Edexcel - Pure Year 2 exam questions

Arancha Ruiz

Ch. 3: Sequences and Series

June 2022 Question 15 Paper 2

15. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Given that the first three terms of a geometric series are

$$12\cos\theta \qquad 5 + 2\sin\theta \qquad \text{and} \qquad 6\tan\theta$$

(a) show that

$$4\sin^2\theta - 52\sin\theta + 25 = 0$$

(3)

Given that θ is an obtuse angle measured in radians,

(b) solve the equation in part (a) to find the exact value of θ

(2)

(c) show that the sum to infinity of the series can be expressed in the form

$$k\left(1 - \sqrt{3}\right)$$

where k is a constant to be found.

(5)

ANSWER

Question	Scheme	Marks	AOs
15(a)	Uses the common ratio $\frac{5+2\sin\theta}{12\cos\theta}=\frac{6\tan\theta}{5+2\sin\theta}$ o.e.	M1	3.1a
	Cross multiplies and uses $\tan\theta\times\cos\theta=\sin\theta$ $(5+2\sin\theta)^2=6\times12\sin\theta$	dM1	1.1b
	Proceeds to given answer $25+20\sin\theta+4\sin^2\theta=72\sin\theta$ $\Rightarrow 4\sin^2\theta-52\sin\theta+25=0$ *	A1*	2.1
		(3)	
(b)	$4\sin^2\theta-52\sin\theta+25=0\Rightarrow\sin\theta=\frac{1}{2}\left(,\frac{25}{2}\right)$	M1	1.1b
	$\theta=\frac{5\pi}{6}$	A1	1.2
		(2)	
(c)	Attempts a value for either a or r e.g. $a=12\cos\theta=12\times-\frac{\sqrt{3}}{2}$ **or** $r=\frac{5+2\sin\theta}{12\cos\theta}=\frac{5+2\times\frac{1}{2}}{12\times-\frac{\sqrt{3}}{2}}$	M1	3.1a
	$"a"=-6\sqrt{3}$ **and** $"r"=-\frac{1}{\sqrt{3}}$ o.e.	A1	1.1b
	Uses $S_\infty=\frac{a}{1-r}=\frac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}$	dM1	2.1
	Rationalises denominator $S_\infty=\frac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}=\frac{-18}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$	ddM1	1.1b
	$(S_\infty=)\,9\left(1-\sqrt{3}\right)$	A1	2.1
		(5)	
		(10 marks)	

June 2022 Question 13 Paper 1

13. (i) In an arithmetic series, the first term is a and the common difference is d.

Show that

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

(3)

(ii) James saves money over a number of weeks to buy a printer that costs £64

He saves £10 in week 1, £9.20 in week 2, £8.40 in week 3 and so on, so that the weekly amounts he saves form an arithmetic sequence.

Given that James takes n weeks to save exactly £64

(a) show that

$$n^2 - 26n + 160 = 0$$

(2)

(b) Solve the equation

$$n^2 - 26n + 160 = 0$$

(1)

(c) Hence state the number of weeks James takes to save enough money to buy the printer, giving a brief reason for your answer.

(1)

ANSWER

Question	Scheme	Marks	AOs
13 (i)	States that $S = a+(a+d)+\ldots\ldots\ldots\ldots+(a+(n-1)d)$	B1	1.1a
	$S = a + \quad . \quad (a+d)+ \quad (a+(n-1)d)$ $S = (a+(n-1)d)+(a+(n-2)d)+\ldots\ldots\ldots\ldots+a$	M1	3.1a
	Reaches $2S = n\times(2a+(n-1)d)$ And so proves that $S=\frac{n}{2}[2a+(n-1)d]$ *	A1*	2.1
		(3)	
(ii)	(a) $S = 10+9.20+8.40+\ldots$		
	$64=\frac{n}{2}(20-0.8\,(n-1))$ o.e	M1	3.1b
	$128 = 20n-0.8n^2+0.8n$ $0.8n^2-20.8n+128=0$ $n^2-26n+160=0$ *	A1*	2.1
		(2)	
	(b) $n=10,16$	B1	1.1b
		(1)	
	(c) 10 weeks with a minimal correct reason. E.g. • He has saved up the amount by 10 weeks so he would not save for another 6 weeks • You would choose the smaller number • He starts saving negative amounts (in week 14) so 16 does not make sense	B1	2.3
		(1)	
			(7 marks)

Video solution:

https://youtu.be/0c3gXTSNnBI

November 2021 Question 3 Paper 1

3. The sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = k - \frac{24}{u_n} \qquad u_1 = 2$$

where k is an integer.

