Pure Year 2 exam questions (A Level maths)-AQA

NOTE:
Please be aware that in the Year 12 collections, you will find questions from the Year 13 papers. However, these questions are intentionally included because they align with Year 12 content and topics. That's the reason why you are missing some questions here, please have a look at the Year 12 collection!
Happy studying!

Ch 11 Parametric equations

June 2022 Question 1 Paper 1

1 A curve is defined by the parametric equations

$$x = \cos\theta \quad \text{and} \quad y = \sin\theta \qquad \text{where } 0 \leq \theta \leq 2\pi$$

Which of the options shown below is a Cartesian equation for this curve?

Circle your answer.

[1 mark]

$\frac{y}{x} = \tan\theta$ $\qquad x^2 + y^2 = 1$ $\qquad x^2 - y^2 = 1$ $\qquad x^2y^2 = 1$

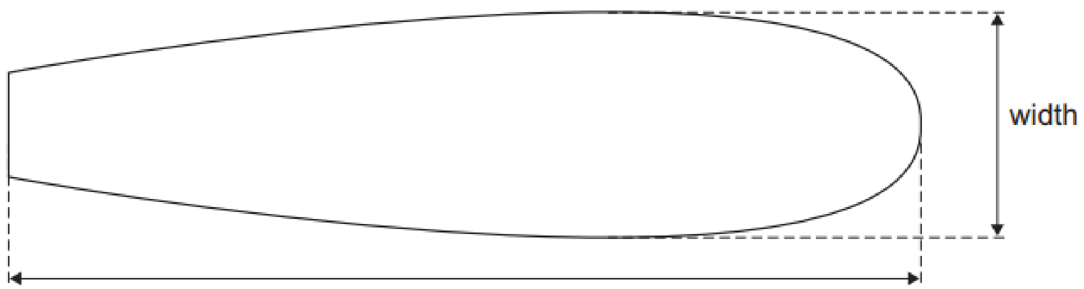
ANSWER

Q	Marking instructions	AO	Marks	Typical solution
1	Circles the correct answer	1.2	B1	$x^2 + y^2 = 1$
	Question 1 Total		**1**	

June 2022 Question 6 Paper 3

6 A design for a surfboard is shown in **Figure 1**.

Figure 1



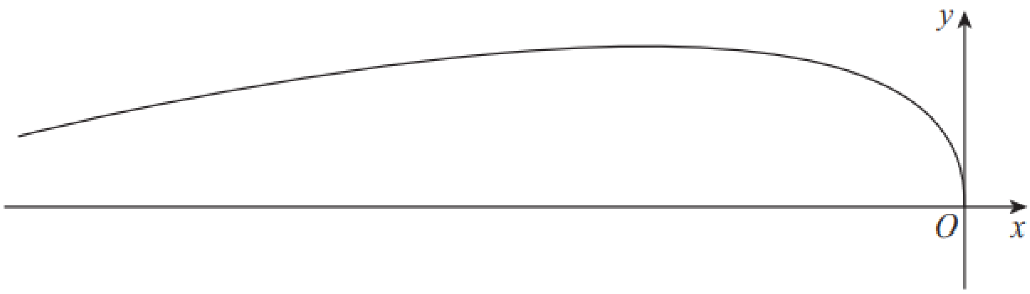
The curve of the **top half** of the surfboard can be modelled by the parametric equations

$$x = -2t^2$$

$$y = 9t - 0.7t^2$$

for $0 \leq t \leq 9.5$ as shown in **Figure 2**, where x and y are measured in centimetres.

Figure 2



6 (a) Find the length of the surfboard.

[2 marks]

6 (b) (i) Find an expression for $\frac{dy}{dx}$ in terms of t.

[3 marks]

6 (b) (ii) Hence, show that the width of the surfboard is approximately one third of its length.

