is **sufficient evidence** to **suggest** that the **mean** working **hours** have **increased** To be awarded R1, marks M1A1M1A1 must be scored as the minimum	3.2a	R1	
	Question 17 Total		**6**	

June 2022 Question 18 Paper 3

18 In a particular year, the height of a male athlete at the Summer Olympics has a mean 1.78 metres and standard deviation 0.23 metres.

The heights of 95% of male athletes are between 1.33 metres and 2.22 metres.

18 (a) Comment on whether a normal distribution may be suitable to model the height of a male athlete at the Summer Olympics in this particular year.

[3 marks]

18 (b) You may assume that the height of a male athlete at the Summer Olympics may be modelled by a normal distribution with mean 1.78 metres and standard deviation 0.23 metres.

18 (b) (i) Find the probability that the height of a randomly selected male athlete is 1.82 metres.

[1 mark]

18 (b) (ii) Find the probability that the height of a randomly selected male athlete is between 1.70 metres and 1.90 metres.

[1 mark]

18 (b) (iii) Two male athletes are chosen at random.

Calculate the probability that **both** of their heights are between 1.70 metres and 1.90 metres.

[1 mark]

18 (c) The summarised data for the heights, h metres, of a random sample of 40 male athletes at the Winter Olympics is given below.

$$\sum h = 69.2 \qquad \sum (h - \bar{h})^2 = 2.81$$

Use this data to calculate estimates of the mean and standard deviation of the heights of male athletes at the Winter Olympics.

[3 marks]

18 (d) Using your answers from **part (c)**, compare the heights of male athletes at the Summer Olympics and male athletes at the Winter Olympics.

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
18(a)	Calculates 1.78 ± 2 × 0.23 or 1.78 ± 1.96 × 0.23 or calculates $P(1.33 < x < 2.22)$ PI by 0.9469 or 0.947 or calculates $\frac{1.33-1.78}{0.23}$ and $\frac{2.22-1.78}{0.23}$	3.1b	M1	$1.78 - 2 \times 0.23 = 1.32$ $1.78 + 2 \times 0.23 = 2.24$ $1.32 \approx 1.33$ $2.24 \approx 2.22$ Height is continuous data and 95% of heights lies within two standard deviations of the mean so normal may be a suitable model.
	Obtains 1.32 and 2.24 and states they are approximately 1.33 and 2.22 or obtains 1.33 and 2.23 and states they are approximately 1.33 and 2.22 or obtains 0.9469 or 0.947 and states is approximately 0.95 or obtains -1.96 and 1.91 and states are approximately -2 and 2	2.4	A1	
	Infers that the normal distribution may be suitable because **height** is **continuous** data and **95%** of heights lies within two standard deviations of the mean or Infers that the normal distribution may be suitable because **height** is **continuous** data and **94.69%** or **94.7%** of heights lies between 1.33 and 2.22	2.2b	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
18(b)(i)	States 0	1.2	B1	0
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
18(b)(ii)	Calculates the correct probability AWFW [0.335, 0.34]	1.1b	B1	0.335
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
18(b)(iii)	Finds the value of their answer to **(b)(ii)** squared Their answer must be correct to at least 2sf	3.1b	B1F	$0.335^2 = 0.112$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
18(c)	Obtains 1.73 CAO Ignore missing or incorrect units	1.1b	B1	Mean = 1.73 Standard deviation $= \sqrt{\frac{2.81}{40}}$ $= 0.265$
	Uses the correct formula for standard deviation eg $s = \sqrt{\frac{2.81}{39}}$ Do not allow variance $= \sqrt{\frac{2.81}{40}}$	1.1a	M1	
	Obtains the correct standard deviation AWFW [0.265, 0.27] Allow if not labelled but if labelled, must be correct Ignore missing or incorrect units	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
18(d)	Uses their mean and 1.78 to compare heights Comparison must include on average. Follow through their answer to part **(c)** Do not allow 'general' Allow statement that they're about the same on average	2.2b	E1F	Summer athletes are taller on average than Winter athletes. Summer athletes' heights are less varied than the heights of Winter athletes.
	Uses their standard deviation and 0.23 to compare heights Comparison must include 'varies', 'spread' 'disperse' 'more variation' or 'consistent' Follow through their answer to part **(c)** Allow statement that they are about the same Do not allow comparison that includes 'range' or 'variety'	2.2b	E1F	
	Subtotal		**2**	

November 2021 Question 15 Paper 3

15 A team game involves solving puzzles to escape from a room.

Using data from the past, the mean time to solve the puzzles and escape from one of these rooms is 65 minutes with a standard deviation of 11.3 minutes.

