Pure Year 2 exam questions (A Level maths)-AQA

NOTE:
Please be aware that in the Year 12 collections, you will find questions from the Year 13 papers. However, these questions are intentionally included because they align with Year 12 content and topics. That's the reason why you are missing some questions here, please have a look at the Year 12 collection!
Happy studying!

Ch 10 Integration

June 2022 Question 5 Paper 2

5 The binomial expansion of $(2+5x)^4$ is given by

$$(2+5x)^4 = A + 160x + Bx^2 + 1000x^3 + 625x^4$$

5 (a) Find the value of A and the value of B.

[2 marks]

5 (b) Show that

$$(2+5x)^4 - (2-5x)^4 = Cx + Dx^3$$

where C and D are constants to be found.

[2 marks]

5 (c) Hence, or otherwise, find

$$\int \left((2+5x)^4 - (2-5x)^4 \right) \mathrm{d}x$$

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Obtains 16 Not incorrectly labelled	1.1b	B1	$(2+5x)^4=16+160x+600x^2$ $+1000x^3+625x^4$ A = 16 B = 600
	Obtains 600 Not incorrectly labelled	1.1b	B1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)	Obtains the expansion of $(2-5x)^4=$ $A-160x+Bx^2-1000x^3+625x^4$ Accept A and B unsubstituted or their A and B Or Uses a valid method and obtains one of $C=320$ or $D=2000$	1.1a	M1	$(2+5x)^4-(2-5x)^4$ $=16+160x+600x^2+1000x^3+625x^4$ $-(16-160x+600x^2-1000x^3+625x^4)$ $=320x+2000x^3$
	Completes reasoned argument to show $(2+5x)^4-(2-5x)^4=320x+2000x^3$ Accept A and B unsubstituted or their A and B Must finish with $320x+2000x^3$ don't accept just C=320 and D = 2000	2.1	R1F	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(c)	Integrates one term correctly Accept C and D unsubstituted or their C and D Or Uses reverse of chain rule to obtain at least one term of the form $P(2\pm 5x)^5$, $P=\pm\frac{1}{5}$ or $\pm\frac{1}{25}$	1.1a	M1	$\int\left((2+5x)^4-(2-5x)^4\right)\mathrm{d}x$ $=\int\left(320x+2000x^3\right)\mathrm{d}x$ $=160x^2+500x^4+c$
	Obtains $\frac{320}{2}x^2+\frac{2000}{4}x^4+c$ FT C and D unsubstituted or their C and D Or $\frac{(2+5x)^5}{5\times5}+\frac{(2-5x)^5}{5\times5}+c$ Condone missing $+c$	1.1b	A1F	
	Subtotal		**2**	

June 2022 Question 15 Paper 1

15 (a) Given that

$$y = \operatorname{cosec} \theta$$

15 (a) (i) Express y in terms of $\sin \theta$.

[1 mark]

15 (a) (ii) Hence, prove that

$$\frac{dy}{d\theta} = -\operatorname{cosec} \theta \cot \theta$$

[3 marks]

15 (a) (iii) Show that

$$\frac{\sqrt{y^2 - 1}}{y} = \cos \theta \qquad \text{for } 0 < \theta < \frac{\pi}{2}$$

[3 marks]

15 (b) (i) Use the substitution

$$x = 2 \operatorname{cosec} u$$

to show that

$$\int \frac{1}{x^2\sqrt{x^2 - 4}} dx \qquad \text{for } x > 2$$

can be written as

$$k \int \sin u \, du$$

where k is a constant to be found.

[6 marks]

15 (b) (ii) Hence, show

$$\int \frac{1}{x^2\sqrt{x^2 - 4}} dx = \frac{\sqrt{x^2 - 4}}{4x} + c \qquad \text{for } x > 2$$

where c is a constant.

[3 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
15(a)(i)	States $(\sin\theta)^{-1}$ or $\frac{1}{\sin\theta}$ $\sin^{-1}\theta$ scores B0 Ignore $\sin^{-1}\theta$ if a correct expression has already been written	1.2	B1	$\frac{1}{\sin\theta}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
15(a)(ii)	Uses chain rule or quotient rule to obtain $\pm k(\sin\theta)^{-2}\cos\theta$ OE or Multiplies and uses product rule and implicit differentiation to obtain $\pm k(\sin\theta)^{-2}\cos\theta$ This mark can be awarded for using $\frac{1}{\cos\theta}$ and differentiating to obtain $\pm k(\cos\theta)^{-2}\sin\theta$ OE Ignore wrong or missing angles	3.1a	M1	
	Obtains $-(\sin\theta)^{-2}\cos\theta$ OE	1.1b	A1	$\frac{dy}{d\theta} = -(\sin\theta)^{-2}\cos\theta$
	Completes **rigorous** argument to show the given result. Must either see separated fractions before final line or sight of $-\frac{\cot\theta}{\sin\theta}$ or $-\frac{1}{\tan\theta\sin\theta}$ or $-\frac{\cos\theta}{\sin\theta}\times\operatorname{cosec}\theta$ or Makes clear use of stated identities as part of the solution At some point the solution must have included $\frac{dy}{d\theta} =$ AG Condone change of order of functions at the end	2.1	R1	$= -\frac{\cos\theta}{\sin\theta}\times\frac{1}{\sin\theta}$ $= -\operatorname{cosec}\theta\cot\theta$
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
15(a)(iii)	Substitutes $y = \text{cosec}\,\theta$ OE or Draws a right angled triangle labelling hypotenuse as y and opposite as 1 PI by obtaining y^2 or $\frac{1}{y^2}$ in terms of $\cos\theta$	1.1b	B1	$\frac{\sqrt{\text{cosec}^2\,\theta - 1}}{\text{cosec}\,\theta}$
	Uses $\text{cosec}^2\,\theta - 1 = \cot^2\theta$ OE or Uses Pythagoras theorem to find missing adjacent side in the right angled triangle or Obtains $\cos^2\theta$ in terms of y	1.1a	M1	$= \frac{\sqrt{\cot^2\theta}}{\text{cosec}\,\theta}$ $= \frac{\cot\theta}{\text{cosec}\,\theta}$
	Completes **rigorous** argument to show the given result This must include **clear replacement** of $\text{cosec}\,\theta$ and $\cot\theta$ **within the solution** using only sine and cosine functions to complete the argument prior to obtaining the answer given AG	2.1	R1	$= \frac{\cos\theta}{\sin\theta} \times \sin\theta$ $= \cos\theta$
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
15(b)(i)	Obtains $\frac{dx}{du} = -2\operatorname{cosec} u \cot u$ OE	1.1b	B1	$\frac{dx}{du} = -2\operatorname{cosec} u \cot u$ $dx = -2\operatorname{cosec} u \cot u \;\; du$ $= \int \frac{-2\operatorname{cosec} u \cot u}{4\operatorname{cosec}^2 u\sqrt{4\operatorname{cosec}^2 u - 4}} du$ $= \int \frac{-2\operatorname{cosec} u \cot u}{4\operatorname{cosec}^2 u\sqrt{4\cot^2 u}} du$ $= \int \frac{-2\operatorname{cosec} u \cot u}{4\operatorname{cosec}^2 u \times 2\cot u} du$ $= \int -\frac{1}{4\operatorname{cosec} u} du$ $= -\frac{1}{4}\int \sin u \;\; du$
	Makes complete substitution to obtain integrand of the form $\frac{P\operatorname{cosec} u \cot u}{Q\operatorname{cosec}^2 u\sqrt{R\operatorname{cosec}^2 u - 4}}$ OE or $\frac{P\operatorname{cosec} u \cot u}{Q\operatorname{cosec}^3 u\sqrt{1 - R\sin^2 u}}$ OE Ignore wrong or missing angles	1.1a	M1	
	Obtains correct integrand $\frac{-2\operatorname{cosec} u \cot u}{4\operatorname{cosec}^2 u\sqrt{4\operatorname{cosec}^2 u - 4}}$ or $\frac{-2\operatorname{cosec} u \cot u}{8\operatorname{cosec}^3 u\sqrt{1 - \sin^2 u}}$ OE	1.1b	A1	
	Uses appropriate Pythagorean-trig identity under the square root. Either $1 + \cot^2 u = \operatorname{cosec}^2 u$ or $1 - \sin^2 u = \cos^2 u$ Ignore wrong or missing angles	3.1a	M1	
	Obtains $k\int \sin u \;\; du$ with no errors seen in any trig identities Must have u and du	1.1b	A1F	
	Obtains $k = -\frac{1}{4}$ OE CSO	2.1	R1	
	Subtotal		**6**	

