Pure Year 2 exam questions (A Level maths)-AQA

NOTE:
Please be aware that in the Year 12 collections, you will find questions from the Year 13 papers. However, these questions are intentionally included because they align with Year 12 content and topics. That's the reason why you are missing some questions here, please have a look at the Year 12 collection!
Happy studying!

Ch 1 Proof

June 2022 Question 4 Paper 1

4 Millie is attempting to use proof by contradiction to show that the result of multiplying an irrational number by a non-zero rational number is always an irrational number.

Select the assumption she should make to start her proof.

Tick (✓) **one** box.

[1 mark]

Every irrational multiplied by a non-zero rational is irrational. ☐

Every irrational multiplied by a non-zero rational is rational. ☐

There exists a non-zero rational and an irrational whose product is irrational. ☐

There exists a non-zero rational and an irrational whose product is rational. ☐

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
4	Ticks the correct box	2.1	R1	There exists a non-zero rational and an irrational whose product is rational.
	Total		**1**	

June 2022 Question 6 Paper 2

6 (a) Asif notices that $24^2 = 576$ and $2 + 4 = 6$ gives the last digit of 576

He checks two more examples:

$27^2 = 729$	$29^2 = 841$
$2 + 7 = 9$	$2 + 9 = 11$
Last digit 9	Last digit 1

Asif concludes that he can find the last digit of any square number greater than 100 by adding the digits of the number being squared.

Give a counter example to show that Asif's conclusion is **not** correct.

[2 marks]

6 (b) Claire tells Asif that he should look only at the last digit of the number being squared.

$27^2 = 729$	$24^2 = 576$
$7^2 = 49$	$4^2 = 16$
Last digit 9	Last digit 6

Using Claire's method determine the last digit of 23456789^2

[1 mark]

6 (c) Given Claire's method is correct, use proof by exhaustion to show that no square number has a last digit of 8

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Squares a number with two or more digits and adds its digits. Must be explicit	1.1a	M1	$12^2 = 144$ $1+2=3$ $3 \neq 4$
	Completes argument to show that Asif's method is incorrect Must compare sum of digits with last digit of square number	2.3	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)	Obtains 1	1.1b	B1	1
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
6(c)	Lists at least four single digits and their squares Or Explains why odd digits do not need to be considered	1.1a	M1	$0^2=0$, $1^2=1$, $2^2=4$, $3^2=9$ $4^2=16$, $5^2=25$, $6^2=36$, $7^2=49$ $8^2=64$, $9^2=81$ Therefore, there can be no square number which has a last digit of 8
	Completes rigorous argument to prove that no square number has a last digit of 8 OE CSO	2.1	R1	
	Subtotal		**2**	

June 2022 Question 9 Paper 3

9 Assume that a and b are integers such that

$$a^2 - 4b - 2 = 0$$

9 (a) Prove that a is even.

[2 marks]

9 (b) Hence, prove that $2b + 1$ is even and explain why this is a contradiction.

[3 marks]

9 (c) Explain what can be deduced about the solutions of the equation

$$a^2 - 4b - 2 = 0$$

[1 mark]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Begins argument with either of: • Factorises $-4b-2$ or $4b+2$ correctly to 2b+1 as a factor • States $4b$ and 2 are both even or begins proof by contradiction by assuming a is odd therefore a^2 is odd	2.1	M1	$a^2-4b-2=0$ $a^2=4b+2$ $a^2=2(2b+1)$ Hence a^2 must be even, which means that a must be even
	Completes reasoned argument by deducing that a^2 must be even or has a factor of 2, which means that a must be even or Completes reasoned argument by deducing that a^2 = 4b+2 which is even because $4b$ and 2 are both even hence a^2 is even which is a contradiction OE	2.2a	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)	Uses $2p$ and obtains $(2p)^2$ PI by $4p^2$ Allow any letter for p except a and b	1.1a	M1	$a = 2p \Rightarrow (2p)^2 = 4p^2$ $4p^2 = 2(2b+1)$ $2p^2 = 2b+1$ Hence $2b+1$ must be even $2b+1$ is an odd number which is a contradiction
	Obtains either $4p^2 = 2(2b+1)$ or $4p^2 = 4b+2$ and followed by $2p^2 = 2b+1$ or $4p^2 = 2(2b+1)$ or $4p^2 = 4b+2$ and followed by $2 \times 2p^2 = 2(2b+1)$	3.1a	A1	
	Complete reasoned argument by deducing that $2b + 1$ is even hence contradiction as $2b+1$ is an odd number or Complete reasoned argument by deducing that $2b + 1$ is even hence contradiction as b cannot be an integer	2.2a	R1	
	Subtotal		**3**	

November 2021 Question 4 Paper 1

4 Millie is attempting to use proof by contradiction to show that the result of multiplying an irrational number by a non-zero rational number is always an irrational number.