After recent changes to the puzzles in the room, it is claimed that the mean time to solve the puzzles and escape has changed.

To test this claim, a random sample of 100 teams is selected.

The total time to solve the puzzles and escape for the 100 teams is 6780 minutes.

Assuming that the times are normally distributed, test at the 2% level the claim that the mean time has changed.

[7 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
15	States both hypotheses correctly for two-tailed test Accept **population mean** for μ	2.5	B1	X = times to solve in minutes $H_0: \mu = 65$ $H_1: \mu \neq 65$
	Calculates mean of the sample **PI** in equation	1.1b	B1	$\overline{x} = \frac{6780}{100} = 67.8$
	Formulates the test statistic or uses the correct distribution of their sample mean **PI** by correct test statistic value or calculates probability or identifies acceptance region Condone $65 - 67.8$	3.3	M1	$\text{Test statistic} = \frac{67.8 - 65}{11.3/\sqrt{100}}$ $= 2.48$
	Obtains the correct value of the test statistic [2.47, 2.5] or obtains the correct probability [0.0066, 0.007] or [0.0132, 0.014] or obtains the correct acceptance region of [62.3, 67.7]	1.1b	A1	Critical value = 2.33 2.48 > 2.33 Reject H_0 There is sufficient evidence at the 2% level to suggest that mean escape time has changed
	Compares their value of test statistic [2.47, 2.5] with their critical value 2.33 Allow critical value [-4, 4] except ±0.02 or ±0.01 or compares their probability [0.0066, 0.007] with 0.01 or compares their probability [0.0132, 0.014] with 0.02 or compares their sample mean 67.8 with their acceptance region [62.3, 67.7]	1.1b	M1	
	Compares correct values and correctly infers H_0 is rejected **CSO** Allow reference to H_1	2.2b	A1	
	Concludes correctly in context that there is **sufficient evidence** to suggest that the **mean** escape **time** has **changed** **CSO**	3.2a	R1	
	Total		**7**	

November 2020 Question 14 Paper 3

14 It is known that a hospital has a mean waiting time of 4 hours for its Accident and Emergency (A&E) patients.

After some new initiatives were introduced, a random sample of 12 patients from the hospital's A&E Department had the following waiting times, in hours.

4.25	3.90	4.15	3.95	4.20	4.15
5.00	3.85	4.25	4.05	3.80	3.95

Carry out a hypothesis test at the 10% significance level to investigate whether the mean waiting time at this hospital's A&E department has changed.

You may assume that the waiting times are normally distributed with standard deviation 0.8 hours.

[7 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
14	States both hypotheses correctly for two-tailed test Accept **population mean** for μ	2.5	B1	X = waiting times, in hours $H_0: \mu = 4$ $H_1: \mu \neq 4$ $\overline{x} = \frac{49.5}{12} = 4.125$ $\text{Test statistic} = \frac{4.125 - 4}{0.8/\sqrt{12}}$ $= 0.541$ Critical value 1.65 0.541 < 1.65 Accept H_0 There is insufficient evidence to suggest that the mean waiting time at this hospital's A&E department has changed
	Calculates mean of the sample **PI** in equation Accept $\overline{x}$ = [4.12, 4.13]	1.1a	M1	
	Formulates the test statistic or uses the correct distribution of their sample mean **PI** by correct test statistic value or calculates probability or identifies acceptance region Condone $4 - 4.125$	1.1a	M1	
	Obtains the correct value of the test statistic [0.519, 0.563] or obtains the correct probability [0.286,0.302] or [0.57, 0.604] or obtains the correct acceptance region of [3.62, 4.38]	1.1b	A1	
	Compares their value of test statistic [0.519, 0.563] with their critical value [1.64, 1.65] Allow critical value [-4, 4] except ±0.1 or ±0.05 or compares their probability [0.286,0.302] with 0.05 or compares their probability [0.57, 0.604] with 0.10 or compares their sample mean [4.12, 4.13] with their acceptance region [3.62, 4.38] Do not allow negative region	1.1b	B1F	
	Compares values correctly and infers H_0 is not rejected **CSO** Allow reference to H_1	2.2b	A1	
	Concludes correctly in context that there is **insufficient evidence** to suggest that the **mean** waiting **time** at this hospital's A&E department has **changed**	3.2a	R1	

November 2020 Question 16 Paper 3

16 An educational expert found that the correlation coefficient between the hours of revision and the scores achieved by 25 students in their A-level exams was 0.379

Her data came from a bivariate normal distribution.

