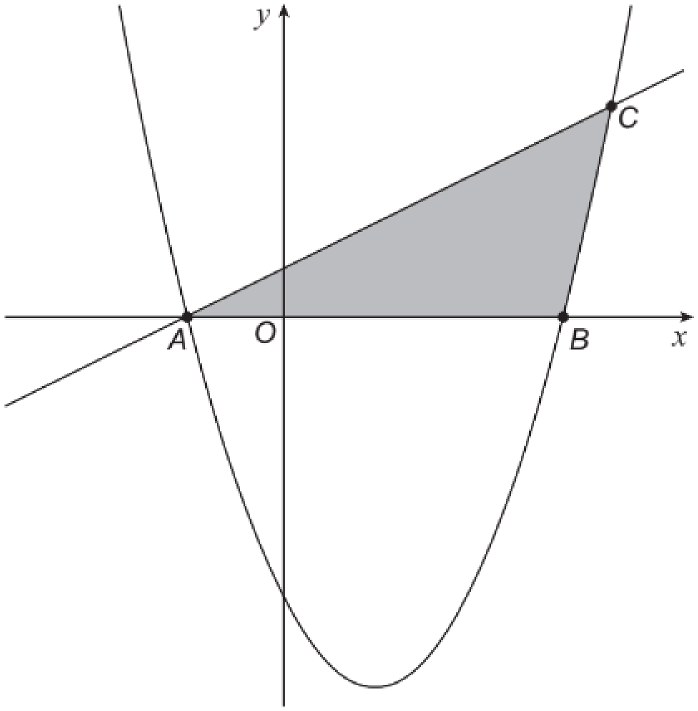
Pure Year 1 exam questions (AS maths)-AQA

NOTE:
Please be aware that in the Year 12 collections, you will find questions from the A-Level (Year 13) papers. However, these questions are intentionally included because they align with Year 12 content and topics.
Happy studying!

Ch 11 Integration

June 2022 Question 9 Paper 2

9 The diagram below shows the graphs of $y = x^2 - 4x - 12$ and $y = x + 2$



9 (a) Write down three inequalities which together describe the shaded region.

[2 marks]

9 (b) Find the coordinates of the points A, B and C.

[4 marks]

9 (c) Find the exact area of the shaded region.

Fully justify your answer.

[6 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Deduces one correct inequality related to the sloping line or the curve. Condone strict inequalities	2.2a	B1	$y \le x + 2$ $y \ge x^2 - 4x - 12$ $y \ge 0$
	Deduces the other two correct inequalities Condone strict inequalities	2.2a	B1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)	States x coordinate of A is -2	1.1b	B1	A is $(-2, 0)$ B is $(6, 0)$ $x + 2 = x^2 - 4x - 12$ $x^2 - 5x - 14 = 0$ $(x + 2)(x - 7) = 0$ C is point $(7, 9)$
	States x coordinate of B is 6	1.1b	B1	
	Eliminates y to obtain x coordinate of $C = 7$	1.1a	M1	
	Obtains correct y coordinates of A, B and C	1.1b	A1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
9(c)	Obtains correct value for area under *AC*	1.1b	B1	Area of triangle under *AC* = 0.5 × 9 × 9 = 40.5
	Integrates a quadratic expression with $\frac{x^3}{3}$ term correct PI by $\frac{13}{3}$ ACF	1.1a	M1	Area below *BC* $= \int_6^7 (x^2 - 4x - 12)\,dx$
	Integrates $x^2 - 4x - 12$ completely correct Condone inclusion of + c here PI by $\frac{13}{3}$ ACF Condone integration of $x^2 - 5x - 14$ correctly	1.1b	A1	$= \left[\frac{x^3}{3} - 2x^2 - 12x\right]_6^7$
	Substitutes a pair of limits into their integrated quadratic, must be three terms, including subtraction. PI by $\frac{13}{3}$ ACF	1.1a	M1	$= \frac{343}{3} - 98 - 84 - 72 + 72 + 72$
	Uses a correct method to combine areas that lead to the exact area of the shaded region	3.1a	M1	Shaded area = 40.5 - $4\frac{1}{3}$
	Obtains $36\frac{1}{6}$ or $\frac{217}{6}$ ISW	2.1	R1	$= 36\frac{1}{6}$
	Subtotal		**6**	

June 2022 Question 7 Paper 1

7 A curve has equation $y = a^2 - x^2$, where $a > 0$

The area enclosed between the curve and the x-axis is 36 units.