[4 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Substitutes t = 9.5 into $x=-2t^2$ or $2t^2$	3.4	M1	$t=9.5 \Rightarrow x=-2\times 9.5^2=-180.5$ Length = 180.5 cm
	Obtains 180.5 Condone incorrect or missing units ISW	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)(i)	Obtains 9 – 1.4t or –4t OE Ignore labels	1.1b	B1	$\frac{dy}{dt}=9-1.4t$ $\frac{dx}{dt}=-4t$
	Uses chain rule to obtain $\frac{dy}{dx}$ Condone sign error	3.1a	M1	$\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$
	Obtains a correct expression Do not ISW	1.1b	A1	$\frac{dy}{dx}=\frac{9-1.4t}{-4t}$
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)(ii)	Equates their $\frac{\mathrm{d}y}{\mathrm{d}x}$ or their $\frac{\mathrm{d}y}{\mathrm{d}t}$ or their numerator of their $\frac{\mathrm{d}y}{\mathrm{d}x}$ to 0 PI by correct t from correct $\frac{\mathrm{d}y}{\mathrm{d}x}$	3.1a	M1	$\frac{9-1.4t}{-4t}=0$ $t=\frac{45}{7}=6.43$ $y=9\times 6.43-0.7\times(6.43)^2$ $=28.9$ Width of surfboard = 58 cm $180.5\div 3=60.2\approx 58$ Hence the width is approximately one third of the length
	Obtains correct value for t ACF eg t = 6.4 or $\frac{9}{1.4}$ Must come from correct $\frac{\mathrm{d}y}{\mathrm{d}x}$	1.1b	A1	
	Substitutes their value for t into the model for y and obtains a value for y provided $0 < t < 9.5$	3.4	M1	
	Compares correct width and correct length and $\frac{1}{3}$ or 3 with a correct concluding statement OE CSO Allow 180 for length	3.2a	R1	
	Subtotal		**4**	

Video solution:

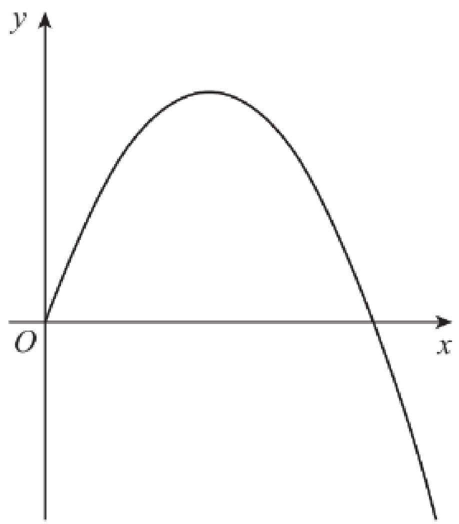
https://youtu.be/YwuaSzct-Gc?feature=shared

November 2021 Question 14 Paper 1

14 The curve C is defined for $t \geq 0$ by the parametric equations

$$x = t^2 + t \quad \text{and} \quad y = 4t^2 - t^3$$

C is shown in the diagram below.



14 (a) Find the gradient of C at the point where it intersects the positive x-axis.

[5 marks]

14 (b) (i) The area A enclosed between C and the x-axis is given by

$$A = \int_0^b y \, \mathrm{d}x$$

Find the value of b.

[1 mark]

14 (b) (ii) Use the substitution $y = 4t^2 - t^3$ to show that

$$A = \int_0^4 (4t^2 + 7t^3 - 2t^4) \, \mathrm{d}t$$

[3 marks]

14 (b) (iii) Find the value of A.

[1 mark]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
14(a)	Uses y = 0 to obtain a non-zero value of t	3.1a	M1	$y=0\Rightarrow 4t^2-t^3=0$ $t=0\ or\ 4$ $\frac{dy}{dt}=8t-3t^2$ $\frac{dx}{dt}=2t+1$ $t=4\Rightarrow\frac{dy}{dx}=-\frac{16}{9}$
	Obtains $\frac{dy}{dt}=8t-3t^2$ or $\frac{dy}{dt}=-16$	1.1b	B1	
	Obtains $\frac{dx}{dt}=2t+1$ or $\frac{dx}{dt}=9$	1.1b	B1	
	Uses their $\frac{dy}{dt}\div$ their $\frac{dx}{dt}=\frac{dy}{dx}$ and their non-zero value of t to find a numerical expression or value for $\frac{dy}{dx}$	3.1a	M1	
	Obtains $-\frac{16}{9}$ OE	1.1b	A1	
	Subtotal		**5**	