Q	Marking instructions	AO	Marks	Typical solution
15(b)(ii)	Integrates $\int \sin u \; \mathrm{d}u$ to obtain $-\cos u$	1.1b	B1	$-\frac{1}{4}\int \sin u \; \mathrm{d}u = \frac{1}{4}\cos u + c$ $= \frac{1}{4}\frac{\sqrt{\left(\frac{x}{2}\right)^2 - 1}}{\left(\frac{x}{2}\right)} + c$ $= \frac{\sqrt{\frac{x^2-4}{4}}}{2x} + c$ $= \frac{\sqrt{x^2-4}}{4x} + c$
	Deduces $\cos u = \frac{\sqrt{\left(\frac{x}{2}\right)^2 - 1}}{\left(\frac{x}{2}\right)}$ OE	2.2a	M1	
	Completes **reasoned** argument to show given result Must have $+c$ throughout Validation by starting with $\frac{\sqrt{x^2-4}}{4x}$ and replacing x with $2\operatorname{cosec} u$ to achieve $\frac{1}{4}\cos u$ scores a maximum of B1M1R0	2.1	R1	
	Subtotal		**3**	

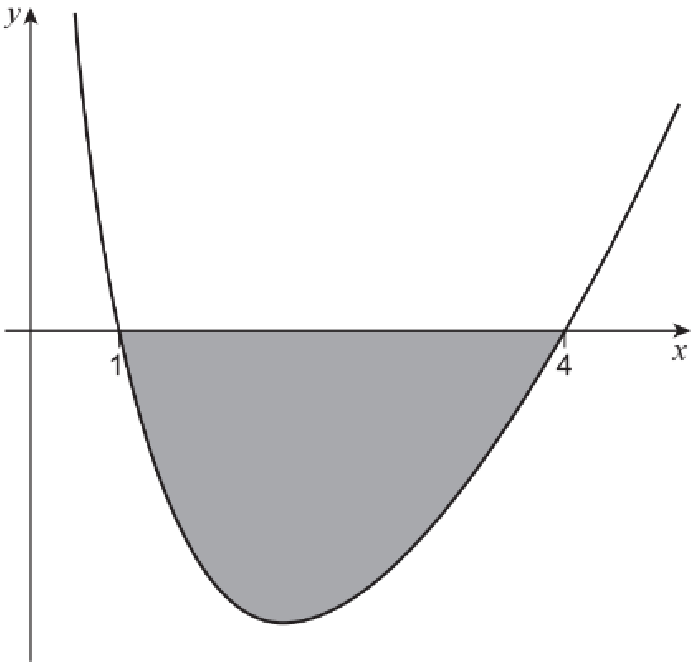
	Question 15 Total		**16**	

June 2022 Question 14 Paper 1

14 The region bounded by the curve

$$y = (2x - 8)\ln x$$

and the x-axis is shaded in the diagram below.



14 (a) Use the trapezium rule with **5 ordinates** to find an estimate for the area of the shaded region.

Give your answer correct to three significant figures.

[3 marks]

14 (b) Show that the exact area is given by

$$32\ln 2 - \frac{33}{2}$$

Fully justify your answer.

[6 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
14(a)	Finds positive or negative y-values for 5 x-values with h = 0.75 PI by AWRT 5.28 or AWRT – 5.28 or Uses 6 x-values and obtains AWRT 5.42 or AWRT – 5.42 In this case maximum mark is M1A0 A0	1.1a	M1	<table><tr><th>x_n</th><th>y_n</th></tr><tr><td>1</td><td>0</td></tr><tr><td>1.75</td><td>-2.51827</td></tr><tr><td>2.5</td><td>-2.74887</td></tr><tr><td>3.25</td><td>-1.76798</td></tr><tr><td>4</td><td>0</td></tr></table> $\frac{0.75}{2}(0+0+2(-2.51827-2.74887-1.76798))$ Area ≈ 5.28
	Uses the trapezium rule correctly with h = 0.75 and correct y-values Accept rounded or truncated values to 3 significant figures.	1.1b	A1	
	Obtains AWRT 5.28 Condone AWRT – 5.28	3.2a	A1	
	Subtotal		**3**	

June 2022 Question 5 Paper 2 (A-Level)

5 The binomial expansion of $(2+5x)^4$ is given by

$$(2+5x)^4 = A + 160x + Bx^2 + 1000x^3 + 625x^4$$

5 (a) Find the value of A and the value of B.

[2 marks]

5 (b) Show that

$$(2+5x)^4 - (2-5x)^4 = Cx + Dx^3$$

where C and D are constants to be found.