Given that $u_1 + 2u_2 + u_3 = 0$

(a) show that

$$3k^2 - 58k + 240 = 0$$

(3)

(b) Find the value of k, giving a reason for your answer.

(2)

(c) Find the value of u_3

(1)

ANSWER

Question	Scheme	Marks	AOs
3(a)	$u_2 = k - 12,\ u_3 = k - \frac{24}{k-12}$	M1	1.1b
	$u_1 + 2u_2 + u_3 = 0 \Rightarrow 2 + 2(k-12) + k - \frac{24}{k-12} = 0$	dM1	1.1b
	$\Rightarrow 3k - 22 - \frac{24}{k-12} = 0 \Rightarrow (3k-22)(k-12) - 24 = 0$ $\Rightarrow 3k^2 - 36k - 22k + 264 - 24 = 0$ $\Rightarrow 3k^2 - 58k + 240 = 0$ *	A1*	2.1
		(3)	
(b)	$k = 6, \left(\frac{40}{3}\right)$	M1	1.1b
	$k = 6$ as k must be an integer	A1	2.3
		(2)	
(c)	$(u_3 =)10$	B1	2.2a
		(1)	
		(6 marks)	

November 2021 Question 5 Paper 1

5. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A company made a profit of £20 000 in its first year of trading, Year 1

A model for future trading predicts that the yearly profit will increase by 8% each year, so that the yearly profits will form a geometric sequence.

According to the model,

(a) show that the profit for Year 3 will be £23 328

(1)

(b) find the first year when the yearly profit will exceed £65 000

(3)

(c) find the total profit for the first 20 years of trading, giving your answer to the nearest £1000

(2)

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ANSWER

Question	Scheme	Marks	AOs
5(a)	$u_3 = £20000 \times 1.08^2 = (£)23328*$	B1*	1.1b
		(1)	
(b)	$20000 \times 1.08^{n-1} > 65000$	M1	1.1b
	$1.08^{n-1} > \frac{13}{4} \Rightarrow n-1 > \frac{\ln(3.25)}{\ln(1.08)}$ or e.g. $1.08^{n-1} > \frac{13}{4} \Rightarrow n-1 > \log_{1.08}\left(\frac{13}{4}\right)$	M1	3.1b
	Year 17	A1	3.2a
		(3)	
(c)	$S_{20} = \frac{20000(1-1.08^{20})}{1-1.08}$	M1	3.4
	Awrt (£) 915 000	A1	1.1b
		(2)	
			(6 marks)

November 2021 Question 1 Paper 2

1. In an arithmetic series

- the first term is 16
- the 21st term is 24

(a) Find the common difference of the series.

(2)

(b) Hence find the sum of the first 500 terms of the series.

(2)

ANSWER

Question	Scheme	Marks	AOs
1(a)	$16+(21-1)\times d=24 \Rightarrow d=...$	M1	1.1b
	$d=0.4$	A1	1.1b
	Answer only scores both marks.		
		(2)	
(b)	$S_n=\frac{1}{2}n\{2a+(n-1)d\} \Rightarrow S_{500}=\frac{1}{2}\times 500\{2\times 16+499\times "0.4"\}$	M1	1.1b
	$=57900$	A1	1.1b
	Answer only scores both marks		
		(2)	

November 2021 Question 9 Paper 2

9. Show that

$$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n \cos(180n)^\circ = \frac{9}{28}$$

(3)