Select the assumption she should make to start her proof.

Tick (✓) **one** box.

[1 mark]

Every irrational multiplied by a non-zero rational is irrational. ☐

Every irrational multiplied by a non-zero rational is rational. ☐

There exists a non-zero rational and an irrational whose product is irrational. ☐

There exists a non-zero rational and an irrational whose product is rational. ☐

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
4	Ticks the correct box	2.1	R1	There exists a non-zero rational and an irrational whose product is rational.
	Total		**1**	

November 2020 Question 5 Paper 1

5 Prove that, for integer values of n such that $0 \leq n < 4$

$$2^{n+2} > 3^n$$

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
5	Selects and begins to use a suitable method of proof. **Exhaustion:** Must check at least two correct values for n in the range $0 \leq n < 4$ and make at least two correct comparisons. Comparisons are implied by ticks/crosses or use of true/false **Direct proof:** Takes logs to any base of both sides and uses a law of logs correctly once **Contradiction:** Must be clear they are attempting contradiction with "$0 \leq n < 4$ and $2^{n+2} \leq 3^n$" assumed explicitly. Condone strict inequality	3.1a	M1	n | 2^{n+2} | 3^n | 0 | 4 | 1 | 4>1 1 | 8 | 3 | 8>3 2 | 16 | 9 | 16>9 3 | 32 | 27 | 32>27 Hence $2^{n+2} > 3^n$ for integer values of n such that $0 \leq n < 4$
	Completes a reasoned mathematical argument, proving $2^{n+2} > 3^n$ when n is an integer and $0 \leq n < 4$. Must include a fully correct concluding statement which refers to 'integer' or lists the four integers If using direct proof or contradiction they must use the laws of logs correctly to remove n from the exponent. Condone use of equality if direct proof used	2.1	R1	
	Total		**2**	

Typical solution table:

n	2^{n+2}	3^n	
0	4	1	4>1
1	8	3	8>3
2	16	9	16>9
3	32	27	32>27

November 2020 Question 7 Paper 2

7 a and b are two positive irrational numbers.

The sum of a and b is rational.

The product of a and b is rational.

Caroline is trying to prove $\frac{1}{a} + \frac{1}{b}$ is rational.

Here is her proof:

Step 1 $\quad \frac{1}{a} + \frac{1}{b} = \frac{2}{a+b}$

Step 2 $\quad$ 2 is rational and $a + b$ is non-zero and rational.

Step 3 $\quad$ Therefore $\frac{2}{a+b}$ is rational.

Step 4 $\quad$ Hence $\frac{1}{a} + \frac{1}{b}$ is rational.

7 (a) (i) Identify Caroline's mistake.

[1 mark]

7 (a) (ii) Write down a correct version of the proof.

[2 marks]

7 (b) Prove by contradiction that the difference of any rational number and any irrational number is irrational.

