Pure Year 1 exam questions (AS maths)-AQA

NOTE:
Please be aware that in the Year 12 collections, you will find questions from the Year 13 papers. However, these questions are intentionally included because they align with Year 12 content and topics.
Happy studying!

Ch 1 Problem solving

June 2022 Question 9 Paper 1

9 Integers m and n are both odd.

Prove that $m^2 + n^2$ is a multiple of 2 but **not** a multiple of 4

[5 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
9	States algebraic expressions for two distinct non-consecutive odd numbers.	3.1a	M1	Let $n = 2p + 1 \quad m = 2q + 1$ Where p and q are integers $m^2 + n^2 = (2p + 1)^2 + (2q + 1)^2$ $= 4p^2 + 4p + 1 + 4q^2 + 4q + 1$ $= 2(2p^2 + 2q^2 + 2p + 2q + 1)$ Factor 2 shows it is a multiple of 2
	Expands their two-termed expression for m and n in $m^2 + n^2$	1.1a	M1	
	Obtains their correct expanded expression. Do not allow if substitutions define the same odd number.	1.1b	A1F	
	Concludes correctly that the expression is a multiple of 2 Do not allow if substitutions define consecutive odd numbers or substitutions which generate the same odd number.	2.4	E1	
	Completes a reasoned argument to conclude correctly the expression is not a multiple of 4. CAO OE Must not have used substitutions which involve m or n or define consecutive odd numbers or which generate the same odd number. CAO	2.1	R1	Factor $(2p^2 + 2q^2 + 2p + 2q + 1)$ is 1 more than a multiple of 2 so $m^2 + n^2$ is not a multiple of 4
	Question 9 Total		**5**	

June 2022 Question 6 Paper 2 (A-Level)

6 (a) Asif notices that $24^2 = 576$ and $2 + 4 = 6$ gives the last digit of 576

He checks two more examples:

$27^2 = 729$	$29^2 = 841$
$2 + 7 = 9$	$2 + 9 = 11$
Last digit 9	Last digit 1

Asif concludes that he can find the last digit of any square number greater than 100 by adding the digits of the number being squared.

Give a counter example to show that Asif's conclusion is **not** correct.

[2 marks]

6 (b) Claire tells Asif that he should look only at the last digit of the number being squared.

$27^2 = 729$	$24^2 = 576$
$7^2 = 49$	$4^2 = 16$
Last digit 9	Last digit 6

Using Claire's method determine the last digit of 23456789^2

[1 mark]

6 (c) Given Claire's method is correct, use proof by exhaustion to show that no square number has a last digit of 8

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Squares a number with two or more digits and adds its digits. Must be explicit	1.1a	M1	$12^2 = 144$ $1 + 2 = 3$ $3 \neq 4$
	Completes argument to show that Asif's method is incorrect Must compare sum of digits with last digit of square number	2.3	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)	Obtains 1	1.1b	B1	1
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
6(c)	Lists at least four single digits and their squares Or Explains why odd digits do not need to be considered	1.1a	M1	$0^2 = 0$, $4^2 = 16$, $8^2 = 64$ $1^2 = 1$, $5^2 = 25$, $9^2 = 81$ $2^2 = 4$, $6^2 = 36$ $3^2 = 9$, $7^2 = 49$
	Completes rigorous argument to prove that no square number has a last digit of 8 OE CSO	2.1	R1	Therefore, there can be no square number which has a last digit of 8
	Subtotal		**2**	

November 2021 Question 9 Paper 2

9 (a) Express $n^3 - n$ as a product of three factors.

[1 mark]

9 (b) Given that n is a positive integer, prove that $n^3 - n$ is a multiple of 6

[3 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
9(a)	States correct factorisation	1.1b	B1	$n^3 - n = n(n^2 - 1)$ $= n(n - 1)(n + 1)$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)	States that $(n - 1)$, n, $(n + 1)$ are 3 consecutive integers. Or States that all integers are either a multiple of 3 or 1 more or less than a multiple of 3	3.1a	E1	$(n - 1)\ n\ (n + 1)$ is the product of three consecutive integers So one must be a multiple of 3 And at least one must be a multiple of 2 So the product has factors of 2 and 3, so is a multiple of 2 × 3 = 6
	Deduces that one of these factors must be a multiple of 3 (PI) Or Deduces that one of these factors must be a multiple of 2. Condone 'even' for implied multiple of 2.	2.2a	E1	
	States that at least one must be a multiple of 2 (even), one must be a multiple of 3 and draws correct conclusion.	2.1	R1	
	Subtotal		**3**	

November 2021 Question 8 Paper 2 (A-Level)

8 Kai is proving that $n^3 - n$ is a multiple of 3 for all positive integer values of n.

Kai begins a proof by exhaustion.

