Edexcel - Pure Year 2 exam questions

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Ch. 8: Parametric Equations

November 2021 Question 13 Paper 1

13. A curve C has parametric equations

$$x = \frac{t^2+5}{t^2+1} \qquad y = \frac{4t}{t^2+1} \qquad t \in \mathbb{R}$$

Show that all points on C satisfy

$$(x-3)^2 + y^2 = 4$$

(3)

ANSWER

Question	Scheme	Marks	AOs
13	$(x-3)^2+y^2=\left(\frac{t^2+5}{t^2+1}-3\right)^2+\left(\frac{4t}{t^2+1}\right)^2$	M1	3.1a
	$=\frac{(2-2t^2)^2+16t^2}{(t^2+1)^2}=\frac{4+8t^2+4t^4}{(t^2+1)^2}$	dM1	1.1b
	$\frac{4(t^4+2t^2+1)}{(t^2+1)^2}=\frac{4(t^2+1)^2}{(t^2+1)^2}=4*$	A1*	2.1
		(3)	

June 2019 Question 4 Paper 2

4.

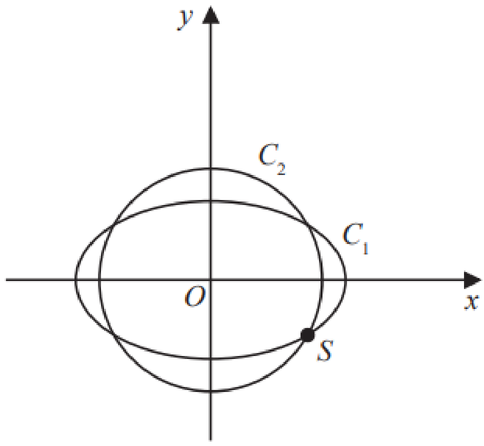


Figure 2

The curve C_1 with parametric equations

$$x = 10\cos t, \quad y = 4\sqrt{2}\sin t, \quad 0 \leqslant t < 2\pi$$

meets the circle C_2 with equation

$$x^2 + y^2 = 66$$

at four distinct points as shown in Figure 2.

Given that one of these points, S, lies in the 4th quadrant, find the Cartesian coordinates of S.

(6)

ANSWER

Question	Scheme		Marks	AOs
4	$C_1: x = 10\cos t,\ y = 4\sqrt{2}\sin t,\ \ 0 \le t < 2\pi;\ \ C_2: x^2 + y^2 = 66$			
Way 1	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$		M1	3.1a
	$100(1-\sin^2 t) + 32\sin^2 t = 66$	$100\cos^2 t + 32(1-\cos^2 t) = 66$	M1	2.1
			A1	1.1b
	$100 - 68\sin^2 t = 66 \Rightarrow \sin^2 t = \frac{1}{2}$ $\Rightarrow \sin t = ...$	$68\cos^2 t + 32 = 66 \Rightarrow \cos^2 t = \frac{1}{2}$ $\Rightarrow \cos t = ...$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the x-coordinate and value of the corresponding y-coordinate. **Note:** These may not be in the correct quadrant		M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$		A1	3.2a
			(6)	
Way 2	$\{\cos^2 t + \sin^2 t = 1 \Rightarrow\}\ \left(\frac{x}{10}\right)^2 + \left(\frac{y}{4\sqrt{2}}\right)^2 = 1\ \ \{\Rightarrow 32x^2 + 100y^2 = 3200\}$		M1	3.1a
	$\frac{x^2}{100} + \frac{66-x^2}{32} = 1$	$\frac{66-y^2}{100} + \frac{y^2}{32} = 1$	M1	2.1
			A1	1.1b
	$32x^2 + 6600 - 100x^2 = 3200$ $x^2 = 50 \Rightarrow x = ...$	$2112 - 32y^2 + 100y^2 = 3200$ $y^2 = 16 \Rightarrow y = ...$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the corresponding x-coordinate or y-coordinate. **Note:** These may not be in the correct quadrant		M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$		A1	3.2a
			(6)	

June 2018 Question 14 Paper 1

14. A curve C has parametric equations

$$x = 3 + 2\sin t, \quad y = 4 + 2\cos 2t, \quad 0 \leqslant t < 2\pi$$

(a) Show that all points on C satisfy $y = 6 - (x - 3)^2$

(2)

(b) (i) Sketch the curve C.

(ii) Explain briefly why C does not include all points of $y = 6 - (x - 3)^2, \quad x \in \mathbb{R}$

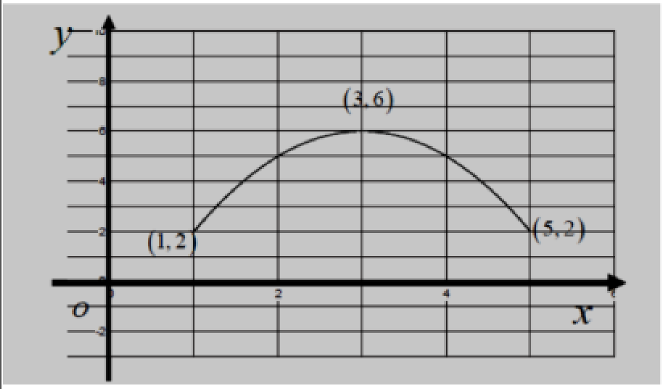
(3)

The line with equation $x + y = k$, where k is a constant, intersects C at two distinct points.

(c) State the range of values of k, writing your answer in set notation.

(5)

ANSWER

Question	Scheme	Marks	AOs
14(a)	Attempts to use $\cos 2t = 1 - 2\sin^2 t \Rightarrow \frac{y-4}{2} = 1 - 2\left(\frac{x-3}{2}\right)^2$	M1	2.1
	$\Rightarrow y - 4 = 2 - 4 \times \frac{(x-3)^2}{4} \Rightarrow y = 6 - (x-3)^2$ *	A1*	1.1b
		(2)	
(b)	 $\cap$ shaped parabola	M1	1.1b
	Fully correct with 'ends' at (1,2) & (5,2)	A1	1.1b
	Suitable reason : Eg states as $x = 3 + 2\sin t$, $1 \leqslant x \leqslant 5$	B1	2.4
		(3)	
(c)	Either finds the lower value for $k = 7$ or deduces that $k < \frac{37}{4}$	B1	2.2a

	Finds where $x+y=k$ meets $y=6-(x-3)^2$ $\Rightarrow k-x=6-(x-3)^2$ and proceeds to 3TQ in x or y	M1	3.1a
	Correct 3TQ in x $x^2-7x+(k+3)=0$ Or y $y^2+(7-2k)y+(k^2-6k+3)=0$	A1	1.1b
	Uses $b^2-4ac=0 \Rightarrow$ $49-4\times1\times(k+3)=0 \Rightarrow k=\left(\frac{37}{4}\right)$ or $(7-2k)^2-4\times1\times(k^2-6k+3)=0 \Rightarrow k=\left(\frac{37}{4}\right)$	M1	2.1
	Range of values for $k=\left\{k:7\leqslant k<\frac{37}{4}\right\}$	A1	2.5
		(5)	
			(10 marks)

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