[2 marks]

5 (c) Hence, or otherwise, find

$$\int \left((2+5x)^4 - (2-5x)^4 \right) \mathrm{d}x$$

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Obtains 16 Not incorrectly labelled	1.1b	B1	$(2+5x)^4 = 16+160x+600x^2 +1000x^3+625x^4$ A = 16 B = 600
	Obtains 600 Not incorrectly labelled	1.1b	B1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)	Obtains the expansion of $(2-5x)^4 =$ $A-160x+Bx^2-1000x^3+625x^4$ Accept A and B unsubstituted or their A and B Or Uses a valid method and obtains one of $C=320$ or $D=2000$	1.1a	M1	$(2+5x)^4-(2-5x)^4$ $=16+160x+600x^2+1000x^3+625x^4$ $-\left(16-160x+600x^2-1000x^3+625x^4\right)$ $=320x+2000x^3$
	Completes reasoned argument to show $(2+5x)^4-(2-5x)^4=320x+2000x^3$ Accept A and B unsubstituted or their A and B Must finish with $320x+2000x^3$ don't accept just C=320 and D = 2000	2.1	R1F	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(c)	Integrates one term correctly Accept C and D unsubstituted or their C and D Or Uses reverse of chain rule to obtain at least one term of the form $P(2\pm5x)^5$, $P=\pm\frac{1}{5}$ or $\pm\frac{1}{25}$	1.1a	M1	$\int\left((2+5x)^4-(2-5x)^4\right)\mathrm{d}x$ $=\int\left(320x+2000x^3\right)\mathrm{d}x$ $=160x^2+500x^4+c$
	Obtains $\frac{320}{2}x^2+\frac{2000}{4}x^4+c$ FT C and D unsubstituted or their C and D Or $\frac{(2+5x)^5}{5\times5}+\frac{(2-5x)^5}{5\times5}+c$ Condone missing $+c$	1.1b	A1F	
	Subtotal		**2**	

June 2022 Question 10 Paper 2

10 A gardener has a greenhouse containing 900 tomato plants.

The gardener notices that some of the tomato plants are damaged by insects.

Initially there are 25 damaged tomato plants.

The number of tomato plants damaged by insects is increasing by 32% each day.

10 (a) The total number of plants damaged by insects, x, is modelled by

$$x = A \times B^t$$

where A and B are constants and t is the number of days after the gardener first noticed the damaged plants.

10 (a) (i) Use this model to find the total number of plants damaged by insects 5 days after the gardener noticed the damaged plants.

[3 marks]

10 (a) (ii) Explain why this model is not realistic in the long term.

[2 marks]

10 (b) A refined model assumes the rate of increase of the number of plants damaged by insects is given by

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{x(900 - x)}{2700}$$

10 (b) (i) Show that

$$\int \left(\frac{A}{x} + \frac{B}{900 - x} \right) \mathrm{d}x = \int \mathrm{d}t$$

where A and B are positive integers to be found.

[3 marks]

10 (b) (ii) Hence, find t in terms of x.

[5 marks]

10 (b) (iii) Hence, find the number of days it takes from when the damage is first noticed until half of the plants are damaged by the insects.

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
10(a)(i)	Forms correct model Or applies repeated percentage increase 4 times PI by AWRT 75.9	3.3	B1	$x = 25 \times 1.32^t$ $t = 5 \Rightarrow x = 25 \times 1.32^5 = 100.18...$
	Substitutes t = 5 into their model Or Applies repeated percentage increase 5 times	3.4	M1	
	Obtains 101 Condone 100 CAO	3.2a	A1	x = 101
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
10(a)(ii)	Explains that the **model** grows **exponentially** Must refer to model and exponential	3.5b	E1	The value predicted by the exponential model will grow without limit.
	Refers to 900 plants. eg this can't be true as there are only 900 tomato plants Condone reference to "tomato(es)" or "plants" in place of "tomato plants"	3.5a	E1	This can't be true as there are only 900 tomato plants in the greenhouse.
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)(i)	Rearranges to obtain one of the following: $\frac{P}{x(900-x)}\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{1}{Q}$ $\frac{P}{x(900-x)}\mathrm{d}x=\frac{1}{Q}\mathrm{d}t$ $\frac{P}{x(900-x)}=\frac{1}{Q}\frac{\mathrm{d}t}{\mathrm{d}x}$ where $P\times Q=2700$ If their P=2700 no need to see explicit $\frac{1}{Q}$ with dt, or $\frac{\mathrm{d}t}{\mathrm{d}x}$ May include integral signs	3.1a	B1	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{x(900-x)}{2700}$ $\frac{2700}{x(900-x)}\frac{\mathrm{d}x}{\mathrm{d}t}=1$ $\int\left(\frac{A}{x}+\frac{B}{900-x}\right)dx=\int dt$ $\frac{2700}{x(900-x)}\equiv\frac{A}{x}+\frac{B}{900-x}$ $2700\equiv A(900-x)+Bx$ $x=0\Rightarrow A=\frac{2700}{900}=3$ $x=900\Rightarrow B=3$ $\therefore$ $\int\left(\frac{3}{x}+\frac{3}{900-x}\right)dx=\int dt$
	Forms partial fraction equation with correct denominators and uses an appropriate method to find their numerators PI by correct A and B without incorrect working	3.1a	M1	
	Obtains correct A and B and concludes with $\int\left(\frac{3}{x}+\frac{3}{900-x}\right)dx=\int dt$ Accept $\int 1dt$ Condone missing brackets	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)(ii)	Integrates to obtain $\ln x$ or $\pm\ln(900-x)$ Condone missing brackets for this mark	3.1a	M1	
	Integrates to obtain $\ln x$ and $\pm\ln(900-x)$ Condone missing brackets for this mark	1.1a	M1	
	Integrates to obtain $3(\ln x-\ln(900-x))+c=t$ OE Condone missing $+c$	1.1b	A1	$3(\ln x-\ln(900-x))+c=t$ $3(\ln 25-\ln(900-25))+c=0$ $c=10.67$
	Uses t = 0, x= 25, to obtain a value for c	3.4	M1	$t=3(\ln x-\ln(900-x))+10.67$
	Obtains correct equation for t in terms of x ACF If c given as a decimal accept AWRT 11 eg $t=3(\ln x-\ln(900-x))+3\ln 35$ $t=3\ln\left(\dfrac{35x}{900-x}\right)$	1.1b	A1	
	Subtotal		**5**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)(iii)	Substitutes x = 450 into their model from part (b)(ii)	3.4	M1	$3(ln\,450-ln(450))+10.67$ $=10.67\ldots$
	Obtains 11 CAO	3.2a	A1	It takes 11 days from when the damage is first noticed until half of the plants are damaged by insects
	Subtotal		**2**	

November 2021 Question 11 Paper 1

11 A curve, C, passes through the point with coordinates (1, 6)

The gradient of C is given by

$$\frac{dy}{dx} = \frac{1}{6}(xy)^2$$

Show that C intersects the coordinate axes at exactly one point and state the coordinates of this point.

Fully justify your answer.