Step 1		$n^3 - n = n(n^2 - 1)$
Step 2	When $n = 3m$, where m is a non-negative integer	$n^3 - n = 3m(9m^2 - 1)$ which is a multiple of 3
Step 3	When $n = 3m + 1$,	$n^3 - n = (3m+1)((3m+1)^2 - 1)$
Step 4		$= (3m+1)(9m^2)$ $= 3(3m+1)(3m^2)$ which is a multiple of 3
Step 5	Therefore $n^3 - n$ is a multiple of 3 for all positive integer values of n	

8 (a) Explain the two mistakes that Kai has made after Step 3.

[2 marks]

8 (b) Correct Kai's argument from Step 4 onwards.

[4 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
8(a)	Identifies the algebraic mistake	2.3	E1	Kai has not expanded the brackets correctly
	Identifies the mistake that all cases have not been exhausted	2.3	E1	Kai has not considered numbers of the form $3m$+2
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)	Completes manipulation correctly in Step 4 to obtain $(3m+1)(9m^2+6m)$ OE	2.1	B1	Step 4: = $(3m+1)(9m^2+6m+1-1)$ = $(3m+1)(9m^2+6m)$ = $3(3m+1)(3m^2+2m)$ which is a multiple of 3 Step 5: when $n=3m+2$, $n^3-n=(3m+2)((3m+2)^2-1)$ $=(3m+2)(9m^2+12m+3)$ $=3(3m+2)(3m^2+4m+1)$ which is a multiple of 3 n^3-n is always a multiple of 3 for all positive integer values of n
	Manipulates expression with a third substitution using either $n=3m+2$ or $n=3m-1$	1.1a	M1	
	Manipulates correct expression to convincingly show multiple of 3 Equivalent working for $n=3m-1$ n^3-n $=(3m-1)((3m-1)^2-1)$ $=(3m-1)(9m^2-6m)$ $=3(3m-1)(3m^2-2m)$ OE	1.1b	A1	
	Completes rigorous argument by clearly showing the factor of 3 in the two cases required and concludes appropriately	2.1	R1	
	Subtotal		**4**	

November 2020 Question 9 Paper 1

9 Chloe is attempting to write $\dfrac{2x^2+x}{(x+1)(x+2)^2}$ as partial fractions, with constant numerators.

Her incorrect attempt is shown below.

Step 1 $\dfrac{2x^2+x}{(x+1)(x+2)^2} \equiv \dfrac{A}{x+1} + \dfrac{B}{(x+2)^2}$

Step 2 $2x^2+x \equiv A(x+2)^2 + B(x+1)$

Step 3 Let $x=-1 \Rightarrow A=1$
Let $x=-2 \Rightarrow B=-6$

Answer $\dfrac{2x^2+x}{(x+1)(x+2)^2} \equiv \dfrac{1}{x+1} - \dfrac{6}{(x+2)^2}$

9 (a) (i) By using a counter example, show that the answer obtained by Chloe cannot be correct.

[2 marks]

9 (a) (ii) Explain her mistake in Step 1.

[1 mark]

9 (b) Write $\dfrac{2x^2+x}{(x+1)(x+2)^2}$ as partial fractions, with constant numerators.

[4 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
9(a)(i)	Deduces an appropriate value for x and substitutes into at least one side of the given identity Any value of $x \neq -2, -1$	2.2a	M1	$\frac{2x^2+x}{(x+1)(x+2)^2} \equiv \frac{1}{x+1} - \frac{6}{(x+2)^2}$ Let $x = 0 \Rightarrow LHS = 0$ $RHS = \frac{1}{1} - \frac{6}{4} = -\frac{1}{2} \neq 0$ ∴Chloe's answer must be incorrect
	Shows that LHS≠RHS and concludes that Chloe's answer must be incorrect Accept $\frac{2x^2+x}{(x+1)(x+2)^2} \neq \frac{1}{x+1} - \frac{6}{(x+2)^2}$	2.1	R1	
	Subtotal		**2**	
9(a)(ii)	Explains that Chloe should have included an additional term with $x+2$ in the denominator or Explains that Chloe should have included (Bx + C) as the numerator for $(x+2)^2$	2.3	E1	Chloe should have included $\frac{C}{x+2}$
	Subtotal		**1**	
9(b)	Writes an identity of the correct form Condone use of equals signs	1.1a	M1	$\frac{2x^2+x}{(x+1)(x+2)^2} \equiv \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$ $2x^2 + x \equiv A(x+2)^2 + B(x+1)(x+2) + C(x+1)$ $x = -1 \Rightarrow A = 1$ $x = -2 \Rightarrow C = -6$ $x^2: A + B = 2 \Rightarrow B = 1$ $\frac{2x^2+x}{(x+1)(x+2)^2} \equiv \frac{1}{x+1} + \frac{1}{x+2} - \frac{6}{(x+2)^2}$
	Uses a suitable method to obtain all three of '*their*' constants. For example by substituting or comparing coefficients Only award the M1 if the identity used results from correctly removing fractions from '*their*' chosen partial fraction form	3.1a	M1	
	Obtains any two correct constants If $Bx + C$ is used, then B = 1 and C = - 4	1.1b	A1	
	Obtains all three correct values for the constant numerators	1.1b	A1	
	Subtotal		**4**	
	Question Total		**7**	

June 2019 Question 7 Paper 1

7 Given that $y \in \mathbb{R}$, prove that

$$(2 + 3y)^4 + (2 - 3y)^4 \geq 32$$

Fully justify your answer.