Q	Marking instructions	AO	Mark	Typical solution
14(b)(i)	Deduces b = 20 (FT t^2+t for their value of t)	2.2a	B1F	b = 20
	Subtotal		**1**	

Q	Marking instructions	AO	Mark	Typical solution
14(b)(ii)	Substitutes their $dx=(2t+1)dt$	3.1a	M1	$\frac{dx}{dt}=2t+1\Rightarrow dx=(2t+1)dt$ $A=\int_0^{20}y\,dx$ $=\int_0^4(4t^2-t^3)(2t+1)dt$ $=\int_0^4 8t^3+4t^2-2t^4-t^3dt$ $=\int_0^4 4t^2+7t^3-2t^4dt$
	Completes correct substitution for y and dx Condone incorrect or omission of limits.	1.1b	A1F	
	Completes rigorous argument, to show given result. t = 4 when x = 20 must be justified either here or in part (b)(i)	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Mark	Typical solution
14(b)(iii)	Evaluates A = 1856/15 or AWRT 124	1.1b	B1	$A = \frac{1856}{15}$
	Subtotal		**1**	

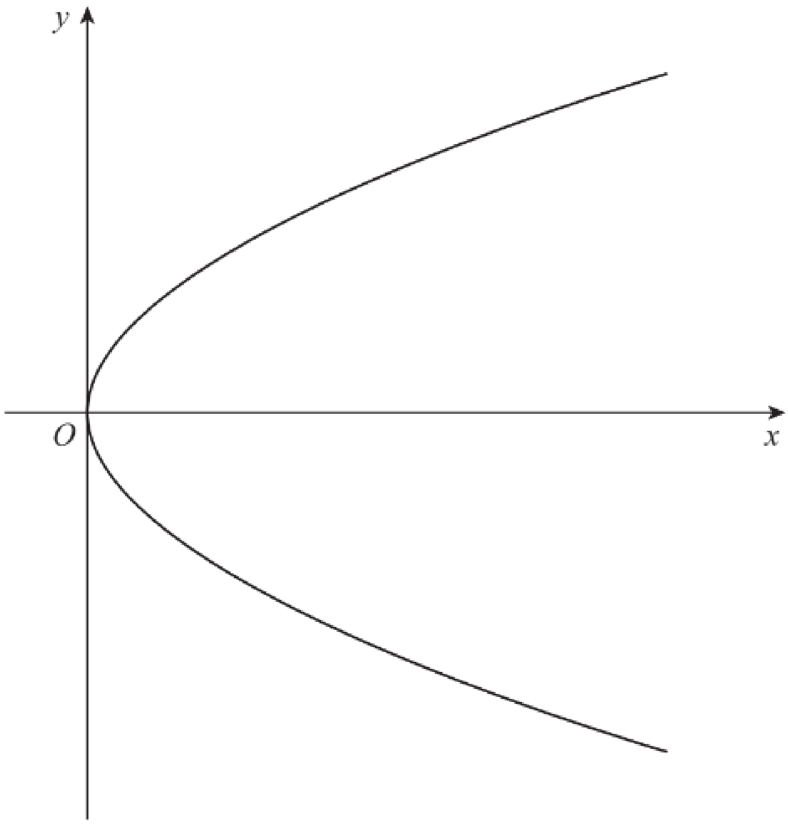
November 2020 Question 8 Paper 2

8 The curve defined by the parametric equations

$$x = t^2 \text{ and } y = 2t \qquad -\sqrt{2} \le t \le \sqrt{2}$$

is shown in **Figure 1** below.