Carry out a hypothesis test at the 1% significance level to determine if there is a positive correlation between the hours of revision and the scores achieved by students in their A-level exams.

The critical value of the correlation coefficient is 0.4622

[4 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
16	States both hypotheses correctly for one-tailed test Do not accept $r = 0$ and $r > 0$ Accept ρ in words eg. H_0:No correlation/coefficient is 0 H_1:Positive correlation/coefficient > 0	2.5	B1	H_0: $\rho = 0$ H_1: $\rho > 0$ $0.379 < 0.4622$ Accept H_0 There is insufficient evidence to suggest that there is a positive correlation between the hours of revision and the scores achieved by them in their A level exams
	Compares given critical value 0.4622 with the given correlation 0.379 coefficient Condone 0.462 or 0.46 or 0.5 for 0.4622 Condone 0.38 or 0.4 for 0.379	3.5a	M1	
	Compares given critical value 0.4622 with the given correlation 0.379 coefficient correctly and infers H_0 is not rejected Allow reference to H_1	2.2b	A1	
	Concludes correctly in context that there is **insufficient evidence** to suggest that there is **positive correlation** between the **hours** of revision and the **scores** achieved by them in their A level exams	3.2a	R1	
	Total		**4**	

June 2019 Question 15 Paper 3

15 Jamal, a farmer, claims that the larger the rainfall, the greater the yield of wheat from his farm.

He decides to investigate his claim, at the 5% level of significance.

He measures the rainfall in centimetres and the yield in kilograms for a random sample of ten years.

He correctly calculates the product moment correlation coefficient between rainfall and yield for his sample to be 0.567

The table below shows the critical values for correlation coefficients for a sample size of 10 for different significance levels, for both 1- and 2-tailed tests.

1-tailed test significance level	5%	2.5%	1%	0.5%
2-tailed test significance level	10%	5%	2%	1%
Critical value	0.549	0.632	0.716	0.765

Determine what Jamal's conclusion to his investigation should be, justifying your answer.

[3 marks]

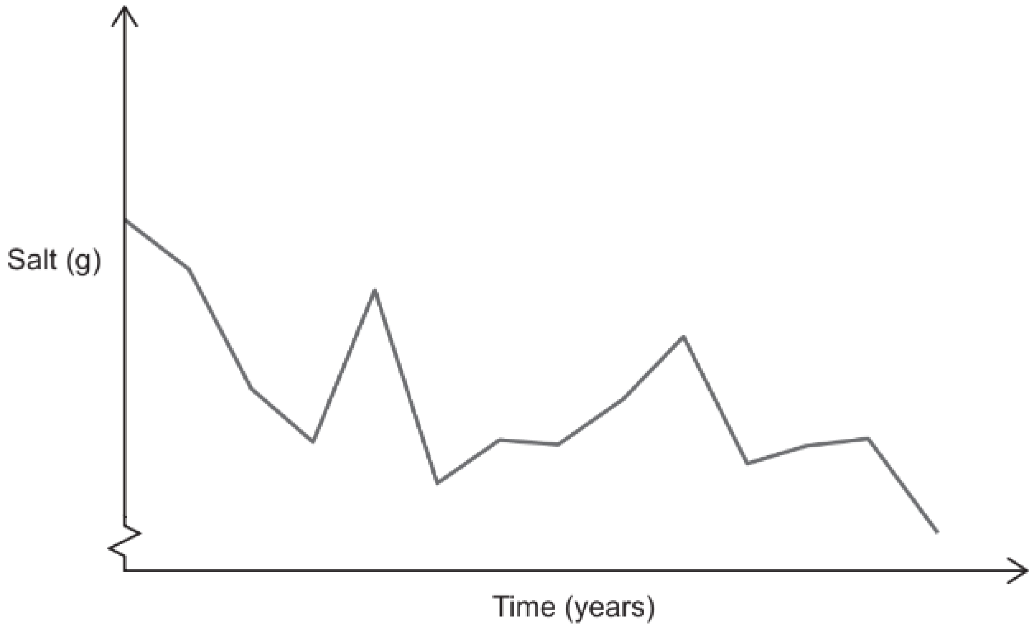
ANSWER

Q	**Marking Instructions**	**AO**	**Mark**	**Typical Solution**
15	Identifies critical value = 0.549	1.1b	B1	$0.567 > 0.549$ There is sufficient evidence that the larger the rainfall, the greater the yield.
	Compares 0.567 correctly to their critical value chosen from the table	3.5a	M1	
	Makes correct inference eg there is sufficient/significant evidence that the larger the rainfall, the greater the yield/positive correlation **between** the two **FT** their critical value chosen from the table	2.2b	R1F	
	Total		**3**	