[8 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
11	Separates variables. To obtain an equation of the form $\int \frac{A}{y^2}\mathrm{d}y = \int Bx^2\mathrm{d}x$ Or $\int \frac{A}{y^2}\frac{\mathrm{d}y}{\mathrm{d}x}\mathrm{d}x = \int Bx^2\mathrm{d}x$ Must have integral signs, and consistent dy and dx Condone x instead of x^2	3.1a	M1	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{6}(xy)^2$ $\frac{1}{y^2}\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{6}x^2$ $\int \frac{1}{y^2}\mathrm{d}y = \int \frac{1}{6}x^2\mathrm{d}x$ $-y^{-1} = \frac{x^3}{18} + c$ $(1,6) \Rightarrow -\frac{1}{6} = \frac{1}{18} + c$
	Integrates one of their integrals of the form above correctly.	1.1a	M1	$\Rightarrow c = -\frac{2}{9}$
	Obtains correct integrated equation. Condone missing +c	1.1b	A1	y cannot equal zero in
	Substitutes $(1,6)$ to determine their constant of integration.	1.1a	M1	$-y^{-1} = \frac{x^3}{18} - \frac{2}{9}$
	Explains why y cannot equal zero follow through their equation in which y cannot be zero.	2.4	E1F	as y^{-1} is undefined at $y = 0$ therefore C does not intersect the x-axis
	Substitutes $x = 0$ into their integrated equation to obtain the y-intercept.	3.1a	M1	
	Obtains y = 4.5 Correctly deduces and states that that C intersects the axes at (exactly/only) one point. Also, states the coordinate $(0, 4.5)$ Also, must have stated that C does not intersect the x-axis CSO	1.1b 2.2a	A1 R1	$x = 0 \Rightarrow -y^{-1} = -\frac{2}{9} \Rightarrow y = \frac{9}{2}$ Hence the curve crosses the y-axis at $(0, 4.5)$
	Total		**8**	

November 2021 Question 15 Paper 1

15 (a) Show that

$$\sin x - \sin x \cos 2x \approx 2x^3$$

for small values of x.

[3 marks]

15 (b) Hence, show that the area between the graph with equation

$$y = \sqrt{8(\sin x - \sin x \cos 2x)}$$

the positive x-axis and the line $x = 0.25$ can be approximated by

$$\text{Area} \approx 2^m \times 5^n$$

where m and n are integers to be found.

[4 marks]

15 (c) (i) Explain why

$$\int_{6.3}^{6.4} 2x^3 \, dx$$

is **not** a suitable approximation for

$$\int_{6.3}^{6.4} (\sin x - \sin x \cos 2x) \, dx$$

[1 mark]

15 (c) (ii) Explain how

$$\int_{6.3}^{6.4} (\sin x - \sin x \cos 2x) \, dx$$

may be approximated by

$$\int_{a}^{b} 2x^3 \, dx$$

for suitable values of a and b.

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
15(a)	Uses small angle approximation for sine at least once.	1.1b	B1	$\sin x - \sin x\cos 2x \approx x - x\left(1-\frac{(2x)^2}{2}\right)$ $\approx x - x + x\frac{4x^2}{2}$ $\approx 2x^3$
	Replaces $\cos 2x$ with $1-\frac{(2x)^2}{2}$ Or Used double angle identity and small angle approximations Condone a sign error or missing brackets.	3.1a	M1	
	Completes rigorous argument to show the given result. Condone "=" instead of "$\approx$"	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
15(b)	Forms an integral of the form $\int_0^{0.25} y\,\mathrm{d}x$ or better where y is their $\sqrt{8\times 2x^3}$. Condone missing limits and $\mathrm{d}x$.	3.1a	M1	$Area \approx \int_0^{0.25} \sqrt{8\times 2x^3}\,\mathrm{d}x$ $= 4\int_0^{0.25} x^{3/2}\,\mathrm{d}x$ $= 4\left[\frac{2x^{5/2}}{5}\right]_0^{0.25}$ $= \frac{8}{5}\times 0.25^{5/2}$ $= \frac{8}{5}\times\left(\frac{1}{2}\right)^5$ $= 2^{-2}\times 5^{-1}$
	Simplifies integrand to $Bx^{3/2}$	1.1a	M1	
	Integrates their integrand of the form $Bx^{3/2}$ correctly	1.1b	A1F	
	Substitutes correct limits and completes argument to obtain correct approximation in correct form.	2.1	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
15(c)(i)	Explains that the limits or 6.4 and 6.3 are not small.	2.4	E1	The approximation is only valid for small values of x and 6.3 and 6.4 are not small.
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
15(c)(ii)	Explains how the limits can be changed. Examples of reasoning could include: $\sin x - \sin x \cos 2x$ is periodic OE (has a period of 2π PI) evaluating the integral over a different interval will result in the same value. Reduce/shift the limits by 2π The graph repeats Uses a substitution to bring the limits within an acceptable interval.	2.4	E1	$\sin x - \sin x \cos 2x$ repeats so evaluate the integral over a different interval. Use small values $a = 6.3 - 2\pi$ and $b = 6.4 - 2\pi$ to obtain a valid approximation.
	Deduces $a = 6.3 - 2\pi$ = AWRT 0.017 and $b = 6.4 - 2\pi$ = AWRT 0.117	2.2a	R1	
	Subtotal		**2**	

November 2021 Question 8 Paper 3

8 Given that

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} x\cos x \, \mathrm{d}x = a\pi + b$$

find the exact value of a and the exact value of b.

Fully justify your answer.

[6 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
8	Uses integration by parts with $u = x$ and $v' = \cos x$ **PI** by $x\sin x + \cos x$	3.1a	B1	$u = x \qquad u' = 1$ $v' = \cos x \qquad v = \sin x$ $\int x\cos x\,\mathrm{d}x = x\sin x - \int \sin x\,\mathrm{d}x$ $= x\sin x + \cos x$ $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} x\cos x\,\mathrm{d}x = \left[x\sin x + \cos x\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ $= \frac{\pi}{3}\sin\frac{\pi}{3} + \cos\frac{\pi}{3} - \left(\frac{\pi}{4}\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right)$ $= \pi\frac{\sqrt{3}}{6} + \frac{1}{2} - \left(\pi\frac{\sqrt{2}}{8} + \frac{\sqrt{2}}{2}\right)$ $= \left(\frac{4\sqrt{3} - 3\sqrt{2}}{24}\right)\pi + \left(\frac{1-\sqrt{2}}{2}\right)$
	Applies integration by parts formula correctly by substituting their u, u', v and v' **PI** by $x\sin x + \cos x$	1.1a	M1	
	Obtains $x\sin x + \cos x$ **CAO**	1.1b	A1	
	Substitutes limits correctly into their integrated expression **PI** by correct a and b	1.1a	M1	
	Uses correct exact value for any one of $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ or $\cos\frac{\pi}{3} = \frac{1}{2}$ or $\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ or $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ **PI** by correct a or b	1.2	B1	
	Obtains correct exact values of a and b **ACF** Ignore if 0.14(…) seen subsequently	1.1b	A1	
	Total		**6**	

Arancha Ruiz

November 2020 Question 6 Paper 1

6 Four students, Tom, Josh, Floella and Georgia are attempting to complete the indefinite integral

$$\int \frac{1}{x}\,\mathrm{d}x \qquad \text{for } x > 0$$

Each of the students' solutions is shown below:

Tom	$\int \frac{1}{x}\,\mathrm{d}x = \ln x$
Josh	$\int \frac{1}{x}\,\mathrm{d}x = k\ln x$
Floella	$\int \frac{1}{x}\,\mathrm{d}x = \ln Ax$
Georgia	$\int \frac{1}{x}\,\mathrm{d}x = \ln x + c$

6 (a) (i) Explain what is wrong with Tom's answer.