Figure 1



8 (a) Find a Cartesian equation of the curve in the form $y^2 = \mathrm{f}(x)$

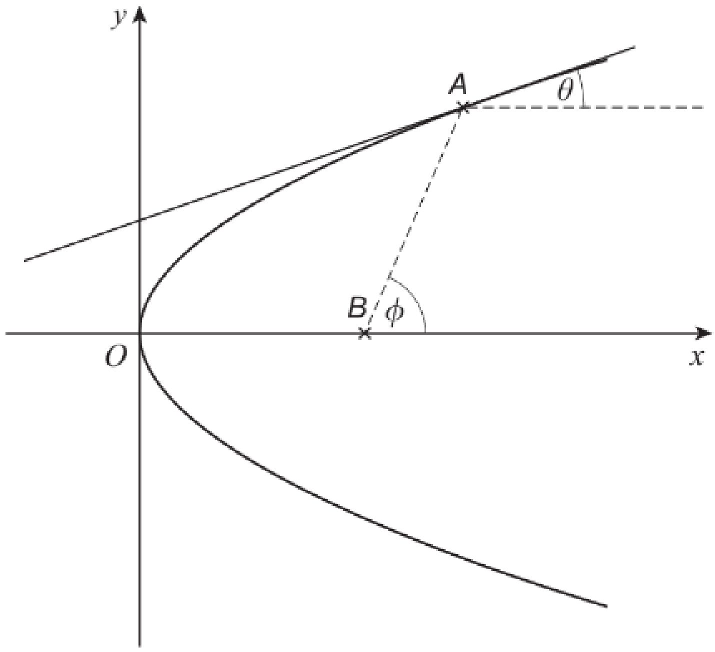
[2 marks]

8 (b) The point A lies on the curve where $t = a$

The tangent to the curve at A is at an angle θ to a line through A parallel to the x-axis.

The point B has coordinates (1, 0)

The line AB is at an angle ϕ to the x-axis.



8 (b) (i) By considering the gradient of the curve, show that

$$\tan\theta = \frac{1}{a}$$

[3 marks]

8 (b) (ii) Find $\tan\phi$ in terms of a.

[2 marks]

8 (b) (iii) Show that $\tan 2\theta = \tan\phi$

[3 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
8(a)	Eliminates t	1.1a	M1	$\frac{y}{2} = t, \quad x = \frac{y^2}{4}$
	Writes the Cartesian equation in the required form	1.1b	A1	$y^2 = 4x$
	Subtotal		**2**	
8(b)(i)	Differentiates both $\frac{dx}{dt} = 2t$, $\frac{dy}{dt} = 2$ with at least one correct Or Differentiates their $y^2 = 4x$ to obtain $y\frac{\mathrm{d}y}{\mathrm{d}x} = A$ Or rearranges and differentiates $y = 2\sqrt{x}$ and obtains $\frac{\mathrm{d}y}{\mathrm{d}x} = Ax^{-\frac{1}{2}}$ OE	3.1a	M1	$\frac{dx}{dt} = 2t, \quad \frac{dy}{dt} = 2$ $\frac{dy}{dx} = \frac{2}{2a} = \frac{1}{a}$ The gradient of a line is equal to the tangent of the angle between the line and the horizontal hence $\tan\theta = \frac{1}{a}$
	Obtains correct $\frac{dy}{dx}$ at $t = a$	1.1b	A1	
	Explains that the gradient of a line is the tangent of the angle between the line and the horizontal or shows on right-angled triangle on diagram and links to $\tan\theta$ **and** concludes $\tan\theta = \frac{1}{a}$	2.4	E1	
	Subtotal		**3**	
8(b)(ii)	Uses formula for gradient of straight line with points A and B Must have $a^2 - 1$ or $1 - a^2$ in denominator	1.1a	M1	$\tan\phi = \frac{2a - 0}{a^2 - 1}$ $= \frac{2a}{a^2 - 1}$
	Obtains correct $\tan\phi$ OE	1.1b	A1	
	Subtotal		**2**	

