Pure Year 2 exam questions (A Level maths)-AQA

NOTE:
Please be aware that in the Year 12 collections, you will find questions from the Year 13 papers. However, these questions are intentionally included because they align with Year 12 content and topics. That's the reason why you are missing some questions here, please have a look at the Year 12 collection!
Happy studying!

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Ch 1 Proof

June 2022 Question 4 Paper 1

4 Millie is attempting to use proof by contradiction to show that the result of multiplying an irrational number by a non-zero rational number is always an irrational number.

Select the assumption she should make to start her proof.

Tick (✓) **one** box.

[1 mark]

Every irrational multiplied by a non-zero rational is irrational. ☐

Every irrational multiplied by a non-zero rational is rational. ☐

There exists a non-zero rational and an irrational whose product is irrational. ☐

There exists a non-zero rational and an irrational whose product is rational. ☐

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
4	Ticks the correct box	2.1	R1	There exists a non-zero rational and an irrational whose product is rational.
	Total		**1**	

June 2022 Question 6 Paper 2

6 (a) Asif notices that $24^2 = 576$ and $2 + 4 = 6$ gives the last digit of 576

He checks two more examples:

$27^2 = 729$	$29^2 = 841$
$2 + 7 = 9$	$2 + 9 = 11$
Last digit 9	Last digit 1

Asif concludes that he can find the last digit of any square number greater than 100 by adding the digits of the number being squared.

Give a counter example to show that Asif's conclusion is **not** correct.

[2 marks]

6 (b) Claire tells Asif that he should look only at the last digit of the number being squared.

$27^2 = 729$	$24^2 = 576$
$7^2 = 49$	$4^2 = 16$
Last digit 9	Last digit 6

Using Claire's method determine the last digit of 23456789^2

[1 mark]

6 (c) Given Claire's method is correct, use proof by exhaustion to show that no square number has a last digit of 8

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Squares a number with two or more digits and adds its digits. Must be explicit	1.1a	M1	$12^2 = 144$ $1+2=3$ $3 \neq 4$
	Completes argument to show that Asif's method is incorrect Must compare sum of digits with last digit of square number	2.3	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)	Obtains 1	1.1b	B1	1
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
6(c)	Lists at least four single digits and their squares Or Explains why odd digits do not need to be considered	1.1a	M1	$0^2=0$, $1^2=1$, $2^2=4$, $3^2=9$, $4^2=16$, $5^2=25$, $6^2=36$, $7^2=49$, $8^2=64$, $9^2=81$
	Completes rigorous argument to prove that no square number has a last digit of 8 OE CSO	2.1	R1	Therefore, there can be no square number which has a last digit of 8
	Subtotal		**2**	

June 2022 Question 9 Paper 3

9 Assume that a and b are integers such that

$$a^2 - 4b - 2 = 0$$

9 (a) Prove that a is even.

[2 marks]

9 (b) Hence, prove that $2b + 1$ is even and explain why this is a contradiction.

[3 marks]

9 (c) Explain what can be deduced about the solutions of the equation

$$a^2 - 4b - 2 = 0$$

[1 mark]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Begins argument with either of: • Factorises $-4b-2$ or $4b+2$ correctly to 2b+1 as a factor • States $4b$ and 2 are both even or begins proof by contradiction by assuming a is odd therefore a^2 is odd	2.1	M1	$a^2 - 4b - 2 = 0$ $a^2 = 4b + 2$ $a^2 = 2(2b+1)$ Hence a^2 must be even, which means that a must be even
	Completes reasoned argument by deducing that a^2 must be even or has a factor of 2, which means that a must be even or Completes reasoned argument by deducing that a^2 = 4b+2 which is even because $4b$ and 2 are both even hence a^2 is even which is a contradiction OE	2.2a	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)	Uses $2p$ and obtains $(2p)^2$ PI by $4p^2$ Allow any letter for p except a and b	1.1a	M1	$a = 2p \Rightarrow (2p)^2 = 4p^2$ $4p^2 = 2(2b+1)$ $2p^2 = 2b+1$ Hence $2b+1$ must be even $2b+1$ is an odd number which is a contradiction
	Obtains either $4p^2 = 2(2b+1)$ or $4p^2 = 4b+2$ and followed by $2p^2 = 2b+1$ or $4p^2 = 2(2b+1)$ or $4p^2 = 4b+2$ and followed by $2 \times 2p^2 = 2(2b+1)$	3.1a	A1	
	Complete reasoned argument by deducing that $2b + 1$ is even hence contradiction as $2b+1$ is an odd number or Complete reasoned argument by deducing that $2b + 1$ is even hence contradiction as b cannot be an integer	2.2a	R1	
	Subtotal		**3**	

November 2021 Question 4 Paper 1

4 Millie is attempting to use proof by contradiction to show that the result of multiplying an irrational number by a non-zero rational number is always an irrational number.

Select the assumption she should make to start her proof.

Tick (✓) **one** box.

[1 mark]

Every irrational multiplied by a non-zero rational is irrational. ☐

Every irrational multiplied by a non-zero rational is rational. ☐

There exists a non-zero rational and an irrational whose product is irrational. ☐

There exists a non-zero rational and an irrational whose product is rational. ☐

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
4	Ticks the correct box	2.1	R1	There exists a non-zero rational and an irrational whose product is rational.
	Total		**1**	

November 2020 Question 5 Paper 1

5 Prove that, for integer values of n such that $0 \le n < 4$

$$2^{n+2} > 3^n$$

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
5	Selects and begins to use a suitable method of proof. **Exhaustion:** Must check at least two correct values for n in the range $0 \le n < 4$ and make at least two correct comparisons. Comparisons are implied by ticks/crosses or use of true/false **Direct proof:** Takes logs to any base of both sides and uses a law of logs correctly once **Contradiction:** Must be clear they are attempting contradiction with "$0 \le n < 4$ and $2^{n+2} \le 3^n$" assumed explicitly. Condone strict inequality	3.1a	M1	n | 2^{n+2} | 3^n | 0 | 4 | 1 | 4>1 1 | 8 | 3 | 8>3 2 | 16 | 9 | 16>9 3 | 32 | 27 | 32>27 Hence $2^{n+2} > 3^n$ for integer values of n such that $0 \le n < 4$
	Completes a reasoned mathematical argument, proving $2^{n+2} > 3^n$ when n is an integer and $0 \le n < 4$. Must include a fully correct concluding statement which refers to 'integer' or lists the four integers If using direct proof or contradiction they must use the laws of logs correctly to remove n from the exponent. Condone use of equality if direct proof used	2.1	R1	
	Total		**2**	

November 2020 Question 7 Paper 2

7 a and b are two positive irrational numbers.

The sum of a and b is rational.

The product of a and b is rational.

Caroline is trying to prove $\frac{1}{a}+\frac{1}{b}$ is rational.

Here is her proof:

Step 1 $\frac{1}{a}+\frac{1}{b}=\frac{2}{a+b}$

Step 2 2 is rational and $a+b$ is non-zero and rational.

Step 3 Therefore $\frac{2}{a+b}$ is rational.

Step 4 Hence $\frac{1}{a}+\frac{1}{b}$ is rational.

7 (a) (i) Identify Caroline's mistake.

[1 mark]

7 (a) (ii) Write down a correct version of the proof.

[2 marks]

7 (b) Prove by contradiction that the difference of any rational number and any irrational number is irrational.

[4 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
7(a)(i)	Identifies the error lies in step 1 without contradiction.	2.3	E1	Mistake is $\frac{1}{a}+\frac{1}{b}=\frac{2}{a+b}$
	Subtotal		**1**	
7(a)(ii)	Recalls correct addition Accept $\frac{b}{ab}+\frac{a}{ab}$	1.1b	M1	$\frac{1}{a}+\frac{1}{b}=\frac{a+b}{ab}$ a + b is rational and ab is rational and therefore $\frac{1}{a}+\frac{1}{b}$ is rational.
	Completes rigorous argument to complete proof. Must state that ab is rational (and non-zero) and $a+b$ is rational and conclude that $\frac{1}{a}+\frac{1}{b}$ or $\frac{a+b}{ab}$ is rational	2.1	R1	
	Subtotal		**2**	
7(b)	States assumption to begin proof by contradiction may PI by $\frac{a}{b}-x=\frac{c}{d}$ or $x-\frac{a}{b}=\frac{c}{d}$	3.1a	M1	Assume that the difference between a rational and an irrational number is rational. $\frac{a}{b}-x=\frac{c}{d}$
	Uses language and notation correctly to state initial assumptions: States their a, b, c and d are integers **and** x is irrational do not accept the irrational written as a fraction Condone missing $b, d \neq 0$	2.5	A1	Where a, b, c and d are integers, $b, d \neq 0$ and x is irrational
	Demonstrates that x can be expressed as a rational number by obtaining $x=\frac{ad-cb}{bd}$ OE	1.1b	M1	$x=\frac{a}{b}-\frac{c}{d}$ $=\frac{ad}{bd}-\frac{cb}{bd}$ $=\frac{ad-cb}{bd}$
	Completes rigorous argument to prove the required result, clearly explaining where the contradiction lies with **ALL** assumptions correct at the start (including $b, d \neq 0$)	2.1	R1	Hence x is rational. This is a contradiction hence the difference of any rational number and any irrational number is irrational.
	Subtotal		**4**	
	Question Total		**7**	

June 2019 Question 9 Paper 1

9 Prove that the sum of a rational number and an irrational number is always irrational.

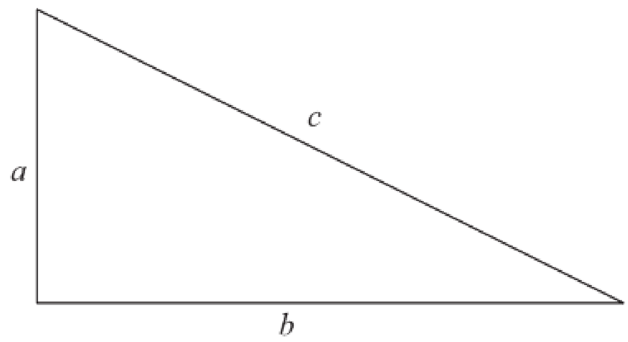
[5 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
9	Begins proof by contradiction. This may be evidenced by: stating assumption at the start "the sum is rational" Or Sight of "contradiction" later as part of argument.	3.1a	M1	Assume m is rational and n is irrational and their sum is rational.
	Forms an equation of the form rational + irrational = rational with the rationals written algebraically $\frac{a}{b}+n=\frac{c}{d}$ n must clearly be irrational and not written as an algebraic fraction and not a specific value.	2.5	M1	Then $\frac{a}{b}+n=\frac{c}{d}$ Where a, b, c and d are all integers. $n=\frac{c}{d}-\frac{a}{b}$ $=\frac{bc-ad}{bd}$
	Manipulates their equation to show that n is rational	1.1b	A1	$\therefore$ n is rational, which is a contradiction.
	Explains or demonstrates why there is a contradiction	2.4	E1	So the original statement is false and the sum of a rational and irrational must be irrational.
	Completes rigorous argument to prove the required result including correct initial assumptions Where a, b, c and d are all integers.	2.1	R1	
	Total		**5**	

June 2018 Question 6 Paper 3

6 The three sides of a right-angled triangle have lengths a, b and c, where $a, b, c \in \mathbb{Z}$



6 (a) State an example where a, b and c are all even.

[1 mark]

6 (b) Prove that it is **not** possible for all of a, b and c to be odd.

[3 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
6(a)	States an appropriate even Pythagorean triple	2.2a	B1	$a = 6$ $b = 8$ $c = 10$
6(b)	Begins an appropriate method of proof assuming at least two sides are odd eg states 'assume a, b odd' or defines a, b (or c) algebraically with different unknowns	3.1a	B1	Assume a and b are odd so $a = 2m + 1$ and $b = 2n + 1$ $(2m+1)^2 + (2n+1)^2$ $= 4m^2 + 4m + 1 + 4n^2 + 4n + 1$ $= 2(2m^2 + 2m + 2n^2 + 2n + 1)$ which is even, so c^2 is even, so c is even. Therefore it is not possible for all three to be odd.
	Uses Pythagoras' theorem with at least two odd sides either in words or algebraically	1.1a	M1	
	Completes rigorous argument to prove the required result **CSO**	2.1	R1	
	Total		**4**	

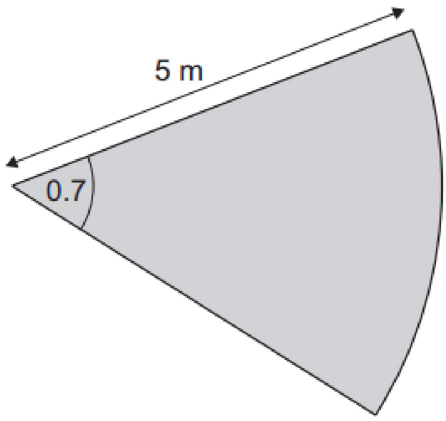
November 2021 Question 5 Paper 3

5 A gardener is creating flowerbeds in the shape of sectors of circles.

The gardener uses an edging strip around the perimeter of each of the flowerbeds.

The cost of the edging strip is £1.80 per metre and can be purchased for any length.

One of the flowerbeds has a radius of 5 metres and an angle at the centre of 0.7 radians as shown in the diagram below.



5 (a) (i) Find the area of this flowerbed.

[2 marks]

5 (a) (ii) Find the cost of the edging strip required for this flowerbed.

[3 marks]

5 (b) A flowerbed is to be made with an area of 20 m^2

5 (b) (i) Show that the cost, £C, of the edging strip required for this flowerbed is given by

$$C = \frac{18}{5}\left(\frac{20}{r} + r\right)$$

where r is the radius measured in metres.

[3 marks]

5 (b) (ii) Hence, show that the minimum cost of the edging strip for this flowerbed occurs when $r \approx 4.5$

Fully justify your answer.

[5 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
5(a)(i)	Uses formula correctly for area of sector	1.1a	M1	$A=\frac{1}{2}\times 5^2\times 0.7$ $=8.75\,\text{m}^2$
	Obtains 8.75 Condone incorrect or missing units	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)(ii)	Uses formula for arc length	1.1b	B1	$P=5\times 0.7+2\times 5$ $=13.5$ $Cost=13.5\times 1.80$ $=£24.30$
	Obtains the perimeter by adding twice the radius to their arc length and multiplies their perimeter by 1.80	3.1b	M1	
	Obtains correct cost £24.30 **CAO**	3.2a	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)(i)	Forms at least one correct equation for area or perimeter May be embedded in the formulae for C	3.3	M1	$P=r\theta+2r$ $\frac{1}{2}r^2\theta=20$ $\Rightarrow\theta=\frac{40}{r^2}$ $P=\frac{40}{r}+2r$ $C=\frac{40\times 1.8}{r}+2\times 1.8r$ $=\frac{72}{r}+\frac{18}{5}r$ $=\frac{18}{5}\left(\frac{20}{r}+r\right)$
	Eliminates θ from two fully correct equations for area and perimeter to obtain an expression for P in terms of r	3.1b	A1	
	Completes argument to show the required result Accept 3.6 for $\frac{18}{5}$	2.1	R1	
	Subtotal		**3**	

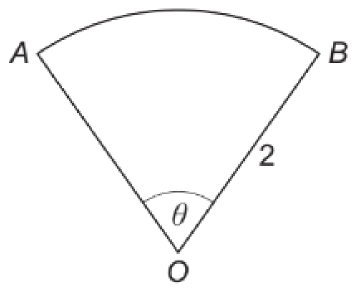
Q	Marking instructions	AO	Marks	Typical solution
5(b)(ii)	Recognises the use differentiation in the model **PI** if $\frac{dC}{dr}$ seen	3.4	B1	$C = \frac{72}{r} + \frac{18}{5}r$ $\frac{dC}{dr} = -\frac{72}{r^2} + \frac{18}{5}$ Minimum occurs when $\frac{dC}{dr} = 0$ $-\frac{72}{r^2} + \frac{18}{5} = 0$ $r^2 = 20$ $r = \sqrt{20} \approx 4.472..$ Hence $r \approx 4.5$ $\frac{d^2C}{dr^2} = \frac{144}{r^3}$ When $r = \sqrt{20}$, $\frac{d^2C}{dr^2} > 0$ Therefore minimum at $r \approx 4.5$
	Differentiates given model with at least one term correct Condone sign error **OE**	1.1b	M1	
	Explains that a minimum/stationary/turning point occurs when $\frac{dC}{dr} = 0$	2.4	E1	
	Solves $\frac{dC}{dr} = 0$ to find correct exact value or decimal value for r to at least two decimal places	1.1b	A1	
	Uses a gradient test or second derivative or sketches graph to determine nature of stationary point Completes argument to show minimum occurs when $r \approx 4.5$ Must have shown $r \approx 4.5$ in previous step	2.1	R1	
	Subtotal		**5**	

Video solution:

https://youtu.be/26igHZSYDts

November 2020 Question 3 Paper 1

3 The diagram shows a sector *OAB* of a circle with centre *O* and radius 2



The angle *AOB* is θ radians and the perimeter of the sector is 6

Find the value of θ

Circle your answer.

[1 mark]

1 $\sqrt{3}$ 2 3

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
3	Circles the correct answer	2.2a	R1	1
	Total		**1**	

November 2020 Question 4 Paper 2

4 Using small angle approximations, show that for small, non-zero, values of x

$$\frac{x \tan 5x}{\cos 4x - 1} \approx A$$

where A is a constant to be determined.

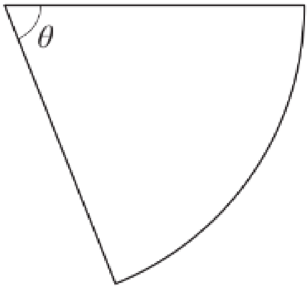
[4 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
4	Uses or states small angle approximation for $\tan 5x \approx 5x$	1.1b	B1	$\frac{x\tan 5x}{\cos 4x - 1} \approx \frac{x \times 5x}{1 - \frac{(4x)^2}{2} - 1}$ $\approx \frac{5x^2}{-8x^2}$ $\approx -\frac{5}{8}$
	Uses or states small angle approximation for $\cos 4x \approx 1 - \frac{(4x)^2}{2}$ Condone omission of bracket	1.1b	B1	
	Substitutes their expressions Of the form $\tan 5x \approx mx$ and $\cos 4x \approx 1 - \frac{nx^2}{2}$ into $\frac{x\tan 5x}{\cos 4x - 1}$ Condone correct extra terms	1.1b	M1	
	Deduces $A = -\frac{5}{8}$ from a reasoned argument CSO	2.2a	R1	
	Total		**4**	

June 2019 Question 3 Paper 1

3 The diagram below shows a sector of a circle.



The radius of the circle is 4 cm and $\theta = 0.8$ radians.

Find the area of the sector.

Circle your answer.

[1 mark]

1.28 cm^2 3.2 cm^2 6.4 cm^2 12.8 cm^2

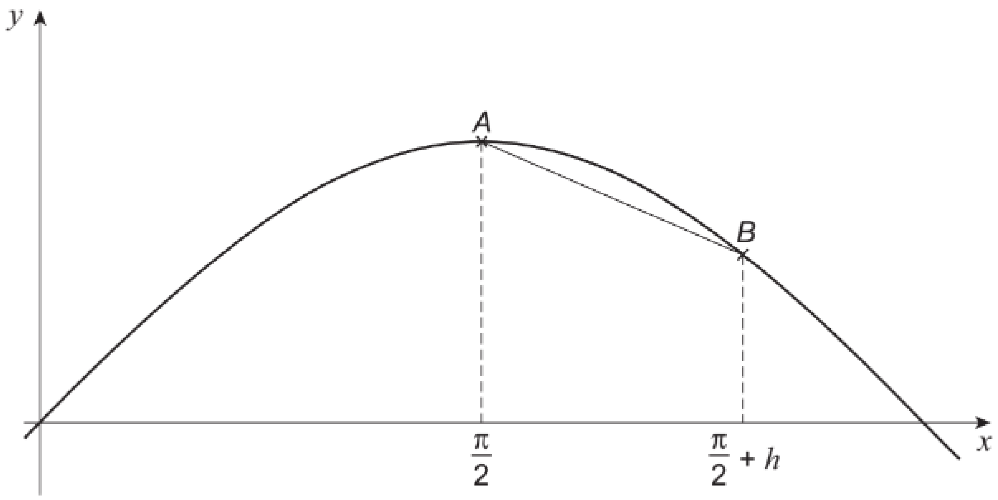
ANSWER

Q	Marking instructions	AO	Mark	Typical solution
3	Circles the correct response	1.1b	B1	6.4 cm^2
	Total		**1**	

June 2019 Question 11 Paper 1

11 Jodie is attempting to use differentiation from first principles to prove that the gradient of $y = \sin x$ is zero when $x = \frac{\pi}{2}$

Jodie's teacher tells her that she has made mistakes starting in Step 4 of her working. Her working is shown below.



Step 1 Gradient of chord $AB = \dfrac{\sin\left(\frac{\pi}{2} + h\right) - \sin\left(\frac{\pi}{2}\right)}{h}$

Step 2 $= \dfrac{\sin\left(\frac{\pi}{2}\right)\cos(h) + \cos\left(\frac{\pi}{2}\right)\sin(h) - \sin\left(\frac{\pi}{2}\right)}{h}$

Step 3 $= \sin\left(\frac{\pi}{2}\right)\left(\dfrac{\cos(h) - 1}{h}\right) + \cos\left(\frac{\pi}{2}\right)\dfrac{\sin(h)}{h}$

Step 4 For gradient of curve at A,

let $h = 0$ then

$\dfrac{\cos(h) - 1}{h} = 0$ and $\dfrac{\sin(h)}{h} = 0$

Step 5 Hence the gradient of the curve at A is given by

$\sin\left(\frac{\pi}{2}\right) \times 0 + \cos\left(\frac{\pi}{2}\right) \times 0 = 0$

Complete Steps 4 and 5 of Jodie's working below, to correct her proof.

[4 marks]

Step 4 For gradient of curve at *A*,

Step 5 Hence the gradient of the curve at *A* is given by

ANSWER

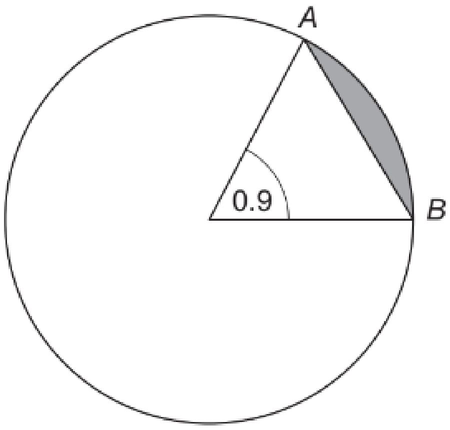
Q	Marking instructions	AO	Mark	Typical solution
11	Replaces $h=0$ with $h \to 0$ or better seen anywhere	2.3	M1	For gradient of curve at *A*, let $h \to 0$ then
	Uses limit notation fully correctly Accept $\frac{\sin(h)}{h} \to 0$ here Accept full limit notation here	2.5	A1	$\frac{\cos(h)-1}{h} \to 0$ and $\frac{\sin(h)}{h} \to 1$
	$\frac{\sin(h)}{h} = 1$ seen OE eg $\sin(h) = h$	2.3	B1	
	Writes last line explicitly as $\sin\left(\frac{\pi}{2}\right) \times 0 + \cos\left(\frac{\pi}{2}\right) \times 1 = 0$ Accept $1 \times 0 + 0 \times 1 = 0$	2.2a	B1	Hence the gradient of the curve at A is given by $\sin\left(\frac{\pi}{2}\right) \times 0 + \cos\left(\frac{\pi}{2}\right) \times 1 = 0$
	Total		**4**	

June 2019 Question 5 Paper 3

5 A circle has equation $x^2 + y^2 - 6x - 8y = 264$

AB is a chord of the circle.

The angle at the centre of the circle, subtended by *AB*, is 0.9 radians, as shown in the diagram below.



Find the area of the minor segment shaded on the diagram.

Give your answer to three significant figures.

[5 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
5	Uses appropriate method to find radius eg complete the square by using 3^2 or 4^2 on LHS or RHS **PI** by correct radius 17 or 289	3.1a	M1	$(x-3)^2-9+(y-4)^2-16=264$ $(x-3)^2+(y-4)^2=289$ $\frac{1}{2}\times17^2\times0.9=130.05$ $\frac{1}{2}\times17^2\sin0.9=113.19$ Area of segment = 16.9
	Deduces correct radius or radius squared or fully correct completed square form seen	2.2a	A1	
	Uses appropriate method to find area of sector using radius 17 or their stated value of radius or value of radius clearly shown on diagram	1.1a	M1	
	Uses appropriate method to find area of triangle using radius 17 or their stated radius	1.1a	M1	
	Obtains area correct to at least 3 significant figures **AWRT** 16.9	1.1b	A1	
	Total		**5**	

June 2018 Question 5 Paper 3

5 Show that, for small values of x, the graph of $y = 5 + 4\sin\frac{x}{2} + 12\tan\frac{x}{3}$ can be approximated by a straight line.

[3 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
5	Uses small angle approximation for $\sin x$ or $\tan x$ Condone $y = 5 + 4x + 12x$ for this mark	AO1.1a	M1	$y = 5 + 4\sin\frac{x}{2} + 12\tan\frac{x}{3}$ $\sin x \approx x, \tan x \approx x$ $y \approx 5 + 4\left(\frac{x}{2}\right) + 12\left(\frac{x}{3}\right)$ $y \approx 6x + 5$ which is the equation of a straight line.
	Obtains correct equation Allow unsimplified form	AO1.1b	A1	
	Concludes that the graph can be approximated by a straight line. Requires simplification of equation (condone equals) and statement.	AO2.1	R1	
	Total		**3**	

Ch 3 Sequences and series

June 2022 Question 3 Paper 1

3 A geometric sequence has a sum to infinity of -3

A second sequence is formed by multiplying each term of the original sequence by -2

What is the sum to infinity of the new sequence?

Circle your answer.

[1 mark]

The sum to infinity does not exist	-6	-3	6

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
3	Circles correct answer	2.2a	R1	6
	Total		**1**	

June 2022 Question 12 Paper 1

12 (a) A geometric sequence has first term 1 and common ratio $\frac{1}{2}$

12 (a) (i) Find the sum to infinity of the sequence.