[1 mark]

6 (a) (ii) Explain what is wrong with Josh's answer.

[1 mark]

6 (b) Explain why Floella and Georgia's answers are equivalent.

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
6(a)(i)	Explains that Tom's solution does not include an arbitrary constant Accept Tom forgot the +c There is no constant on the RHS	2.4	E1	Tom's solution has no constant of integration
	Subtotal		**1**	
6(a)(ii)	Explains that the constant is in the wrong place or Explains that the k should not be there or that $k = 1$ or Shows that differentiating does not give $\frac{1}{x}$ or The constant has been multiplied instead of being added or It should be $\ln kx$ not $k \ln x$	2.4	E1	Although there is a constant, it is in the wrong place
	Subtotal		**1**	
6(b)	Rewrites $ln\, Ax$ as $\ln A + \ln x$ Condone use of any letter for A to demonstrate the log rule used Condone use of log without a specified base	1.1a	M1	ln Ax = $\ln A + \ln x$ This is equivalent as c = ln A
	Deduces explicitly that c = ln A clearly demonstrating equivalence OE	2.2a	R1	
	Subtotal		**2**	
	Question Total		**4**	

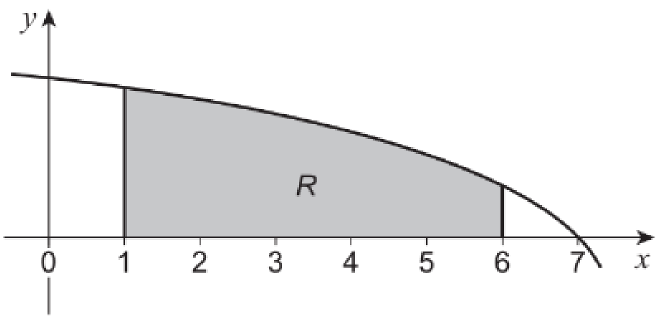
November 2020 Question 11 Paper 1

11 The region R enclosed by the lines $x = 1$, $x = 6$, $y = 0$ and the curve

$$y = \ln(8 - x)$$

is shown shaded in **Figure 3** below.

Figure 3



All distances are measured in centimetres.

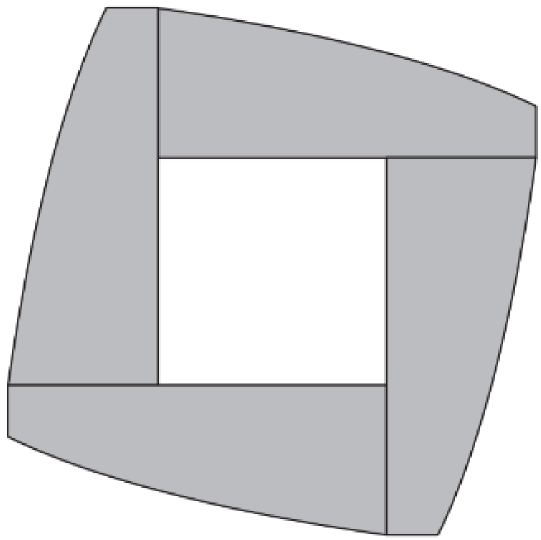
11 (a) Use a single trapezium to find an approximate value of the area of the shaded region, giving your answer in cm^2 to two decimal places.

[2 marks]

Arancha Ruiz

11 (b) Shape *B* is made from four copies of region *R* as shown in **Figure 4** below.

Figure 4



Shape *B* is cut from metal of thickness 2 mm

The metal has a density of 10.5 g/cm^3

Use the trapezium rule with **six** ordinates to calculate an approximate value of the mass of Shape *B*.

Give your answer to the nearest gram.

[5 marks]

11 (c) Without further calculation, give one reason why the mass found in part **(b)** may be:

11 (c) (i) an underestimate.

[1 mark]

11 (c) (ii) an overestimate.

[1 mark]

ANSWER

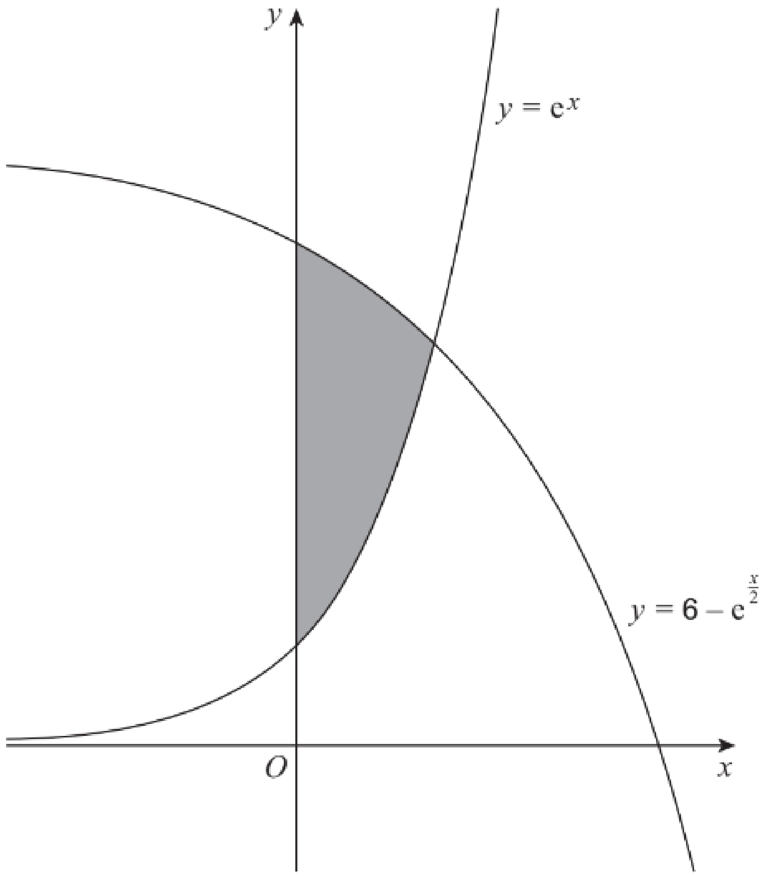
<table>
<tr><th>Q</th><th>Marking Instructions</th><th>AO</th><th>Marks</th><th>Typical Solution</th></tr>
<tr><td rowspan="2">11(a)</td><td>Evaluates f(1) and f(6) using exact logs or decimals
Award if seen embedded in calculations using more than one trapezium</td><td>1.1a</td><td>M1</td><td rowspan="2">f(1) = 1.945910149..
f(6)= 0.69314718..
A = $\frac{5}{2}$ (1.9459+0.6931)
= 6.5976…
=6.60 cm^2</td></tr>
<tr><td>Evaluates an approximate value of the area of R
AWRT 6.60
Condone omission of units</td><td>1.1b</td><td>A1</td></tr>
<tr><td></td><td>Subtotal</td><td></td><td>2</td><td></td></tr>
<tr><td rowspan="5">11(b)</td><td>Writes or uses the six ordinates as ln 7, ln 6, ln 5, ln 4, ln 3 ln 2
or
Obtains the values of the correct six ordinates in decimal form</td><td>1.1b</td><td>B1</td><td rowspan="2"><table><tr><th>x</th><th>f(x)</th></tr><tr><td>1</td><td>1.9459</td></tr><tr><td>2</td><td>1.7918</td></tr><tr><td>3</td><td>1.6094</td></tr><tr><td>4</td><td>1.3863</td></tr><tr><td>5</td><td>1.0986</td></tr><tr><td>6</td><td>0.6931</td></tr></table>Area = $\frac{1}{2}$x1x(1.9459+0.6931+2 (1.7918+1.6094+1.3863+1.0986))</td></tr>
<tr><td>Uses the correct formula for the trapezium rule with their six ordinates and h = 1
Award this mark if seven ordinates used with $h = \frac{5}{6}$
Answer for seven = 7.2145648..</td><td>1.1a</td><td>M1</td></tr>
<tr><td>Evaluates an approximate value for the area of R.
Must have used six ordinates
AWRT 7.2
PI by correct final answer</td><td>1.1b</td><td>A1</td><td rowspan="3">Area= 7.205633 cm^2

