Edexcel - Pure Year 2 exam questions

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Ch. 4: Binomial Expansion

June 2022 Question 7 Paper 2

7. (a) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$\sqrt{4-9x}$$

writing each term in simplest form.

(4)

A student uses this expansion with $x = \frac{1}{9}$ to find an approximation for $\sqrt{3}$

Using the answer to part (a) and without doing any calculations,

(b) state whether this approximation will be an overestimate or an underestimate of $\sqrt{3}$ giving a brief reason for your answer.

(1)

ANSWER

Question	Scheme	Marks	AOs
7(a)	$\sqrt{4-9x} = 2(1 \pm ...)^{\frac{1}{2}}$	B1	1.1b
	$\left(1-"\frac{9x}{4}"\right)^{\frac{1}{2}} = ... + \dfrac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\left("-\frac{9x}{4}"\right)^2}{2!}$ or $... + \dfrac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)\left("-\frac{9x}{4}"\right)^3}{3!}$	M1	1.1b
	$1+\frac{1}{2}\times\left(-\frac{9x}{4}\right)+\dfrac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\left(-\frac{9x}{4}\right)^2}{2!}+\dfrac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)\left(-\frac{9x}{4}\right)^3}{3!}$	A1	1.1b
	$\sqrt{4-9x} = 2 - \frac{9x}{4} - \frac{81x^2}{64} - \frac{729x^3}{512}$	A1	1.1b
		(4)	
(b)	States that the approximation will be an **overestimate** since all terms (after the first one) in the expansion are negative (since $x > 0$)	B1	3.2b
		(1)	
			(5 marks)

November 2021 Question 9 Paper 1

9.

$$f(x) = \frac{50x^2 + 38x + 9}{(5x+2)^2(1-2x)} \qquad x \neq -\frac{2}{5} \quad x \neq \frac{1}{2}$$

Given that f(x) can be expressed in the form

$$\frac{A}{5x+2} + \frac{B}{(5x+2)^2} + \frac{C}{1-2x}$$

where A, B and C are constants

(a) (i) find the value of B and the value of C

(ii) show that $A = 0$

(4)

(b) (i) Use binomial expansions to show that, in ascending powers of x

$$f(x) = p + qx + rx^2 + \ldots$$

where p, q and r are simplified fractions to be found.

(ii) Find the range of values of x for which this expansion is valid.

(7)

ANSWER

Question	Scheme	Marks	AOs
9(a)(i)	$50x^2+38x+9 \equiv A(5x+2)(1-2x)+B(1-2x)+C(5x+2)^2$ $\Rightarrow B=\ldots$ or $C=\ldots$	M1	1.1b
	$B=1$ and $C=2$	A1	1.1b
(a)(ii)	E.g. $x=0$ $x=0\Rightarrow 9=2A+B+4C$ $\Rightarrow 9=2A+1+8\Rightarrow A=\ldots$	M1	2.1
	$A=0$*	A1*	1.1b
		(4)	
(b)(i)	$\frac{1}{(5x+2)^2}=(5x+2)^{-2}=2^{-2}\left(1+\frac{5}{2}x\right)^{-2}$ or $(5x+2)^{-2}=2^{-2}+\ldots$	M1	3.1a
	$\left(1+\frac{5}{2}x\right)^{-2}=1-2\left(\frac{5}{2}x\right)+\frac{-2(-2-1)}{2!}\left(\frac{5}{2}x\right)^2+\ldots$	M1	1.1b
	$2^{-2}\left(1+\frac{5}{2}x\right)^{-2}=\frac{1}{4}-\frac{5}{4}x+\frac{75}{16}x^2+\ldots$	A1	1.1b
	$\frac{1}{(1-2x)}=(1-2x)^{-1}=1+2x+\frac{-1(-1-1)}{2!}(2x)^2+.$	M1	1.1b
	$\frac{1}{(5x+2)^2}+\frac{2}{1-2x}=\frac{1}{4}-\frac{5}{4}x+\frac{75}{16}x^2+\ldots+2+4x+8x^2+\ldots$	dM1	2.1
	$=\frac{9}{4}+\frac{11}{4}x+\frac{203}{16}x^2+\ldots$	A1	1.1b
(b)(ii)	$\|x\|<\frac{2}{5}$	B1	2.2a
		(7)	
			(11 marks)

November 2020 Question 1 Paper 1

1. (a) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$(1+8x)^{\frac{1}{2}}$$

giving each term in simplest form.

(3)

(b) Explain how you could use $x = \dfrac{1}{32}$ in the expansion to find an approximation for $\sqrt{5}$

There is no need to carry out the calculation.