[2 marks]

12 (a) (ii) Hence, or otherwise, evaluate

$$\sum_{n=1}^{\infty} (\sin 30°)^n$$

[2 marks]

12 (b) Find the smallest positive exact value of θ, in **radians**, which satisfies the equation

$$\sum_{n=0}^{\infty} (\cos\theta)^n = 2 - \sqrt{2}$$

[4 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
12(a)(i)	Uses $\frac{a}{1-r}$	1.1a	M1	$S_\infty = \frac{1}{1-\frac{1}{2}} = 2$
	Obtains 2	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
12(a)(ii)	Deduces $a = \frac{1}{2}$ and $r = \frac{1}{2}$ or Deduces $\sum_{n=1}^{\infty}\left(\sin 30^\circ\right)^n$ = their part **(a)(i)** -1 or Deduces the answer is half of their answer in part **(a)(i)**	2.2a	M1	$\sum_{n=1}^{\infty}\left(\sin 30^\circ\right)^n = \frac{1}{2} + \frac{1}{4} + \ldots$ $= \frac{\frac{1}{2}}{1-\frac{1}{2}}$ $= 1$
	Obtains 1	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
12(b)	Forms equation $\frac{a}{1-r}=2-\sqrt{2}$ If the above is not seen then condone $\frac{\cos\theta}{1-\cos\theta}=2-\sqrt{2}$ or Condone use of a numerical value for a where $a > 0$	3.1a	M1	$\sum_{n=0}^{\infty}(\cos\theta)^n=2-\sqrt{2}$ $\frac{a}{1-r}=2-\sqrt{2}$ $a=1$ $r=\cos\theta$ $1-\cos\theta=\frac{1}{2-\sqrt{2}}$ $\cos\theta=1-\frac{1}{2-\sqrt{2}}$ $\theta=\frac{3\pi}{4}$
	Uses $a=1$ and $r=\cos\theta$	1.1b	B1	
	Obtains either $r=1-\frac{1}{2-\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$ or $\cos\theta=1-\frac{1}{2-\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$ ACF	1.1b	A1	
	Deduces $\theta=\frac{3\pi}{4}$	2.2a	R1	
	Subtotal		**4**	

June 2022 Question 2 Paper 1

2 A periodic sequence is defined by

$$U_n = (-1)^n$$

State the period of the sequence.

Circle your answer.

[1 mark]

-1 0 1 2

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
2	Circles the correct answer	1.1b	B1	2
	Question 2 Total		**1**	

June 2022 Question 9 Paper 1

9 The first three terms of an arithmetic sequence are given by

$$2x + 5 \qquad 5x + 1 \qquad 6x + 7$$

9 (a) Show that $x = 5$ is the only value which gives an arithmetic sequence.

[3 marks]

9 (b) (i) Write down the value of the first term of the sequence.

[1 mark]

9 (b) (ii) Find the value of the common difference of the sequence.

[1 mark]

9 (c) The sum of the first N terms of the arithmetic sequence is S_N where

$$S_N < 100\,000$$

$$S_{N+1} > 100\,000$$

Find the value of N.

[4 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Forms an appropriate equation in x only by either using the differences of at least one pair of terms or Using the mean of the first and third term = the second term Condone missing brackets or Forms two simultaneous equations in x and d or Substitutes x = 5 and demonstrates that the three terms obtained, 15, 26 and 37 have a common difference of 11 or Shows that the sum formula for an arithmetic series works when x = 5 The approaches that substitute x = 5 score a maximum of M1 A0 R0	3.1a	M1	$5x+1-(2x+5)=6x+7-(5x+1)$ $3x-4=x+6$ $x=5$ Therefore $x=5$ is the only solution
	Obtains a correct equation or Obtains two correct simultaneous equations in x and d Need not be simplified	1.1b	A1	
	Solves to conclude that $x=5$ is the only solution Must include the word 'only' OE	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)(i)	Obtains 15	1.1b	B1	$a=15$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)(ii)	Obtains 11	1.1b	B1	$d=11$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
9(c)	Forms an expression for the sum to N or $N+1$ terms using their a and d values Need not be simplified Condone missing brackets or use of n or Uses a trial and improvement method obtaining sums for two different values of n	3.1a	M1	$S_N = \frac{N}{2}(2\times15+11(N-1))$ $\frac{N}{2}(2\times15+11(N-1))=100000$ $N=133.9...$ $N=133$
	Forms an equation or inequality using their expression and $100000\pm k$ where $0\le k\le11$ or Uses trial and improvement to obtain one sum below 100000 and one sum above 100000 for consecutive integers	1.1a	M1	
	Obtains either 133.9.. or 132.9.. or $N>132$ or $N<134$ or Obtains the sum of 98553 when n = 133 and obtains the sum of 100031 when n = 134	1.1b	A1	
	Obtains 133 having solved a correct quadratic This mark can be recovered if N = 133 and N = 134 are correctly checked	3.2a	A1	
	Subtotal		**4**	

	Question 9 Total		**9**	

June 2022 Question 1 Paper 3

1 State the range of values of x for which the binomial expansion of

$$\sqrt{1 - \frac{x}{4}}$$

is valid.

Circle your answer.

[1 mark]

$|x| < \frac{1}{4}$ $|x| < 1$ $|x| < 2$ $|x| < 4$

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
1	Circles correct answer	1.1b	B1	$\|x\| < 4$
	Question 1 Total		**1**	

November 2021 Question 3 Paper 2

3 A sequence is defined by

$$u_1 = a \text{ and } u_{n+1} = -1 \times u_n$$

Find $\sum_{n=1}^{95} u_n$

Circle your answer.

[1 mark]

$-a$ $\quad$ 0 $\quad$ a $\quad$ $95a$

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
3	Circles correct answer	2.2a	B1	a
	Total		**1**	

November 2021 Question 6 Paper 1

6 (a) The ninth term of an arithmetic series is 3

The sum of the first n terms of the series is S_n and $S_{21} = 42$

Find the first term and common difference of the series.

[4 marks]

6 (b) A second arithmetic series has first term -18 and common difference $\frac{3}{4}$

The sum of the first n terms of this series is T_n

Find the value of n such that $T_n = S_n$

[3 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Obtains $a+8d=3$ OE	1.1b	B1	$a+8d=3$ $\frac{21}{2}(2a+20d)=42$ $a+10d=2$ $a=7$ $d=-0.5$
	Obtains $\frac{21}{2}(2a+20d)=42$ OE	1.1b	B1	
	Begins to solve their $a+8d=3$ $\frac{21}{2}(2a+20d)=42$ with elimination of one variable or better. For their equations condone only the following slips $a+9d=3$ $\frac{21}{2}(2a+20d)=21$ PI correct a and d	3.1a	M1	
	Obtains correct a and d	1.1b	A1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)	Obtains at least one correct (unsimplified) expression for S_n or T_n FT their non-zero values of a and d for S_n PI by simplified correct equation.	1.1b	B1F	$\frac{n}{2}(14-0.5(n-1))=\frac{n}{2}(-36+0.75(n-1))$ $n=0\,or\,41$ Hence n = 41
	Equates their expressions S_n and T_n with at least one correct. FT their non-zero values of a and d for S_n And finds a non-zero value of n PI by n = 41	3.1a	M1	
	Deduces correct value of n = 41	2.2a	R1	
	Subtotal		**3**	

November 2021 Question 7 Paper 3

7 A building has a leaking roof and, while it is raining, water drips into a 12 litre bucket.

When the rain stops, the bucket is one third full.

Water continues to drip into the bucket from a puddle on the roof.

In the first minute after the rain stops, 30 millilitres of water drips into the bucket.

In each subsequent minute, the amount of water that drips into the bucket reduces by 2%.

During the nth minute after the rain stops, the volume of water that drips into the bucket is W_n millilitres.

7 (a) Find W_2

[1 mark]

7 (b) Explain why

$$W_n = A \times 0.98^{n-1}$$

and state the value of A.

[2 marks]

7 (c) Find the increase in the water in the bucket 15 minutes after the rain stops.

Give your answer to the nearest millilitre.

[2 marks]

7 (d) Assuming it does not start to rain again, find the maximum amount of water in the bucket.

[3 marks]

7 (e) After several hours the water has stopped dripping.

Give **two** reasons why the amount of water in the bucket is not as much as the answer found in part **(d)**.

[2 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
7(a)	Obtains the correct volume **AWRT** 29 Condone incorrect or missing units	1.1b	B1	$W_2 = 29.4$
	Subtotal		**1**	

Q	Marking instructions	AO	Mark	Typical solution
7(b)	States $A = 30$ **PI** by building up of a sequence to three terms or $W_n = 30 \times 0.98^{n-1}$ seen	1.1b	B1	W_n is the nth term of a geometric sequence, a 2% reduction gives a common ratio of 0.98 A = 30
	Explains W_n is (the nth term of) a geometric sequence explaining that a 2% reduction gives a common ratio of 0.98 **PI** by building up of a sequence to three terms	3.3	E1	
	Subtotal		**2**	

Q	Marking instructions	AO	Mark	Typical solution
7(c)	Uses geometric model with their value of A substituted to find S_{15}	3.4	M1	$S_{15} = \frac{30\left(1-0.98^{15}\right)}{1-0.98}$ $= 392$
	Obtains their correct value of S_{15} **FT** their value of A Condone unrounded answers	1.1b	A1F	
	Subtotal		**2**	

Q	Marking instructions	AO	Mark	Typical solution
7(d)	Uses sum to infinity formula with their value of A substituted	3.4	M1	$S_\infty = \dfrac{30}{1-0.98}$ $= 1500$ 1.5 + 4 = 5.5 litres
	Obtains their correct value of sum to infinity	1.1b	A1F	
	Obtains 5.5 litres **CAO** Accept answer in litres or millilitres	3.2a	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Mark	Typical solution
7(e)	Explains that the model used assumes the drips continue indefinitely which is unrealistic	3.5b	E1	The sum to infinity was used but this assumes there are infinite drips, but they have stopped
	States a relevant environmental factor eg water has evaporated or wind affected water level or water consumed by animals	3.5a	E1	Water will evaporate over several hours
	Subtotal		**2**	

Video solution:

https://youtu.be/PIc6hDTeuqg

November 2020 Question 7 Paper 1

7 Consecutive terms of a sequence are related by

$$u_{n+1} = 3 - (u_n)^2$$

7 (a) In the case that $u_1 = 2$

7 (a) (i) Find u_3

[2 marks]

7 (a) (ii) Find u_{50}

[1 mark]

7 (b) State a different value for u_1 which gives the same value for u_{50} as found in part **(a)(ii)**.

[1 mark]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
7(a)(i)	Substitutes 2 into formula correctly to obtain $u_2 = -1$ PI by correct $u_3 = 2$	1.1a	M1	$u_2 = -1$ $u_3 = 2$
	Obtains correct $u_3 = 2$ and no further working resulting in a contradictory value for u_3	1.1b	A1	
	Subtotal		**2**	
7(a)(ii)	Deduces correct $u_{50} = -1$	2.2a	B1	$u_{50} = -1$
	Subtotal		**1**	
7(b)	Deduces correct $u_1 = -2$ Accept any correct value eg $\sqrt{2}$ or $-\sqrt{2}$ Condone if ± 2 seen	2.2a	B1	$u_1 = -2$
	Subtotal		**1**	
	Question Total		**4**	

November 2020 Question 10 Paper 1

10 (a) An arithmetic series is given by

$$\sum_{r=5}^{20} (4r + 1)$$

10 (a) (i) Write down the first term of the series.

[1 mark]

10 (a) (ii) Write down the common difference of the series.

[1 mark]

10 (a) (iii) Find the number of terms of the series.

[1 mark]

10 (b) A **different** arithmetic series is given by

$$\sum_{r=10}^{100} (br + c)$$

where b and c are constants.

The sum of this series is 7735

10 (b) (i) Show that $55b + c = 85$

[4 marks]

10 (b) (ii) The 40th term of the series is 4 times the 2nd term.

Find the values of b and c.

[4 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
10(a)(i)	Obtains correct first term	1.1b	B1	21
	Subtotal		**1**	
10(a)(ii)	Obtains correct common difference	1.1b	B1	4
	Subtotal		**1**	
10(a)(iii)	Obtains correct number of terms	1.1b	B1	16
	Subtotal		**1**	
10(b)(i)	Finds or uses at least one of the first term, the common difference, the last term or the number of terms correctly or Expresses given series as a difference of two series using n= 1 to 100 and n = 1 to 9. Either $\sum_{n=1}^{100}(br+c)-\sum_{n=1}^{n=9}(br+c)$ or $b\sum_{n=1}^{100}r+100c-b\sum_{n=1}^{n=9}r-9c$	1.1b	B1	$n=91$ $a=10b+c$ $d=b$ $\mathrm{L}=100b+c$ $\frac{91}{2}(2(10b+c)+90b)=7735$ $91(55b+c)=7735$ $55b+c=85$
	Forms an equation in terms of b and c for the sum of n terms using '*their*' first term, '*their*' number of terms and either '*their*' common difference or '*their*' last term Alternative $\frac{100}{2}[2b+2c+99b]-\frac{9}{2}[2b+2c+8b]$	3.1a	M1	
	Obtains correct equation ACF Alternative 5050b +100c - 45b - 9c = 7735 or 5005b + 91c = 7735	1.1b	A1	
	Completes rigorous argument to show the required result. This must include at least one single step of correct working between the initial correct formula and the given answer AG	2.1	R1	
	Subtotal		**4**	

10(b)(ii)	Uses or writes down $a + 39d$ or $a + d$ with '*their*' expressions for a and d Must be in terms of b and c	3.1a	B1	$4(11b + c) = 49b + c$ $5b - 3c = 0$ $b = 1.5$ $c = 2.5$
	Uses '*their*' $a + 39d$ and $a + d$ consistently to form 'their' equation $u_{40} = 4u_2$ in terms of b and c. Condone use of $50b + c$ for the fortieth term Condone $11b + c = 4(49b + c)$ OE with '*their*' a and d in terms of b and c	1.1a	M1	
	Solves $55b + c = 85$ with '*their*' other equation involving b and c PI by obtaining correct values of b and c or Obtains $b = -12.75$ and $c = 786.25$ from using $11b + c = 4(49b + c)$	1.1a	M1	
	Obtains correct values of b and c	1.1b	A1	
	Subtotal		**4**	
	Question Total		**11**	

November 2020 Question 8 Paper 3

8 The sum to infinity of a geometric series is 96

The first term of the series is less than 30

The second term of the series is 18

8 (a) Find the first term and common ratio of the series.

[5 marks]

8 (b) (i) Show that the nth term of the series, u_n, can be written as

$$u_n = \frac{3^n}{2^{2n-5}}$$

[4 marks]

8 (b) (ii) Hence show that

$$\log_3 u_n = n(1 - 2\log_3 2) + 5\log_3 2$$

[3 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
8(a)	Forms an equation for sum to infinity or 2nd term **PI** by correct answer	1.1a	M1	$ar = 18$ $\frac{a}{1-r} = 96$ $r - r^2 = \frac{18}{96}$ $r = \frac{1}{4}$ or $\frac{3}{4}$ $a = 72$ or 24 Since a < 30 $r = \frac{3}{4}$ $a = 24$
	Obtains both correct equations	1.1b	A1	
	Solves their two equations to find two values of a or r **PI** by correct values of a or r	3.1a	M1	
	Obtains two correct values of a = 72 or 24 or obtains two correct values of $r = \frac{1}{4}$ or $\frac{3}{4}$	1.1b	A1	
	Deduces correct pair of a and r Follow through their values of a and r Must have one value of a > *30*	2.2a	A1F	
8(b)(i)	Substitutes their a and r into the expression $u_n = ar^{n-1}$	1.1b	B1F	$u_n = 24 \times \left(\frac{3}{4}\right)^{n-1}$ $= 3 \times 2^3 \times \frac{3^{n-1}}{2^{2(n-1)}}$ $= \frac{3 \times 3^{n-1}}{2^{-3} \times 2^{2(n-1)}}$ $= \frac{3^n}{2^{2n-5}}$
	Writes their a or r in terms of prime numbers fully	3.1a	M1	
	Deduces that $a = 24$ can be written as $2^3 \times 3$ and $r = \frac{3}{4}$ as $\frac{3}{2^2}$ **PI** by expressing all terms in powers of 2 and 3	2.2a	A1	
	Completes reasoned argument by expressing all terms in powers of 2 and 3 and simplifies to show required result **AG**	2.1	R1	
8(b)(ii)	Applies logarithmic subtraction or addition law correctly	1.1a	M1	
	Applies logarithmic power law to obtain either $n \log_3 3$ or $(2n-5) \log_3 2$ Condone omission of brackets	1.1a	M1	

	Completes reasoned argument by using $n \log_3 3$ and $(2n-5) \log_3 2$ to show required result **AG** Do not allow recovery of omitted brackets	2.1	R1	$\log_3 u_n = \log_3 \frac{3^n}{2^{2n-5}}$ $= \log_3 3^n - \log_3 2^{2n-5}$ $= n - (2n-5)\log_3 2$ $= n + (5-2n)\log_3 2$ $= n - 2n\log_3 2 + 5\log_3 2$ $= n(1-2\log_3 2) + 5\log_3 2$
	Total		**12**	

June 2019 Question 5 Paper 1

5 An arithmetic sequence has first term a and common difference d.

The sum of the first 16 terms of the sequence is 260

5 (a) Show that $4a + 30d = 65$

[2 marks]

5 (b) Given that the sum of the first 60 terms is 315, find the sum of the first 41 terms.

[3 marks]

5 (c) S_n is the sum of the first n terms of the sequence.

Explain why the value you found in part **(b)** is the maximum value of S_n

[2 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
5(a)	Uses S_n =260 for arithmetic sequence with *n*=16 to form a correct equation PI by $8(2a+15d)=260$	1.1a	M1	$\frac{16}{2}(2a+(16-1)d)=260$ $8(2a+15d)=260$ $2(2a+15d)=65$ $4a+30d=65$
	Completes rigorous argument with correct algebraic manipulation to show required result **Must** see at least one line of simplification after $8(2a+15d)=260$ before given answer.	2.1	R1	
5(b)	Forms a second equation in ***a*** and ***d*** using $S_{60}=315$ and solves simultaneously to find ***a*** or ***d***	3.1a	M1	$30(2a+59d)=315$ $20a+590d=105$ $a=20$ $d=-0.5$ $S_{41}=\frac{41}{2}(2\times 20-40\times 0.5)=410$
	Obtains correct ***a*** and ***d***	1.1b	A1	
	Uses their ***a*** and ***d*** to obtain their value of $S_{41}=41a+820d$ Follow through provided one of their a or d is correct.	1.1b	A1F	
5(c)	Explains that values of U_n are positive n < 41 Or Explains that values of U_n are negative for $n>41$ Or Uses quadratic manipulation or differentiation of formula for S_n to obtain n = 40.5 CSO	2.4	M1	The terms before the 41st term are all positive. The terms after the 41st term are all negative so the sum of the first 41 terms must be a maximum value.
	Completes a valid argument explaining all terms positive before 41 and negative after 41 Or Completes argument linking 40.5 with the sum to 40 terms and the sum to 41 terms. CSO	2.1	R1	
	Total		**7**	

June 2019 Question 8 Paper 1

8 $\mathrm{P}(n) = \sum_{k=0}^{n} k^3 - \sum_{k=0}^{n-1} k^3$ where n is a positive integer.

8 (a) Find P(3) and P(10)

[2 marks]

8 (b) Solve the equation $\mathrm{P}(n) = 1.25 \times 10^8$

[2 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
8(a)	Obtains one correct value	1.1b	B1	$P(3)=27$ $P(10)=1000$
	Obtains both correct values	1.1b	B1	
8(b)	Forms cubic equation replacing $P(n)=n^3$ (condone k^3) PI correct answer	2.5	M1	$n^3 = 1.25\times10^8$ $n=500$
	Obtains 500 CSO	1.1b	A1	
	Total		**4**	

Arancha Ruiz

June 2019 Question 3 Paper 3

3 Given $u_1 = 1$, determine which one of the formulae below defines an increasing sequence for $n \geq 1$

Circle your answer.

[1 mark]

$u_{n+1} = 1 + \frac{1}{u_n}$ $\qquad$ $u_n = 2 - 0.9^{n-1}$ $\qquad$ $u_{n+1} = -1 + 0.5u_n$ $\qquad$ $u_n = 0.9^{n-1}$

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
3	Circles the correct response	2.2a	B1	$u_n = 2 - 0.9^{n-1}$
				$u_n = 2 - 0.9^{n-1}$
	Total		**1**	

June 2018 Question 3 Paper 1

3 A periodic sequence is defined by $U_n = \sin\left(\frac{n\pi}{2}\right)$

State the period of this sequence.

Circle your answer.

[1 mark]

8 2π 4 π

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
3	Circles correct answer	AO1.1b	B1	4
	Total		**1**	

June 2018 Question 9 Paper 1

9 An arithmetic sequence has first term a and common difference d.

The sum of the first 36 terms of the sequence is equal to the square of the sum of the first 6 terms.

9 (a) Show that $4a + 70d = 4a^2 + 20ad + 25d^2$

[4 marks]

9 (b) Given that the sixth term of the sequence is 25, find the smallest possible value of a.

[5 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
9(a)	Uses S_n for arithmetic sequence with n = 6 or n = 36	AO1.1a	M1	$S_6 = 3(2a+5d)$ $= 6a+15d$
	Finds correct expressions for S_6 and S_{36}	AO1.1b	A1	$S_{36} = 18(2a+35d)$ $= 36a+630d$
	Forms equation in a and d using *their* S_{36} = (*their* S_6)^2	AO3.1a	M1	
	Expands quadratic and collects like terms to obtain printed answer Only award for completely correct solution with no errors	AO2.1	R1	$36a+630d = (6a+15d)^2$ $36a+630d = 36a^2+90ad+90ad+225d^2$ $4a+70d = 4a^2+20ad+25d^2$
(b)	Uses u_n for arithmetic sequence with n = 6	AO1.1b	B1	$a+5d = 25 \Rightarrow d = \dfrac{25-a}{5}$
	Eliminates a or d using *their* 'a+5d = 25' and the printed result in part (a) to obtain a quadratic in one variable	AO1.1a	M1	$4a+70\left(\dfrac{25-a}{5}\right) = 4a^2+20a\left(\dfrac{25-a}{5}\right)+25\left(\dfrac{25-a}{5}\right)^2$ $4a+350-14a = 4a^2+100a-4a^2+625-50a+a^2$
	Obtains correct quadratic equation Need not be simplified	AO1.1b	A1	350-10a=100a+625-50a+ a^2
	Solves their quadratic a =-5, a = -55 (*or* d=6, d=16)	AO1.1a	M1	a^2+60a+275=0
	Deduces min value a=-55 **NMS** a=-55 5/5	AO3.2a	A1	a =-5, a = -55 (*or* d=6, d=16) a = -55
	Total		**9**	

Ch 4 Functions

June 2022 Question 3 Paper 1

3 The curve

$$y = \log_4 x$$

is transformed by a stretch, scale factor 2, parallel to the y-axis.

State the equation of the curve after it has been transformed.

Circle your answer.

[1 mark]

$y = \frac{1}{2}\log_4 x$ $\quad$ $y = 2\log_4 x$ $\quad$ $y = \log_4 2x$ $\quad$ $y = \log_8 x$

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
3	Circles the correct answer	2.2a	R1	$y = 2\log_4 x$
	Question 3 Total		**1**	

June 2022 Question 3 Paper 3

3 The function f is defined by

$$f(x) = 2x + 1$$

Solve the equation

$$f(x) = f^{-1}(x)$$

Circle your answer.

[1 mark]

$x = -1$ $x = 0$ $x = 1$ $x = 2$

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
3	Circles correct answer	3.1a	B1	$x = -1$
	Question 3 Total		**1**	

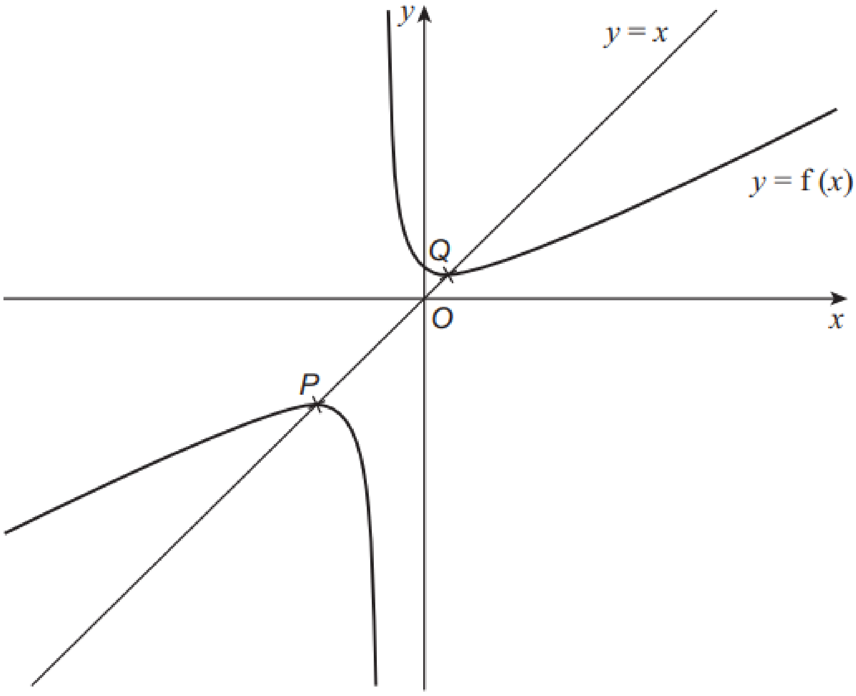
June 2022 Question 10 Paper 3

10 The function f is defined by

$$f(x) = \frac{x^2 + 10}{2x + 5}$$

where f has its maximum possible domain.

The curve $y = f(x)$ intersects the line $y = x$ at the points P and Q as shown below.



10 (b) Explain how you know that the function f is many-to-one.

[2 marks]

10 (c) (i) Show that the x-coordinates of P and Q satisfy the equation

$$x^2 + 5x - 10 = 0$$

[2 marks]

10 (c) (ii) Hence, find the exact x-coordinate of P and the exact x-coordinate of Q.

[1 mark]

10 (d) Show that P and Q are stationary points of the curve.

Fully justify your answer.

[5 marks]

10 (e) Using set notation, state the range of f.