Volume of Shape B
= 4 x 7.205633 x 0.2
=5.7645… cm^3

Mass of Shape B =
5.7645cm^3 x 10.5 g/cm^3
=60.52731 g
=61 g</td></tr>
<tr><td>Forms an expression for the mass of either one section or all four sections using ‘their’ area and consistent units
PI by correct final answer</td><td>3.1b</td><td>M1</td></tr>
<tr><td>Obtains an approximate value for the correct mass of Shape B
Must state units
If seven ordinates used this mark can be awarded as answer would be 61g
CAO</td><td>3.2a</td><td>A1</td></tr>
<tr><td></td><td>Subtotal</td><td></td><td>5</td><td></td></tr>
<tr><td>11(c)(i)</td><td>Explains that the trapezia are all below the curve
or
Explains that the curve is concave
or
Draws a diagram and indicates the gaps</td><td>3.5a</td><td>E1</td><td>The trapezia are all below the curve</td></tr>
<tr><td></td><td>Subtotal</td><td></td><td>1</td><td></td></tr>
<tr><td>11(c)(ii)</td><td>Explains that numbers have been rounded</td><td>3.5a</td><td>E1</td><td>Numbers in the calculation have been rounded</td></tr>
<tr><td></td><td>Subtotal</td><td></td><td>1</td><td></td></tr>
<tr><td></td><td>Question Total</td><td></td><td>9</td><td></td></tr>
</table>

November 2020 Question 14 Paper 1

15 The region enclosed between the curves $y = e^x$, $y = 6 - e^{\frac{x}{2}}$ and the line $x = 0$ is shown shaded in the diagram below.



Show that the exact area of the shaded region is

$$6 \ln 4 - 5$$

Fully justify your answer.

[10 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
15	Forms a single equation eliminating x or y	3.1a	M1	$6 - e^{\frac{x}{2}} = e^x$
	Obtains a correct rearranged quadratic equation. Either $e^x + e^{\frac{x}{2}} - 6 = 0$ or $\left(e^{\frac{x}{2}} + 3\right)\left(e^{\frac{x}{2}} - 2\right) = 0$ or $e^x + e^{\frac{x}{2}} + \frac{1}{4} = \frac{25}{4}$ OE	1.1b	A1	$e^x + e^{\frac{x}{2}} - 6 = 0$ $\left(e^{\frac{x}{2}} + 3\right)\left(e^{\frac{x}{2}} - 2\right) = 0$ $e^{\frac{x}{2}} = -3 \text{ or } 2$
	Solves 'their' quadratic Must be a quadratic in $e^{\frac{x}{2}}$ or If squaring is used then it must be a quadratic in e^x or Obtains x = 1.386	1.1a	M1	$e^{\frac{x}{2}} > 0$ so -3 is not a valid solution $\frac{x}{2}$ = ln 2 $x = 2\ln 2 = \ln 4$
	Explains that $e^{\frac{x}{2}} = -3$ is not valid as $e^{\frac{x}{2}} > 0$ or If squaring is used they must clearly check both solutions by substituting and conclude that $\ln 9$ is not valid OE	2.4	E1F	$\int_0^{\ln 4} (6 - e^{\frac{x}{2}} - e^x)\, dx$ = $\left[6x - 2e^{\frac{x}{2}} - e^x\right]_0^{\ln 4}$
	Obtains $x = 2\ln 2$ or $x = \ln 4$	1.1b	A1	= $\left(6\ln 4 - 2e^{\frac{\ln 4}{2}} - e^{\ln 4}\right)$ - (- 2 - 1)
	Forms any definite integral which would contribute to finding the required area This could be $\int_0^{\ln 4} (6 - e^{\frac{x}{2}} - e^x)\, dx$ or $\int_0^{\ln 4} (6 - e^{\frac{x}{2}})\, dx$ or $\int_0^{\ln 4} e^x dx$ or $\int_0^{\ln 4} (e^x + e^{\frac{x}{2}} - 6)\, dx$ Follow through '*their*' value of x for the upper limit	1.1a	M1	= 6 ln 4 – 4 – 4 + 3 = 6 ln 4 – 5
	Forms a fully correct definite integral (or integrals) which would lead to evaluating the correct area Follow through '*their*' incorrect upper limit	3.1a	A1F	

	Integrates '*their*' expressions involving exponentials fully correctly Follow through their exponential expressions – but must have integrated both e^x and $e^{\frac{x}{2}}$ terms Condone missing/incorrect limits	1.1b	B1F	
	Substitutes 0 and '*their*' upper limits into '*their*' integrated expression Must correctly use F (their upper limit) – F (0) for each integral	1.1a	M1	
	Completes rigorous argument by showing explicit evaluation of exponential terms before obtaining final answer AG This mark can be achieved without achieving the E1 mark	2.1	R1	
	Total		**10**	

Video solution:

https://youtu.be/j7A5g7Qcf1A

November 2020 Question 5 Paper 2

5 Use integration by substitution to show that

$$\int_{-\frac{1}{4}}^{6} x\sqrt{4x+1}\,dx = \frac{875}{12}$$

Fully justify your answer.