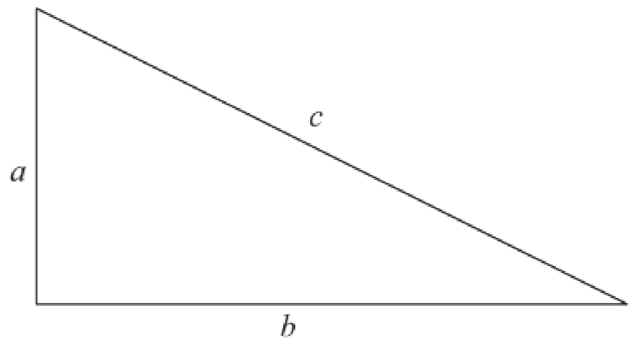
[6 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
7	Expands at least one bracket – must reach a quartic	3.1a	M1	$(2+3y)^4 = 2^4 + 4\times2^3\times(3y) + 6\times2^2\times(3y)^2 + 4\times2\times(3y)^3 + (3y)^4$
	Obtains at least one correct expansion – not necessarily simplified	1.1b	A1	$= 16 + 96y + 216y^2 + 216y^3 + 81y^4$
	Obtains two correct expansions – not necessarily simplified	1.1b	A1	$(2-3y)^4 = 2^4 + 4\times2^3\times(-3y) + 6\times2^2\times(-3y)^2 + 4\times2\times(-3y)^3 + (-3y)^4$ $= 16 - 96y + 216y^2 - 216y^3 + 81y^4$
	Combines their expansions to obtain a sum containing only even power terms.	1.1a	M1	$(2+3y)^4 + (2-3y)^4$ $= 32 + 432y^2 + 162y^4$
	Explains that y^2 and y^4 are always positive **or zero** (for $y \in \mathbb{R}$) Or finds minimum value using calculus and justifies this as not just a local minimum	2.4	E1	$y^2 \geq 0$ and $y^4 \geq 0$ for all y
	Reaches correct conclusion. Sets out a well-constructed mathematical argument. R1 can be awarded if E1 not given.	2.1	R1	$(2+3y)^4 + (2-3y)^4 \geq 32$ for all y
	Total		**6**	

June 2019 Question 6 Paper 3 (A-Level)

6 The three sides of a right-angled triangle have lengths a, b and c, where $a, b, c \in \mathbb{Z}$



6 (a) State an example where a, b and c are all even.

[1 mark]

6 (b) Prove that it is **not** possible for all of a, b and c to be odd.

[3 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
6(a)	States an appropriate even Pythagorean triple	2.2a	B1	$a = 6$ $b = 8$ $c = 10$
6(b)	Begins an appropriate method of proof assuming at least two sides are odd eg states 'assume a, b odd' or defines a, b (or c) algebraically with different unknowns	3.1a	B1	Assume a and b are odd so $a = 2m + 1$ and $b = 2n + 1$ $(2m+1)^2 + (2n+1)^2$ $= 4m^2 + 4m + 1 + 4n^2 + 4n + 1$ $= 2(2m^2 + 2m + 2n^2 + 2n + 1)$ which is even, so c^2 is even, so c is even. Therefore it is not possible for all three to be odd.
	Uses Pythagoras' theorem with at least two odd sides either in words or algebraically	1.1a	M1	
	Completes rigorous argument to prove the required result **CSO**	2.1	R1	
	Total		**4**	

June 2018 Question 7 Paper 2

7 Prove that

$$n \text{ is a prime number greater than } 5 \Rightarrow n^4 \text{ has final digit } 1$$

[5 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical solution
7	Investigates last digit of n. Allow M1 for investigation of $2k + 1$	AO3.1a	M1	Last digit of n determines last digit of n^4
	Deduces that only need to investigate numbers ending in 1, 3, 7, 9 Condone inclusion of 5 at this stage	AO2.2a	M1	All even numbers divide by 2, so are not prime Any number ending in 5 is a multiple of 5 so is not prime Primes > 5 end in 1, 3, 7 or 9
	Considers each in turn to show that n^4 will end in a 1	AO1.1a	M1	If n ends in 1, 1^4 is 1 so n^4 ends in a 1 If n ends in 3, 3^4 is 81 so n^4 ends in a 1 If n ends in 7, 7^4 is 2401 so n^4 ends in a 1 If n ends in 9, 9^4 is 6561 so n^4 ends in a 1
	Provides evidence that 1^4, 3^4, 7^4, 9^4 all end in a 1	AO1.1b	A1	
	Constructs rigorous mathematical argument to show the required result Only award if they have a completely correct solution, which is clear, easy to follow and contains no slips. Must include clear statement that final digit of n determines final digit of n^4	AO2.1	R1	Statement proved by exhaustion
	Total		**5**	