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
10(a)	States -2.5 OE	2.2a	B1	-2.5
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)	Explains that many-to-one function is when distinct values of x give the same value for y	2.4	E1	Many-to-one function is when two or more x values give the same y value. This graph is many-to-one because you can draw a horizontal line and it will cross the graph twice.
	Uses the shape of the graph to justify their answer or gives an example of two x values eg $\mathrm{f}(0)=\mathrm{f}(4)$ or states turning or minimum or maximum points indicate many-to-one	2.4	E1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(c)(i)	Equates x and $\frac{x^2+10}{2x+5}$	3.1a	M1	$x=\frac{x^2+10}{2x+5}$ $x(2x+5)=x^2+10$ $2x^2+5x=x^2+10$ $x^2+5x-10=0$
	Rearranges with at least one intermediate step to obtain quadratic equation AG Condone $0 = x^2 + 5x - 10$	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(c)(ii)	Obtains $\frac{-5\pm\sqrt{65}}{2}$ Ignore any labels ISW	1.1b	B1	$x=\frac{-5\pm\sqrt{65}}{2}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
10(d)	Uses quotient rule to obtain an expression in the form of $\frac{Ax(2x+5)+B(x^2+10)}{(2x+5)^2}$ or uses product rule to obtain an expression in the form of $Cx(2x+5)^{-1}+D(x^2+10)(2x+5)^{-2}$ or uses implicit differentiation to obtain an equation of the form $Ax\frac{dy}{dx}+By+C\frac{dy}{dx}=Dx$ A, B, C and D can be any values but not 0 Condone missing brackets	3.1a	M1	$f'(x)=\frac{2x(2x+5)-2(x^2+10)}{(2x+5)^2}$ $=\frac{2x^2+10x-20}{(2x+5)^2}$ $f'(x)=0 \Leftrightarrow 2x^2+10x-20=0$ $x^2+5x-10=0$ This is the same equation solved in part c(i) so P and Q must be stationary points.
	Obtains fully correct $f'(x)$ or obtains $2x\frac{dy}{dx}+2y+5\frac{dy}{dx}=2x$ ACF May be unsimplified	1.1b	A1	

	Equates their $f'(x)$ or their numerator of $f'(x)$ to 0 or sets $\frac{dy}{dx} = 0$	1.1a	M1	
	Rearranges to obtain $x^2 + 5x - 10 = 0$ or $2x^2 + 10x - 20 = 0$ and links it to the equation in part **c(i)** or their answer to **c(ii)** or solves their quadratic $f'(x) = 0$ correctly or deduces $y = x$ and substitutes to get $x = \frac{x^2 + 10}{2x + 5}$ then rearranges to get $x^2 + 5x - 10 = 0$	1.1a	M1	

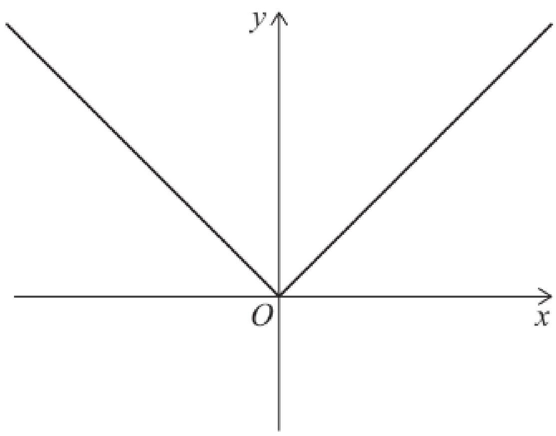
	Completes a reasoned argument by using $x = \frac{-5 \pm \sqrt{65}}{2}$ to conclude that P and Q are stationary points CSO Must have brackets correct throughout	2.1	R1	
	Subtotal		**5**	

Q	Marking instructions	AO	Marks	Typical solution
10(e)	Deduces critical regions from their answer to **c(ii)** condone strict inequalities or poor notation or decimal values	2.2a	M1	$x \le \frac{-5-\sqrt{65}}{2}$ and $x \ge \frac{-5+\sqrt{65}}{2}$ $\left\{x : x \le \frac{-5-\sqrt{65}}{2}\right\} \cup \left\{x : x \ge \frac{-5+\sqrt{65}}{2}\right\}$
	Writes correct range in correct set notation eg $\left(-\infty, \frac{-5-\sqrt{65}}{2}\right] \cup \left[\frac{-5+\sqrt{65}}{2}, \infty\right)$ Accept other letters for x or using $f(x)$ provided consistent throughout set Follow through their answer to **c(ii)**	2.5	A1F	
	Subtotal		**2**	

November 2021 Question 4 Paper 2

4 **Figure 1** shows the graph of $y = |2x|$

Figure 1



4 (a) On **Figure 1** add a sketch of the graph of

$$y = |3x - 6|$$

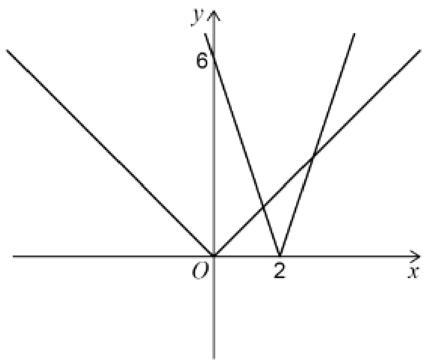
[2 marks]

4 (b) Find the coordinates of the points of intersection of the two graphs.

Fully justify your answer.

[4 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
4(a)	Sketches any V shaped graph with the apex on the positive x axis	1.1a	M1	
	Sketches a roughly symmetrical v-shaped graph touching the positive x-axis and intersecting $y = \|2x\|$ twice in the first quadrant Condone missing or incorrect labels on the axes	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
4(b)	Forms the equation $\|3x - 6\| = \|2x\|$ and selects an appropriate method to begin removing modulus signs For example Squares both sides to obtain $9x^2 - 36x + 36 = 4x^2$ or Considers $3x - 6 = 2x$ or $3x - 6 = -2x$	3.1a	M1	$\|3x - 6\| = \|2x\|$ $3x - 6 = 2x$ $x = 6$ $-3x + 6 = 2x$ $x = 1.2$ when $x = 6$ $y = 12$ when $x = 1.2$ $y = 2.4$
	Obtains $x = 6$	1.1b	A1	
	Obtains $x = 1.2$ OE	1.1b	A1	
	Obtains $y = 12$ and $y = 2.4$	1.1b	A1	
	Subtotal		**4**	

November 2021 Question 10 Paper 2

10 The function h is defined by

$$h(x) = \frac{\sqrt{x}}{x - 3}$$

where h has its maximum possible domain.

10 (a) Find the domain of h.

Give your answer using set notation.

[3 marks]

10 (b) Alice correctly calculates

$$h(1) = -0.5 \quad \text{and} \quad h(4) = 2$$

She then argues that since there is a change of sign there must be a value of x in the interval $1 < x < 4$ that gives $h(x) = 0$

Explain the error in Alice's argument.

[2 marks]

10 (c) By considering any turning points of h, determine whether h has an inverse function.

Fully justify your answer.

[6 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
10(a)	Obtains a domain excluding negatives or excluding 3 Condone $x > 0$	1.1a	M1	$\{x: x \geq 0, x \neq 3\}$
	Deduces both $x \geq 0$ and $x \neq 3$ with no extras Condone $x > 0$	2.2a	A1	
	Obtains correct domain correctly stated in set notation	2.5	R1	
	Subtotal		**3**	

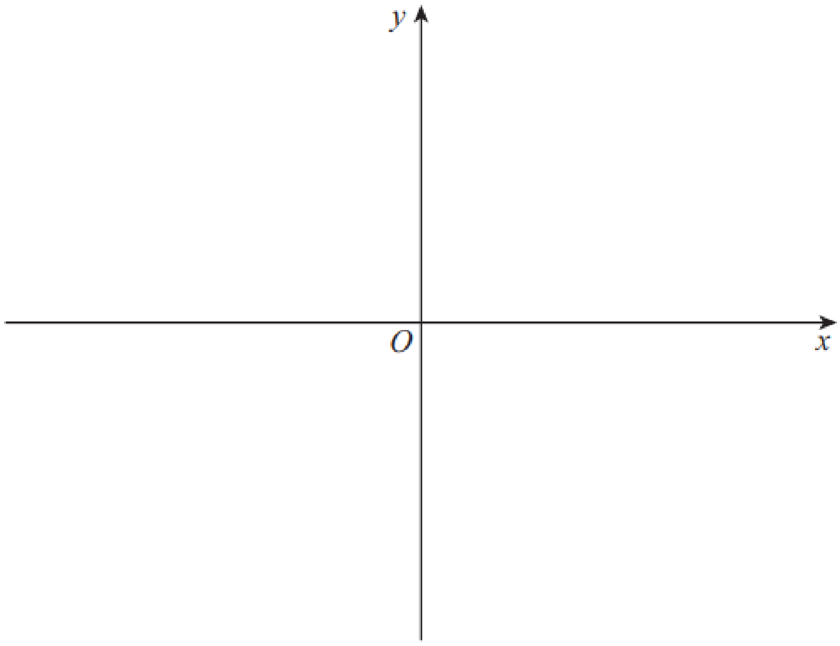
Q	Marking instructions	AO	Marks	Typical solution
10(b)	States that $h(x)$ has a discontinuity/asymptote at x = 3 or in the interval (1 , 4) OE	2.4	M1	$h(x)$ is not continuous at x = 3
	Explains that the discontinuity is at x = 3 and this is in the interval $(1, 4)$	2.3	A1	This means that a change of sign between x = 1 and 4 does not imply a root
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(c)	Selects an appropriate method to differentiate and reaches $h'(x)$ of the form $ax^{-\frac{1}{2}}(x-3)^{-1} + bx^{\frac{1}{2}}(x-3)^{-2}$ OE	3.1a	M1	$h(x) = \dfrac{\sqrt{x}}{x-3}$ $h'(x) = \dfrac{\frac{1}{2}x^{-\frac{1}{2}}(x-3) - x^{\frac{1}{2}}}{(x-3)^2}$
	Obtains correct $h'(x)$ ACF	1.1b	A1	
	Equates their $h'(x)$ to 0	1.1a	M1	$h'(x) = 0 \Rightarrow \dfrac{\frac{1}{2}x^{-\frac{1}{2}}(x-3) - x^{\frac{1}{2}}}{(x-3)^2} = 0$
	Obtains $x = -3$	1.1b	A1	$\dfrac{1}{2}x^{-\frac{1}{2}}(x-3) - x^{\frac{1}{2}} = 0$
	Explains that a continuous function with no turning points is one to one and therefore the inverse exists. Condone omission of 'continuous'	2.4	E1	$\dfrac{(x-3)}{2\sqrt{x}} - \sqrt{x} = 0$ $x - 3 - 2x = 0$ $x = -3$
	Completes a reasoned argument to correctly show that the function is one to one and deduces that $h(x)$ has an inverse Must explain that $x = -3$ is not in the domain and therefore there are no turning points and considers the sign of $h(x)$ either side of x = 3	2.1	R1	$x = -3$ is not in the domain of h hence the function has no turning points $h(x) > 0$ for $x > 3$ $h(x) < 0$ for $x < 3$ Hence function is one to one and has an inverse
	Subtotal		**6**	

November 2020 Question 4 Paper 1

4 (a) Sketch the graph of

$$y = 4 - |2x - 6|$$



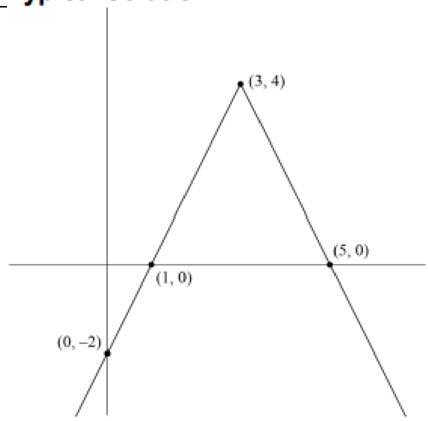
[3 marks]

4 (b) Solve the inequality

$$4 - |2x - 6| > 2$$

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
4(a)	Sketches an inverted V shape graph Condone lack of symmetry	1.1a	M1	
	Sketches an inverted V shape in the correct quadrants Condone lack of symmetry or absence of curve to the left of (0 , - 2)	1.1b	A1	
	Correctly labels all three intersections with coordinate axis. Accept the coordinates of each point or x values on x axis and y value on y axis Ignore any other values	1.1b	A1	
	Total		**3**	
4(b)	Obtains at least one correct critical value using a correct method. Can be read off graph or calculator Condone use of equals or incorrect inequality sign	1.1a	M1	$2 < x < 4$
	Writes correct solution in a correct form Accept $x > 2$, $x < 4$ or (2 , 4)	1.1b	A1	
	Subtotal		**2**	
	Question Total		**5**	

November 2020 Question 13 Paper 1

13 The function f is defined by

$$\mathrm{f}(x) = \frac{2x+3}{x-2} \qquad x \in \mathbb{R},\ x \neq 2$$

13 (a) (i) Find f^{-1}

[3 marks]

13 (a) (ii) Write down an expression for $\mathrm{ff}(x)$.

[1 mark]

13 (b) The function g is defined by

$$\mathrm{g}(x) = \frac{2x^2 - 5x}{2} \qquad x \in \mathbb{R},\ 0 \leq x \leq 4$$

13 (b) (i) Find the range of g.

[3 marks]

13 (b) (ii) Determine whether g has an inverse.

Fully justify your answer.

[2 marks]

13 (c) Show that

$$\mathrm{gf}(x) = \frac{48 + 29x - 2x^2}{2x^2 - 8x + 8}$$

[4 marks]

13 (d) It can be shown that fg is given by

$$\mathrm{fg}(x) = \frac{4x^2 - 10x + 6}{2x^2 - 5x - 4}$$

with domain $\{x \in \mathbb{R} : 0 \leq x \leq 4,\ x \neq a\}$

Find the value of a.

Fully justify your answer.

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
13(a)(i)	Rearranges to make x the subject by isolating x terms or Swaps x and y and isolates y terms	1.1a	M1	$y = \dfrac{2x+3}{x-2}$ $xy - 2y = 2x + 3$ $xy - 2x = 2y + 3$ $x(y-2) = 2y + 3$ $x = \dfrac{2y+3}{y-2}$ $f^{-1}(x) = \dfrac{2x+3}{x-2}\ x \neq 2$
	Obtains correct rearrangement and factorises ACF PI by final correct answer	1.1b	A1	
	Obtains $f^{-1}(x)$ and states domain Must use fully correct notation	2.5	R1	
	Subtotal		**3**	
13(a)(ii)	Obtains any valid expression in x for $ff(x)$ Can be left unsimplified ISW	1.1b	B1	$ff(x) = x$
	Subtotal		**1**	
13(b)(i)	Deduces the greatest value of g by evaluating g (4)	2.2a	B1	$g(4) = 6$ Vertex at (1.25 , -1.5625) $\{y : -1.5625 \leq y \leq 6\}$
	Obtains the minimum value of g	3.1a	B1	
	States the range using their finite greatest value and finite minimum value using set notation or interval notation Accept $[-1.5625, 6]$ in interval notation For set notation - use of none curly brackets or commas scores R0	2.5	R1F	
	Subtotal		**3**	
13(b)(ii)	Demonstrates that g is a many to one function by using an appropriate method eg Sketches the function Or Evaluates $g(x)$ at two points that give the same answer.	2.4	E1	$g(0) = 0 = g(2.5)$ g is many to one so it does not have an inverse.
	Deduces that g is many to one and states that g has no inverse Or Explains that g is not one to one and states that g has no inverse	2.2a	E1	

13(c)	Substitutes $f(x)$ into $g(x)$ correctly	1.1a	M1	$gf(x) = \frac{2\left(\frac{2x+3}{x-2}\right)^2 - 5\left(\frac{2x+3}{x-2}\right)}{2}$ $= \frac{2(2x+3)^2 - 5(2x+3)(x-2)}{2(x-2)^2}$ $= \frac{2(4x^2+12x+9)-5(2x^2-x-6)}{2(x^2-4x+4)}$ $= \frac{48+29x-2x^2}{2x^2-8x+8}$
	Obtains common denominator of $2(x-2)^2$ or $(x-2)^2$ correctly The fraction(s) must have the fully correct structure	1.1b	A1	
	Expands at least two quadratics correctly	1.1a	M1	
	Completes rigorous argument to show the required result Must have expanded all three quadratics correctly Terms in the numerator and denominator can be in any order AG	2.1	R1	
	Subtotal		**4**	

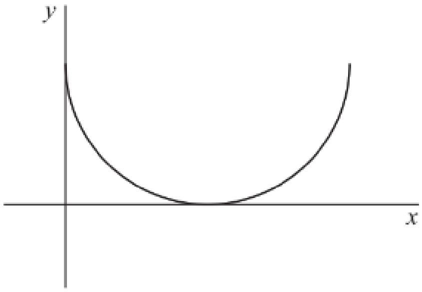
13(d)	States $g(x) = 2$ or States $2x^2 - 5x - 4 = 0$ PI by solving correct quadratic PI by sight of $\frac{5+\sqrt{57}}{4}$ or $\frac{5-\sqrt{57}}{4}$	3.1a	M1	$2x^2 - 5x - 4 = 0$ $x = \frac{5 \pm \sqrt{57}}{4}$ $a > 0$ since $0 \le x \le 4$ $a = \frac{5+\sqrt{57}}{4}$
	Determines the exact value of a giving a clear reason for the rejection of the negative root	2.4	R1	
	Subtotal		**2**	
	Question Total		**15**	

November 2020 Question 3 Paper 3

3 Determine which one of these graphs does **not** represent y as a function of x.

Tick (✓) **one** box.

[1 mark]



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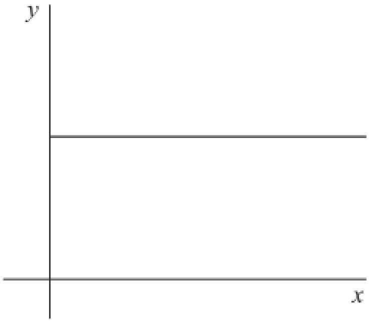

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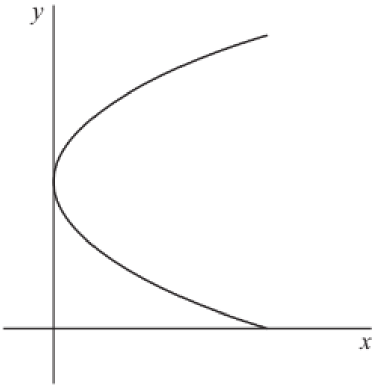

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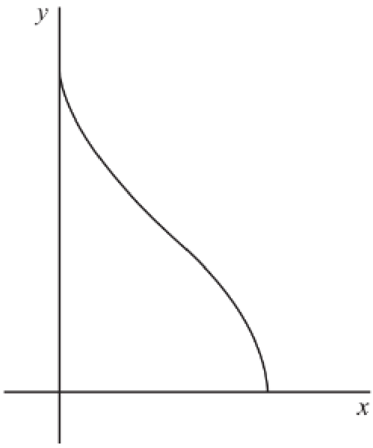

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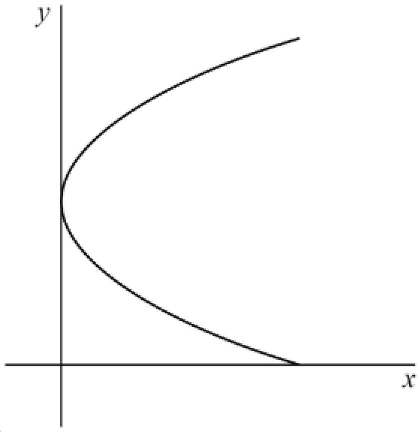
ANSWER

Q	Marking instructions	AO	Marks	Typical solution
3	Ticks correct box	1.2	B1	
	Total		**1**	

June 2019 Question 6 Paper 1

6 The function f is defined by

$$f(x) = \frac{1}{2}(x^2 + 1),\ x \geq 0$$

6 (a) Find the range of f.

[1 mark]

6 (b) (i) Find $f^{-1}(x)$

[3 marks]

6 (b) (ii) State the range of $f^{-1}(x)$

[1 mark]

6 (c) State the transformation which maps the graph of $y = f(x)$ onto the graph of $y = f^{-1}(x)$

[1 mark]

6 (d) Find the coordinates of the point of intersection of the graphs of $y = f(x)$ and $y = f^{-1}(x)$

[2 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)	Deduces the range of f Accept $\mathrm{f}(x) \geq \frac{1}{2}$, $y \geq \frac{1}{2}$ *or* $[0.5, \infty)$ OE	AO2.2a	**B1**	$\{y : y \geq \frac{1}{2}\}$
6(b)(i)	Rearranges formula, isolating squared term with at least one correct step seen.	1.1a	**M1**	$y = \frac{1}{2}(x^2 + 1)$ $2y = x^2 + 1$ $2y - 1 = x^2$ $x = \sqrt{2y-1}$ $\mathrm{f}^{-1}(x) = \sqrt{2x-1}$ $x \geq \frac{1}{2}$
	Obtains inverse function in any correct form.	1.1b	**A1**	
	Obtains correct inverse function using $\mathrm{f}^{-1}(x) = \ldots$ and states correct domain	2.5	**A1**	
6 (b)(ii)	States correct range Accept $\mathrm{f}^{-1}(x) \geq 0$ OE	1.1b	**B1**	$\{y : y \geq 0\}$
6(c)	Recalls correct transformation	1.2	**B1**	Reflection in $y = x$
6(d)	Forms equation using two of the three expressions $x = \frac{x^2+1}{2} = \sqrt{2x-1}$ allow their $\sqrt{2x-1}$ PI by correct answer	3.1a	**M1**	$x = \frac{x^2+1}{2}$ (1, 1)
	Obtains x=1 and y=1 CSO	1.1b	**A1**	
	Total		**8**	

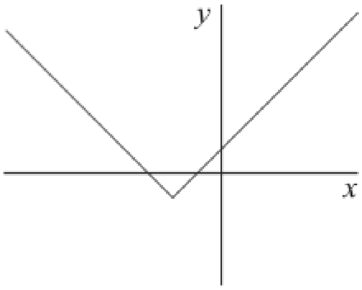
June 2019 Question 1 Paper 2

1 Identify the graph of $y = 1 - |x + 2|$ from the options below.

Tick (✓) **one** box.

[1 mark]

A



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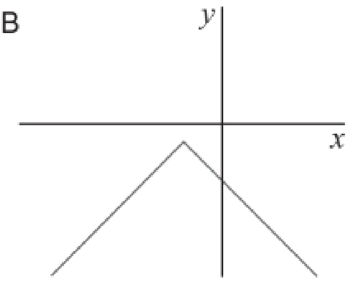

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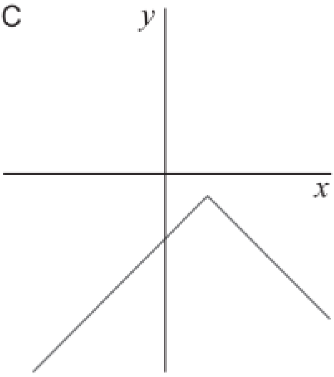

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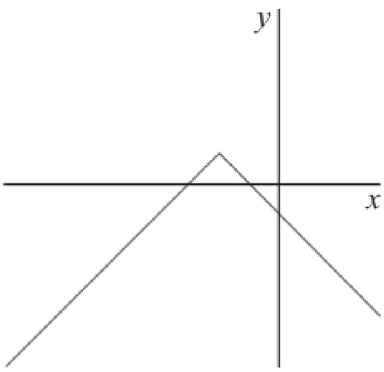

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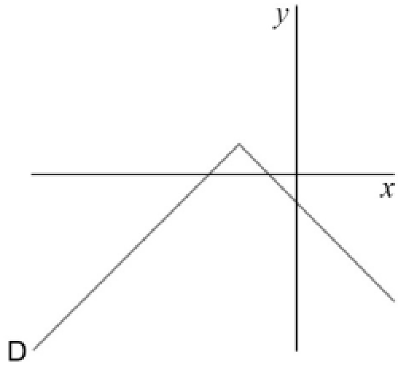
ANSWER

Q	Marking instructions	AO	Mark	Typical solution
1	Ticks the correct response	2.2a	R1	
	Total		**1**	

June 2019 Question 3 Paper 2

3 Each of these functions has domain $x \in \mathbb{R}$

Which function does **not** have an inverse?

Circle your answer.

[1 mark]

$f(x) = x^3$ $\quad$ $f(x) = 2x + 1$ $\quad$ $f(x) = x^2$ $\quad$ $f(x) = e^x$

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
3	Circles the correct response	1.2	B1	$f(x) = x^2$
	Total		**1**	

June 2018 Question 4 Paper 1

4 The function f is defined by $f(x) = e^{x-4}$, $x \in \mathbb{R}$

Find $f^{-1}(x)$ and state its domain.

[3 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
4	Takes logs of an equation. Must be correct use of logs.	AO1.1a	M1	$y = e^{x-4}$ $\ln y = x - 4$ $4 + \ln y = x$ $f^{-1}(x) = 4 + \ln x, x > 0$
	Obtains correct inverse function in any correct form	AO1.1b	A1	
	Deduces correct domain	AO2.2a	B1	
	Total		**3**	

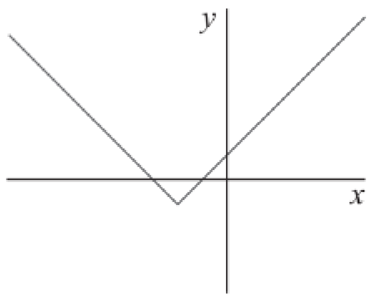
June 2018 Question 1 Paper 2

1 Identify the graph of $y = 1 - |x + 2|$ from the options below.

Tick (✓) **one** box.

[1 mark]

A



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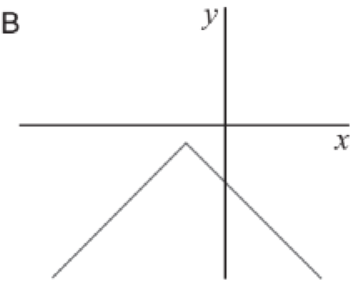
B



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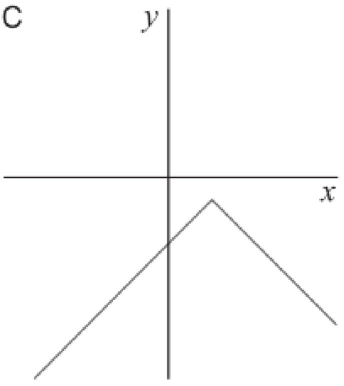
C



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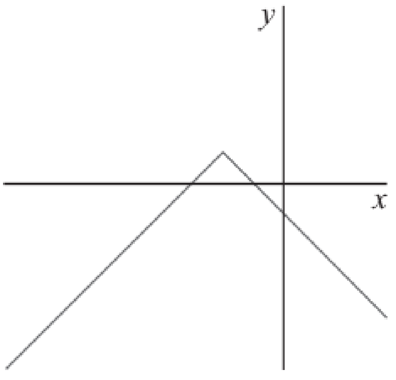
D



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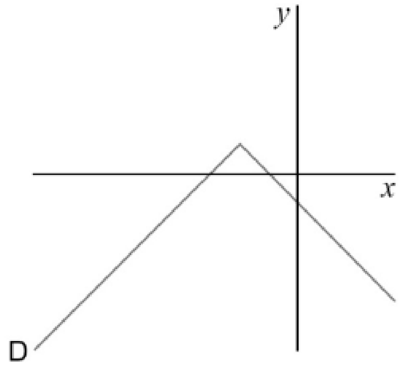
ANSWER

Q	Marking instructions	AO	Mark	Typical solution
1	Ticks the correct response	2.2a	R1	
	Total		**1**	

June 2018 Question 3 Paper 2

3 Each of these functions has domain $x \in \mathbb{R}$

Which function does **not** have an inverse?

Circle your answer.

[1 mark]

$f(x) = x^3$ $\quad$ $f(x) = 2x + 1$ $\quad$ $f(x) = x^2$ $\quad$ $f(x) = e^x$

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
3	Circles the correct response	1.2	B1	$f(x) = x^2$
	Total		**1**	

June 2022 Question 4 Paper 1

4 The graph of

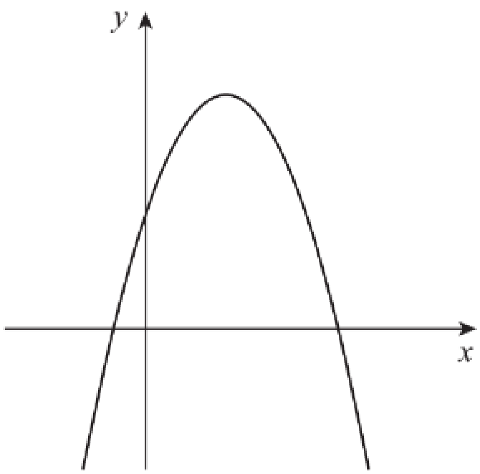
$$y = f(x)$$

where

$$f(x) = ax^2 + bx + c$$

is shown in **Figure 1**.

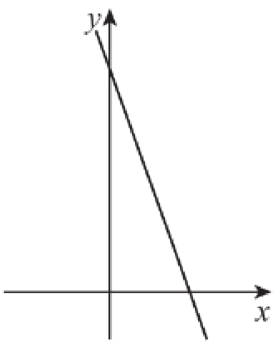
Figure 1



Which of the following shows the graph of $y = f'(x)$?