Find the value of a.

Fully justify your answer.

[6 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
7	Integrates with at least one term correct.	3.1a	M1	
	Obtains correct integral.	1.1b	A1	$a^2 - x^2 = 0$ $(a + x)(a - x) = 0$
	Obtains $x = -a$ or a	1.1b	B1	$x = -a$ or a
	Substitutes their limits into their two-term integrated expression Do not allow any limits involving x	1.1a	M1	$\int_{-a}^{a} (a^2 - x^2)dx = 36$ $\left[a^2x - \frac{x^3}{3}\right]_{-a}^{a} = 36$
	Equates their expression in terms of a With limits $-a$ to a, to 36 Or With limits 0 to a or $-a$ to 0, to 18	3.1a	M1	$a^3 - \frac{a^3}{3} + a^3 - \frac{a^3}{3} = 36$ $\frac{4a^3}{3} = 36$ $a^3 = 27$
	Completes a reasoned argument to obtain $a = 3$	2.1	R1	$a = 3$
	Question 7 Total		**6**	

June 2022 Question 5 Paper 2 (A-Level)

5 The binomial expansion of $(2+5x)^4$ is given by

$$(2+5x)^4 = A + 160x + Bx^2 + 1000x^3 + 625x^4$$

5 (a) Find the value of A and the value of B.

[2 marks]

5 (b) Show that

$$(2+5x)^4 - (2-5x)^4 = Cx + Dx^3$$

where C and D are constants to be found.

[2 marks]

5 (c) Hence, or otherwise, find

$$\int \left((2+5x)^4 - (2-5x)^4\right) \mathrm{d}x$$

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Obtains 16 Not incorrectly labelled	1.1b	B1	$(2+5x)^4 = 16+160x+600x^2$ $+1000x^3+625x^4$ A = 16 B = 600
	Obtains 600 Not incorrectly labelled	1.1b	B1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)	Obtains the expansion of $(2-5x)^4 =$ $A-160x+Bx^2-1000x^3+625x^4$ Accept A and B unsubstituted or their A and B Or Uses a valid method and obtains one of $C=320$ or $D=2000$	1.1a	M1	$(2+5x)^4-(2-5x)^4$ $=16+160x+600x^2+1000x^3+625x^4$ $-(16-160x+600x^2-1000x^3+625x^4)$ $=320x+2000x^3$
	Completes reasoned argument to show $(2+5x)^4-(2-5x)^4=320x+2000x^3$ Accept A and B unsubstituted or their A and B Must finish with $320x+2000x^3$ don't accept just C=320 and D = 2000	2.1	R1F	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(c)	Integrates one term correctly Accept C and D unsubstituted or their C and D Or Uses reverse of chain rule to obtain at least one term of the form $P(2\pm5x)^5$, $P=\pm\frac{1}{5}$ or $\pm\frac{1}{25}$	1.1a	M1	$\int\left((2+5x)^4-(2-5x)^4\right)\mathrm{d}x$ $=\int\left(320x+2000x^3\right)\mathrm{d}x$ $=160x^2+500x^4+c$
	Obtains $\frac{320}{2}x^2+\frac{2000}{4}x^4+c$ FT C and D unsubstituted or their C and D Or $\frac{(2+5x)^5}{5\times5}+\frac{(2-5x)^5}{5\times5}+c$ Condone missing $+c$	1.1b	A1F	
	Subtotal		**2**	

June 2022 Question 4 Paper 3 (A-Level)

4 Find

$$\int \left(x^2 + x^{\frac{1}{2}}\right) \mathrm{d}x$$

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
4	Integrates one x term correctly Accept unsimplified terms	1.1a	M1	$\frac{x^3}{3}+\frac{2x^{\frac{3}{2}}}{3}+c$
	Obtains correct answer ACF Must include + c Accept unsimplified terms Do not ISW	1.1b	A1	
	Question 4 Total		**2**	

November 2021 Question 3 Paper 2

3 It is given that

$$\frac{dy}{dx} = \sqrt{x}$$

Find an expression for y.