[4 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
7(a)(i)	Identifies the error lies in step 1 without contradiction.	2.3	E1	Mistake is $\frac{1}{a}+\frac{1}{b}=\frac{2}{a+b}$
	Subtotal		**1**	
7(a)(ii)	Recalls correct addition Accept $\frac{b}{ab}+\frac{a}{ab}$	1.1b	M1	$\frac{1}{a}+\frac{1}{b}=\frac{a+b}{ab}$ a + b is rational and ab is rational and therefore $\frac{1}{a}+\frac{1}{b}$ is rational.
	Completes rigorous argument to complete proof. Must state that ab is rational (and non-zero) and $a+b$ is rational and conclude that $\frac{1}{a}+\frac{1}{b}$ or $\frac{a+b}{ab}$ is rational	2.1	R1	
	Subtotal		**2**	
7(b)	States assumption to begin proof by contradiction may PI by $\frac{a}{b}-x=\frac{c}{d}$ or $x-\frac{a}{b}=\frac{c}{d}$	3.1a	M1	Assume that the difference between a rational and an irrational number is rational. $\frac{a}{b}-x=\frac{c}{d}$
	Uses language and notation correctly to state initial assumptions: States their a, b, c and d are integers **and** x is irrational do not accept the irrational written as a fraction Condone missing $b, d \neq 0$	2.5	A1	Where a, b, c and d are integers, $b, d \neq 0$ and x is irrational
	Demonstrates that x can be expressed as a rational number by obtaining $x=\frac{ad-cb}{bd}$ OE	1.1b	M1	$x=\frac{a}{b}-\frac{c}{d}$ $=\frac{ad}{bd}-\frac{cb}{bd}$ $=\frac{ad-cb}{bd}$
	Completes rigorous argument to prove the required result, clearly explaining where the contradiction lies with **ALL** assumptions correct at the start (including $b, d \neq 0$)	2.1	R1	Hence x is rational. This is a contradiction hence the difference of any rational number and any irrational number is irrational.
	Subtotal		**4**	
	Question Total		**7**	

June 2019 Question 9 Paper 1

9 Prove that the sum of a rational number and an irrational number is always irrational.

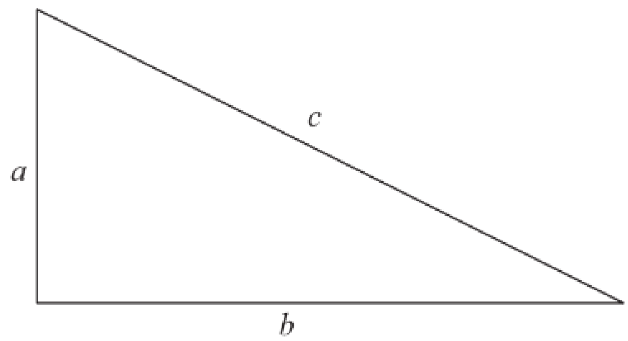
[5 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
9	Begins proof by contradiction. This may be evidenced by: stating assumption at the start "the sum is rational" Or Sight of "contradiction" later as part of argument.	3.1a	M1	Assume m is rational and n is irrational and their sum is rational. Then
	Forms an equation of the form rational + irrational = rational with the rationals written algebraically $\frac{a}{b}+n=\frac{c}{d}$ n must clearly be irrational and not written as an algebraic fraction and not a specific value.	2.5	M1	$\frac{a}{b}+n=\frac{c}{d}$ Where a, b, c and d are all integers. $n=\frac{c}{d}-\frac{a}{b}$ $=\frac{bc-ad}{bd}$
	Manipulates their equation to show that n is rational	1.1b	A1	$\therefore$ n is rational, which is a contradiction.
	Explains or demonstrates why there is a contradiction	2.4	E1	So the original statement is false and the sum of a rational and irrational must be irrational.
	Completes rigorous argument to prove the required result including correct initial assumptions Where a, b, c and d are all integers.	2.1	R1	
	Total		**5**	

June 2018 Question 6 Paper 3

6 The three sides of a right-angled triangle have lengths a, b and c, where $a, b, c \in \mathbb{Z}$



6 (a) State an example where a, b and c are all even.

[1 mark]

6 (b) Prove that it is **not** possible for all of a, b and c to be odd.

[3 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
6(a)	States an appropriate even Pythagorean triple	2.2a	B1	$a=6$ $b=8$ $c=10$
6(b)	Begins an appropriate method of proof assuming at least two sides are odd eg states 'assume a, b odd' or defines a, b (or c) algebraically with different unknowns	3.1a	B1	Assume a and b are odd so $a = 2m + 1$ and $b = 2n + 1$ $(2m+1)^2 + (2n+1)^2$ $= 4m^2 + 4m + 1 + 4n^2 + 4n + 1$ $= 2(2m^2 + 2m + 2n^2 + 2n + 1)$ which is even, so c^2 is even, so c is even. Therefore it is not possible for all three to be odd.
	Uses Pythagoras' theorem with at least two odd sides either in words or algebraically	1.1a	M1	
	Completes rigorous argument to prove the required result **CSO**	2.1	R1	
	Total		**4**	