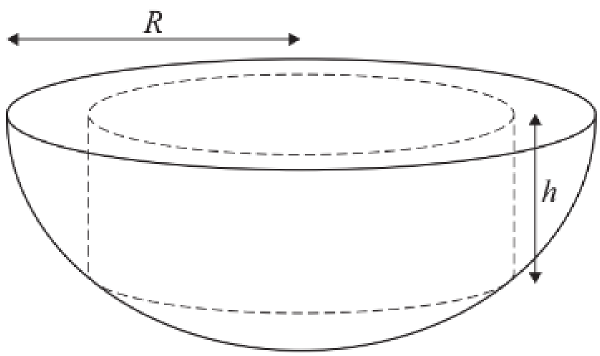
8(b)(iii)	States double angle formula for $\tan 2\theta$	1.2	B1	$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$ $= \dfrac{2\times\frac{1}{a}}{1-\left(\frac{1}{a}\right)^2}$ $= \dfrac{2a}{a^2-1}$ $= \tan\phi$
	Substitutes $\tan\theta = \dfrac{1}{a}$ into their $\tan 2\theta = \dfrac{2\tan\theta}{1\pm\tan^2\theta}$	1.1a	M1	
	Simplifies and completes argument to show required result	2.1	R1	
	Subtotal		**3**	
	Question Total		**10**	

November 2020 Question 9 Paper 2

9 A cylinder is to be cut out of the circular face of a solid hemisphere.

The cylinder and the hemisphere have the same axis of symmetry.

The cylinder has height h and the hemisphere has a radius of R.



9 (a) Show that the volume, V, of the cylinder is given by

$$V = \pi R^2 h - \pi h^3$$

[3 marks]

9 (b) Find the maximum volume of the cylinder in terms of R.

Fully justify your answer.

[7 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Identifies and clearly defines variable for radius of cylinder. Can be shown on diagram or can be implied by use in $V = \pi r^2 h$	2.5	B1	Radius of cylinder = r $h^2 + r^2 = R^2$ $V = \pi r^2 h$
	Uses Pythagoras to connect h, r and R	3.1a	M1	
	Eliminates the radius variable to form an expression for the volume of the cylinder in terms of h, completing argument to show given result. Condone undefined r	2.1	R1	$V = \pi\left(R^2 - h^2\right)h$ $= \pi R^2 h - \pi h^3$
	Subtotal		**3**	

9(b)	Differentiates the expression for volume w.r.t. h with at least one term correct.	3.1a	M1	$\frac{\mathrm{d}V}{\mathrm{d}h} = \pi R^2 - 3\pi h^2$
	Obtains correct $\frac{\mathrm{d}V}{\mathrm{d}h}$	1.1b	A1	For maximum volume $\frac{\mathrm{d}V}{\mathrm{d}h} = 0$
	Explains that their derivative w.r.t h equals zero for a maximum or stationary point	2.4	E1	$\Rightarrow R^2 - 3h^2 = 0$
	Equates volume derivative w.r.t. h to zero and correctly obtains a value for h in terms of R	1.1a	M1	$h^2 = \frac{R^2}{3} \Rightarrow h = \frac{R}{\sqrt{3}}$

Substitutes their h, in terms of R, from derivative w.r.t. h into volume formula.	1.1a	M1	Hence volume $V = \pi R^2 \frac{R}{\sqrt{3}} - \pi\left(\frac{R}{\sqrt{3}}\right)$ $= \frac{2\sqrt{3}\pi R^3}{9}$
Obtains the correct max volume in the form $kR^3 - pR^3$ or better	3.2a	A1	
Justifies correct volume in the form $kR^3 - pR^3$ or better form is the maximum eg: • V = 0 when h=0 or R and V>0 in between. • Sketches shape of graph passing through the origin with (min on negative side) and max on positive side • Obtains $\frac{d^2V}{dh^2} = -6\pi h$ <0 NB R1 can be awarded even if E1 is not awarded.	2.1	R1	$\frac{d^2V}{dh^2} = -6\pi h$ When $h = \frac{R}{\sqrt{3}}$ $\frac{d^2V}{dh^2} < 0$ Therefore maximum

Video solution:

https://youtu.be/xIm9q8Q_Ccg

June 2019 Question 15 Paper 1

15 (a) At time t hours **after a high tide**, the height, h metres, of the tide and the velocity, v knots, of the tidal flow can be modelled using the parametric equations

$$v = 4 - \left(\frac{2t}{3} - 2\right)^2$$

$$h = 3 - 2\sqrt[3]{t - 3}$$

High tides and low tides occur alternately when the velocity of the tidal flow is zero.

A high tide occurs at 2 am.

15 (a) (i) Use the model to find the height of this high tide.

[1 mark]

15 (a) (ii) Find the time of the first **low** tide after 2 am.