June 2018 Question 16 Paper 3

* The Large Data Set they mention in this question is the previous one. So you don't need to know about the Family food data set anymore.

16 (a) The graph below shows the amount of salt, in grams, purchased per person per week in England between 2001–02 and 2014, based upon the Large Data Set.



Meera and Gemma are arguing about what this graph shows.

Meera believes that the amount of salt consumed by people decreased greatly during this period.

Gemma says that this is not the case.

Using your knowledge of the Large Data Set, give **two** reasons why Gemma may be correct.

[2 marks]

16 (b) It is known that the mean amount of sugar purchased per person in England in 2014 was 78.9 grams, with a standard deviation of 25.0 grams.

In 2018, a sample of 918 people had a mean of 80.4 grams of sugar purchased per person.

Investigate, at the 5% level of significance, whether the mean amount of sugar purchased per person in England has changed between 2014 and 2018.

Assume that the survey data is a random sample taken from a normal distribution and that the standard deviation has remained the same.

[6 marks]

16 (c) Another test is performed to determine whether the mean amount of fat purchased per person has changed between 2014 and 2018.

At the 10% significance level, the null hypothesis is rejected.

With reference to the 10% significance level, explain why it is not necessarily true that there has been a change.

[2 marks]

ANSWER

Q	Marking Instructions	AO	Mark	Typical Solution
16(a)	States correct first reason involving the y-axis Accept there is no scale on the *y*-axis **or** graph does not start at 0	2.3	E1	Scale on y-axis does not start at zero.
	States correct second reason involving salt purchased and consumed implying data not comparable	2.3	E1	Data is for salt purchased as separate food stuff, not consumed
16(b)	States both hypotheses correctly for two-tailed test Accept H_0: **population** mean is 78.9	2.5	B1	$H_0: \mu = 78.9$ $H_1: \mu \neq 78.9$
	Formulates the test statistic or uses the correct distribution of the sample mean **PI** by correct test statistic value or probability or acceptance region Condone $78.9 - 80.4$ If region used, condone any z = (- 4, 4)	1.1a	M1	$\text{Test statistic} = \dfrac{80.4 - 78.9}{25.0/\sqrt{918}}$ $= 1.82$ Critical z value 1.96 1.82 < 1.96
	Obtains the correct value of the test statistic 1.82 or obtains the correct probability 0.0345 or 0.0691 obtains acceptance region of [77.3, 80.5]	1.1b	A1	Accept H_0 - there is insufficient evidence to suggest that the mean amount of sugar purchased has changed

	Compares their 1.82 with 1.96 or compares their 0.0345 with 0.025 compares their 0.0691 with 0.05 or compares 80.4 with their region [77.3, 80.5]	1.1a	M1	
	Infers H_0 accepted **CSO** Must refer to H_0	2.2b	A1	
	Correctly concludes in context that there is **insufficient evidence** to suggest that the **mean** amount of **sugar purchased** has **changed** **CSO**	3.2a	E1	
16(c)	Explains role of significance level in rejecting null hypothesis in error Accept Type I error	2.3	E1	There is a 10% chance of rejecting null hypothesis in error
	Explains that there is 10 % chance for this to occur Reference to 10 % chance the conclusion is incorrect scores E0E1	2.3	E1	
	Total		**10**	

June 2018 Question 17 Paper 3

17 Suzanne is a member of a sports club.

For each sport she competes in, she wins half of the matches.

17 (a) After buying a new tennis racket Suzanne plays 10 matches and wins 7 of them.

Investigate, at the 10% level of significance, whether Suzanne's new racket has made a difference to the probability of her winning a match.

[7 marks]

17 (b) After buying a new squash racket, Suzanne plays 20 matches. Find the minimum number of matches she must win for her to conclude, at the 10% level of significance, that the new racket has improved her performance.