Tick (✓) **one** box.

[1 mark]



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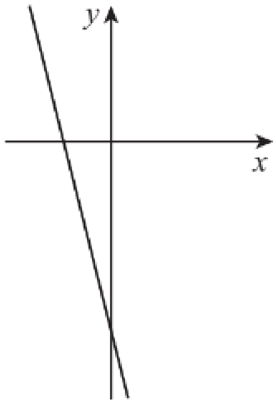
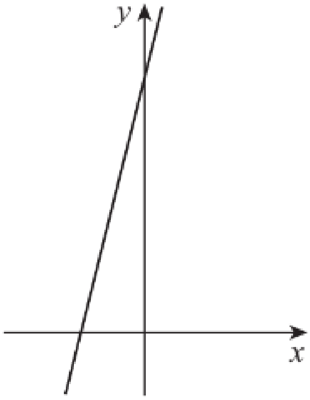
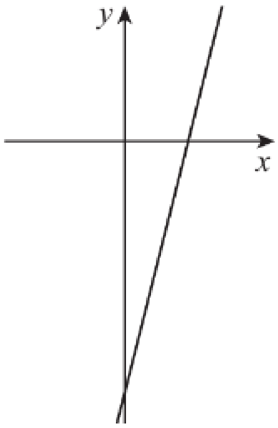


y
x

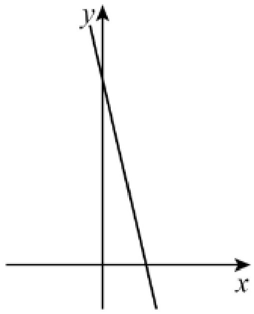


y
x



y
x

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
4	Ticks the correct box	2.2a	R1	
	Question 4 Total		**1**	

June 2022 Question 5 Paper 1

5 Find an equation of the tangent to the curve

$$y = (x - 2)^4$$

at the point where $x = 0$

[3 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
5	Differentiates to obtain a correct derivative either $4(x-2)^3$ OE or $4x^3 - 24x^2 + 48x - 32$ PI by -32 obtained with no errors seen in evaluating $\frac{dy}{dx}$	1.1b	B1	
	Substitutes x = 0 into their $\frac{dy}{dx}$ to obtain a numerical value or PI by constant from their $\frac{dy}{dx}$ or PI by -32 obtained with no errors seen in evaluating $\frac{dy}{dx}$	1.1a	M1	$\frac{dy}{dx} = 4(x-2)^3$ When x = 0 $\frac{dy}{dx} = -32$ $y = 16$ $y = -32x + 16$
	Obtains $y = -32x + 16$ ACF Award the mark at the first opportunity and ISW any incorrect rearrangement No errors seen	1.1b	A1	
	Question 5 Total		**3**	

June 2022 Question 2 Paper 2

2 State the value of

$$\lim_{h \to 0} \frac{\sin(\pi + h) - \sin \pi}{h}$$

Circle your answer.

[1 mark]

$\cos h$ $\quad -1 \quad$ 0 $\quad 1$

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
2	Circles correct answer	2.2a	R1	–1
	Question 2 Total		**1**	

November 2021 Question 9 Paper 1

9 A function f is defined for all real values of x as

$$f(x) = x^4 + 5x^3$$

The function has exactly two stationary points when $x = 0$ and $x = -\frac{15}{4}$

9 (a) (i) Find $f''(x)$

[2 marks]

9 (a) (ii) Determine the nature of the stationary points.

Fully justify your answer.

[4 marks]

9 (b) State the range of values of x for which

$$f(x) = x^4 + 5x^3$$

is an increasing function.

[1 mark]

9 (c) A second function g is defined for all real values of x as

$$g(x) = x^4 - 5x^3$$

9 (c) (i) State the single transformation which maps f onto g.

[1 mark]

9 (c) (ii) State the range of values of x for which g is an increasing function.

[1 mark]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
9(a)(i)	Differentiates f(x) at least one correct term May be unsimplified	1.1a	M1	$f'(x) = 4x^3 + 15x^2$ $f''(x) = 12x^2 + 30x$
	Obtains $f''(x) = 12x^2 + 30x$	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)(ii)	Substitutes $x = -\frac{15}{4}$ into their $f''(x)$ or uses gradient test both sides of $x = -\frac{15}{4}$	1.1a	M1	$f''\left(-\frac{15}{4}\right) = 12\left(-\frac{15}{4}\right)^2 + 30\left(-\frac{15}{4}\right)$ $= \frac{225}{4} > 0$ Hence there is a minimum at $x = -\frac{15}{4}$ $f''(0) = 0$ $f''(1) = 12 + 30 > 0$ and f''(-1) = 12 – 30 < 0 hence point of inflection at $x = 0$
	Completes rigorous justification for minimum at $x = -\frac{15}{4}$ This must be correctly deduced using shape of graph or $f''\left(-\frac{15}{4}\right) = \frac{225}{4} > 0$	2.1	R1	
	Substitutes two values either side of $x = 0$ into their $f''(x)$ or uses gradient test both sides of $x = 0$ or argues using the shape of a quartic curve with two stationary points	1.1a	M1	
	Completes rigorous justification for point of inflection at $x = 0$ This must be correctly deduced using the shape of the graph or a completely correct test both sides of the point Other explanation eg quartic with two stationary points, one of the points must be a point of inflection	2.2a	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)	Deduces $x > -\frac{15}{4}$ **OE** Condone use of ' $\geq$'	2.2a	B1	$x > -\frac{15}{4}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
9(c)(i)	Deduces the transformation is a reflection in the y-axis **OE**	2.2a	B1	Reflection in the y-axis
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
9(c)(ii)	Deduces $x > \frac{15}{4}$ Condone use of ' $\geq$' **FT** their answer in part (b) only if their value in (b) is negative	2.2a	B1F	$x > \frac{15}{4}$
	Subtotal		**1**	

June 2019 Question 13 Paper 1

13 A curve, C, has equation

$$y = \frac{e^{3x-5}}{x^2}$$

Show that C has exactly one stationary point.

Fully justify your answer.

[7 marks]

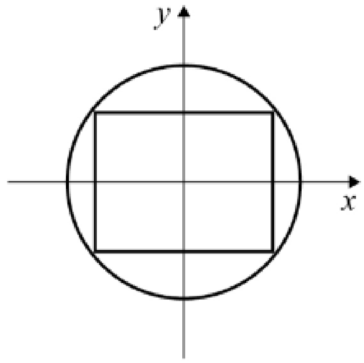
ANSWER

Q	Marking instructions	AO	Mark	Typical solution
13	Chooses an appropriate technique to differentiate accept any evidence of product rule or quotient rule	3.1a	M1	$x \neq 0$ as y is undefined $\frac{dy}{dx} = \frac{3e^{3x-5}x^2 - 2xe^{3x-5}}{x^4}$ At a turning point $\frac{dy}{dx} = 0$ $\Rightarrow \frac{3e^{3x-5}x^2 - 2xe^{3x-5}}{x^4} = 0$ $\Rightarrow 3e^{3x-5}x^2 - 2xe^{3x-5} = 0$ $\Rightarrow (3x-2)xe^{3x-5} = 0$ $\Rightarrow x = \frac{2}{3}, \quad e^{3x-5} = 0$ $e^{3x-5} \neq 0$ $\therefore$ there is only one stationary point.
	Differentiates e^{3x-5} correctly	1.1b	B1	
	Obtains correct $\frac{dy}{dx}$ ACF	1.1b	A1	
	Explains that stationary points occur when $\frac{dy}{dx} = 0$	2.4	E1	
	Equates their $\frac{dy}{dx}$ to zero and solves their equation with at least one correct line of correct rearrangement. Resulting in a value for x.	1.1a	M1	
	Deduces their factor $e^{3x-5} \neq 0$	2.2a	B1F	
	Completes argument to show exactly one stationary point at $x = \frac{2}{3}$. Must include consideration of $x \neq 0$ somewhere.	2.1	R1	
	Total		**7**	

June 2018 Question 13 Paper 1

13 A company is designing a logo. The logo is a circle of radius 4 inches with an inscribed rectangle. The rectangle must be as large as possible.

The company models the logo on an x-y plane as shown in the diagram.



Use calculus to find the maximum area of the rectangle.

Fully justify your answer.

[10 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
13	Identifies and clearly defines consistent variables for length and width. Can be shown on diagram.	AO3.1b	B1	Width of rectangle = $2x$ Length of rectangle = $2y$
	Models the area of rectangle with an expression of the correct dimensions	AO3.3	M1	$A = 4xy$
	Eliminates either variable to form a model for the area in one variable.	AO1.1a	M1	$x^2 + y^2 = 16$
	Obtains a correct equation to model the area in one variable	AO1.1b	A1	$A = 4x\sqrt{16 - x^2}$
	Differentiates their expression for area. Condone one error	AO3.4	M1	$\frac{dA}{dx} = 4\sqrt{16 - x^2} - \frac{4x^2}{\sqrt{16-x^2}}$ $\frac{dA}{dx} = \frac{64-8x^2}{\sqrt{16-x^2}}$ For maximum point $\frac{\mathrm{d}A}{\mathrm{d}x} = 0$
	Explains that their derivative equals zero for a maximum or stationary point.	AO2.4	E1	$\frac{64-8x^2}{\sqrt{16-x^2}} = 0$ $x = 2\sqrt{2}$
	Equates area derivative to zero and obtains correct value for either variable. CAO	AO1.1b	A1	When x = 2.8, $\frac{\mathrm{d}A}{\mathrm{d}x} = 0.448$ When $x = 2.9$, $\frac{\mathrm{d}A}{\mathrm{d}x} = -1.191$
	Completes a gradient test or uses second derivative of their area function to determine nature of stationary point	AO1.1a	M1	Therefore maximum
	Deduces that the area is a maximum at $x = 2\sqrt{2}$ or $\theta = \frac{\pi}{4}$ Values need not be exact	AO2.2a	R1	The maximum area is 32 sq in
	Obtains maximum area with correct units AWRT 32	AO3.2a	B1	
	Total		**10**	

June 2018 Question 9 Paper 3

9 A curve has equation

$$x^2y^2 + xy^4 = 12$$

9 (a) Prove that the curve does not intersect the coordinate axes.

[2 marks]

9 (b) (i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2xy + y^3}{2x^2 + 4xy^2}$

[5 marks]

9 (b) (ii) Prove that the curve has no stationary points.

[4 marks]

9 (b) (iii) In the case when $x > 0$, find the equation of the tangent to the curve when $y = 1$

[4 marks]

ANSWER

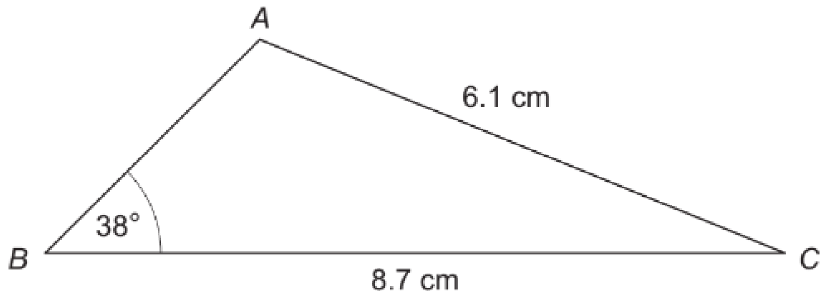
Q	Marking instructions	AO	Mark	Typical solution
9(a)	Demonstrates by substitution that $x=0$ or $y=0$ leads to value on the LHS = 0	2.4	E1	When x = 0 $0^2y^2 + 0y^4 = 0$ When y = 0 $x^2 0^2 + x0^4 = 0$ This is a contradiction because $x^2y^2 + xy^4 = 12$ so the curve does not intersect either axis
	Completes rigorous argument to show required result	2.1	R1	
9 (b)(i)	Uses implicit differentiation	3.1a	M1	$2xy^2 + 2x^2y\frac{dy}{dx} + y^4 + 4xy^3\frac{dy}{dx} = 0$ $\frac{dy}{dx} = -\frac{2xy^2 + y^4}{2x^2y + 4xy^3}$ $= -\frac{y\,(2xy+y^3)}{y(2x^2+4xy^2)}$ $= -\frac{2xy + y^3}{2x^2 + 4xy^2}$,
	Product rule used LHS (at least one pair of terms correct)	1.1a	M1	
	Differentiates equation of curve fully correctly	1.1b	A1	
	Collects their $\frac{dy}{dx}$ terms in an equation and factorises	3.1a	M1	
	Completes convincing argument to obtain required result by **factorising** then simplifying y **AG**	2.1	R1	

9 (b)(ii)	Begins argument by setting $\frac{dy}{dx}=0$ to form an equation for x and y **PI** by $2xy+y^3=0$	2.1	M1	For stationary points $\frac{dy}{dx}=0$ $\Rightarrow 2xy+y^3=0$ $\Rightarrow y^2=-2x$ $\Rightarrow x^2y^2+x(-2x)y^2=12$ $\Rightarrow -x^2y^2=12$ Since $-x^2y^2<0$ there can be no stationary points.
	Obtains $y^2=-2x$ or $y=\sqrt{-2x}$ or $x=\frac{-y^2}{2}$	1.1b	A1	
	Substitutes $y^2=-2x$ or $x=\frac{-y^2}{2}$ into equation for curve	1.1a	M1	
	Completes convincing argument to deduce the required result	2.2a	R1	
9 (b)(iii)	Substitutes y = 1 into equation of curve to obtain correct quadratic **ACF**	3.1a	M1	$y=1\Rightarrow x^2+x-12=0$ $\Rightarrow x=3 \quad (x>0)$ $\Rightarrow \frac{dy}{dx}=-\frac{7}{30}$ $y-1=-\frac{7}{30}(x-3)$
	Deduces x = 3 **PI** by substituting their x in their dy/dx	2.2a	R1	
	Substitutes their x and y = 1 in their dy/dx	1.1a	M1	
	Obtains correct equation of tangent **ACF** **ISW**	1.1b	A1	

Ch 6 Trigonometric functions

June 2022 Question 4 Paper 2

4



The diagram shows a triangle *ABC*.

AB is the shortest side. The lengths of *AC* and *BC* are 6.1 cm and 8.7 cm respectively.

The size of angle *ABC* is 38°

Find the size of the largest angle.

Give your answer to the nearest degree.

[3 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
4	Uses the sine rule Or Substitutes correctly into the cosine rule	1.1a	M1	$\frac{\sin\theta}{8.7} = \frac{\sin 38}{6.1}$ $\theta = 61.4$ $A = 180 - 61.4$ $= 118.58...$ $= 119^{\circ}$
	Obtains a value of 61 or 61.410964… rounded or truncated Condone answer (in radians) of 1.0718… or 0.4364… PI by correct obtuse angle or 81 Or Obtains correct length AB = 3.9367… Or $\text{AB}^2 = 15.4998...$	1.1b	A1	
	Deduces the largest angle is 119 AWRT CAO	2.2a	A1	
	Question 4 Total		**3**	

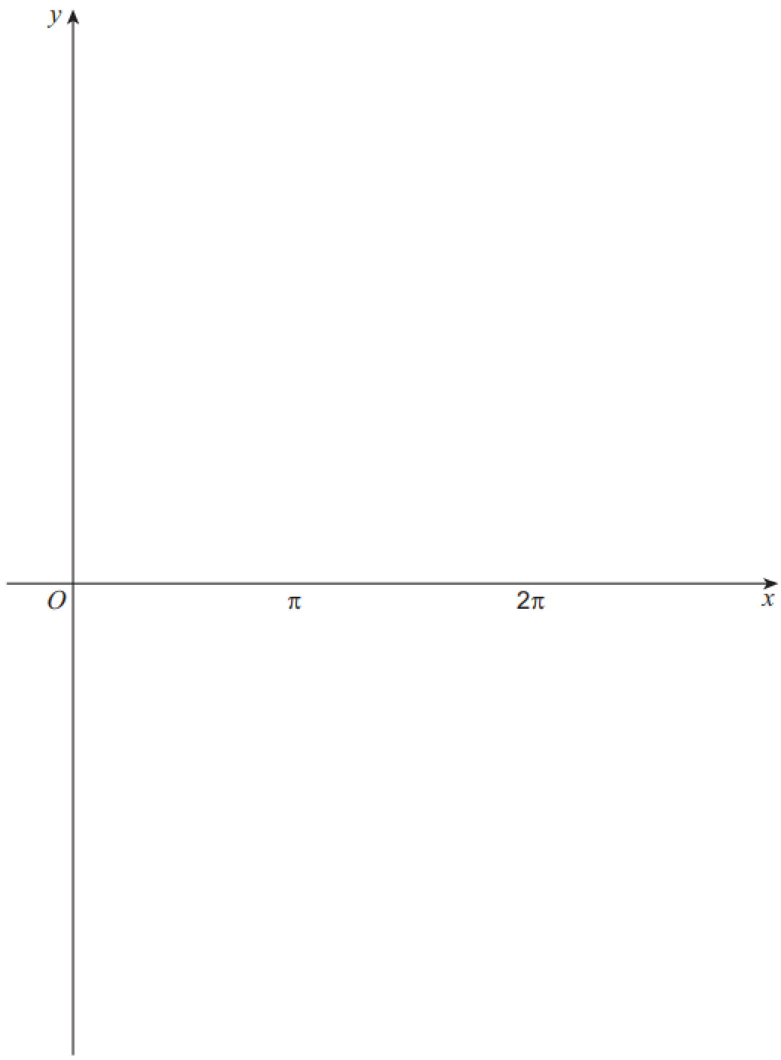
June 2022 Question 7 Paper 1

7 Sketch the graph of

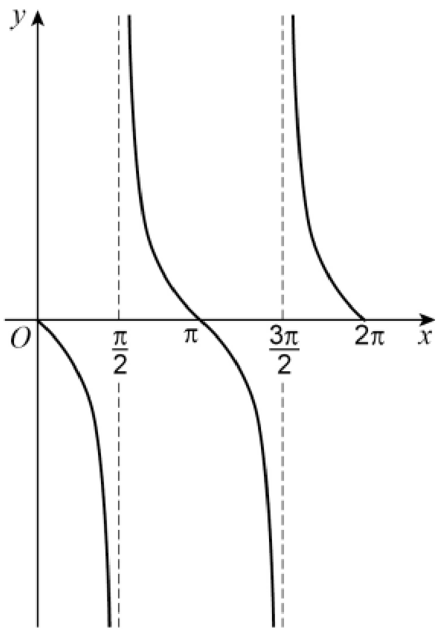
$$y = \cot\left(x - \frac{\pi}{2}\right)$$

for $0 \le x \le 2\pi$

[3 marks]



ANSWER

Q	Marking instructions	AO	Marks	Typical solution
7	Sketches one of the sections of the curve shown below Either  or  Condone translations Do not allow the end points of the curve turning to intersect the asymptote	1.2	B1	
	Sketches the three branches within the interval from 0 to 2π Condone overlapping branches Asymptotes need not be drawn	1.1a	M1	
	Completes fully correct sketch with asymptotes drawn at approximately the correct positions Labelling not required and can be ignored Ignore anything after 2π or to the left of O	1.1b	A1	
	Question 7 Total		**3**	

November 2021 Question 9 Paper 2

9 A robotic arm which is attached to a flat surface at the origin *O*, is used to draw a graphic design.

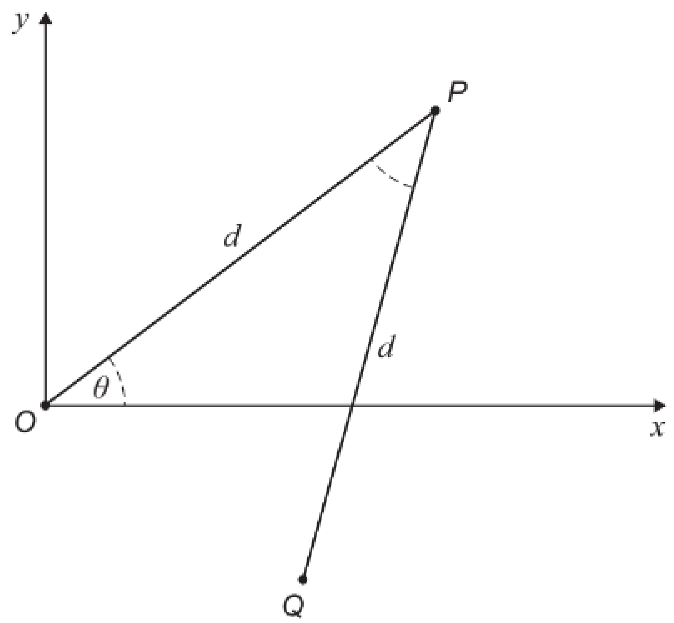
The arm is made from two rods *OP* and *PQ*, each of length d, which are joined at *P*.

A pen is attached to the arm at *Q*.

The coordinates of the pen are controlled by adjusting the angle *OPQ* and the angle θ between *OP* and the x-axis.

For this particular design the pen is made to move so that the two angles are always equal to each other with $0 \le \theta \le \frac{\pi}{2}$ as shown in **Figure 2**.

Figure 2



9 (a) Show that the x-coordinate of the pen can be modelled by the equation

$$x = d\left(\cos\theta + \sin\left(2\theta - \frac{\pi}{2}\right)\right)$$

[2 marks]

9 (b) Hence, show that

$$x = d(1 + \cos\theta - 2\cos^2\theta)$$

[2 marks]

9 (c) It can be shown that

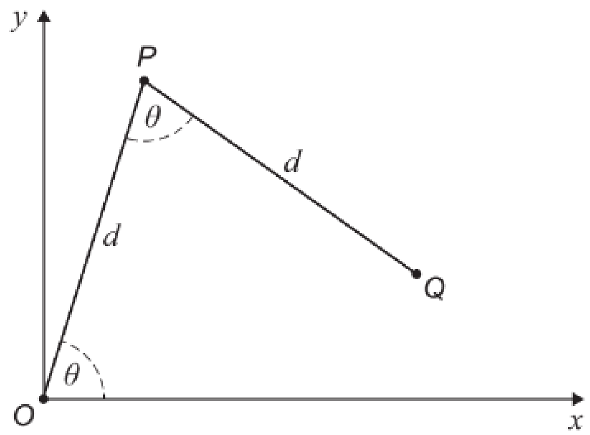
$$x = \frac{9d}{8} - d\left(\cos\theta - \frac{1}{4}\right)^2$$

State the greatest possible value of x and the corresponding value of $\cos\theta$

[2 marks]

9 (d) **Figure 3** below shows the arm when the x-coordinate is at its greatest possible value.

Figure 3



Find, in terms of d, the exact distance OQ.

[3 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Begins to find the required horizontal distance by considering an appropriate horizontal distance in a right-angled triangle For example: $d\cos\theta$, $d\cos 2\theta$ or $d\sin\left(\frac{\pi}{2}-2\theta\right)$ Award for correctly identifying at least one of the horizontal components required. This can be done on a sketch or by clearly stating which distance they are referring to	3.3	M1	QP makes angle of 2θ with horizontal $x = d\cos\theta - d\cos 2\theta$ $x = d\cos\theta - d\sin\left(\frac{\pi}{2} - 2\theta\right)$ $x = d\left(\cos\theta + \sin\left(2\theta - \frac{\pi}{2}\right)\right)$
	Completes correct manipulation to show required result AG	2.1	R1	
	Subtotal		**2**	

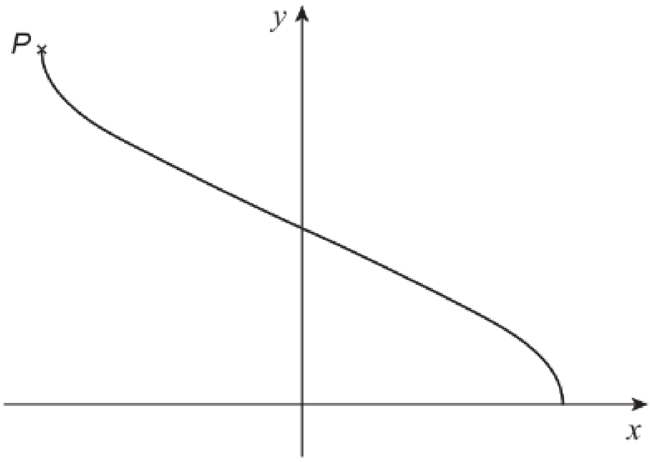
Q	Marking instructions	AO	Marks	Typical solution
9(b)	Uses a compound angle formula and expands $\sin\left(2\theta - \frac{\pi}{2}\right)$ or Uses complementary angles to obtain the formula without $\frac{\pi}{2}$ or States $x = d(\cos\theta - \cos 2\theta)$	3.1a	M1	$x = d\left(\cos\theta + \sin\left(2\theta - \frac{\pi}{2}\right)\right)$ $x = d(\cos\theta - \cos 2\theta)$ $x = d(\cos\theta - (2\cos^2\theta - 1)$ $x = d(1 + \cos\theta - 2\cos^2\theta)$
	Uses $\cos 2\theta = 2\cos^2\theta - 1$ to show the required result AG	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(c)	States greatest value = $\frac{9d}{8}$	1.1b	B1	Greatest value = $\frac{9d}{8}$ $\cos\theta = \frac{1}{4}$
	States $\cos\theta = \frac{1}{4}$	1.1b	B1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(d)	Begins to find OQ by either using the cosine rule or the sine rule with d and θ Accept either $OQ^2 = d^2 + d^2 - 2d^2 cos\theta$ or $\frac{OQ}{sin\theta} = \frac{d}{sin\alpha}$ Note $\alpha = \frac{\pi-\theta}{2}$	3.1a	M1	$OQ^2 = d^2 + d^2 - 2d^2 cos\theta$ $= 2d^2 - 2d^2 \times \frac{1}{4}$ $= \frac{3d^2}{2}$ $OQ = \frac{\sqrt{6}}{2}d$
	Substitutes their exact value for $cos\theta$ into the cosine rule or Finds and substitutes their corresponding exact values for $sin\theta$ and $cos\frac{\theta}{2}$ into the sine rule Note When $cos\,\theta = \frac{1}{4}$, $sin\,\theta = \frac{\sqrt{15}}{4}$ and $cos\frac{\theta}{2} = \sqrt{\frac{5}{8}}$,	1.1a	M1	
	Obtains the correct exact value of OQ ACF	1.1b	A1	
	Subtotal		**3**	

November 2021 Question 1 Paper 3

1 The graph of $y = \arccos x$ is shown.



State the coordinates of the end point *P*.

Circle your answer.