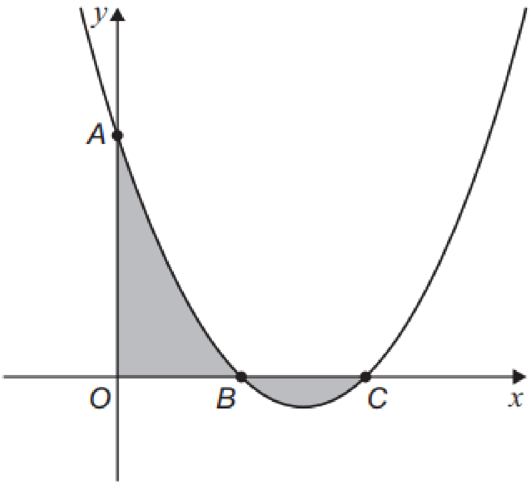
[3 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
3	Rewrites $\sqrt{x}$ as $x^{\frac{1}{2}}$	1.1b	B1	$\frac{dy}{dx} = x^{\frac{1}{2}}$ $y = \frac{2}{3}x^{\frac{3}{2}} + c$
	Integrates their x^k for non-integer k to obtain x^{k+1}	1.1a	M1	
	Obtains $y = \frac{2}{3}x^{\frac{3}{2}} + c$ Condone no constant of integration or numerical value for c used	1.1b	A1	
	Total		**3**	

November 2021 Question 7 Paper 2

7 The diagram below shows the graph of the curve that has equation $y = x^2 - 3x + 2$ along with two shaded regions.



7 (a) State the coordinates of the points *A*, *B* and *C*.

[2 marks]

7 (b) Katy is asked by her teacher to find the total area of the two shaded regions.

Katy uses her calculator to find $\int_0^2 (x^2 - 3x + 2)\,dx$ and gets the answer $\frac{2}{3}$

Katy's teacher says that her answer is incorrect.

7 (b) (i) Show that the total area of the two shaded regions is 1

Fully justify your answer.

[5 marks]

7 (b) (ii) Explain why Katy's method was not valid.

[1 mark]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
7(a)	States correct coordinates of *B* or *C* Or States the correct x coordinates of B **and** C	1.1a	M1	*A* is (0, 2) *B* is (1, 0) *C* is (2, 0)
	Obtains A(0 , 2), B(1 , 0) and C(2 , 0)	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)(i)	Integrates in two parts with limits *O* to *B* and *B* to *C*	3.1a	M1	$\int_0^1 (x^2 - 3x + 2)\,\mathrm{d}x$ $= [\frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x]_0^1$ $= [\frac{1}{3} - \frac{3}{2} + 2] = \frac{5}{6}$
	Integrates quadratic function with at least one term correct	1.1a	M1	
	Integrates completely correctly	1.1b	A1	$\int_1^2 (x^2 - 3x + 2)\,\mathrm{d}x$
	Substitutes the two sets of their limits and subtracts the *BC* value from the *OB* value or uses the modulus for the *BC* value	1.1a	M1	$[\frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x]_1^2$ $[\frac{1}{3} \times 8 - \frac{3}{2} \times 4 + 2 \times 2] - (\frac{5}{6})$ $= -\frac{1}{6}$
	Completes calculation of total area convincingly to given answer. AG	2.1	R1	Total area $= \frac{5}{6} - (-\frac{1}{6}) = 1$
	Subtotal		**5**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)(ii)	Explains that the area between B and C is treated as negative (OE)	2.3	E1	The calculator treats the area between B and C as negative.
	Subtotal		**1**	

November 2020 Question 9 Paper 2

9 (a) (i) Find

$$\int (4x - x^3)\,\mathrm{d}x$$

[2 marks]

9 (a) (ii) Evaluate

$$\int_{-2}^{2} (4x - x^3)\,\mathrm{d}x$$

[1 mark]

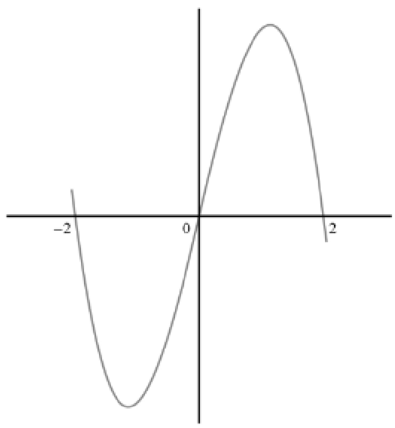
9 (b) Using a sketch, explain why the integral in part **(a)(ii)** does **not** give the area enclosed between the curve $y = 4x - x^3$ and the x-axis.