(2)

ANSWER

Question	Scheme	Marks	AOs
1 (a)	$(1+8x)^{\frac{1}{2}} = 1+\frac{1}{2}\times 8x+\frac{\frac{1}{2}\times -\frac{1}{2}}{2!}\times(8x)^2+\frac{\frac{1}{2}\times -\frac{1}{2}\times -\frac{3}{2}}{3!}\times(8x)^3$	M1 A1	1.1b 1.1b
	$=1+4x-8x^2+32x^3+\ldots$	A1	1.1b
		(3)	
(b)	Substitutes $x=\frac{1}{32}$ into $(1+8x)^{\frac{1}{2}}$ to give $\frac{\sqrt{5}}{2}$	M1	1.1b
	Explains that $x=\frac{1}{32}$ is substituted into $1+4x-8x^2+32x^3$ and you multiply the result by 2	A1ft	2.4
		(2)	
			(5 marks)

June 2019 Question 4 Paper 1

4. (a) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$\frac{1}{\sqrt{4-x}}$$

giving each coefficient in its simplest form.

(4)

The expansion can be used to find an approximation to $\sqrt{2}$
Possible values of x that could be substituted into this expansion are:

- $x = -14$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{18}} = \dfrac{\sqrt{2}}{6}$
- $x = 2$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$
- $x = -\dfrac{1}{2}$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{\dfrac{9}{2}}} = \dfrac{\sqrt{2}}{3}$

(b) Without evaluating your expansion,

(i) state, giving a reason, which of the three values of x should not be used

(1)

(ii) state, giving a reason, which of the three values of x would lead to the most accurate approximation to $\sqrt{2}$

(1)

ANSWER

4 (a)	$\frac{1}{\sqrt{4-x}}=(4-x)^{-\frac{1}{2}}=4^{-\frac{1}{2}}\times(1\pm......$	M1	2.1
	Uses a "correct" binomial expansion for their $(1+ax)^n=1+nax+\frac{n(n-1)}{2}a^2x^2+$	M1	1.1b
	$\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}=1+\left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right)+\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\left(-\frac{x}{4}\right)^2$	A1	1.1b
	$\frac{1}{\sqrt{4-x}}=\frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2$	A1	1.1b
		(4)	
(b) (i)	States $x=-14$ and gives a valid reason. Eg explains that the expansion is not valid for $\lvert x\rvert>4$	B1	2.4
		(1)	
(b)(ii)	States $x=-\frac{1}{2}$ and gives a valid reason. Eg. explains that it is closest to zero	B1	2.4
		(1)	
			(6 marks)

June 2018 Question 11 Paper 1

11. (a) Use binomial expansions to show that $\sqrt{\frac{1+4x}{1-x}} \approx 1 + \frac{5}{2}x - \frac{5}{8}x^2$ **(6)**

A student substitutes $x = \frac{1}{2}$ into both sides of the approximation shown in part (a) in an attempt to find an approximation to $\sqrt{6}$

(b) Give a reason why the student **should not** use $x = \frac{1}{2}$ **(1)**

(c) Substitute $x = \frac{1}{11}$ into

$$\sqrt{\frac{1+4x}{1-x}} = 1 + \frac{5}{2}x - \frac{5}{8}x^2$$

to obtain an approximation to $\sqrt{6}$. Give your answer as a fraction in its simplest form. **(3)**

ANSWER

Question	Scheme	Marks	AOs
11 (a)	$\sqrt{\dfrac{1+4x}{1-x}}=(1+4x)^{0.5}\times(1-x)^{-0.5}$	B1	3.1a
	$(1+4x)^{0.5}=1+0.5\times(4x)+\dfrac{0.5\times-0.5}{2}\times(4x)^2$	M1	1.1b
	$(1-x)^{-0.5}=1+(-0.5)(-x)+\dfrac{(-0.5)\times(-1.5)}{2}(-x)^2$	M1	1.1b
	$(1+4x)^{0.5}=1+2x-2x^2$ and $(1-x)^{-0.5}=1+0.5x+0.375x^2$ oe	A1	1.1b
	$(1+4x)^{0.5}\times(1-x)^{-0.5}=\left(1+2x-2x^2......\right)\times\left(1+\dfrac{1}{2}x+\dfrac{3}{8}x^2...\right)$ $=1+\dfrac{1}{2}x+\dfrac{3}{8}x^2+2x+x^2-2x^2+...$ $=A+Bx+Cx^2$	dM1	2.1
	$=1+\dfrac{5}{2}x-\dfrac{5}{8}x^2......$ *	A1*	1.1b
		(6)	
(b)	Expression is valid $\lvert x\rvert<\dfrac{1}{4}$ Should not use $x=\dfrac{1}{2}$ as $\dfrac{1}{2}>\dfrac{1}{4}$	B1	2.3
		(1)	
(c)	Substitutes $x=\dfrac{1}{11}$ into $\sqrt{\dfrac{1+4x}{1-x}}\approx1+\dfrac{5}{2}x-\dfrac{5}{8}x^2$	M1	1.1b
	$\sqrt{\dfrac{3}{2}}=\dfrac{1183}{968}$	A1	1.1b
	(so $\sqrt{6}$ is) $\dfrac{1183}{484}$ or $\dfrac{2904}{1183}$	A1	2.1
		(3)	
			(10 marks)