[1 mark]

$(-\pi,\ 1)$ $(-1,\ \pi)$ $\left(-\frac{\pi}{2},\ 1\right)$ $\left(-1,\ \frac{\pi}{2}\right)$

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
1	Circles correct answer	1.2	B1	$(-1, \pi)$
	Total		**1**	

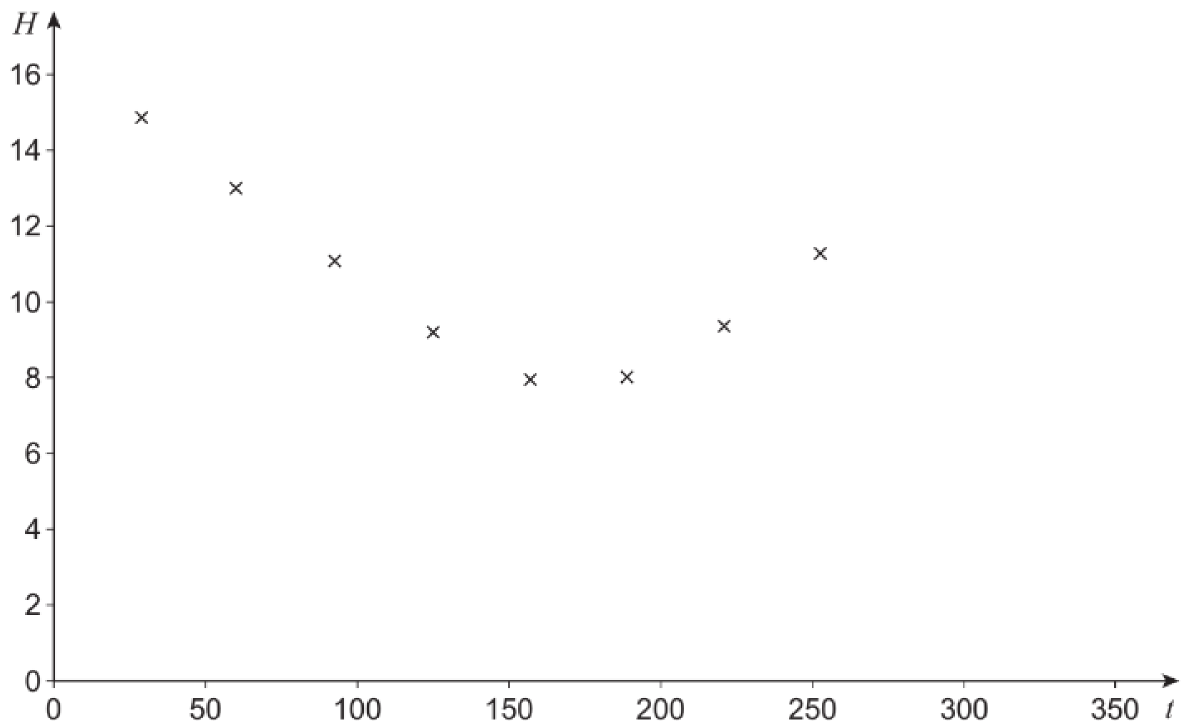
November 2020 Question 8 Paper 1

8 Mike, an amateur astronomer who lives in the South of England, wants to know how the number of hours of darkness changes through the year.

On various days between February and September he records the length of time, H hours, of darkness along with t, the number of days after 1 January.

His results are shown in **Figure 1** below.

Figure 1



Mike models this data using the equation

$$H = 3.87 \sin\left(\frac{2\pi(t + 101.75)}{365}\right) + 11.7$$

8 (a) Find the minimum number of hours of darkness predicted by Mike's model. Give your answer to the nearest minute.

[2 marks]

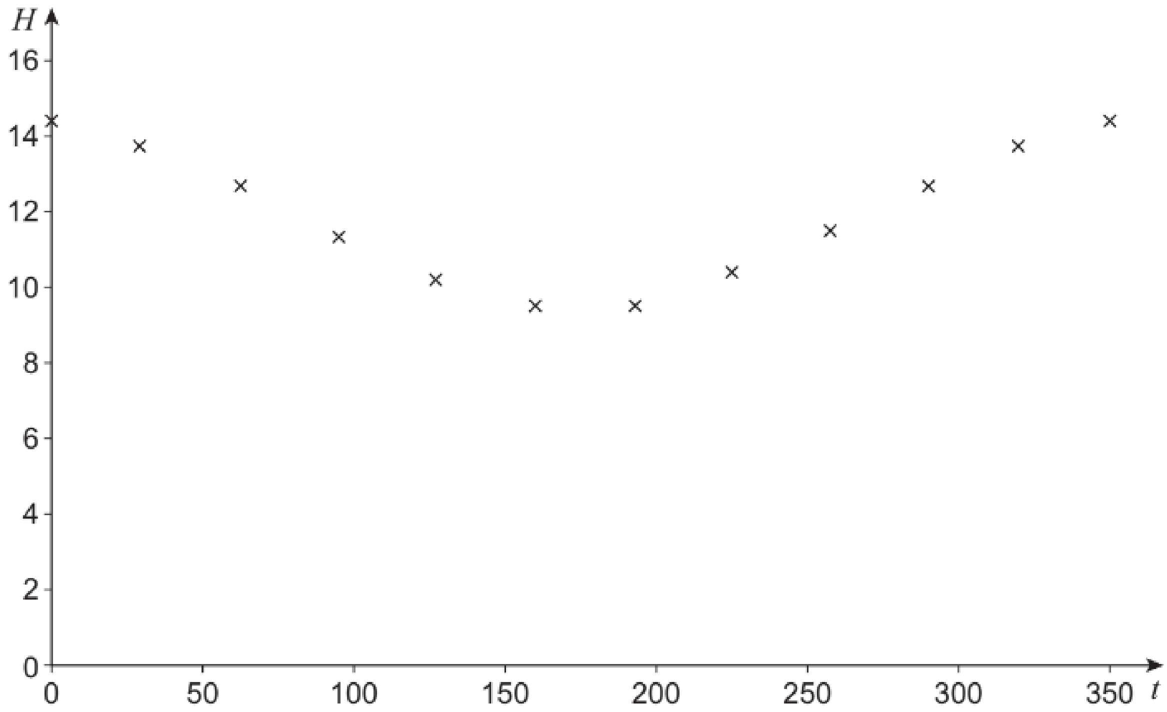
8 (b) Find the maximum number of consecutive days where the number of hours of darkness predicted by Mike's model exceeds 14

[3 marks]

8 (c) Mike's friend Sofia, who lives in Spain, also records the number of hours of darkness on various days throughout the year.

Her results are shown in **Figure 2** below.

Figure 2



Sofia attempts to model her data by refining Mike's model.

She decides to increase the 3.87 value, leaving everything else unchanged.

Explain whether Sofia's refinement is appropriate.

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
8(a)	Uses sin = -1 in the model to obtain -3.87 + 11.7 If a *t* value is used then the sine must evaluate to -1 or Differentiates, sets the derivative equal to 0 and obtains a value for *t* which they substitute back into the formula	3.4	M1	$sin\left(\frac{2\pi(t+101.75)}{365}\right) = -1$ $-3.87 + 11.7 = 7.83$ 7hours 50mins
	Obtains correct answer Accept 470 minutes, $\frac{47}{6}$ or $7\frac{5}{6}$ hours	3.2a	A1	
	Subtotal		**2**	
8(b)	Uses model to form equation or inequality with H =14 Condone incorrect inequality	3.4	M1	$3.87\,sin\left(\frac{2\pi(t+101.75)}{365}\right) + 11.7 = 14$
	Solves equation to obtain at least two correct values of t Can be rounded or truncated Eg -64.77, 43.779, 300.22, 408.77	1.1b	A1	$t = 300.22$ or $t = 408.77$ $408 - 300 = 108$
	Subtracts an appropriate pair of t values to obtain number of consecutive days Condone any rounding to the nearest whole number or truncation of their pair of values Accept 109 or 107 Alternative method = 43 + (365 – 300) = 108	3.2a	A1	
	Subtotal		**3**	
8(c)	Explains that Sofia's refinement would increase the amplitude of the graph Accept The range of the graph would increase It would increase the fluctuation of the graph	3.3	M1	Sofia's refinement would increase the range of the graph Sofia's graph suggests this is not the case, so the refinement is not appropriate
	Explains that Sofia's refinement is not appropriate as her data/graph suggests a lower amplitude OE	3.5c	A1	
	Subtotal		**2**	
	Question Total		**7**	

November 2020 Question 2 Paper 2

2 Which one of the following equations has no real solutions?

Tick (✓) **one** box.

[1 mark]

$\cot x = 0$ ☐

$\ln x = 0$ ☐

$|x + 1| = 0$ ☐

$\sec x = 0$ ☐

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
2	Ticks correct box	2.2a	B1	$\sec x = 0$
	Total		**1**	

June 2019 Question 12 Paper 1

12 (a) Show that the equation

$$2\cot^2 x + 2\operatorname{cosec}^2 x = 1 + 4\operatorname{cosec} x$$

can be written in the form

$$a\operatorname{cosec}^2 x + b\operatorname{cosec} x + c = 0$$

[2 marks]

12 (b) Hence, given x is obtuse and

$$2\cot^2 x + 2\operatorname{cosec}^2 x = 1 + 4\operatorname{cosec} x$$

find the exact value of $\tan x$

Fully justify your answer.

[5 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
12(a)	Uses appropriate trig identity to form quadratic equation **in single trigonometrical** term. Condone $2(\pm 1 \pm \operatorname{cosec}^2 x) + 2\operatorname{cosec}^2 x = 1 + 4\operatorname{cosec} x$	1.1a	M1	$2\cot^2 x + 2\operatorname{cosec}^2 x = 1 + 4\operatorname{cosec} x$ $2(\operatorname{cosec}^2 x - 1) + 2\operatorname{cosec}^2 x = 1 + 4\operatorname{cosec} x$ $4\operatorname{cosec}^2 x - 4\operatorname{cosec} x - 3 = 0$
	Completes rigorous argument to show the required result	2.1	R1	
12(b)	Solves quadratic and Obtains one of $\operatorname{cosec} x = \frac{3}{2}$ or $\operatorname{cosec} x = -\frac{1}{2}$ OE	1.1b	B1	$\operatorname{cosec} x = \frac{3}{2}$ or $\operatorname{cosec} x = -\frac{1}{2}$ reject since $\lvert\operatorname{cosec} x\rvert \geq 1$ $\cot^2 x = \left(\frac{3}{2}\right)^2 - 1 = \frac{5}{4}$ $\tan x = -\frac{2\sqrt{5}}{5}$ Since *x* is obtuse
	Explains why their spurious solution(s) is rejected referring to the range of cosec or sine with explicit comparison to ± 1 May accept later rejection for valid reason ie sq root of negative OE	2.4	E1F	
	Uses trig identity or right-angled triangle/Pythagoras or given equation with their exact value of cosec *x or sin x to obtain an exact value of tan x* value used must satisfy $\lvert\operatorname{cosec} x\rvert \geq 1$ OE	1.1a	M1	
	Completes rigorous argument to find correct exact magnitude of tan *x* *ACF*	2.1	R1	
	Deduces tan x is negative. May be seen anywhere without contradiction by a positive final answer.	2.2a	B1	
	Total		**7**	

Video solution:

https://youtu.be/_geLVXvTYs0

June 2018 Question 12 Paper 1

12 $p(x) = 30x^3 - 7x^2 - 7x + 2$

12 (a) Prove that $(2x + 1)$ is a factor of $p(x)$

[2 marks]

12 (b) Factorise $p(x)$ completely.

[3 marks]

12 (c) Prove that there are no real solutions to the equation

$$\frac{30\sec^2 x + 2\cos x}{7} = \sec x + 1$$

[5 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
12(a)	Begins a proof using a valid method Eg. Factor theorem, algebraic division, multiplication of correct factors	AO1.1a	M1	$p\left(-\frac{1}{2}\right)=30\times\left(-\frac{1}{2}\right)^3-7\left(-\frac{1}{2}\right)^2-7\left(-\frac{1}{2}\right)+2$ $=0$ $\therefore 2x+1$ is a factor of $p(x)$
	Constructs rigorous mathematical proof. To achieve this mark: Factor theorem the student must clearly substitute and state that p(-1/2)=0 **and** clearly state that this implies that $2x$ + 1 is a factor Algebraic division OR Multiplication of correct factors The method must be completely correct with a concluding statement	AO2.1	R1	
(b)	Obtains quadratic factor PI	AO1.1a	M1	$p(x)=(2x+1)(15x^2-11x+2)$ $=(2x+1)(5x-2)(3x-1)$
	Obtains second linear factor	AO1.1b	A1	
	Writes $p(x)$ as the product of the correct three linear factors. NMS correct answer 3/3	AO1.1b	A1	
(c)	Rearranges to achieve a cubic equation in $\sec x$ (or $\cos x$)	AO3.1a	M1	$\frac{30\sec^2 x+2\cos x}{7}=\sec x+1$ $\Rightarrow 30\sec^2 x+2\cos x=7\sec x+7$ $\Rightarrow 30\sec^3 x+2=7\sec^2 x+7\sec x$
	Equates to zero and uses result from (b) or factorises	AO1.1a	M1	
	Deduces that if solutions exist they must be of the form $\sec x$ = -½, $\sec x$ =1/3 or $\sec x$ = 2/5 OE	AO2.2a	A1	$30\sec^3 x-7\sec^2 x-7\sec x+2=0$ $\Rightarrow(2\sec x+1)(5\sec x-2)(3\sec x-1)=0$ $\Rightarrow \sec x=-\frac{1}{2},\frac{1}{3},\frac{2}{5}$
	Explains that the range of $\sec x$ is $(-\infty,-1]\cup[1,\infty)$ OE OE for $\cos x$	AO2.4	E1	These values do not fall within the range of $\sec x$ as they are between -1 and 1
	Completes argument explaining that there cannot be any real solutions as values are outside of the function's range.	AO2.1	R1	$\therefore \frac{30\sec^2 x+2\cos x}{7}=\sec x+1$ has no real solutions.
	Total		**10**	

June 2018 Question 1 Paper 3

1 $f(x) = \arcsin x$

State the maximum possible domain of f

Tick (✓) **one** box.

[1 mark]

$\{x \in \mathbb{R} : -1 \le x \le 1\}$ ☐

$\left\{x \in \mathbb{R} : -\frac{\pi}{2} \le x \le \frac{\pi}{2}\right\}$ ☐

$\{x \in \mathbb{R} : -\pi \le x \le \pi\}$ ☐

$\{x \in \mathbb{R} : -90 \le x \le 90\}$ ☐

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
1	Ticks the correct response	1.2	B1	$\{x \in \mathbb{R} : -1 \le x \le 1\}$
	Total		**1**	

Ch 7 Further algebra

June 2022 Question 6 Paper 1

6 (a) Find the first two terms, in ascending powers of x, of the binomial expansion of

$$\left(1-\frac{x}{2}\right)^{\frac{1}{2}}$$

[2 marks]

6 (b) Hence, for small values of x, show that

$$\sin 4x + \sqrt{\cos x} \approx A + Bx + Cx^2$$

where A, B and C are constants to be found.

[4 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Expands to obtain the first two terms Can be unsimplified Condone sign error	1.1a	M1	$\left(1-\frac{x}{2}\right)^{\frac{1}{2}} \approx 1+\left(\frac{1}{2}\right)\left(-\frac{x}{2}\right)$ $\approx 1-\frac{1}{4}x$
	Obtains $1-\frac{1}{4}x$ OE Accept if listed as two separate terms. Ignore any extra terms	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)	States or uses at least one small angle approximation correctly either $\sin kx \approx kx$ or $\sqrt{\cos x} \approx \sqrt{1-\frac{x^2}{2}}$	3.1a	M1	$\sin(4x)+\sqrt{\cos x} \approx 4x+\sqrt{1-\frac{x^2}{2}}$ $\approx 4x+\left(1-\frac{x^2}{4}\right)$ $\approx 1+4x-\frac{1}{4}x^2$
	Uses both small angle approximations correctly for sine and cosine $\sin kx \approx kx$ and $\sqrt{\cos x} \approx \sqrt{1-\frac{x^2}{2}}$ Must have eliminated all trig expressions Inconsistent variables for angles must eventually be consistent to be awarded A1	1.1b	A1	
	Uses their expansion from **(a)** Must have replaced x with x^2 or Applies binomial theorem correctly to $\left(1-\frac{x^2}{2}\right)^{\frac{1}{2}}$ ignore any extra terms	3.1a	M1	
	Completes argument to obtain $4x+\left(1-\frac{x^2}{4}\right)$ or $1+4x-\frac{1}{4}x^2$ Accept any order of terms Ignore higher powers of x Must be in terms of x Do not ISW	2.1	R1	
	Subtotal		**4**	

November 2021 Question 5 Paper 2

5 Express

$$\frac{5(x-3)}{(2x-11)(4-3x)}$$

in the form

$$\frac{A}{(2x-11)}+\frac{B}{(4-3x)}$$

where A and B are integers.

[3 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
5	Forms the identity/equation $5(x-3) \equiv A(4-3x)+B(2x-11)$ and either Compares coefficients or Substitutes a value for x PI by correct A or B	1.1a	M1	$\frac{5(x-3)}{(2x-11)(4-3x)}=\frac{A}{(2x-11)}+\frac{B}{(4-3x)}$ $5(x-3)=A(4-3x)+B(2x-11)$ $x=\frac{4}{3} \Longrightarrow B=1$ $x=\frac{11}{2} \Rightarrow A=-1$
	Obtains $A=-1$	1.1b	A1	
	Obtains $B=1$	1.1b	A1	
	Total		**3**	

November 2021 Question 2 Paper 3

2 Simplify fully

$$\frac{(x+3)(6-2x)}{(x-3)(3+x)} \qquad \text{for } x \neq \pm 3$$

Circle your answer.

[1 mark]

-2 $\qquad$ 2 $\qquad$ $\frac{(6-2x)}{(x-3)}$ $\qquad$ $\frac{(2x-6)}{(x-3)}$

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
2	Circles correct answer	1.1b	B1	–2
	Total		**1**	

November 2021 Question 6 Paper 3

6 Given that $x > 0$ and $x \neq 25$, fully simplify

$$\frac{10 + 5x - 2x^{\frac{1}{2}} - x^{\frac{3}{2}}}{5 - \sqrt{x}}$$

Fully justify your answer.

[4 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
6	Begins to solve the problem using an appropriate technique eg factorising or grouping terms in numerator or writing $y = \sqrt{x}$ **PI** if 2 + x or 25 – x or 5 – $x^{1/2}$ seen or multiplies by $\frac{5+\sqrt{x}}{5+\sqrt{x}}$	3.1a	M1	$\frac{10+5x-2x^{\frac{1}{2}}-x^{\frac{3}{2}}}{5-\sqrt{x}} \times \frac{5+\sqrt{x}}{5+\sqrt{x}}$ $= \frac{50+25x-10x^{\frac{1}{2}}-5x^{\frac{3}{2}}+10\sqrt{x}+5x\sqrt{x}-2x-x^{\frac{3}{2}}\sqrt{x}}{25-x}$ $= \frac{50+23x-x^2}{25-x}$ $= \frac{(25-x)(2+x)}{25-x}$ $= 2+x$
	Obtains one correct common factor in numerator eg 2 + x or 25 – x or 5 – $x^{1/2}$ or expands numerator condone one error may be unsimplified	1.1a	M1	
	Obtains second correct common factor in numerator or obtains correct simplified numerator and denominator **PI** in long division	1.1a	M1	
	Completes manipulation by cancelling common factor to obtain 2 + x	1.1b	A1	
	Total		**4**	

November 2020 Question 1 Paper 1

1 The first three terms, in ascending powers of x, of the binomial expansion of $(9+2x)^{\frac{1}{2}}$ are given by

$$(9+2x)^{\frac{1}{2}} \approx a + \frac{x}{3} - \frac{x^2}{54}$$

where a is a constant.

1 (a) State the range of values of x for which this expansion is valid.

Circle your answer.

[1 mark]

$|x| < \frac{2}{9}$ $\qquad |x| < \frac{2}{3}$ $\qquad |x| < 1$ $\qquad |x| < \frac{9}{2}$

1 (b) Find the value of a.

Circle your answer.

[1 mark]

1 2 3 9

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
1a	Circles the correct answer	1.1b	B1	$\lvert x\rvert < \frac{9}{2}$
	Subtotal		**1**	
1b	Circles the correct answer	1.1b	B1	3
	Subtotal		**1**	
	Question Total		**2**	

June 2019 Question 9 Paper 2

9 (a) Show that the first two terms of the binomial expansion of $\sqrt{4-2x^2}$ are

$$2-\frac{x^2}{2}$$

[2 marks]

9 (b) State the range of values of x for which the expansion found in part **(a)** is valid.

[2 marks]

9 (c) Hence, find an approximation for

$$\int_0^{0.4} \sqrt{\cos x}\, \mathrm{d}x$$

giving your answer to five decimal places.

Fully justify your answer.

[4 marks]

9 (d) A student decides to use this method to find an approximation for

$$\int_0^{1.4} \sqrt{\cos x}\, \mathrm{d}x$$

Explain why this may not be a suitable method.

[1 mark]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
9(a)	Write in a form to which the binomial expansion can be applied Must be of form $a\left(1-\frac{x^2}{2}\right)^{\frac{1}{2}}$	3.1a	M1	$\sqrt{4-2x^2}=2\left(1-\frac{x^2}{2}\right)^{\frac{1}{2}}$ $\approx 2\left(1+\frac{1}{2}\left(-\frac{x^2}{2}\right)\right)$ $\approx 2-\frac{x^2}{2}$
	Completes rigorous argument to obtain correct expansion **AG**	2.1	R1	
9(b)	Compares their $\frac{x^2}{2}$ to 1 Condone incorrect inequality **PI** by $\left\|-2x^2\right\|<4$	1.1a	M1	$\left\|-\frac{x^2}{2}\right\|<1$ $\Rightarrow \|x\|<\sqrt{2}$
	Obtains correct range of values **ACF**	1.1b	A1	
9(c)	Explains that as 0.4 radians is small therefore $\cos x\approx 1-\frac{x^2}{2}$ **Must** refer to 0.4 and small angle approximation for cos x	2.4	E1	As 0.4 is small $\cos x\approx 1-\frac{x^2}{2}$ $\int_0^{0.4}\sqrt{\cos x}\,dx\approx\int_0^{0.4}\sqrt{1-\frac{x^2}{2}}\,dx$ $\approx\frac{1}{2}\int_0^{0.4}2-\frac{x^2}{2}\,dx$ $\approx\int_0^{0.4}1-\frac{x^2}{4}\,dx$ $\approx\left[x-\frac{x^3}{12}\right]_0^{0.4}$ $\approx 0.4-\frac{0.4^3}{12}$ ≈ 0.39467
	Uses half of their expansion from **9(a)** as the integrand	1.1a	M1	
	Integrates their expression with at least one term correct	1.1a	M1	
	Obtains correct value must be at least five decimal places Condone $\frac{148}{375}$ **CAO**	1.1b	A1	
9(d)	States that 1.4 radians is not a small angle so the approximation is not valid **Must** refer to small angle approximation **and** 1.4 or State invalid as 1.4 is bigger than 0.664 **NB** 0.664 is the limiting value for approximation to be valid	2.4	E1	Since 1.4 is not a small angle the approximation is not suitable
	Total		**9**	

June 2018 Question 6 Paper 1

6 (a) Find the first three terms, in ascending powers of x, of the binomial expansion of $\dfrac{1}{\sqrt{4+x}}$

[3 marks]

6 (b) Hence, find the first three terms of the binomial expansion of $\dfrac{1}{\sqrt{4-x^3}}$

[2 marks]

6 (c) Using your answer to part **(b)**, find an approximation for $\displaystyle\int_0^1 \frac{1}{\sqrt{4-x^3}}\,\mathrm{d}x$, giving your answer to seven decimal places.

[3 marks]

6 (d) (i) Edward, a student, decides to use this method to find a more accurate value for the integral by increasing the number of terms of the binomial expansion used.

Explain clearly whether Edward's approximation will be an overestimate, an underestimate, or if it is impossible to tell.

[2 marks]

6 (d) (ii) Edward goes on to use the expansion from part **(b)** to find an approximation for $\displaystyle\int_{-2}^0 \frac{1}{\sqrt{4-x^3}}\,\mathrm{d}x$

Explain why Edward's approximation is invalid.

[2 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)	Writes in a form to which the binomial expansion can be applied Accept $A(1+\frac{x}{4})^{-\frac{1}{2}}$	AO3.1a	M1	$\frac{1}{\sqrt{4+x}}=\frac{1}{2}(1+\frac{x}{4})^{-\frac{1}{2}}$ $\approx\frac{1}{2}\left[1+\left(-\frac{1}{2}\right)\frac{x}{4}+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(\frac{x}{4}\right)^2}{2!}\right]$ $\approx\frac{1}{2}\left[1-\frac{x}{8}+\frac{3x^2}{128}\right]$ $\approx\frac{1}{2}-\frac{1}{16}x+\frac{3}{256}x^2$
	Uses binomial expansion for their $(1+kx)^{\pm\frac{1}{2}}$ with at least two terms correct (can be unsimplified)	AO1.1a	M1	
	Obtains correct simplified answer No need to expand brackets CAO	AO1.1b	A1	
(b)	Substitutes $-x^3$ in their three term expansion from part (a)	AO1.1a	M1	$\frac{1}{\sqrt{4-x^3}}\approx\frac{1}{2}-\frac{1}{16}\left(-x^3\right)+\frac{3}{256}\left(-x^3\right)^2$ $\approx\frac{1}{2}+\frac{x^3}{16}+\frac{3x^6}{256}$
	Obtains correct expansion. FT their (a)	AO1.1b	A1F	
(c)	Uses their three term expansion as the integrand ignore limits PI by next mark	AO1.1a	M1	$\int_0^1\frac{1}{\sqrt{4-x^3}}dx\approx\int_0^1\frac{1}{2}+\frac{x^3}{16}+\frac{3x^6}{256}dx$ $\approx\left[\frac{x}{2}+\frac{x^4}{64}+\frac{3x^7}{1792}\right]_0^1$ $\approx\frac{1}{2}+\frac{1}{64}+\frac{3}{1792}$ ≈ 0.5172991
	Integrates (at least two terms correct)	AO1.1a	M1	
	Obtains correct value CAO	AO1.1b	A1	
(d)(i)	Explains that each term in the expansion is positive	AO2.4	E1	Each term in the expansion is positive. So increasing the terms will increase the estimated value hence the value must be an underestimate.
	Deduces that increasing the number of terms will increase the estimated value and that the value must be an underestimate. (Condone inference if evidence given ie value calculated numerically and compared)	AO2.2a	R1	
(d)(ii)	States the validity of their binomial expansion for part (b) Provided their $k\neq\pm1$	AO3.1a	B1F	The binomial expansion is valid for $\lvert x\rvert<\sqrt[3]{4}$
	Compares integral lower limit with validity of correct expansion CAO	AO2.3	E1	$2>\sqrt[3]{4}$
	Total		**12**	

June 2018 Question 9 Paper 2

9 (a) Show that the first two terms of the binomial expansion of $\sqrt{4-2x^2}$ are

$$2-\frac{x^2}{2}$$

[2 marks]

9 (b) State the range of values of x for which the expansion found in part **(a)** is valid.

[2 marks]

9 (c) Hence, find an approximation for

$$\int_0^{0.4} \sqrt{\cos x}\,\mathrm{d}x$$

giving your answer to five decimal places.

Fully justify your answer.

[4 marks]

9 (d) A student decides to use this method to find an approximation for

$$\int_0^{1.4} \sqrt{\cos x}\,\mathrm{d}x$$

Explain why this may not be a suitable method.