[2 marks]

9 (c) Find the area enclosed between the curve $y = 4x - x^3$ and the x-axis.

[2 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
9(a)(i)	Integrates to obtain terms in x^2 and x^4	1.1a	M1	$2x^2 - \frac{x^4}{4} + c$
	Obtains fully correct integral (ISW) Condone omission of $+c$	1.1b	A1	
	Subtotal		**2**	
9(a)(ii)	Obtains answer of 0 *nb* correct answer can be obtained directly from calculator, if working shown then CSO	1.1b	B1	0
	Subtotal		**1**	
9(b)	Shows curve with three zeros with correct orientation	1.1b	B1	 The area between -2 and 0 lies below the axis so its integral has a negative value
	Explains that the integral for the area below the axis has a negative value (PI) (Allow even if the graph is incorrectly drawn or omitted)	2.4	E1	
	Subtotal		**2**	
9(c)	Uses 2 × $\int_0^2 4x - x^3\, dx$ (OE) **Or** Obtains values of 4, and -4 or 4 from two separate integrals	1.1a	M1	2 × $\int_0^2 4x - x^3\, dx$ $2\left[2x^2 - \frac{x^4}{4}\right]_0^2$ $= 8$
	Obtains correct area	1.1b	A1	
	Subtotal		**2**	
	Question Total		**7**	

November 2020 Question 1 Paper 3 (A-Level)

1 Given that

$$\int_0^{10} f(x)\,dx = 7$$

deduce the value of

$$\int_0^{10} \left(f(x) + 1\right) dx$$

Circle your answer.

[1 mark]

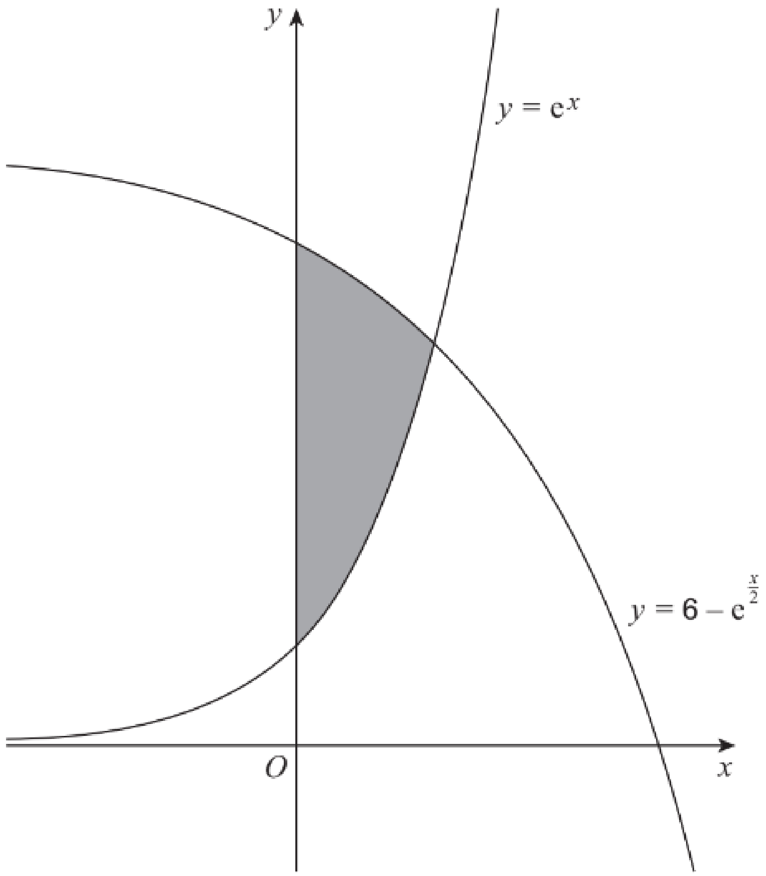
-3 $\quad$ 7 $\quad$ 8 $\quad$ 17

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
1	Circles correct answer	2.2a	B1	17
	Total		**1**	

November 2020 Question 15 Paper 1 (A-Level)

15 The region enclosed between the curves $y = e^x$, $y = 6 - e^{\frac{x}{2}}$ and the line $x = 0$ is shown shaded in the diagram below.



Show that the exact area of the shaded region is

$$6 \ln 4 - 5$$

Fully justify your answer.