ANSWER

Question	Scheme	Marks	AOs
9	$a=\left(\frac{3}{4}\right)^2$ or $a=\frac{9}{16}$ or $r=-\frac{3}{4}$	B1	2.2a
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n \cos(180n)^\circ = \dfrac{\frac{9}{16}}{1-\left(-\frac{3}{4}\right)} = \ldots$	M1	3.1a
	$=\frac{9}{28}$ *	A1*	1.1b
		(3)	

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November 2020 Question 5 Paper 1

5. A car has six forward gears.

The fastest speed of the car

- in 1^{st} gear is 28 km h^{-1}
- in 6^{th} gear is 115 km h^{-1}

Given that the fastest speed of the car in successive gears is modelled by an **arithmetic sequence**,

(a) find the fastest speed of the car in 3^{rd} gear.

(3)

Given that the fastest speed of the car in successive gears is modelled by a **geometric sequence**,

(b) find the fastest speed of the car in 5^{th} gear.

(3)

ANSWER

Question	Scheme	Marks	AOs
5 (a)	Uses $115 = 28 + 5d \Rightarrow d = (17.4)$	M1	3.1b
	Uses $28 + 2 \times "17.4" = \ldots$	M1	3.4
	$= 62.8$ (km h^{-1})	A1	1.1b
		(3)	
(b)	Uses $115 = 28r^5 \Rightarrow r = (1.3265)$	M1	3.1b
	Uses $28 \times "1.3265^4" = \ldots$ or $\dfrac{115}{"1.3265"}$	M1	3.4
	$= 86.7$ (km h^{-1})	A1	1.1b
		(3)	
			(6 marks)

Video solution:

https://youtu.be/TI827wheRr0

November 2020 Question 13 Paper 1

13. A sequence of numbers $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = \frac{k(a_n + 2)}{a_n} \qquad n \in \mathbb{N}$$

where k is a constant.

Given that

- the sequence is a periodic sequence of order 3
- $a_1 = 2$

(a) show that

$$k^2 + k - 2 = 0$$

(3)

(b) For this sequence explain why $k \neq 1$

(1)

(c) Find the value of

$$\sum_{r=1}^{80} a_r$$

(3)

ANSWER

Question	Scheme	Marks	AOs
13 (a)	Uses the sequence formula $a_{n+1} = \dfrac{k(a_n+2)}{a_n}$ once with $a_1 = 2$	M1	1.1b
	$(a_1=2),\, a_2 = 2k,\, a_3 = k+1,\, a_4 = \dfrac{k(k+3)}{k+1}$ Finds four consecutive terms and sets a_4 equal to a_1 (oe)	M1	3.1a
	$\dfrac{k(k+3)}{k+1} = 2 \Rightarrow k^2+3k = 2k+2 \Rightarrow k^2+k-2=0$ *	A1*	2.1
		(3)	
(b)	States that when $k=1$, all terms are the same and concludes that the sequence does not have a period of order 3	B1	2.3
		(1)	
(c)	Deduces the repeating terms are $a_{1/4}=2,\, a_{2/5}=-4,\, a_{3/6}=-1,$	B1	2.2a
	$\sum_{n=1}^{80} a_k = 26\times(2+-4+-1)+2+-4$	M1	3.1a
	$=-80$	A1	1.1b
		(3)	
			(7 marks)

November 2020 Question 15 Paper 2

15. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A geometric series has common ratio r and first term a.

Given $r \neq 1$ and $a \neq 0$

(a) prove that

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

(4)

Given also that S_{10} is four times S_5

(b) find the exact value of r.