[1 mark]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
9(a)	Write in a form to which the binomial expansion can be applied Must be of form $a\left(1-\frac{x^2}{2}\right)^{\frac{1}{2}}$	3.1a	M1	$\sqrt{4-2x^2}=2\left(1-\frac{x^2}{2}\right)^{\frac{1}{2}}$ $\approx 2\left(1+\frac{1}{2}\left(-\frac{x^2}{2}\right)\right)$ $\approx 2-\frac{x^2}{2}$
	Completes rigorous argument to obtain correct expansion **AG**	2.1	R1	
9(b)	Compares their $\frac{x^2}{2}$ to 1 Condone incorrect inequality **PI** by $\left\|-2x^2\right\|<4$	1.1a	M1	$\left\|-\frac{x^2}{2}\right\|<1$ $\Rightarrow \|x\|<\sqrt{2}$
	Obtains correct range of values **ACF**	1.1b	A1	
9(c)	Explains that as 0.4 radians is small therefore $\cos x\approx 1-\frac{x^2}{2}$ **Must** refer to 0.4 and small angle approximation for cos x	2.4	E1	As 0.4 is small $\cos x\approx 1-\frac{x^2}{2}$ $\int_0^{0.4}\sqrt{\cos x}\,dx\approx\int_0^{0.4}\sqrt{1-\frac{x^2}{2}}\,dx$ $\approx\frac{1}{2}\int_0^{0.4}2-\frac{x^2}{2}\,dx$ $\approx\int_0^{0.4}1-\frac{x^2}{4}\,dx$ $\approx\left[x-\frac{x^3}{12}\right]_0^{0.4}$ $\approx 0.4-\frac{0.4^3}{12}$ ≈ 0.39467
	Uses half of their expansion from **9(a)** as the integrand	1.1a	M1	
	Integrates their expression with at least one term correct	1.1a	M1	
	Obtains correct value must be at least five decimal places Condone $\frac{148}{375}$ **CAO**	1.1b	A1	
9(d)	States that 1.4 radians is not a small angle so the approximation is not valid **Must** refer to small angle approximation **and** 1.4 or State invalid as 1.4 is bigger than 0.664 **NB** 0.664 is the limiting value for approximation to be valid	2.4	E1	Since 1.4 is not a small angle the approximation is not suitable
	Total		**9**	

Ch 8 Trigonometric identities

November 2021 Question 8 Paper 1

8 (a) Given that

$$9\sin^2\theta + \sin 2\theta = 8$$

show that

$$8\cot^2\theta - 2\cot\theta - 1 = 0$$

[4 marks]

8 (b) Hence, solve

$$9\sin^2\theta + \sin 2\theta = 8$$

in the interval $0 < \theta < 2\pi$

Give your answers to two decimal places.

[3 marks]

8 (c) Solve

$$9\sin^2\left(2x - \frac{\pi}{4}\right) + \sin\left(4x - \frac{\pi}{2}\right) = 8$$

in the interval $0 < x < \frac{\pi}{2}$

Give your answers to one decimal place.

[2 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
8(a)	Recalls and uses $\sin 2\theta = 2\sin\theta\cos\theta$	1.2	B1	$9\sin^2\theta + \sin 2\theta = 8$ $9\sin^2\theta + 2\sin\theta\cos\theta = 8$ $9 + 2\cot\theta = 8\text{cosec}^2\theta$ $9 + 2\cot\theta = 8(\cot^2\theta + 1)$ $8\cot^2\theta - 2\cot\theta - 1 = 0$
	Uses $\cot^2\theta + 1 = \text{cosec}^2\theta$ Or $\tan^2\theta + 1 = \sec^2\theta$ Condone a sign error	1.1a	M1	
	Divides through by $\cos^2\theta$ or $\sin^2\theta$	1.1a	M1	
	Completes rearrangement to achieve given result. **AG**	2.1	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Mark	Typical solution
8(b)	Solves to give values of cot θ or tan θ PI by sight of 2 and – 4 or $-\frac{1}{4}$ and $\frac{1}{2}$ or by two correct answers	1.1a	M1	$\cot\theta = -\frac{1}{4}$ or $\cot\theta = \frac{1}{2}$ tan θ = – 4 or tan θ = 2 θ = 1.82 θ = 1.82 + π = 4.96 θ = 1.11 θ = 1.11 + π = 4.25
	Obtains two correct values of θ. Condone AWRT correct answers.	1.1b	A1	θ = 1.11, 1.82, 4.25, 4.96
	Obtains all four solutions with no additional solutions or errors. Ignore additional solutions outside the interval. AWRT 1.11, 1.82, 4.25, 4.96 CAO	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Mark	Typical solution
8(c)	Sets $2x - \frac{\pi}{4}$ equal to at least one of their solutions. PI by a correct answer	3.1a	M1	$2x - \frac{\pi}{4} = 1.107.., 1.815..$ x = 0.9, 1.3
	Obtains correct AWRT values. Correct values should be rounded from 0.94627… and 1.300058… ISW once correct answers seen. CSO Condone extra values outside of the interval.	1.1b	A1	
	Subtotal		**2**	

November 2020 Question 2 Paper 3

2 Given that

$$6\cos\theta + 8\sin\theta \equiv R\cos(\theta + \alpha)$$

find the value of R.

Circle your answer.

[1 mark]

6 8 10 14

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
2	Circles correct answer	1.1b	B1	10
	Total		1	

November 2020 Question 9 Paper 3

9 (a) For $\cos\theta \neq 0$, prove that

$$\text{cosec}\,2\theta + \cot 2\theta = \cot\theta$$

[4 marks]

9 (b) Explain why

$$\cot\theta \neq \text{cosec}\,2\theta + \cot 2\theta$$

when $\cos\theta = 0$

[1 mark]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
9(a)	Uses cosec$2\theta = \frac{1}{\sin 2\theta}$ and cot2θ=$\frac{\cos 2\theta}{\sin 2\theta}$	1.2	B1	$\frac{1}{\sin 2\theta}+\frac{\cos 2\theta}{\sin 2\theta}$ $=\frac{1+\cos 2\theta}{\sin 2\theta}$ $=\frac{1+\cos^2\theta-\sin^2\theta}{2\sin\theta\cos\theta}$ $=\frac{2\cos^2\theta}{2\sin\theta\cos\theta}$ $=\frac{\cos\theta}{\sin\theta}=\cot\theta$
	Uses the identity for $\sin 2\theta = 2\sin\theta\cos\theta$ or an identity for $\cos 2\theta = \cos^2\theta - \sin^2\theta$ *or* $2\cos^2\theta - 1$ or $1 - 2\sin^2\theta$ to commence proof	2.1	M1	
	Uses the identities for sin2θ and cos2θ in correct proof	1.1b	A1	
	Completes a reasoned argument leading to a single trigonometric fraction to prove given identity **AG**	2.1	R1	
9(b)	Deduces that when $\cos\theta = 0$ then $\cot\theta$ is defined/zero/exists on LHS but cosec2θ or cot2θ or $\frac{1}{2\sin\theta\cos\theta}$ *or* $\frac{1}{\sin 2\theta}$ is undefined on RHS or deduces that LHS is defined but RHS is undefined Must compare both LHS and RHS	2.2a	E1	When $\cos\theta = 0$ the value of $\cot\theta$ = 0 on LHS but because the value of sin2θ =0, cosec2θ and cot2θ are undefined on RHS.
	Total		**5**	

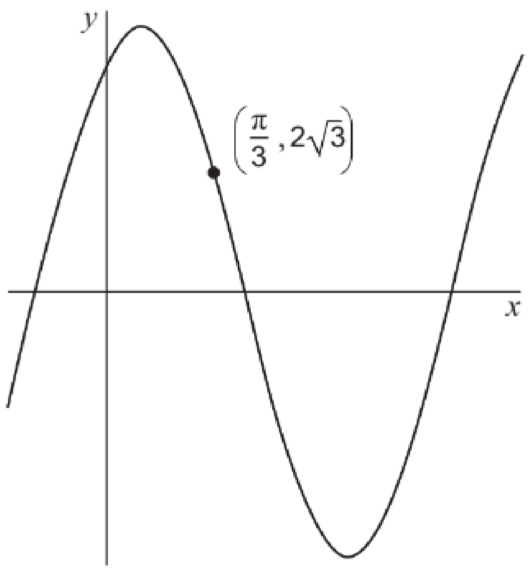
June 2019 Question 6 Paper 2

6 A curve has equation

$$y = a\sin x + b\cos x$$

where a and b are constants.

The maximum value of y is 4 and the curve passes through the point $\left(\frac{\pi}{3}, 2\sqrt{3}\right)$ as shown in the diagram.



Find the exact values of a and b.

[6 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
6	Compares with $R\cos(x \pm \alpha)$ or $R\sin(x \pm \alpha)$ by forming an identity e.g. $R\sin(x+\alpha) \equiv a\sin x + b\cos x$ **OE** or Differentiates correctly and equates to zero **CAO PI** by $a\cos x = b\sin x$ **PI** by *R* = 4 or $a^2+b^2=16$	3.1a	M1	$R\sin(x+\alpha) = a\sin x + b\cos x$ $R = 4$ $4\sin\left(\frac{\pi}{3}+\alpha\right) = 2\sqrt{3}$ $\alpha = \frac{\pi}{3}$ $a = 4\cos\frac{\pi}{3} = 2$ $b = 4\sin\frac{\pi}{3} = 2\sqrt{3}$
	Deduces *R* = 4 or $a^2+b^2=16$	2.2a	A1	
	Forms a correct equation for α **PI** by correct α or Forms the equation shown below $2\sqrt{3} = \frac{a\sqrt{3}}{2} + \frac{b}{2}$ **OE** **Must** substitute correct exact values for the trig functions	1.1b	B1	
	Solves their equation to obtain any correct value of α Correct values are shown below $\alpha = \frac{\pi}{3}$ or 0 for $R\sin(x \pm \alpha)$ $\alpha = \pm\frac{\pi}{6}$ for $R\cos(x \pm \alpha)$ or Eliminates a variable correctly from their two equations – **must** obtain a correct simplified equation	1.1a	M1	
	Deduces $a = 2$	2.2a	R1	
	Deduces $b = 2\sqrt{3}$	2.2a	R1	
	Total		**6**	

June 2019 Question 1 Paper 3

1 $\mathrm{f}(x) = \arcsin x$

State the maximum possible domain of f

Tick (✓) **one** box.

[1 mark]

$\{x \in \mathbb{R} : -1 \leq x \leq 1\}$ ☐

$\left\{x \in \mathbb{R} : -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}\right\}$ ☐

$\{x \in \mathbb{R} : -\pi \leq x \leq \pi\}$ ☐

$\{x \in \mathbb{R} : -90 \leq x \leq 90\}$ ☐

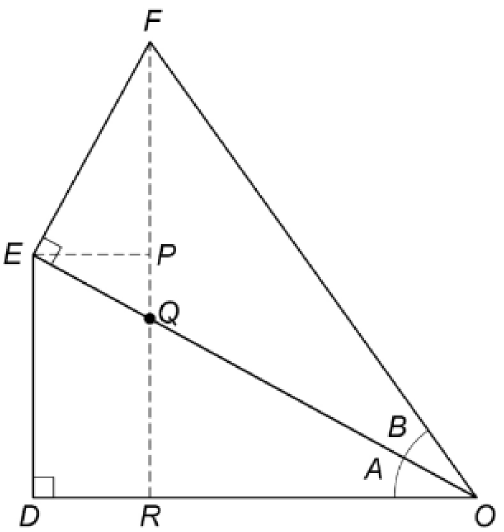
ANSWER

Q	Marking instructions	AO	Mark	Typical solution
1	Ticks the correct response	1.2	B1	$\{x \in \mathbb{R} : -1 \leq x \leq 1\}$
	Total		**1**	

June 2018 Question 14 Paper 1

14 Some students are trying to prove an identity for $\sin(A+B)$.

They start by drawing two right-angled triangles ODE and OEF, as shown.



The students' incomplete proof continues,

Let angle $DOE = A$ and angle $EOF = B$.

In triangle OFR,

Line 1 $\sin(A+B) = \dfrac{RF}{OF}$

Line 2 $= \dfrac{RP + PF}{OF}$

Line 3 $= \dfrac{DE}{OF} + \dfrac{PF}{OF}$ since $DE = RP$

Line 4 $= \dfrac{DE}{.....} \times \dfrac{.....}{OF} + \dfrac{PF}{EF} \times \dfrac{EF}{OF}$

Line 5 $= .... + \cos A \sin B$

14 (a) Explain why $\dfrac{PF}{EF} \times \dfrac{EF}{OF}$ in Line 4 leads to $\cos A \sin B$ in Line 5

[2 marks]

14 (b) Complete Line 4 and Line 5 to prove the identity

Line 4 $$= \frac{DE}{\cdots\cdots} \times \frac{\cdots\cdots}{OF} + \frac{PF}{EF} \times \frac{EF}{OF}$$

Line 5 $= \text{...} + \cos A \sin B$

[1 mark]

14 (c) Explain why the argument used in part **(a)** only proves the identity when A and B are acute angles.

[1 mark]

14 (d) Another student claims that by replacing B with $-B$ in the identity for $\sin(A+B)$ it is possible to find an identity for $\sin(A-B)$.

Assuming the identity for $\sin(A+B)$ is correct for all values of A and B, prove a similar result for $\sin(A-B)$.

[3 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
14(a)	Explains why $\angle EFQ = A$ Must be a fully correct explanation with reasons which may include: Vertically opposite angles and right angle implies similar triangles.	AO2.4	E1	$\angle OQR = \angle FQE$ vertically opposite angles $\angle ORQ = \angle FEQ = 90°$ So $\angle EFQ = A$
	Deduces $\frac{PF}{EF} = \cos(A)$ **AND** $\frac{EF}{OF} = \sin(B)$ Must have at least stated or implied that $\angle EFQ = A$ through similarity	AO2.2a	R1	Since $\angle EFQ = A$ $\frac{PF}{EF} = \cos(A)$ And $\frac{EF}{OF} = \sin(B)$ in triangle OEF
14(b)	Completes proof	AO2.2a	B1	$\frac{DE}{\mathbf{OE}} \times \frac{\mathbf{OE}}{OF} + \frac{PF}{EF} \times \frac{EF}{OF}$ $= \mathbf{\sin A \cos B} + \cos A \sin B$
14(c)	Explains that the proof is based on right angled triangles which limits A and B to acute angles	AO2.3	E1	Since the proof is based on the diagram which uses right-angled triangles it is assumed that A and B are acute. Therefore, the proof only holds for acute angles.
14(d)	Substitutes $-B$ into identity for $\sin(A+B)$ to give $\sin(A-B)$	AO2.1	R1	$\sin(A-B) = \sin A\cos(-B) + \cos A\sin(-B)$
	Recalls at least one of the identities $\sin(-B) = -\sin(B)$ $\cos(-B) = \cos(B)$ Must be explicitly stated	AO1.2	B1	$\sin(-B) = -\sin(B)$ $\cos(-B) = \cos(B)$
	Deduces correct identity with no errors. This must be clearly deduced from a correct argument and not simply stated.	AO2.2a	R1	Hence $\sin(A-B) = \sin A\cos B - \cos A\sin B$
	Total		**7**	

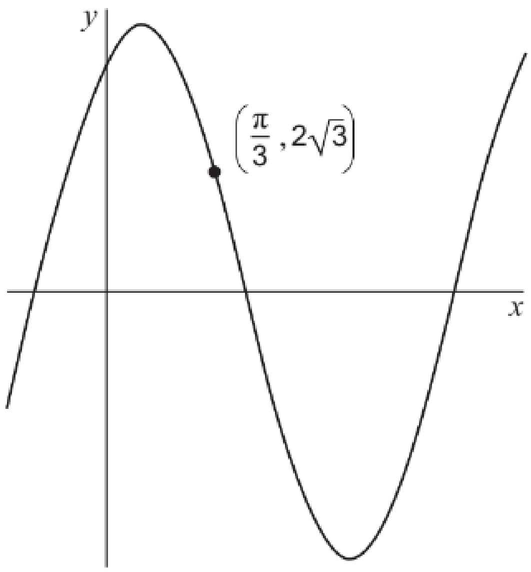
June 2018 Question 6 Paper 2

6 A curve has equation

$$y = a\sin x + b\cos x$$

where a and b are constants.

The maximum value of y is 4 and the curve passes through the point $\left(\frac{\pi}{3}, 2\sqrt{3}\right)$ as shown in the diagram.



Find the exact values of a and b.

[6 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
6	Compares with $R\cos(x\pm\alpha)$ or $R\sin(x\pm\alpha)$ by forming an identity e.g. $R\sin(x+\alpha)\equiv a\sin x+b\cos x$ **OE** or Differentiates correctly and equates to zero **CAO PI** by $a\cos x=b\sin x$ **PI** by R = 4 or $a^2+b^2=16$	3.1a	M1	$R\sin(x+\alpha)=a\sin x+b\cos x$ $R=4$ $4\sin\left(\frac{\pi}{3}+\alpha\right)=2\sqrt{3}$ $\alpha=\frac{\pi}{3}$ $a=4\cos\frac{\pi}{3}=2$ $b=4\sin\frac{\pi}{3}=2\sqrt{3}$
	Deduces R = 4 or $a^2+b^2=16$	2.2a	A1	
	Forms a correct equation for α **PI** by correct α or Forms the equation shown below $2\sqrt{3}=\frac{a\sqrt{3}}{2}+\frac{b}{2}$ **OE** **Must** substitute correct exact values for the trig functions	1.1b	B1	
	Solves their equation to obtain any correct value of α Correct values are shown below $\alpha=\frac{\pi}{3}$ or 0 for $R\sin(x\pm\alpha)$ $\alpha=\pm\frac{\pi}{6}$ for $R\cos(x\pm\alpha)$ or Eliminates a variable correctly from their two equations – **must** obtain a correct simplified equation	1.1a	M1	
	Deduces $a=2$	2.2a	R1	
	Deduces $b=2\sqrt{3}$	2.2a	R1	
	Total		**6**	

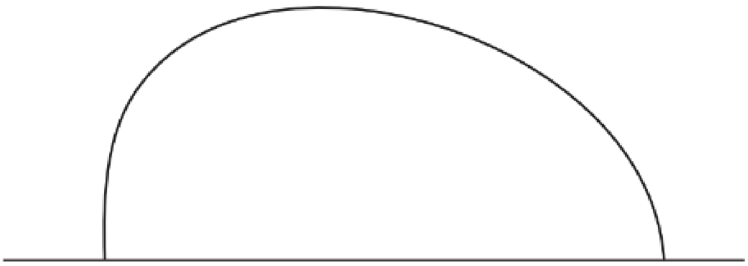
Ch 9 Further differentiation

June 2022 Question 13 Paper 1

13 **Figure 2** shows the approximate shape of the vertical cross section of the entrance to a cave. The cave has a horizontal floor.

The entrance to the cave joins the floor at the points *O* and *P*.

Figure 2



Garry models the shape of the cross section of the entrance to the cave using the equation

$$x^2 + y^2 = a\sqrt{x} - y$$

where a is a constant, and x and y are the horizontal and vertical distances respectively, in metres, measured from *O*.

13 (a) The distance *OP* is 16 metres.

Find the value of a that Garry should use in the model.

[2 marks]

13 (b) Show that the maximum height of the cave above *OP* is approximately 10.5 metres.

[6 marks]

13 (c) Suggest one limitation of the model Garry has used.

[1 mark]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
13(a)	Substitutes y = 0 and x = 16 correctly into $x^2+y^2=a\sqrt{x}-y$	3.4	M1	$x^2+y^2=a\sqrt{x}-y$ $16^2+0^2=a\sqrt{16}-0$ $256=4a$ $a=64$
	Obtains a = 64	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
13(b)	Differentiates implicitly with either $2y\frac{dy}{dx}$ or $-\frac{dy}{dx}$ seen	3.1b	B1	$2x+2y\frac{dy}{dx}=\frac{64}{2}x^{-\frac{1}{2}}-\frac{dy}{dx}$ $\frac{dy}{dx}=0 \Rightarrow 2x=\frac{32}{\sqrt{x}}$ $x^{\frac{3}{2}}=16$ $x=6.3496...$ $(6.3496...)^2+y^2=64\sqrt{6.3496...}-y$ $y=10.51$ Maximum height is approximately 10.5 metres
	Differentiates any two of the four terms correctly. Can be in terms of a or with their a value	1.1a	M1	
	Obtains a fully correct differentiated equation Can be in terms of a Follow through their a value $2x+2y\frac{dy}{dx}=\frac{a}{2}x^{-\frac{1}{2}}-\frac{dy}{dx}$	1.1b	A1F	
	Uses $\frac{dy}{dx}=0$	1.1a	M1	
	Substitutes their numerical x value where 0 < x < 16, into the model with their a value	3.4	M1	

	Obtains a value for y AWRT 10.51 and concludes that the maximum height is approximately 10.5 metres AG Condone equals Must state units CSO	3.2a	R1	
	Subtotal		**6**	

Q	Marking instructions	AO	Marks	Typical solution
13(c)	States or infers that the entrance is unlikely to be a smooth curve Accept: • The cave has dents • Entrance is not perfectly smooth Ignore comments about the floor or the vertical cross section	3.5b	E1	The entrance to the cave is unlikely to be perfectly smooth
	Subtotal		**1**	

	Question 13 Total		**9**	

November 2021 Question 2 Paper 1

2 Given that $y = \ln(5x)$

find $\frac{dy}{dx}$

Circle your answer.

[1 mark]

$\frac{dy}{dx} = \frac{1}{x}$ $\quad$ $\frac{dy}{dx} = \frac{1}{5x}$ $\quad$ $\frac{dy}{dx} = \frac{5}{x}$ $\quad$ $\frac{dy}{dx} = \ln 5$

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
2	Circles correct answer	1.1b	B1	$\frac{dy}{dx} = \frac{1}{x}$
	Total		**1**	

November 2021 Question 10 Paper 1

10 (a) Given that

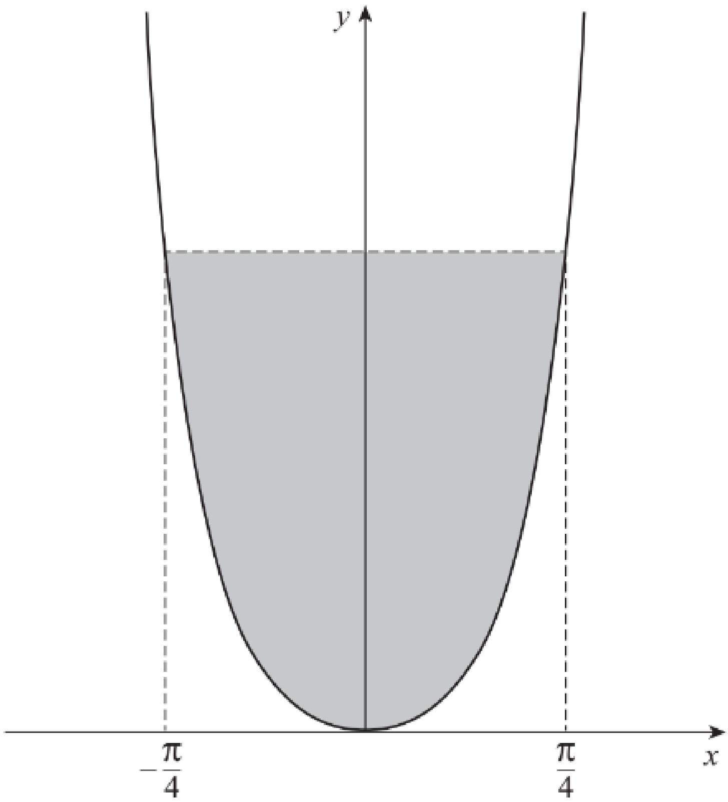
$$y = \tan x$$

use the quotient rule to show that

$$\frac{dy}{dx} = \sec^2 x$$

[3 marks]

10 (b) The region enclosed by the curve $y = \tan^2 x$ and the horizontal line, which intersects the curve at $x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$, is shaded in the diagram below.



Show that the area of the shaded region is

$$\pi - 2$$

Fully justify your answer.

[5 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
10(a)	Recalls $\tan x = \dfrac{\sin x}{\cos x}$	1.2	B1	$\dfrac{d}{dx}(\tan x) = \dfrac{d}{dx}\left(\dfrac{\sin x}{\cos x}\right)$ $= \dfrac{\cos x \cos x - (-\sin x)\sin x}{\cos^2 x}$ $= \dfrac{\sin^2 x + \cos^2 x}{\cos^2 x}$ $= \dfrac{1}{\cos^2 x}$ $= \sec^2 x$
	Uses the correct quotient rule. Condone sign error in differentiation of sin or cosine.	1.1a	M1	
	Completes rigorous argument to show the required result. Use of $\sin^2 x + \cos^2 x = 1$ or $\tan^2 x + 1 = \sec^2 x$ must be explicit. Must include $\dfrac{d}{dx}(\tan x) = \ldots$ or $\dfrac{dy}{dx} = \ldots$	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)	Writes down an integral of the form $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2 x\,dx$, $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (1 - \tan^2 x)\,dx$ Condone missing or incorrect limits and missing dx	3.1a	M1	Area under curve $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2 x\,dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x - 1\,dx$ $= [\tan x - x]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$ $= \left(\tan\frac{\pi}{4} - \frac{\pi}{4}\right) - \left(\tan\left(-\frac{\pi}{4}\right) - -\frac{\pi}{4}\right)$ $= 1 - \frac{\pi}{4} + 1 - \frac{\pi}{4}$ $= 2 - \frac{\pi}{2}$ Area of shaded region $\frac{\pi}{2} - \left(2 - \frac{\pi}{2}\right)$ $= \pi - 2$
	Uses $\tan^2 x + 1 = \sec^2 x$ to write integrand in a form which can be integrated, condone sign error.	3.1a	M1	
	Integrates their expression of the form $A\sec^2 x + B$	1.1b	A1F	
	Forms an expression for or evaluates the area of the relevant rectangle. $2\frac{\pi}{4}\tan^2\frac{\pi}{4}$ or $\frac{\pi}{4}\tan^2\frac{\pi}{4}$ Could be implicit within their integral	1.1b	B1	
	Completes rigorous argument to show the required result. Substitution of consistent limits should be explicit and no slips in algebra. Use of dx is required. AG	2.1	R1	
	Subtotal		**5**	

November 2021 Question 12 Paper 1

12 The equation of a curve is

$$(x+y)^2 = 4y + 2x + 8$$

The curve intersects the positive x-axis at the point P.

12 (a) Show that the gradient of the curve at P is $-\frac{3}{2}$

[6 marks]

12 (b) Find the equation of the normal to the curve at P, giving your answer in the form $ax + by = c$, where a, b and c are integers.