[10 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
15	Forms a single equation eliminating x or y	3.1a	M1	$6 - e^{\frac{x}{2}} = e^x$
	Obtains a correct rearranged quadratic equation. Either $e^x + e^{\frac{x}{2}} - 6 = 0$ or $\left(e^{\frac{x}{2}} + 3\right)\left(e^{\frac{x}{2}} - 2\right) = 0$ or $e^x + e^{\frac{x}{2}} + \frac{1}{4} = \frac{25}{4}$ OE	1.1b	A1	$e^x + e^{\frac{x}{2}} - 6 = 0$ $\left(e^{\frac{x}{2}} + 3\right)\left(e^{\frac{x}{2}} - 2\right) = 0$ $e^{\frac{x}{2}} = -3$ or 2
	Solves 'their' quadratic Must be a quadratic in $e^{\frac{x}{2}}$ or If squaring is used then it must be a quadratic in e^x or Obtains x = 1.386	1.1a	M1	$e^{\frac{x}{2}} > 0$ so -3 is not a valid solution $\frac{x}{2}$ = ln 2 $x = 2\ln 2 = \ln 4$
	Explains that $e^{\frac{x}{2}} = -3$ is not valid as $e^{\frac{x}{2}} > 0$ or If squaring is used they must clearly check both solutions by substituting and conclude that $\ln 9$ is not valid OE	2.4	E1F	$\int_0^{\ln 4} (6 - e^{\frac{x}{2}} - e^x)\, dx$ = $\left[6x - 2e^{\frac{x}{2}} - e^x\right]_0^{\ln 4}$
	Obtains $x = 2\ln 2$ or $x = \ln 4$	1.1b	A1	= $\left(6\ln 4 - 2e^{\frac{\ln 4}{2}} - e^{\ln 4}\right)$ - (- 2 - 1)
	Forms any definite integral which would contribute to finding the required area This could be $\int_0^{\ln 4} (6 - e^{\frac{x}{2}} - e^x)\, dx$ or $\int_0^{\ln 4} (6 - e^{\frac{x}{2}})\, dx$ or $\int_0^{\ln 4} e^x dx$ or $\int_0^{\ln 4} (e^x + e^{\frac{x}{2}} - 6)\, dx$ Follow through '*their*' value of x for the upper limit	1.1a	M1	= 6 ln 4 – 4 – 4 + 3 = 6 ln 4 – 5
	Forms a fully correct definite integral (or integrals) which would lead to evaluating the correct area Follow through '*their*' incorrect upper limit	3.1a	A1F	

	Integrates '*their*' expressions involving exponentials fully correctly Follow through their exponential expressions – but must have integrated both e^x and $e^{\frac{x}{2}}$ terms Condone missing/incorrect limits	1.1b	B1F	
	Substitutes 0 and '*their*' upper limits into '*their*' integrated expression Must correctly use F (their upper limit) – F (0) for each integral	1.1a	M1	
	Completes rigorous argument by showing explicit evaluation of exponential terms before obtaining final answer AG This mark can be achieved without achieving the E1 mark	2.1	R1	
	Total		**10**	

Video solution:

https://youtu.be/j7A5g7Qcf1A

June 2019 Question 9 Paper 1

9 A curve cuts the x-axis at (2, 0) and has gradient function

$$\frac{dy}{dx} = \frac{24}{x^3}$$

9 (a) Find the equation of the curve.

[4 marks]

9 (b) Show that the perpendicular bisector of the line joining $A(-2, 8)$ to $B(-6, -4)$ is the normal to the curve at (2, 0)