(4)

ANSWER

Question	Scheme	Marks	AOs
15(a)	$S_n = a + ar + ar^2 + \ldots\ldots + ar^{n-1}$	B1	1.2
	$rS_n = ar + ar^2 + ar^3 + \ldots\ldots + ar^n \Rightarrow S_n - rS_n = \ldots$	M1	2.1
	$S_n - rS_n = a - ar^n$	A1	1.1b
	$S_n(1-r) = a(1-r^n) \Rightarrow S_n = \dfrac{a(1-r^n)}{(1-r)}$ *	A1*	2.1
		(4)	
(b)	$\dfrac{a(1-r^{10})}{1-r} = 4\times\dfrac{a(1-r^5)}{1-r}$ or $4\times\dfrac{a(1-r^{10})}{1-r} = \dfrac{a(1-r^5)}{1-r}$ Equation in r^{10} and r^5 (and possibly $1-r$)	M1	3.1a
	$1 - r^{10} = 4(1-r^5)$	A1	1.1b
	$r^{10} - 4r^5 + 3 = 0 \Rightarrow (r^5-1)(r^5-3) = 0 \Rightarrow r^5 = \ldots$ or e.g. $1 - r^{10} = 4(1-r^5) \Rightarrow (1-r^5)(1+r^5) = 4(1-r^5) \Rightarrow r^5 = \ldots$	dM1	2.1
	$r = \sqrt[5]{3}$ oe only	A1	1.1b
		(4)	
			(8 marks)

June 2019 Question 11 Paper 1

11. A competitor is running a 20 kilometre race.

She runs each of the first 4 kilometres at a steady pace of 6 minutes per kilometre.
After the first 4 kilometres, she begins to slow down.

In order to estimate her finishing time, the time that she will take to complete each subsequent kilometre is modelled to be 5% greater than the time that she took to complete the previous kilometre.

Using the model,

(a) show that her time to run the first 6 kilometres is estimated to be 36 minutes 55 seconds,
(2)

(b) show that her estimated time, in minutes, to run the rth kilometre, for $5 \leqslant r \leqslant 20$, is

$$6 \times 1.05^{r-4}$$

(1)

(c) estimate the total time, in minutes and seconds, that she will take to complete the race.
(4)

ANSWER

Question	Scheme	Marks	AOs
11 (a)	Total time for 6 km = 24 minutes + $6\times1.05+6\times1.05^2$ minutes	M1	3.4
	$= 36.915$ minutes = 36 minutes 55 seconds *	A1*	1.1b
		(2)	
(b)	**5** $^{\text{th}}$ km is $6\times1.05=6\times1.05^1$ **6**$^{\text{th}}$ km is $6\times1.05\times1.05=6\times1.05^2$ **7**$^{\text{th}}$ km is $6\times1.05\times1.05\times1.05=6\times1.05^3$ Hence the time for the r^{th} km is $6\times1.05^{r-4}$	B1	3.4
		(1)	
(c)	Attempts the total time for the race = Eg. 24 minutes + $\sum_{r=5}^{r=20} 6\times1.05^{r-4}$ minutes	M1	3.1a
	Uses the series formula to find an allowable sum Eg. Time for 5 $^{\text{th}}$ to 20$^{\text{th}}$ km $=\dfrac{6.3(1.05^{16}-1)}{1.05-1}=(149.04)$	M1	3.4
	Correct calculation that leads to the **total time** Eg. Total time = 24 + $\dfrac{6.3(1.05^{16}-1)}{1.05-1}$	A1	1.1b
	Total time = awrt 173 minutes and 3 seconds	A1	1.1b
		(4)	
			(7 marks)

June 2019 Question 8 Paper 2

8. (i) Find the value of

$$\sum_{r=4}^{\infty} 20 \times \left(\frac{1}{2}\right)^r$$

(3)

(ii) Show that

$$\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) = 2$$

(3)