[2 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
12(a)	Substitutes y = 0 to form an equation for x PI x = 4	3.1a	M1	$(x+y)^2 = 4y+2x+8$ $y=0 \Rightarrow x^2 = 8+2x$ $\Rightarrow x = 4 \text{ or } -2$ $x = 4$ at P
	Obtains x = 4 ignore any other value.	1.1b	A1	
	Expands and uses product rule to obtain the derivative of their Axy term. or Uses chain rule to obtain $2(x+y)\left(1+\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ Condone missing brackets.	3.1a	M1	$x^2+2xy+y^2 = 4y+2x+8$ $2x+2y+2x\frac{\mathrm{d}y}{\mathrm{d}x}+2y\frac{\mathrm{d}y}{\mathrm{d}x} = 4\frac{\mathrm{d}y}{\mathrm{d}x}+2$ $x=4, y=0$ $\Rightarrow 8+8\frac{\mathrm{d}y}{\mathrm{d}x} = 4\frac{\mathrm{d}y}{\mathrm{d}x}+2$
	Uses implicit differentiation correctly to obtain the derivative of $4y$ or y^2.	1.1b	B1	$4\frac{\mathrm{d}y}{\mathrm{d}x} = -6$ $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{3}{2}$
	Obtains correct equation from correct differentiation.	1.1b	A1	
	Substitutes x = 4 and y = 0 into $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2-2x-2y}{2x+2y-4}$ OE and obtains $-\frac{3}{2}$ If substituting into an earlier equation must reach $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{3}{2}$ (AG)	2.1	R1	
	Subtotal		**6**	

Q	Marking instructions	AO	Mark	Typical solution
12(b)	Uses $\frac{2}{3}$ and y = 0 and their x = 4 from part (a) to form equation of line.	1.1a	M1	$y = \frac{2}{3}(x-4)$ $2x-3y=8$
	Obtains their equation in correct form.	1.1b	A1F	
	Subtotal		**2**	

November 2020 Question 12 Paper 1

12 A curve C has equation

$$x^3 \sin y + \cos y = Ax$$

where A is a constant.

C passes through the point $P\left(\sqrt{3}, \frac{\pi}{6}\right)$

12 (a) Show that $A = 2$

[2 marks]

12 (b) (i) Show that $\frac{dy}{dx} = \frac{2 - 3x^2 \sin y}{x^3 \cos y - \sin y}$

[5 marks]

12 (b) (ii) Hence, find the gradient of the curve at P.

[2 marks]

12 (b) (iii) The tangent to C at P intersects the x-axis at Q.

Find the exact x-coordinate of Q.

[4 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
12(a)	Substitutes $x = \sqrt{3}$ and $y = \frac{\pi}{6}$ to obtain an equation or an expression for A	1.1a	M1	$(\sqrt{3})^3 \sin\frac{\pi}{6} + \cos\frac{\pi}{6} = A\sqrt{3}$ $\frac{3\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = A\sqrt{3}$ $\frac{3}{2} + \frac{1}{2} = A$ $A = 2$
	Completes argument to show A = 2 Must clearly show use of $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ and $\sin\frac{\pi}{6} = \frac{1}{2}$ AG	2.1	R1	
	Subtotal		**2**	
12(b)(i)	Uses implicit differentiation correctly at least once with sight of $\sin y\frac{dy}{dx}$ or $\cos y\frac{dy}{dx}$ Condone sign error	3.1a	M1	$3x^2 \sin y + x^3 \cos y\frac{dy}{dx} - \sin y\frac{dy}{dx} = 2$ $\frac{dy}{dx}(x^3 \cos y - \sin y) = 2 - 3x^2 \sin y$ $\frac{dy}{dx} = \frac{2 - 3x^2 \sin y}{x^3 \cos y - \sin y}$
	Uses product rule with sight of $Px^2 \sin y \pm x^3 \cos y\frac{dy}{dx}$ Condone omission of $\frac{dy}{dx}$	3.1a	M1	
	Obtains equation of the form $Px^2 \sin y \pm x^3 \cos y\frac{dy}{dx} \pm \sin y\frac{dy}{dx} = 2$	1.1b	A1	
	Obtains completely correct equation	1.1b	A1	
	Isolates $\frac{dy}{dx}$ terms and factorises to complete rigorous argument with no slips to show the given result AG	2.1	R1	
	Subtotal		**5**	
12(b)(ii)	Substitutes $x = \sqrt{3}$ and $y = \frac{\pi}{6}$ to obtain an expression for the gradient	1.1a	M1	$\frac{dy}{dx} = \frac{2 - 3(\sqrt{3})^2 \sin\frac{\pi}{6}}{(\sqrt{3})^3 \cos\frac{\pi}{6} - \sin\frac{\pi}{6}}$ $= -\frac{5}{8}$
	Obtains correct gradient of $-\frac{5}{8}$ OE	1.1b	A1	
	Subtotal		**2**	

12(b)(iii)	Forms equation for the tangent (condone normal) at P using '*their*' gradient and $\left(\sqrt{3}, \frac{\pi}{6}\right)$ ACF or Writes the equation as $y = mx + c$ using '*their*' gradient of tangent (condone normal) and substitutes $\left(\sqrt{3}, \frac{\pi}{6}\right)$ to obtain an equation in c PI by correct exact value for x	3.1a	M1	$y - \frac{\pi}{6} = -\frac{5}{8}(x - \sqrt{3})$ $0 - \frac{\pi}{6} = -\frac{5}{8}(x - \sqrt{3})$ $x = \sqrt{3} + \frac{4\pi}{15}$
	Obtains fully correct equation for the '*their*' **tangent** at P ACF Note $c = \frac{5\sqrt{3}}{8} + \frac{\pi}{6}$ or $c = 1.606..$ Follow through 'their' gradient of tangent from 12(b)(ii) must be to at least 3 dp	1.1b	A1F	
	Substitutes $y = 0$ into '*their*' tangent (condone normal) equation and solves to find the x coordinate of Q Accept decimals	3.1a	M1	
	Obtains $x = \sqrt{3} + \frac{4\pi}{15}$ OE must be exact form Eg $x = \frac{8}{5}(\frac{5\sqrt{3}}{8} + \frac{\pi}{6})$	1.1b	A1	
	Subtotal		**4**	
	Question Total		**13**	

June 2019 Question 10 Paper 1

10 The volume of a spherical bubble is increasing at a constant rate.

Show that the rate of increase of the radius, r, of the bubble is inversely proportional to r^2

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

[4 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
10	Models the rate of change of volume with a differential equation of the correct form. With respect to time, not contradicted.	3.3	B1	$\frac{dv}{dt} = k$
	Obtains $4\pi r^2$ by differentiation.	1.1b	B1	$\frac{dv}{dr} = 4\pi r^2$
	Uses the chain rule to connect rates of change substituting their expressions for dv/dt and dv/dr. Or Integrates to obtain expression for v=kt+c Then differentiates wrt r dv/dr=kdt/dr Substitutes their expression for dv/dr	3.1b	M1	$\frac{dv}{dt} = \frac{dv}{dr} \times \frac{dr}{dt}$
	Completes argument, obtaining a correct expression for $\frac{dr}{dt}$ and concluding that $\frac{dr}{dt} \propto \frac{1}{r^2}$ OE statement	2.1	R1	$k = 4\pi r^2 \times \frac{dr}{dt}$ $\frac{dr}{dt} = \frac{k}{4\pi r^2}$ $\therefore \frac{dr}{dt} \propto \frac{1}{r^2}$
	Total		**4**	

June 2019 Question 9 Paper 3

9 A curve has equation

$$x^2y^2 + xy^4 = 12$$

9 (a) Prove that the curve does not intersect the coordinate axes.

[2 marks]

9 (b) (i) Show that $\dfrac{dy}{dx} = -\dfrac{2xy + y^3}{2x^2 + 4xy^2}$

[5 marks]

9 (b) (ii) Prove that the curve has no stationary points.

[4 marks]

9 (b) (iii) In the case when $x > 0$, find the equation of the tangent to the curve when $y = 1$

[4 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
9(a)	Demonstrates by substitution that $x=0$ or $y=0$ leads to value on the LHS = 0	2.4	E1	When x = 0 $0^2y^2 + 0y^4 = 0$ When y = 0 $x^2 0^2 + x0^4 = 0$ This is a contradiction because $x^2y^2 + xy^4 = 12$ so the curve does not intersect either axis
	Completes rigorous argument to show required result	2.1	R1	
9 (b)(i)	Uses implicit differentiation	3.1a	M1	$2xy^2 + 2x^2y\frac{dy}{dx} + y^4 + 4xy^3\frac{dy}{dx} = 0$ $\frac{dy}{dx} = -\frac{2xy^2 + y^4}{2x^2y + 4xy^3}$ $= -\frac{y\,(2xy + y^3)}{y(2x^2 + 4xy^2)}$ $= -\frac{2xy + y^3}{2x^2 + 4xy^2},$
	Product rule used LHS (at least one pair of terms correct)	1.1a	M1	
	Differentiates equation of curve fully correctly	1.1b	A1	
	Collects their $\frac{dy}{dx}$ terms in an equation and factorises	3.1a	M1	
	Completes convincing argument to obtain required result by **factorising** then simplifying y **AG**	2.1	R1	

9 (b)(ii)	Begins argument by setting $\frac{dy}{dx}=0$ to form an equation for x and y **PI** by $2xy+y^3=0$	2.1	M1	For stationary points $\frac{dy}{dx}=0$ $\Rightarrow 2xy+y^3=0$ $\Rightarrow y^2=-2x$ $\Rightarrow x^2y^2+x(-2x)y^2=12$ $\Rightarrow -x^2y^2=12$ Since $-x^2y^2<0$ there can be no stationary points.
	Obtains $y^2=-2x$ or $y=\sqrt{-2x}$ or $x=\frac{-y^2}{2}$	1.1b	A1	
	Substitutes $y^2=-2x$ or $x=\frac{-y^2}{2}$ into equation for curve	1.1a	M1	
	Completes convincing argument to deduce the required result	2.2a	R1	
9 (b)(iii)	Substitutes y = 1 into equation of curve to obtain correct quadratic **ACF**	3.1a	M1	$y=1\Rightarrow x^2+x-12=0$ $\Rightarrow x=3 \qquad (x>0)$ $\Rightarrow \frac{dy}{dx}=-\frac{7}{30}$ $y-1=-\frac{7}{30}(x-3)$
	Deduces x = 3 **PI** by substituting their x in their dy/dx	2.2a	R1	
	Substitutes their x and y = 1 in their dy/dx	1.1a	M1	
	Obtains correct equation of tangent **ACF** **ISW**	1.1b	A1	
	Total		**15**	

Ch 10 Integration

June 2022 Question 5 Paper 2

5 The binomial expansion of $(2+5x)^4$ is given by

$$(2+5x)^4 = A + 160x + Bx^2 + 1000x^3 + 625x^4$$

5 (a) Find the value of A and the value of B.

[2 marks]

5 (b) Show that

$$(2+5x)^4 - (2-5x)^4 = Cx + Dx^3$$

where C and D are constants to be found.

[2 marks]

5 (c) Hence, or otherwise, find

$$\int \left((2+5x)^4 - (2-5x)^4 \right) \mathrm{d}x$$

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Obtains 16 Not incorrectly labelled	1.1b	B1	$(2+5x)^4 = 16+160x+600x^2$ $+1000x^3+625x^4$ A = 16 B = 600
	Obtains 600 Not incorrectly labelled	1.1b	B1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)	Obtains the expansion of $(2-5x)^4 =$ $A-160x+Bx^2-1000x^3+625x^4$ Accept A and B unsubstituted or their A and B Or Uses a valid method and obtains one of $C=320$ or $D=2000$	1.1a	M1	$(2+5x)^4-(2-5x)^4$ $=16+160x+600x^2+1000x^3+625x^4$ $-(16-160x+600x^2-1000x^3+625x^4)$ $=320x+2000x^3$
	Completes reasoned argument to show $(2+5x)^4-(2-5x)^4=320x+2000x^3$ Accept A and B unsubstituted or their A and B Must finish with $320x+2000x^3$ don't accept just C=320 and D = 2000	2.1	R1F	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(c)	Integrates one term correctly Accept C and D unsubstituted or their C and D Or Uses reverse of chain rule to obtain at least one term of the form $P(2\pm5x)^5$, $P=\pm\frac{1}{5}$ or $\pm\frac{1}{25}$	1.1a	M1	$\int\left((2+5x)^4-(2-5x)^4\right)\mathrm{d}x$ $=\int\left(320x+2000x^3\right)\mathrm{d}x$ $=160x^2+500x^4+c$
	Obtains $\frac{320}{2}x^2+\frac{2000}{4}x^4+c$ FT C and D unsubstituted or their C and D Or $\frac{(2+5x)^5}{5\times5}+\frac{(2-5x)^5}{5\times5}+c$ Condone missing $+c$	1.1b	A1F	
	Subtotal		**2**	

Arancha Ruiz

June 2022 Question 15 Paper 1

15 (a) Given that

$$y = \operatorname{cosec}\theta$$

15 (a) (i) Express y in terms of $\sin\theta$.

[1 mark]

15 (a) (ii) Hence, prove that

$$\frac{\mathrm{d}y}{\mathrm{d}\theta} = -\operatorname{cosec}\theta\cot\theta$$

[3 marks]

15 (a) (iii) Show that

$$\frac{\sqrt{y^2-1}}{y} = \cos\theta \qquad \text{for } 0 < \theta < \frac{\pi}{2}$$

[3 marks]

15 (b) (i) Use the substitution

$$x = 2\operatorname{cosec}u$$

to show that

$$\int \frac{1}{x^2\sqrt{x^2-4}}\,\mathrm{d}x \qquad \text{for } x > 2$$

can be written as

$$k\int \sin u\,\mathrm{d}u$$

where k is a constant to be found.

[6 marks]

15 (b) (ii) Hence, show

$$\int \frac{1}{x^2\sqrt{x^2-4}}\,\mathrm{d}x = \frac{\sqrt{x^2-4}}{4x} + c \qquad \text{for } x > 2$$

where c is a constant.

[3 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
15(a)(i)	States $(\sin\theta)^{-1}$ or $\dfrac{1}{\sin\theta}$ $\sin^{-1}\theta$ scores B0 Ignore $\sin^{-1}\theta$ if a correct expression has already been written	1.2	B1	$\dfrac{1}{\sin\theta}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
15(a)(ii)	Uses chain rule or quotient rule to obtain $\pm k(\sin\theta)^{-2}\cos\theta$ OE or Multiplies and uses product rule and implicit differentiation to obtain $\pm k(\sin\theta)^{-2}\cos\theta$ This mark can be awarded for using $\dfrac{1}{\cos\theta}$ and differentiating to obtain $\pm k(\cos\theta)^{-2}\sin\theta$ OE Ignore wrong or missing angles	3.1a	M1	
	Obtains $-(\sin\theta)^{-2}\cos\theta$ OE	1.1b	A1	$\dfrac{dy}{d\theta}=-(\sin\theta)^{-2}\cos\theta$
	Completes **rigorous** argument to show the given result. Must either see separated fractions before final line or sight of $-\dfrac{\cot\theta}{\sin\theta}$ or $-\dfrac{1}{\tan\theta\sin\theta}$ or $-\dfrac{\cos\theta}{\sin\theta}\times\operatorname{cosec}\theta$ or Makes clear use of stated identities as part of the solution At some point the solution must have included $\dfrac{dy}{d\theta}=$ AG Condone change of order of functions at the end	2.1	R1	$=-\dfrac{\cos\theta}{\sin\theta}\times\dfrac{1}{\sin\theta}$ $=-\operatorname{cosec}\theta\cot\theta$
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
15(a)(iii)	Substitutes $y = \text{cosec}\,\theta$ OE or Draws a right angled triangle labelling hypotenuse as y and opposite as 1 PI by obtaining y^2 or $\frac{1}{y^2}$ in terms of $\cos\theta$	1.1b	B1	$\frac{\sqrt{\text{cosec}^2\theta - 1}}{\text{cosec}\,\theta}$
	Uses $\text{cosec}^2\theta - 1 = \cot^2\theta$ OE or Uses Pythagoras theorem to find missing adjacent side in the right angled triangle or Obtains $\cos^2\theta$ in terms of y	1.1a	M1	$= \frac{\sqrt{\cot^2\theta}}{\text{cosec}\,\theta}$ $= \frac{\cot\theta}{\text{cosec}\,\theta}$
	Completes **rigorous** argument to show the given result This must include **clear replacement** of $\text{cosec}\,\theta$ and $\cot\theta$ **within the solution** using only sine and cosine functions to complete the argument prior to obtaining the answer given AG	2.1	R1	$= \frac{\cos\theta}{\sin\theta} \times \sin\theta$ $= \cos\theta$
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
15(b)(i)	Obtains $\frac{dx}{du} = -2\operatorname{cosec} u \cot u$ OE	1.1b	B1	$\frac{dx}{du} = -2\operatorname{cosec} u \cot u$ $dx = -2\operatorname{cosec} u \cot u \ du$
	Makes complete substitution to obtain integrand of the form $\frac{P\operatorname{cosec} u \cot u}{Q\operatorname{cosec}^2 u\sqrt{R\operatorname{cosec}^2 u - 4}}$ OE or $\frac{P\operatorname{cosec} u \cot u}{Q\operatorname{cosec}^3 u\sqrt{1 - R\sin^2 u}}$ OE Ignore wrong or missing angles	1.1a	M1	
	Obtains correct integrand $\frac{-2\operatorname{cosec} u \cot u}{4\operatorname{cosec}^2 u\sqrt{4\operatorname{cosec}^2 u - 4}}$ or $\frac{-2\operatorname{cosec} u \cot u}{8\operatorname{cosec}^3 u\sqrt{1 - \sin^2 u}}$ OE	1.1b	A1	$= \int \frac{-2\operatorname{cosec} u \cot u}{4\operatorname{cosec}^2 u\sqrt{4\operatorname{cosec}^2 u - 4}} du$ $= \int \frac{-2\operatorname{cosec} u \cot u}{4\operatorname{cosec}^2 u\sqrt{4\cot^2 u}} du$
	Uses appropriate Pythagorean-trig identity under the square root. Either $1 + \cot^2 u = \operatorname{cosec}^2 u$ or $1 - \sin^2 u = \cos^2 u$ Ignore wrong or missing angles	3.1a	M1	$= \int \frac{-2\operatorname{cosec} u \cot u}{4\operatorname{cosec}^2 u \times 2\cot u} du$ $= \int -\frac{1}{4\operatorname{cosec} u} du$ $= -\frac{1}{4}\int \sin u \ du$
	Obtains $k\int \sin u \ du$ with no errors seen in any trig identities Must have u and du	1.1b	A1F	
	Obtains $k = -\frac{1}{4}$ OE CSO	2.1	R1	
	Subtotal		**6**	

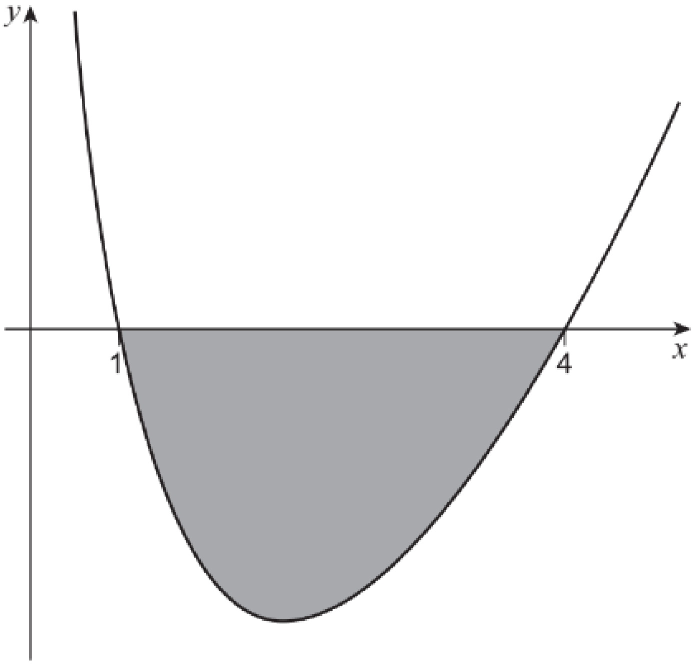
Q	Marking instructions	AO	Marks	Typical solution
15(b)(ii)	Integrates $\int \sin u \, \mathrm{d}u$ to obtain $-\cos u$	1.1b	B1	$-\frac{1}{4}\int \sin u \, \mathrm{d}u = \frac{1}{4}\cos u + c$ $= \frac{1}{4}\frac{\sqrt{\left(\frac{x}{2}\right)^2 - 1}}{\left(\frac{x}{2}\right)} + c$ $= \frac{\sqrt{\frac{x^2-4}{4}}}{2x} + c$ $= \frac{\sqrt{x^2-4}}{4x} + c$
	Deduces $\cos u = \frac{\sqrt{\left(\frac{x}{2}\right)^2 - 1}}{\left(\frac{x}{2}\right)}$ OE	2.2a	M1	
	Completes **reasoned** argument to show given result Must have $+c$ throughout Validation by starting with $\frac{\sqrt{x^2-4}}{4x}$ and replacing x with $2\operatorname{cosec} u$ to achieve $\frac{1}{4}\cos u$ scores a maximum of B1M1R0	2.1	R1	
	Subtotal		**3**	
	Question 15 Total		**16**	

June 2022 Question 14 Paper 1

14 The region bounded by the curve

$$y = (2x - 8)\ln x$$

and the x-axis is shaded in the diagram below.



14 (a) Use the trapezium rule with **5 ordinates** to find an estimate for the area of the shaded region.

Give your answer correct to three significant figures.

[3 marks]

14 (b) Show that the exact area is given by

$$32\ln 2 - \frac{33}{2}$$

Fully justify your answer.

[6 marks]

ANSWER

<table>
<tr><th>Q</th><th>Marking instructions</th><th>AO</th><th>Marks</th><th>Typical solution</th></tr>
<tr><td rowspan="3">14(a)</td><td>Finds positive or negative y-values for 5 x-values with h = 0.75
PI by
AWRT 5.28 or AWRT – 5.28
or
Uses 6 x-values and obtains AWRT 5.42 or AWRT – 5.42
In this case maximum mark is M1A0 A0</td><td>1.1a</td><td>M1</td><td rowspan="3"><table><tr><th>x_n</th><th>y_n</th></tr><tr><td>1</td><td>0</td></tr><tr><td>1.75</td><td>-2.51827</td></tr><tr><td>2.5</td><td>-2.74887</td></tr><tr><td>3.25</td><td>-1.76798</td></tr><tr><td>4</td><td>0</td></tr></table>
$\frac{0.75}{2}(0+0+2(-2.51827-2.74887-1.76798))$

Area $\approx$ 5.28</td></tr>
<tr><td>Uses the trapezium rule correctly with h = 0.75 and correct y-values
Accept rounded or truncated values to 3 significant figures.</td><td>1.1b</td><td>A1</td></tr>
<tr><td>Obtains AWRT 5.28
Condone AWRT – 5.28</td><td>3.2a</td><td>A1</td></tr>
<tr><td></td><td align="right">Subtotal</td><td></td><td>3</td><td></td></tr>
</table>

June 2022 Question 5 Paper 2 (A-Level)

5 The binomial expansion of $(2+5x)^4$ is given by

$$(2+5x)^4 = A + 160x + Bx^2 + 1000x^3 + 625x^4$$

5 (a) Find the value of A and the value of B.

[2 marks]

5 (b) Show that

$$(2+5x)^4 - (2-5x)^4 = Cx + Dx^3$$

where C and D are constants to be found.

[2 marks]

5 (c) Hence, or otherwise, find

$$\int \left((2+5x)^4 - (2-5x)^4 \right) \mathrm{d}x$$

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Obtains 16 Not incorrectly labelled	1.1b	B1	$(2+5x)^4 = 16+160x+600x^2$ $+1000x^3+625x^4$ A = 16 B = 600
	Obtains 600 Not incorrectly labelled	1.1b	B1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)	Obtains the expansion of $(2-5x)^4=$ $A-160x+Bx^2-1000x^3+625x^4$ Accept A and B unsubstituted or their A and B Or Uses a valid method and obtains one of $C=320$ or $D=2000$	1.1a	M1	$(2+5x)^4-(2-5x)^4$ $=16+160x+600x^2+1000x^3+625x^4$ $-(16-160x+600x^2-1000x^3+625x^4)$ $=320x+2000x^3$
	Completes reasoned argument to show $(2+5x)^4-(2-5x)^4=320x+2000x^3$ Accept A and B unsubstituted or their A and B Must finish with $320x+2000x^3$ don't accept just C=320 and D = 2000	2.1	R1F	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(c)	Integrates one term correctly Accept C and D unsubstituted or their C and D Or Uses reverse of chain rule to obtain at least one term of the form $P(2\pm5x)^5$, $P=\pm\frac{1}{5}$ or $\pm\frac{1}{25}$	1.1a	M1	$\int\left((2+5x)^4-(2-5x)^4\right)\mathrm{d}x$ $=\int\left(320x+2000x^3\right)\mathrm{d}x$ $=160x^2+500x^4+c$
	Obtains $\frac{320}{2}x^2+\frac{2000}{4}x^4+c$ FT C and D unsubstituted or their C and D Or $\frac{(2+5x)^5}{5\times5}+\frac{(2-5x)^5}{5\times5}+c$ Condone missing $+c$	1.1b	A1F	
	Subtotal		**2**	

June 2022 Question 10 Paper 2

10 A gardener has a greenhouse containing 900 tomato plants.

The gardener notices that some of the tomato plants are damaged by insects.

Initially there are 25 damaged tomato plants.

The number of tomato plants damaged by insects is increasing by 32% each day.

10 (a) The total number of plants damaged by insects, x, is modelled by

$$x = A \times B^t$$

where A and B are constants and t is the number of days after the gardener first noticed the damaged plants.

10 (a) (i) Use this model to find the total number of plants damaged by insects 5 days after the gardener noticed the damaged plants.

[3 marks]

10 (a) (ii) Explain why this model is not realistic in the long term.

[2 marks]

10 (b) A refined model assumes the rate of increase of the number of plants damaged by insects is given by

$$\frac{dx}{dt} = \frac{x(900 - x)}{2700}$$

10 (b) (i) Show that

$$\int \left(\frac{A}{x} + \frac{B}{900 - x} \right) dx = \int dt$$

where A and B are positive integers to be found.

[3 marks]

10 (b) (ii) Hence, find t in terms of x.

[5 marks]

10 (b) (iii) Hence, find the number of days it takes from when the damage is first noticed until half of the plants are damaged by the insects.