[6 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
5	Uses a suitable substitution $u = 4x+1$ or $u = \sqrt{4x+1}$ OE	3.1a	M1	$u = 4x+1 \Rightarrow \frac{du}{dx} = 4$ $x = -\frac{1}{4} \Rightarrow u = 0$ $x = 6 \Rightarrow u = 25$ $x = \frac{u-1}{4}$ $\int_{-\frac{1}{4}}^{6} x\sqrt{4x+1}\,dx = \int_{0}^{25} \frac{u-1}{4}\sqrt{u}\,\frac{1}{4}du$ $= \frac{1}{16}\int_{1}^{25} u^{\frac{3}{2}} - u^{\frac{1}{2}}du$ $= \frac{1}{16}\left[\frac{2u^{\frac{5}{2}}}{5} - \frac{2u^{\frac{3}{2}}}{3}\right]_0^{25}$ $= \frac{1}{8}\left(\frac{25^{\frac{5}{2}}}{5} - \frac{25^{\frac{3}{2}}}{3}\right)$ $= \frac{875}{12}$
	Differentiates their substitution correctly	1.1b	A1	
	Completes substitution to obtain correct integrand for their suitable substitution. Can be unsimplified.	1.1a	M1	
	Correctly integrates their simplified integrand provided it is of the form $A\left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)$ or $B\left(u^4 - u^2\right)$	1.1a	A1	
	Substitutes correct limits for their substitution or 6 and -1/4 for x	1.1a	M1	
	Completes rigorous argument to show the required result. AG	2.1	A1	
	Total		**6**	

June 2019 Question 16 Paper 1

16 (a) $y = e^{-x}(\sin x + \cos x)$

Find $\dfrac{dy}{dx}$

Simplify your answer.

[3 marks]

16 (b) Hence, show that

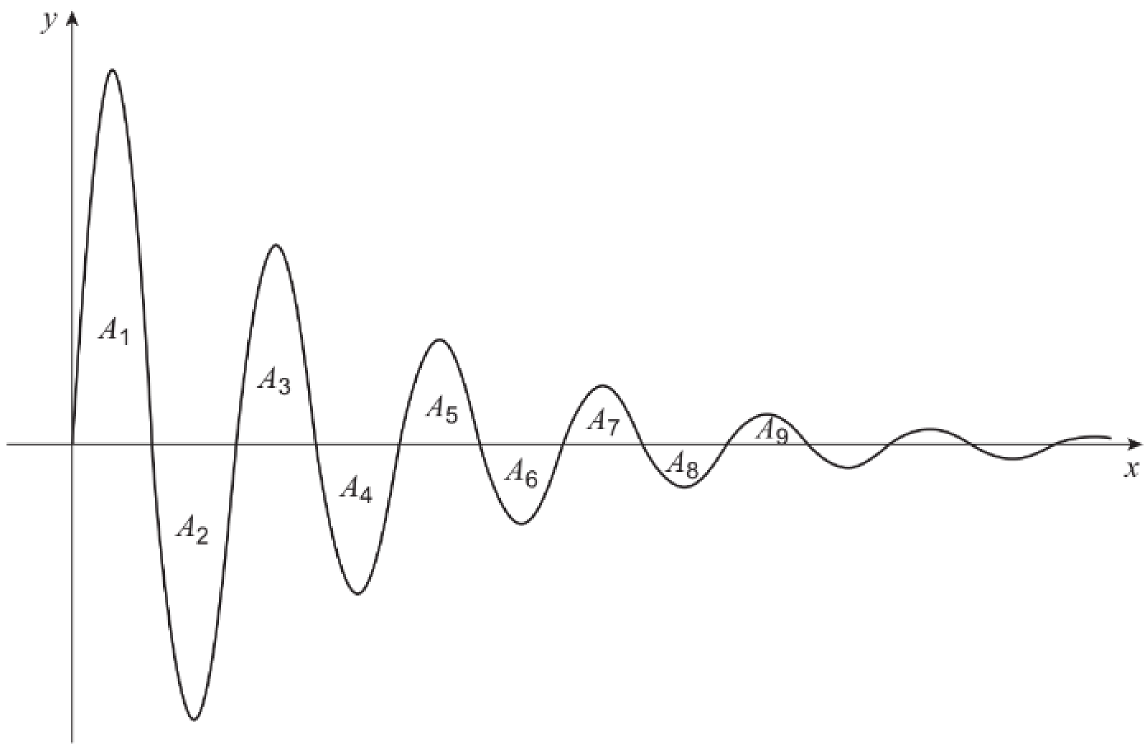
$$\int e^{-x} \sin x \, dx = ae^{-x}(\sin x + \cos x) + c$$

where a is a rational number.

[2 marks]

16 (c) A sketch of the graph of $y = e^{-x} \sin x$ for $x \geq 0$ is shown below.

The areas of the finite regions bounded by the curve and the x-axis are denoted by $A_1, A_2, \ldots, A_n, \ldots$



16 (c) (i) Find the exact value of the area A_1

[3 marks]

16 (c) (ii) Show that

$$\frac{A_2}{A_1} = e^{-\pi}$$

[4 marks]

16 (c) (iii) Given that

$$\frac{A_{n+1}}{A_n} = e^{-\pi}$$

show that the exact value of the total area enclosed between the curve and the x-axis is