ANSWER

Question	Scheme	Marks	AOs
8 (i) Way 1	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = 20\left(\frac{1}{2}\right)^4 + 20\left(\frac{1}{2}\right)^5 + 20\left(\frac{1}{2}\right)^6 + \ldots$		
	$= \dfrac{20(\frac{1}{2})^4}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= (1.25)(2)\} = 2.5$ o.e.	A1	1.1b
		(3)	
(i) Way 2	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = \sum_{r=1}^{\infty} 20\times\left(\frac{1}{2}\right)^r - \sum_{r=1}^{3} 20\times\left(\frac{1}{2}\right)^r$		
	$= \dfrac{10}{1-\frac{1}{2}} - (10+5+2.5)$ or $= \dfrac{10}{1-\frac{1}{2}} - \dfrac{10(1-(\frac{1}{2})^3)}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= 20 - 17.5\} = 2.5$ o.e.	A1	1.1b
		(3)	
(ii) Way 1	$\left\{\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) =\right\}$		
	$= \log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{4}{3}\right) + \ldots\ldots + \log_5\left(\frac{50}{49}\right) = \log_5\left(\frac{3}{2}\times\frac{4}{3}\times\ldots\times\frac{50}{49}\right)$	M1	1.1b
		M1	3.1a
	$= \log_5\left(\frac{50}{2}\right)$ or $\log_5(25) = 2$ *	A1*	2.1
		(3)	
(ii) Way 2	$\left\{\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) =\right\} \sum_{n=1}^{48}(\log_5(n+2) - \log_5(n+1))$	M1	1.1b
	$= (\log_5 3 + \log_5 4 + \ldots\ldots + \log_5 50) - (\log_5 2 + \log_5 3 + \ldots\ldots + \log_5 49)$	M1	3.1a
	$= \log_5 50 - \log_5 2$ or $\log_5\left(\frac{50}{2}\right)$ or $\log_5(25) = 2$*	A1*	2.1
		(3)	
			(6 marks)

June 2018 Question 4 Paper 2

4. (i) Show that $\sum_{r=1}^{16}(3 + 5r + 2^r) = 131\,798$ **(4)**

(ii) A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = \frac{1}{u_n}, \quad u_1 = \frac{2}{3}$$

Find the exact value of $\sum_{r=1}^{100} u_r$ **(3)**

ANSWER

Question	Scheme	Marks	AOs
4	(i) $\sum_{r=1}^{16}\left(3+5r+2^r\right)=131\,798$; (ii) $u_1, u_2, u_3, \ldots : u_{n+1}=\frac{1}{u_n}, u_1=\frac{2}{3}$		
(i) Way 1	$\left\{\sum_{r=1}^{16}\left(3+5r+2^r\right)=\right\} \sum_{r=1}^{16}(3+5r)+\sum_{r=1}^{16}\left(2^r\right)$	M1	3.1a
	$=\frac{16}{2}(2(8)+15(5)) + \frac{2(2^{16}-1)}{2-1}$	M1 M1	1.1b 1.1b
	$=728+131\,070 = 131\,798$ *	A1*	2.1
		(4)	
(i) Way 2	$\left\{\sum_{r=1}^{16}\left(3+5r+2^r\right)=\right\} \sum_{r=1}^{16}3+\sum_{r=1}^{16}(5r)+\sum_{r=1}^{16}\left(2^r\right)$	M1	3.1a
	$=(3\times 16)+\frac{16}{2}(2(5)+15(5)) + \frac{2(2^{16}-1)}{2-1}$	M1 M1	1.1b 1.1b
	$=48+680+131\,070 = 131\,798$ *	A1*	2.1
		(4)	
(i) Way 3	Sum $= 10+17+26+39+60+97+166+299+560+1077+2106$ $+4159+8260+16457+32846+65619=131\,798$ *	M1 M1 M1 A1*	3.1a 1.1b 1.1b 2.1
		(4)	
(ii)	$\left\{u_1=\frac{2}{3}\right\}, u_2=\frac{3}{2}, u_3=\frac{2}{3}, \ldots$ ***(can be implied by later working)***	M1	1.1b
	$\left\{\sum_{r=1}^{100}u_r=\right\}$ $50\left(\frac{2}{3}\right)+50\left(\frac{3}{2}\right)$ or $50\left(\frac{2}{3}+\frac{3}{2}\right)$	M1	2.2a
	$=\frac{325}{3}$ $\left(\text{or } 108\frac{1}{3} \text{ or } 108.\dot{3} \text{ or } \frac{1300}{12} \text{ or } \frac{650}{6}\right)$	A1	1.1b
		(3)	
			(7 marks)