[5 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
17 (a)	States both hypotheses correctly for two-tailed test	AO2.5	B1	X = number of matches won
	States model used **PI**	AO3.3	M1	H_0: p = 0.5 H_1: $p \neq 0.5$
	Calculates $P(X \le 6)$ or $P(X \le 7)$ 0.828(1) or 0.945(3)	AO1.1a	M1	Under null hypothesis X~B(10,0.5)
	Obtains the correct probability for $P(X \ge 7)$	AO1.1b	A1	$P(X \ge 7) = 1 - P(X \le 6)$ $= 1 - 0.8281$ $= 0.172$
	Evaluates Binomial model by comparing $P(X \ge 7)$ with 0.05	AO3.5a	M1	
	Infers H_0 accepted CSO	AO2.2b	A1	$0.172 > 0.05$
	Concludes correctly in context. (FT only available if previous M1 mark scored and B1 scored)	AO3.2a	E1F	Accept H_0 There is not sufficient evidence that Suzanne's new racket has made a difference

17(a)	**(ALTERNATIVE using critical region)** States both hypotheses correctly for two-tailed test	AO2.5	B1	X = number of matches won
	States model used **PI**	AO3.3	M1	H_0: p=0.5 H_1: p≠0.5
	Considers critical region	AO1.1a	M1	
	Identifies critical region	AO1.1b	A1	Under null hypothesis X~B(10,0.5)
	Evaluates Binomial model by comparing $X = 7$ with critical values	AO3.5a	M1	$P(X \le 1) = 0.0107$ *or* 0.0108
	Infers H_0 accepted CSO	AO2.2b	A1	$P(X \ge 9) = 0.0107$ *or* 0.0108
	Correctly concludes in context. 'Not sufficient evidence' or equivalent required.	AO3.2a	E1F	Critical region is $X \le 1$ and $X \ge 9$ $X = 7$ not in critical region Accept H_0 There is not sufficient evidence that Suzanne's new racket has made a difference

Q	Marking Instructions	AO	Marks	Typical Solution
17(b)	States model used **PI**	AO3.3	M1	Y~B(20,0.5)
	Expresses condition in terms of a cumulative probability statement PI by sight of $P(Y \geq c) < 0.1$	AO3.1b	M1	Require $P(Y > y) < 0.1$
	Tests one appropriate value for y	AO3.2b	R1	$P(Y \geq 13) = 0.1316 > 0.1$
	Obtains at least two correct cumulative probabilities	AO1.1b	A1	$P(Y \geq 14) = 0.0577 > 0.1$
	Obtains the correct minimum number of matches CSO	AO3.2a	A1	Minimum number of matches = 14
	Total		**12**	

June 2018 Question 18 Paper 3

18 In a region of England, the government decides to use an advertising campaign to encourage people to eat more healthily.

Before the campaign, the mean consumption of chocolate per person per week was known to be 66.5 g, with a standard deviation of 21.2 g

18 (a) After the campaign, the first 750 available people from this region were surveyed to find out their average consumption of chocolate.

18 (a) (i) State the sampling method used to collect the survey.

[1 mark]

18 (b) A second sample of 750 people revealed that the mean consumption of chocolate per person per week was 65.4 g

Investigate, at the 10% level of significance, whether the advertising campaign has decreased the mean consumption of chocolate per person per week.

Assume that an appropriate sampling method was used and that the consumption of chocolate is normally distributed with an unchanged standard deviation.

[6 marks]

ANSWER

	Marking Instructions	AO	Marks	Typical Solution
18(a)(i)	States opportunistic (sampling). Accept opportunity/convenience.	AO1.2	B1	Opportunistic sampling
18(a)(ii)	Explains that sample is not random.	AO3.5b	E1	The sample is not random.
18(b)	States both hypotheses correctly for one-tailed test	AO2.5	B1	H_0: μ = 66.5 H_1: μ < 66.5
	Formulates the test statistic	AO1.1a	M1	$z = \dfrac{65.4 - 66.5}{21.2/\sqrt{750}}$
	Obtains the correct value of the test statistic	AO1.1b	A1	$= -1.42$
	States the correct critical z-value OE	AO1.1b	B1	Critical z value = -1.28
	Infers H_0 rejected CSO	AO2.2b	A1	-1.42 < -1.28
	Correctly concludes in context. (FT only available if first B1 and M1 scored).	AO3.2a	E1F	Reject H_0 - there is sufficient evidence that the advertising campaign has reduced the consumption of chocolate.
	Total		**8**	