$$\frac{1 + e^{\pi}}{2(e^{\pi} - 1)}$$

[4 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
16(a)	Chooses an appropriate technique to differentiate accept any evidence of product rule or quotient rule	3.1a	M1	$\frac{dy}{dx} = -e^{-x}(\sin x + \cos x) + e^{-x}(\cos x - \sin x)$ $= -2e^{-x}\sin x$
	Differentiates fully correctly	1.1b	A1	
	Obtains fully correct simplified answer.	1.1b	A1	
16(b)	Uses their result from (a) in the form of $Be^{-x}\sin x$ showing an understanding of the fundamental theorem of calculus Condone missing constant.	3.1a	M1	$\int(-2e^{-x}\sin x)dx = e^{-x}(\sin x + \cos x) + k$ $\therefore \int(e^{-x}\sin x)dx = -\frac{1}{2}e^{-x}(\sin x + \cos x) + c$
	Obtains $\frac{1}{B}e^{-x}(\sin x + \cos x)$	2.1	R1F	
16 (c)(i)	Writes the area as $\frac{1}{B}\left[e^{-x}(\sin x + \cos x)\right]_0^{\pi}$ Condone missing limits	3.1a	M1	$\int_0^{\pi}(e^{-x}\sin x)dx = -\frac{1}{2}\left[e^{-x}(\sin x + \cos x)\right]_0^{\pi}$ $= -\frac{1}{2}[e^{-\pi}(\sin \pi + \cos \pi)$ $-e^{0}(\sin 0 + \cos 0)]$ $= \frac{e^{-\pi}+1}{2}$
	Deduces correct limits and substitutes correctly	2.2a	A1	
	Obtains correct exact value from correct answer in part(b) CSO	1.1b	A1	
16 (c)(ii)	Substitutes correct limits for A_2 Into their $\frac{1}{B}\left[e^{-x}(\sin x + \cos x)\right]_0^{\pi}$ Or Writes $A_2 = \pm\int_{\pi}^{2\pi}(e^{-x}\sin x)dx$ and uses the substitution $u = x - \pi$	1.1a	M1	$\int_{\pi}^{2\pi}(e^{-x}\sin x)dx = -\frac{1}{2}\left[e^{-x}(\sin x + \cos x)\right]_{\pi}^{2\pi}$ $= -\frac{e^{-\pi}+1}{2}e^{-\pi}$ Area $= \frac{e^{-\pi}+1}{2}e^{-\pi}$ $\frac{A_2}{A_1} = \frac{\frac{e^{-\pi}+1}{2}e^{-\pi}}{\frac{e^{-\pi}+1}{2}} = e^{-\pi}$
	Obtains correct exact area for $\pm A_2$ CSO Or Makes complete substitution $A_2 = \pm\int_0^{\pi}\left(e^{-(u+\pi)}\sin(u+\pi)\right)du$	1.1b	A1	

	Forms required ratio using their exact A1 and A2, may be unsimplified Or Extracts factor of $e^{-\pi}$ and uses $\sin(u+\pi) = -\sin u$ To obtain $A_2 = -e^{-\pi}\int_0^{\pi} e^{-u}\sin u\, du$	1.1a	M1	
	Completes rigorous argument, with correct limits and negatives handled correctly CSO	2.1	R1	
16 (c)(iii)	Deduces that the areas form a geometric series Accept any indication of this being geometric series	2.2a	B1	$\frac{a}{1-r} = \frac{e^{-\pi}+1}{2} \times \frac{1}{1-e^{-\pi}}$ $= \frac{1+e^{\pi}}{2(e^{\pi}-1)}$
	Uses $\frac{A_1}{1-e^{-\pi}}$	3.1a	M1	
	Obtains a value for the geometric series first term using their (c)(i)	1.1a	B1F	
	Completes rigorous argument to achieve required result in correct form. CSO AG	2.1	R1	
	Total		**16**	

Video solution:

https://youtu.be/RNgva0ZnNlk

June 2019 Question 7 Paper 3

7 (a) Express $\dfrac{4x+3}{(x-1)^2}$ in the form $\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}$

[3 marks]

7 (b) Show that

$$\int_3^4 \frac{4x+3}{(x-1)^2}\,\mathrm{d}x = p + \ln q$$

where p and q are rational numbers.

[5 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
7(a)	Forms $4x+3 \equiv A(x-1)+B$	1.1b	B1	$\frac{4x+3}{(x-1)^2} \equiv \frac{A}{x-1} + \frac{B}{(x-1)^2}$ $4x+3 \equiv A(x-1)+B$
	Uses substitution or comparison of coefficients to find their A or B (must have degree of LHS = degree of RHS)	1.1a	M1	Let x = 1 hence B = 7 Let x = 0 then 3 = B – A and hence A = 4
	Obtains correct A and B	1.1b	A1	A = 4 and B = 7
7(b)	Integrates their expression, at least one term correct	3.1a	M1	$\int_3^4 \left(\frac{4}{x-1} + \frac{7}{(x-1)^2}\right) dx$ = $\left[4\ln(x-1) - \frac{7}{x-1}\right]_3^4$ =$\left[4\ln 3 - \frac{7}{3}\right]$ – $\left[4\ln 2 - \frac{7}{2}\right]$ =4ln$\frac{3}{2}$ + $\frac{7}{6}$ =ln$\frac{3^4}{2^4}$ + $\frac{7}{6}$ = ln$\frac{81}{16}$ + $\frac{7}{6}$
	Integrates their expression fully correctly Must be of the form $A\ln(x-1) - \frac{B}{x-1}$ **OE** **FT** their A and B	1.1b	A1F	
	Substitutes limits correctly into their integrated expression	1.1a	M1	
	Uses at least one law of logs correctly	1.1a	M1	
	Completes argument to obtain **correct exact answer** in correct form or stating p = $\frac{7}{6}$ and q = $\frac{81}{16}$ No subsequent incorrect working	2.1	R1	
	Total		**8**	

June 2018 Question 7 Paper 3

7 (a) Express $\frac{4x+3}{(x-1)^2}$ in the form $\frac{A}{x-1}+\frac{B}{(x-1)^2}$

[3 marks]

7 (b) Show that

$$\int_3^4 \frac{4x+3}{(x-1)^2}\,\mathrm{d}x = p + \ln q$$

where p and q are rational numbers.

[5 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
7(a)	Forms $4x+3 \equiv A(x-1)+B$	1.1b	B1	$\frac{4x+3}{(x-1)^2} \equiv \frac{A}{x-1} + \frac{B}{(x-1)^2}$ $4x+3 \equiv A(x-1)+B$
	Uses substitution or comparison of coefficients to find their A or B (must have degree of LHS = degree of RHS)	1.1a	M1	Let x = 1 hence B = 7 Let x = 0 then 3 = B – A and hence A = 4
	Obtains correct A and B	1.1b	A1	A = 4 and B = 7
7(b)	Integrates their expression, at least one term correct	3.1a	M1	$\int_3^4 \left(\frac{4}{x-1} + \frac{7}{(x-1)^2}\right) dx$
	Integrates their expression fully correctly Must be of the form $A\ln(x-1) - \frac{B}{x-1}$ **OE** **FT** their A and B	1.1b	A1F	= $\left[4\ln(x-1) - \frac{7}{x-1}\right]_3^4$ =$\left[4\ln 3 - \frac{7}{3}\right]$ – $\left[4\ln 2 - \frac{7}{2}\right]$
	Substitutes limits correctly into their integrated expression	1.1a	M1	=4ln$\frac{3}{2}$ + $\frac{7}{6}$
	Uses at least one law of logs correctly	1.1a	M1	=ln$\frac{3^4}{2^4}$ + $\frac{7}{6}$
	Completes argument to obtain **correct exact answer** in correct form or stating p = $\frac{7}{6}$ and q = $\frac{81}{16}$ No subsequent incorrect working	2.1	R1	= ln$\frac{81}{16}$ + $\frac{7}{6}$
	Total		**8**	