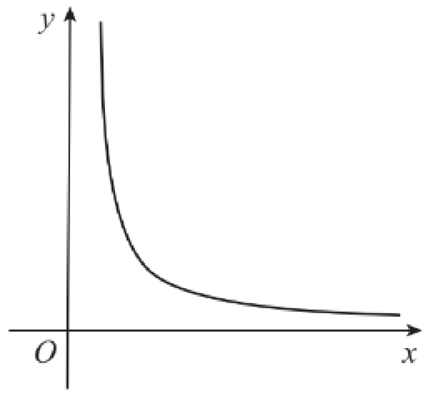
[6 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
9(a)	Decides to integrate $\frac{24}{x^3}$	3.1a	M1	$y = \int \frac{24}{x^3}\,dx$
	Obtains correct integral with or without c	1.1b	A1	$= -\frac{12}{x^2} + c$
	Includes c and substitutes (2, 0) to calculate c or uses definite integral	1.1a	M1	$0 = -\frac{12}{2^2} + c$
	Evaluates c and states correct equation ACF Follow through only on sign error in integral (c = –3)	1.1b	A1F	$c = 3$ $y = -\frac{12}{x^2} + 3$
9(b)	Finds midpoint of AB	1.1b	B1	Midpoint of AB is (–4, 2)
	Finds gradient of AB (unsimplified)	1.1b	B1	Gradient of AB $= \frac{8-(-4)}{-2-(-6)} = 3$
	Finds gradient of curve at (2, 0)	1.1b	B1	Gradient of perpendicular bisector $\frac{-1}{3}$
	Uses perpendicular gradients property	3.1a	M1	
	Finds the correct equation of the perpendicular bisector using (-4, 2), **or** the correct equation of the normal using (2, 0) ACF PI by correct "m & c" pair	3.1a	M1	Equation is $y = \frac{-1}{3}x + \frac{2}{3}$ When $x = 2$, $y = \frac{-1}{3} \times 2 + \frac{2}{3} = 0$
	Completes rigorous argument to show that the perpendicular bisector is the normal. Must include showing normal and bisector have the same gradient and that (2, 0) lies on the bisector, or that (-4, 2) lies on the normal, or that the equations are identical.	2.1	R1	So passes through (2,0) Gradient of curve at (2, 0) is $\frac{24}{2^3} = 3$ Gradient of normal is $\frac{-1}{3}$ So perpendicular bisector is the normal
	Total		**10**	

June 2019 Question 6 Paper 2

6 A curve has equation $y = \dfrac{2}{x\sqrt{x}}$



The region enclosed between the curve, the x-axis and the lines $x = 1$ and $x = a$ has area 3 units.

Given that $a > 1$, find the value of a.

Fully justify your answer.

[5 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
6	Expresses $\frac{1}{x\sqrt{x}}$ as $x^{-\frac{3}{2}}$ or $x^{-1.5}$ or $x^{-1\frac{1}{2}}$ PI completes correct integration Condone incorrect use of '2' **NB $a = 16$ with no justification scores 0/5**	1.1a	M1	$\frac{2}{x\sqrt{x}} = 2x^{-\frac{3}{2}}$ $3 = \int_1^a 2x^{-\frac{3}{2}}\,dx$ $= [-4x^{-\frac{1}{2}}]_1^a$ $3 = -4a^{-\frac{1}{2}} + 4$ $a^{-\frac{1}{2}} = \frac{1}{4}$ $a = 16$
	Carries out correct integration to obtain $-4x^{-\frac{1}{2}}$ OE	1.1b	A1	
	Forms an equation by equating 3 PI by • correct integral $[-4x^{-\frac{1}{2}}]_1^a$ • original expression as integral with powers $\int 2x^{-\frac{3}{2}}\,dx$ • original expression as integral $\int \frac{2}{x\sqrt{x}}\,dx$ • 'Their' integration with limits 1 and a • 'Their' expression after integration and after using limits 1 and a Condone limits interchanged **If assuming $a = 16$ and then trying to verify scores M1A1M1 max**	3.1a	M1	
	Substitutes x = 1 as the lower limit and x = a as the upper limit into 'their' integrated expression and subtracts	1.1a	M1	
	Obtains $a = 16$ CAO	1.1b	A1	
	Total		**5**	

June 2018 Question 5 Paper 2

5 $\quad f'(x) = \left(2x - \frac{3}{x}\right)^2$ and $f(3) = 2$

Find $f(x)$.

[4 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
5	Expands the bracket and obtains any correct form (PI)	AO1.1b	B1	$f'(x) = 4x^2 - 12 + \frac{9}{x^2}$
	Integrates at least one of 'their' terms correctly	AO1.1a	M1	$f(x) = \int 4x^2 - 12 + \frac{9}{x^2}\,dx$ $= \frac{4x^3}{3} - 12x - \frac{9}{x} + c$
	Substitutes $x = 3$ into 'their' integrated expression and equates to 2	AO1.1a	M1	$f(3) = 36 - 36 - 3 + c = 2$ $c = 5$
	Obtains completely correct expression, must be explicitly stated CAO (ACF)	AO1.1b	A1	$f(x) = \frac{4x^3}{3} - 12x - \frac{9}{x} + 5$
	Total		**4**	