[3 marks]

15 (a) (iii) Find the height of this low tide.

[1 mark]

15 (b) Use the model to find the height of the tide when it is flowing with maximum velocity.

[3 marks]

15 (c) Comment on the validity of the model.

[2 marks]

ANSWER

Q	Marking Instructions	AO	Mark	Typical Solution
15 (a)(i)	Uses model with t=0 to find correct value of h *AWRT 5.9*	AO3.4	B1	5.88metres
15 (a)(ii)	Uses v=0 to set up quadratic equation for t	AO3.4	M1	$0 = 4 - (\frac{2t}{3} - 2)^2$
	Obtains t = 6 PI correct answer	AO1.1b	A1	t = 6 8 am
	Interprets their lowest positive solution correctly NMS can score 3	AO3.2a	A1F	
15 (a)(iii)	Obtains correct h for their positive t provided h < 5.88 Accept 0.12 If given to more decimal places AWFW 0.115 to 0.116 FT their t allow negative h	AO3.4	B1F	0.12m
15(b)	Identifies t=3 (for maximum velocity)	AO3.1b	B1	t = 3 $h = 3 - 2\sqrt[3]{3-3}$ h = 3 metres
	Substitutes their t into model for h PI their answer	3.4	M1	
	Finds correct height **with units**	3.2a	A1	
15(c)	Explains that the validity of the model is limited by time.	3.5b	B1	The model breaks down after one cycle of the tide.
	Explains that the height continues to decrease after 6 hours (or after the first cycle or first low tide). Or explains there are no other times when v=0	3.5a	B1	After 6 hours the model shows the height continues to decrease.
	Total		**10**	

June 2018 Question 5 Paper 1

5 A curve is defined by the parametric equations

$$x = 4 \times 2^{-t} + 3$$

$$y = 3 \times 2^{t} - 5$$

5 (a) Show that $\dfrac{dy}{dx} = -\dfrac{3}{4} \times 2^{2t}$

[3 marks]

5 (b) Find the Cartesian equation of the curve in the form $xy + ax + by = c$, where a, b and c are integers.

[3 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
5(a)	Differentiates 2^t or 2^{-t} to obtain $\pm A\ln 2\times 2^{\pm t}$	AO1.1a	M1	
	Obtains $\frac{\mathrm{d}y}{\mathrm{d}t}=(\pm A\ln 2)2^t$ and $\frac{\mathrm{d}x}{\mathrm{d}t}=(\pm B\ln 2)2^{-t}$	AO1.1b	A1	$\frac{\mathrm{d}y}{\mathrm{d}t}=(3\ln 2)2^t$ $\frac{\mathrm{d}x}{\mathrm{d}t}=(-4\ln 2)2^{-t}$
	Uses chain rule with correct $\frac{\mathrm{d}y}{\mathrm{d}t}$ and $\frac{\mathrm{d}x}{\mathrm{d}t}$ and completes rigorous argument to obtain fully correct printed answer	AO2.1	R1	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(3\ln 2)2^t}{(-4\ln 2)2^{-t}}$ $=-\frac{3}{4}\times 2^{2t}$
(b)	Rearranges to write 2^{-t} in terms of x or 2^t in terms of y Or Writes given expression in terms of t	AO3.1a	M1	$2^t=\frac{y+5}{3}$ $2^{-t}=\frac{x-3}{4}$
	Eliminates t Or Compares coefficients PI by a=5 or b=-3	AO1.1a	M1	$1=\left(\frac{y+5}{3}\right)\left(\frac{x-3}{4}\right)$ $12=xy+5x-3y-15$ $xy+5x-3y=27$
	Completes rigorous argument to obtain correct values of a, b and c and write the Cartesian equation in the required form ISW	AO2.1	R1	**ALT** $xy+ax+by=(4\times 2^{-t}+3)(3\times 2^t-5)+a(4\times 2^{-t}+3)+b(3\times 2^t-5)$ $=12-15+(4a-20)2^{-t}+(3b+9)2^t+3a-5b$ $a=5, b=-3$ $xy+5x-3y=-3+15+15$ $=27$
	Total		**6**	