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
10(a)(i)	Forms correct model Or applies repeated percentage increase 4 times PI by AWRT 75.9	3.3	B1	$x = 25 \times 1.32^t$ $t = 5 \Rightarrow x = 25 \times 1.32^5 = 100.18...$
	Substitutes t = 5 into their model Or Applies repeated percentage increase 5 times	3.4	M1	
	Obtains 101 Condone 100 CAO	3.2a	A1	x = 101
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
10(a)(ii)	Explains that the **model** grows **exponentially** Must refer to model and exponential	3.5b	E1	The value predicted by the exponential model will grow without limit.
	Refers to 900 plants. eg this can't be true as there are only 900 tomato plants Condone reference to "tomato(es)" or "plants" in place of "tomato plants"	3.5a	E1	This can't be true as there are only 900 tomato plants in the greenhouse.
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)(i)	Rearranges to obtain one of the following: $\frac{P}{x(900-x)}\frac{dx}{dt}=\frac{1}{Q}$ $\frac{P}{x(900-x)}dx=\frac{1}{Q}dt$ $\frac{P}{x(900-x)}=\frac{1}{Q}\frac{dt}{dx}$ where $P \times Q = 2700$ If their P=2700 no need to see explicit $\frac{1}{Q}$ with dt, or $\frac{dt}{dx}$ May include integral signs	3.1a	B1	$\frac{dx}{dt}=\frac{x(900-x)}{2700}$ $\frac{2700}{x(900-x)}\frac{dx}{dt}=1$ $\int\left(\frac{A}{x}+\frac{B}{900-x}\right)dx=\int dt$ $\frac{2700}{x(900-x)}\equiv\frac{A}{x}+\frac{B}{900-x}$ $2700\equiv A(900-x)+Bx$ $x=0\Rightarrow A=\frac{2700}{900}=3$ $x=900\Rightarrow B=3$ $\therefore$ $\int\left(\frac{3}{x}+\frac{3}{900-x}\right)dx=\int dt$
	Forms partial fraction equation with correct denominators and uses an appropriate method to find their numerators PI by correct A and B without incorrect working	3.1a	M1	
	Obtains correct A and B and concludes with $\int\left(\frac{3}{x}+\frac{3}{900-x}\right)dx=\int dt$ Accept $\int 1dt$ Condone missing brackets	2.1	R1	
	Subtotal		**3**	

Arancha Ruiz

Q	Marking instructions	AO	Marks	Typical solution
10(b)(ii)	Integrates to obtain $\ln x$ or $\pm\ln(900-x)$ Condone missing brackets for this mark	3.1a	M1	
	Integrates to obtain $\ln x$ and $\pm\ln(900-x)$ Condone missing brackets for this mark	1.1a	M1	
	Integrates to obtain $3(\ln x-\ln(900-x))+c=t$ OE Condone missing $+c$	1.1b	A1	$3(\ln x-\ln(900-x))+c=t$ $3(\ln 25-\ln(900-25))+c=0$ $c=10.67$
	Uses t = 0, x= 25, to obtain a value for c	3.4	M1	$t=3(\ln x-\ln(900-x))+10.67$
	Obtains correct equation for t in terms of x ACF If c given as a decimal accept AWRT 11 eg $t=3(\ln x-\ln(900-x))+3\ln 35$ $t=3\ln\left(\frac{35x}{900-x}\right)$	1.1b	A1	
	Subtotal		**5**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)(iii)	Substitutes x = 450 into their model from part (b)(ii)	3.4	M1	$3(ln\,450-ln(450))+10.67$ $=10.67\ldots$
	Obtains 11 CAO	3.2a	A1	It takes 11 days from when the damage is first noticed until half of the plants are damaged by insects
	Subtotal		**2**	

November 2021 Question 11 Paper 1

11 A curve, C, passes through the point with coordinates (1, 6)

The gradient of C is given by

$$\frac{dy}{dx} = \frac{1}{6}(xy)^2$$

Show that C intersects the coordinate axes at exactly one point and state the coordinates of this point.

Fully justify your answer.

[8 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
11	Separates variables. To obtain an equation of the form $\int \frac{A}{y^2}\,dy = \int Bx^2\,dx$ Or $\int \frac{A}{y^2}\frac{dy}{dx}\,dx = \int Bx^2\,dx$ Must have integral signs, and consistent dy and dx Condone x instead of x^2	3.1a	M1	$\frac{dy}{dx} = \frac{1}{6}(xy)^2$ $\frac{1}{y^2}\frac{dy}{dx} = \frac{1}{6}x^2$ $\int \frac{1}{y^2}\,dy = \int \frac{1}{6}x^2\,dx$ $-y^{-1} = \frac{x^3}{18} + c$ $(1,6) \Rightarrow -\frac{1}{6} = \frac{1}{18} + c$ $\Rightarrow c = -\frac{2}{9}$
	Integrates one of their integrals of the form above correctly.	1.1a	M1	
	Obtains correct integrated equation. Condone missing +c	1.1b	A1	y cannot equal zero in $-y^{-1} = \frac{x^3}{18} - \frac{2}{9}$ as y^{-1} is undefined at $y = 0$
	Substitutes $(1,6)$ to determine their constant of integration.	1.1a	M1	
	Explains why y cannot equal zero follow through their equation in which y cannot be zero.	2.4	E1F	therefore C does not intersect the x-axis
	Substitutes $x = 0$ into their integrated equation to obtain the y-intercept.	3.1a	M1	$x = 0 \Rightarrow -y^{-1} = -\frac{2}{9} \Rightarrow y = \frac{9}{2}$
	Obtains y = 4.5 Correctly deduces and states that that C intersects the axes at (exactly/only) one point. Also, states the coordinate $(0, 4.5)$ Also, must have stated that C does not intersect the x-axis CSO	1.1b 2.2a	A1 R1	Hence the curve crosses the y-axis at $(0, 4.5)$
	Total		**8**	

November 2021 Question 15 Paper 1

15 (a) Show that

$$\sin x - \sin x \cos 2x \approx 2x^3$$

for small values of x.

[3 marks]

15 (b) Hence, show that the area between the graph with equation

$$y = \sqrt{8(\sin x - \sin x \cos 2x)}$$

the positive x-axis and the line $x = 0.25$ can be approximated by

$$\text{Area} \approx 2^m \times 5^n$$

where m and n are integers to be found.

[4 marks]

15 (c) (i) Explain why

$$\int_{6.3}^{6.4} 2x^3 \, dx$$

is **not** a suitable approximation for

$$\int_{6.3}^{6.4} (\sin x - \sin x \cos 2x) \, dx$$

[1 mark]

15 (c) (ii) Explain how

$$\int_{6.3}^{6.4} (\sin x - \sin x \cos 2x) \, dx$$

may be approximated by

$$\int_{a}^{b} 2x^3 \, dx$$

for suitable values of a and b.

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
15(a)	Uses small angle approximation for sine at least once.	1.1b	B1	$\sin x - \sin x \cos 2x \approx x - x\left(1 - \frac{(2x)^2}{2}\right)$ $\approx x - x + x\frac{4x^2}{2}$ $\approx 2x^3$
	Replaces $\cos 2x$ with $1 - \frac{(2x)^2}{2}$ Or Used double angle identity and small angle approximations Condone a sign error or missing brackets.	3.1a	M1	
	Completes rigorous argument to show the given result. Condone "=" instead of "$\approx$"	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
15(b)	Forms an integral of the form $\int_0^{0.25} y\,\mathrm{d}x$ or better where y is their $\sqrt{8 \times 2x^3}$. Condone missing limits and dx.	3.1a	M1	$Area \approx \int_0^{0.25} \sqrt{8 \times 2x^3}\,\mathrm{d}x$ $= 4\int_0^{0.25} x^{3/2}\,\mathrm{d}x$ $= 4\left[\frac{2x^{5/2}}{5}\right]_0^{0.25}$ $= \frac{8}{5} \times 0.25^{5/2}$ $= \frac{8}{5} \times \left(\frac{1}{2}\right)^5$ $= 2^{-2} \times 5^{-1}$
	Simplifies integrand to $Bx^{3/2}$	1.1a	M1	
	Integrates their integrand of the form $Bx^{3/2}$ correctly	1.1b	A1F	
	Substitutes correct limits and completes argument to obtain correct approximation in correct form.	2.1	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
15(c)(i)	Explains that the limits or 6.4 and 6.3 are not small.	2.4	E1	The approximation is only valid for small values of x and 6.3 and 6.4 are not small.
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
15(c)(ii)	Explains how the limits can be changed. Examples of reasoning could include: $\sin x - \sin x \cos 2x$ is periodic OE (has a period of 2π PI) evaluating the integral over a different interval will result in the same value. Reduce/shift the limits by 2π The graph repeats Uses a substitution to bring the limits within an acceptable interval.	2.4	E1	$\sin x - \sin x \cos 2x$ repeats so evaluate the integral over a different interval. Use small values $a = 6.3 - 2\pi$ and $b = 6.4 - 2\pi$ to obtain a valid approximation.
	Deduces $a = 6.3 - 2\pi$ = AWRT 0.017 and $b = 6.4 - 2\pi$ = AWRT 0.117	2.2a	R1	
	Subtotal		**2**	

November 2021 Question 8 Paper 3

8 Given that

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} x\cos x \, dx = a\pi + b$$

find the exact value of a and the exact value of b.

Fully justify your answer.

[6 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
8	Uses integration by parts with $u = x$ and $v' = \cos x$ **PI** by $x\sin x + \cos x$	3.1a	B1	$u = x \qquad u' = 1$ $v' = \cos x \qquad v = \sin x$ $\int x\cos x\,dx = x\sin x - \int \sin x\,dx$ $= x\sin x + \cos x$ $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} x\cos x\,dx = \left[x\sin x + \cos x\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ $= \frac{\pi}{3}\sin\frac{\pi}{3} + \cos\frac{\pi}{3} - \left(\frac{\pi}{4}\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right)$ $= \pi\frac{\sqrt{3}}{6} + \frac{1}{2} - \left(\pi\frac{\sqrt{2}}{8} + \frac{\sqrt{2}}{2}\right)$ $= \left(\frac{4\sqrt{3} - 3\sqrt{2}}{24}\right)\pi + \left(\frac{1-\sqrt{2}}{2}\right)$
	Applies integration by parts formula correctly by substituting their u, u', v and v' **PI** by $x\sin x + \cos x$	1.1a	M1	
	Obtains $x\sin x + \cos x$ **CAO**	1.1b	A1	
	Substitutes limits correctly into their integrated expression **PI** by correct a and b	1.1a	M1	
	Uses correct exact value for any one of $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ or $\cos\frac{\pi}{3} = \frac{1}{2}$ or $\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ or $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ **PI** by correct a or b	1.2	B1	
	Obtains correct exact values of a and b **ACF** Ignore if 0.14(…) seen subsequently	1.1b	A1	
	Total		**6**	

November 2020 Question 6 Paper 1

6 Four students, Tom, Josh, Floella and Georgia are attempting to complete the indefinite integral

$$\int \frac{1}{x}\,dx \qquad \text{for } x > 0$$

Each of the students' solutions is shown below:

Tom $\int \frac{1}{x}\,dx = \ln x$

Josh $\int \frac{1}{x}\,dx = k\ln x$

Floella $\int \frac{1}{x}\,dx = \ln Ax$

Georgia $\int \frac{1}{x}\,dx = \ln x + c$

6 (a) (i) Explain what is wrong with Tom's answer.

[1 mark]

6 (a) (ii) Explain what is wrong with Josh's answer.

[1 mark]

6 (b) Explain why Floella and Georgia's answers are equivalent.

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
6(a)(i)	Explains that Tom's solution does not include an arbitrary constant Accept Tom forgot the +c There is no constant on the RHS	2.4	E1	Tom's solution has no constant of integration
	Subtotal		**1**	
6(a)(ii)	Explains that the constant is in the wrong place or Explains that the k should not be there or that k = 1 or Shows that differentiating does not give $\frac{1}{x}$ or The constant has been multiplied instead of being added or It should be $\ln kx$ not $k \ln x$	2.4	E1	Although there is a constant, it is in the wrong place
	Subtotal		**1**	
6(b)	Rewrites $\ln Ax$ as $\ln A + \ln x$ Condone use of any letter for A to demonstrate the log rule used Condone use of log without a specified base	1.1a	M1	ln Ax = $\ln A + \ln x$ This is equivalent as c = ln A
	Deduces explicitly that c = ln A clearly demonstrating equivalence OE	2.2a	R1	
	Subtotal		**2**	
	Question Total		**4**	

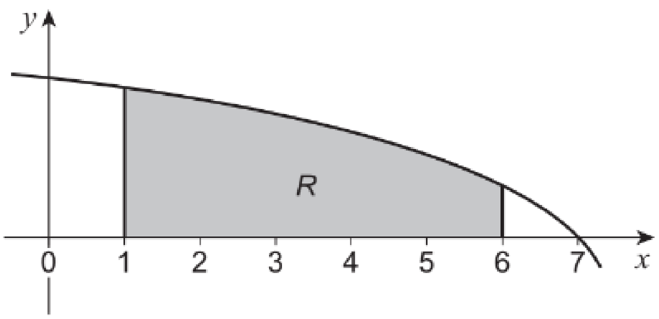
November 2020 Question 11 Paper 1

11 The region R enclosed by the lines $x = 1$, $x = 6$, $y = 0$ and the curve

$$y = \ln(8 - x)$$

is shown shaded in **Figure 3** below.

Figure 3



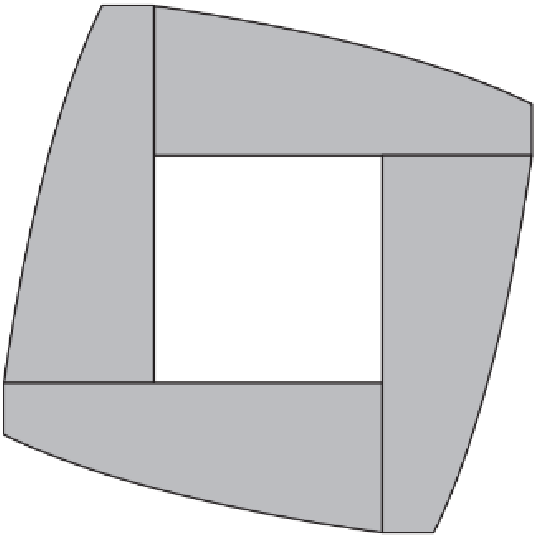
All distances are measured in centimetres.

11 (a) Use a single trapezium to find an approximate value of the area of the shaded region, giving your answer in cm^2 to two decimal places.

[2 marks]

11 (b) Shape B is made from four copies of region R as shown in **Figure 4** below.

Figure 4



Shape B is cut from metal of thickness 2 mm

The metal has a density of 10.5 g/cm^3

Use the trapezium rule with **six** ordinates to calculate an approximate value of the mass of Shape B.

Give your answer to the nearest gram.

[5 marks]

11 (c) Without further calculation, give one reason why the mass found in part **(b)** may be:

11 (c) (i) an underestimate.

[1 mark]

11 (c) (ii) an overestimate.

[1 mark]

ANSWER

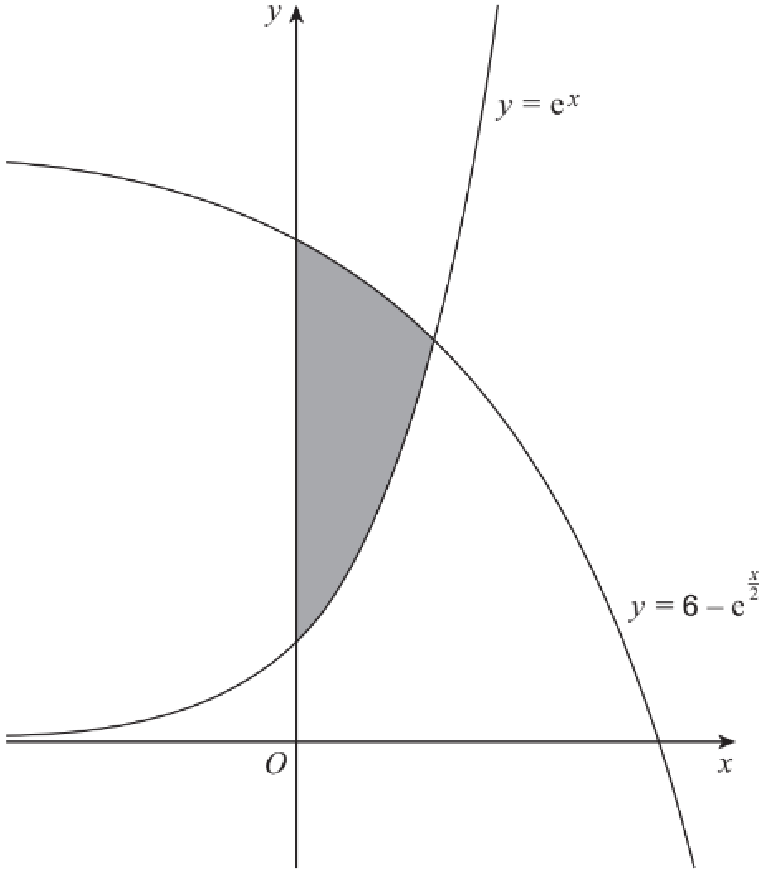
<table>
<tr><th>Q</th><th>Marking Instructions</th><th>AO</th><th>Marks</th><th>Typical Solution</th></tr>
<tr><td>11(a)</td><td>Evaluates f(1) and f(6) using exact logs or decimals
Award if seen embedded in calculations using more than one trapezium</td><td>1.1a</td><td>M1</td><td>f(1) = 1.945910149..
f(6)= 0.69314718..
A = $\frac{5}{2}$(1.9459+0.6931)
= 6.5976…
=6.60 cm^2</td></tr>
<tr><td></td><td>Evaluates an approximate value of the area of R
AWRT 6.60
Condone omission of units</td><td>1.1b</td><td>A1</td><td></td></tr>
<tr><td></td><td>Subtotal</td><td></td><td>2</td><td></td></tr>
<tr><td>11(b)</td><td>Writes or uses the six ordinates as ln 7, ln 6, ln 5, ln 4, ln 3 ln 2
or
Obtains the values of the correct six ordinates in decimal form</td><td>1.1b</td><td>B1</td><td><table><tr><th>x</th><th>f(x)</th></tr><tr><td>1</td><td>1.9459</td></tr><tr><td>2</td><td>1.7918</td></tr><tr><td>3</td><td>1.6094</td></tr><tr><td>4</td><td>1.3863</td></tr><tr><td>5</td><td>1.0986</td></tr><tr><td>6</td><td>0.6931</td></tr></table>Area = $\frac{1}{2}$x1x(1.9459+0.6931+2(1.7918+1.6094+1.3863+1.0986))</td></tr>
<tr><td></td><td>Uses the correct formula for the trapezium rule with their six ordinates and h = 1
Award this mark if seven ordinates used with $h=\frac{5}{6}$
Answer for seven = 7.2145648..</td><td>1.1a</td><td>M1</td><td></td></tr>
<tr><td></td><td>Evaluates an approximate value for the area of R.
Must have used six ordinates
AWRT 7.2
PI by correct final answer</td><td>1.1b</td><td>A1</td><td>Area= 7.205633 cm^2

Volume of Shape B
= 4 x 7.205633 x 0.2
=5.7645… cm^3

Mass of Shape B =
5.7645cm^3 x 10.5 g/cm^3
=60.52731 g
=61 g</td></tr>
<tr><td></td><td>Forms an expression for the mass of either one section or all four sections using ‘their’ area and consistent units
PI by correct final answer</td><td>3.1b</td><td>M1</td><td></td></tr>
<tr><td></td><td>Obtains an approximate value for the correct mass of Shape B
Must state units
If seven ordinates used this mark can be awarded as answer would be 61g
CAO</td><td>3.2a</td><td>A1</td><td></td></tr>
<tr><td></td><td>Subtotal</td><td></td><td>5</td><td></td></tr>
<tr><td>11(c)(i)</td><td>Explains that the trapezia are all below the curve
or
Explains that the curve is concave
or
Draws a diagram and indicates the gaps</td><td>3.5a</td><td>E1</td><td>The trapezia are all below the curve</td></tr>
<tr><td></td><td>Subtotal</td><td></td><td>1</td><td></td></tr>
<tr><td>11(c)(ii)</td><td>Explains that numbers have been rounded</td><td>3.5a</td><td>E1</td><td>Numbers in the calculation have been rounded</td></tr>
<tr><td></td><td>Subtotal</td><td></td><td>1</td><td></td></tr>
<tr><td></td><td>Question Total</td><td></td><td>9</td><td></td></tr>
</table>

November 2020 Question 14 Paper 1

15 The region enclosed between the curves $y = e^x$, $y = 6 - e^{\frac{x}{2}}$ and the line $x = 0$ is shown shaded in the diagram below.



Show that the exact area of the shaded region is

$$6 \ln 4 - 5$$

Fully justify your answer.

[10 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
15	Forms a single equation eliminating x or y	3.1a	M1	$6 - e^{\frac{x}{2}} = e^x$
	Obtains a correct rearranged quadratic equation. Either $e^x + e^{\frac{x}{2}} - 6 = 0$ or $\left(e^{\frac{x}{2}} + 3\right)\left(e^{\frac{x}{2}} - 2\right) = 0$ or $e^x + e^{\frac{x}{2}} + \frac{1}{4} = \frac{25}{4}$ OE	1.1b	A1	$e^x + e^{\frac{x}{2}} - 6 = 0$ $\left(e^{\frac{x}{2}} + 3\right)\left(e^{\frac{x}{2}} - 2\right) = 0$ $e^{\frac{x}{2}} = -3 \text{ or } 2$
	Solves 'their' quadratic Must be a quadratic in $e^{\frac{x}{2}}$ or If squaring is used then it must be a quadratic in e^x or Obtains x = 1.386	1.1a	M1	$e^{\frac{x}{2}} > 0$ so -3 is not a valid solution $\frac{x}{2}$ = ln 2 $x = 2\ln 2 = \ln 4$
	Explains that $e^{\frac{x}{2}} = -3$ is not valid as $e^{\frac{x}{2}} > 0$ or If squaring is used they must clearly check both solutions by substituting and conclude that $\ln 9$ is not valid OE	2.4	E1F	$\int_0^{\ln 4} (6 - e^{\frac{x}{2}} - e^x)\, dx$ = $\left[6x - 2e^{\frac{x}{2}} - e^x\right]_0^{\ln 4}$
	Obtains $x = 2\ln 2$ or $x = \ln 4$	1.1b	A1	= $\left(6\ln 4 - 2e^{\frac{\ln 4}{2}} - e^{\ln 4}\right)$ - $(-2-1)$
	Forms any definite integral which would contribute to finding the required area This could be $\int_0^{\ln 4} (6 - e^{\frac{x}{2}} - e^x)\, dx$ or $\int_0^{\ln 4} (6 - e^{\frac{x}{2}})\, dx$ or $\int_0^{\ln 4} e^x dx$ or $\int_0^{\ln 4} (e^x + e^{\frac{x}{2}} - 6)\, dx$ Follow through '*their*' value of x for the upper limit	1.1a	M1	= 6 ln 4 – 4 – 4 + 3 = 6 ln 4 – 5
	Forms a fully correct definite integral (or integrals) which would lead to evaluating the correct area Follow through '*their*' incorrect upper limit	3.1a	A1F	

	Integrates '*their*' expressions involving exponentials fully correctly Follow through their exponential expressions – but must have integrated both e^x and $e^{\frac{x}{2}}$ terms Condone missing/incorrect limits	1.1b	B1F	
	Substitutes 0 and '*their*' upper limits into '*their*' integrated expression Must correctly use F (their upper limit) – F (0) for each integral	1.1a	M1	
	Completes rigorous argument by showing explicit evaluation of exponential terms before obtaining final answer AG This mark can be achieved without achieving the E1 mark	2.1	R1	
	Total		**10**	

Video solution:

https://youtu.be/j7A5g7Qcf1A

November 2020 Question 5 Paper 2

5 Use integration by substitution to show that

$$\int_{-\frac{1}{4}}^{6} x\sqrt{4x+1}\,\mathrm{d}x = \frac{875}{12}$$

Fully justify your answer.

[6 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
5	Uses a suitable substitution $u = 4x+1$ or $u = \sqrt{4x+1}$ OE	3.1a	M1	$u = 4x+1 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 4$
	Differentiates their substitution correctly	1.1b	A1	$x = -\frac{1}{4} \Rightarrow u = 0$
	Completes substitution to obtain correct integrand for their suitable substitution. Can be unsimplified.	1.1a	M1	$x = 6 \Rightarrow u = 25$ $x = \frac{u-1}{4}$
	Correctly integrates their simplified integrand provided it is of the form $A\left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)$ or $B\left(u^4 - u^2\right)$	1.1a	A1	$\int_{-\frac{1}{4}}^{6} x\sqrt{4x+1}\,\mathrm{d}x = \int_0^{25} \frac{u-1}{4}\sqrt{u}\,\frac{1}{4}\mathrm{d}u$ $= \frac{1}{16}\int_1^{25} u^{\frac{3}{2}} - u^{\frac{1}{2}}\mathrm{d}u$
	Substitutes correct limits for their substitution or 6 and -1/4 for x	1.1a	M1	$= \frac{1}{16}\left[\frac{2u^{\frac{5}{2}}}{5} - \frac{2u^{\frac{3}{2}}}{3}\right]_0^{25}$ $= \frac{1}{8}\left(\frac{25^{\frac{5}{2}}}{5} - \frac{25^{\frac{3}{2}}}{3}\right)$ $= \frac{875}{12}$
	Completes rigorous argument to show the required result. AG	2.1	A1	
	Total		**6**	

June 2019 Question 16 Paper 1

16 (a) $y = e^{-x}(\sin x + \cos x)$

Find $\dfrac{dy}{dx}$

Simplify your answer.

[3 marks]

16 (b) Hence, show that

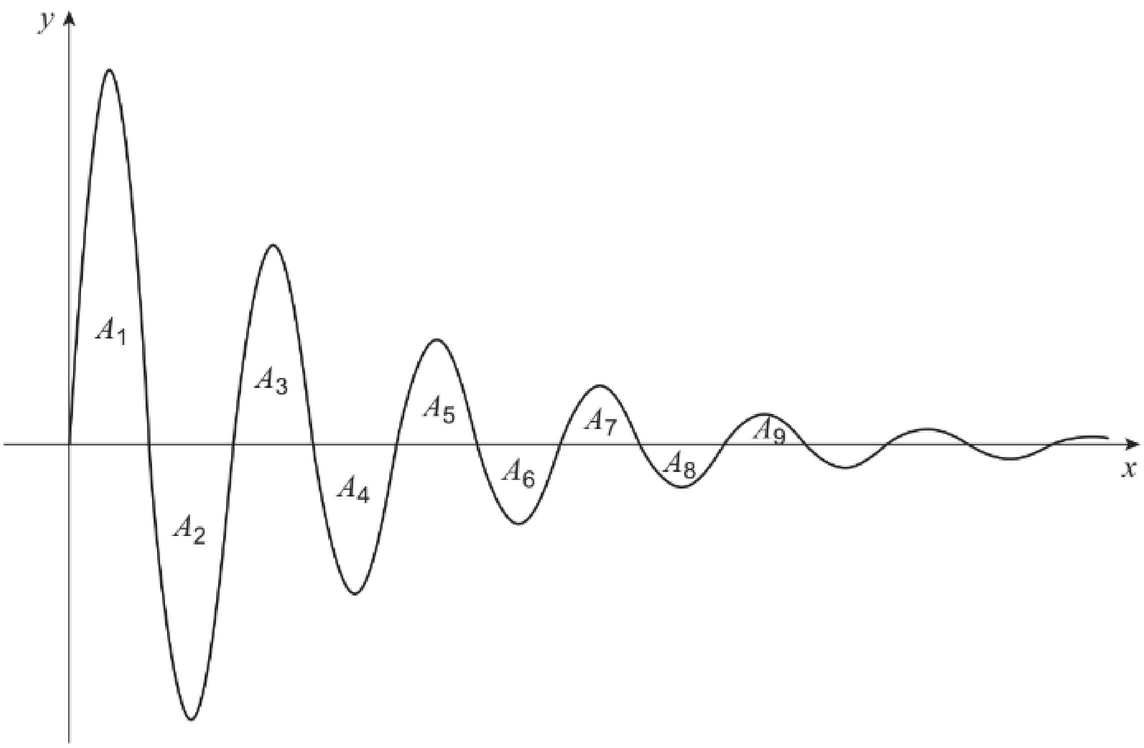
$$\int e^{-x} \sin x \, dx = ae^{-x}(\sin x + \cos x) + c$$

where a is a rational number.

[2 marks]

16 (c) A sketch of the graph of $y = e^{-x} \sin x$ for $x \geq 0$ is shown below.

The areas of the finite regions bounded by the curve and the x-axis are denoted by $A_1, A_2, \ldots, A_n, \ldots$



16 (c) (i) Find the exact value of the area A_1

[3 marks]

16 (c) (ii) Show that

$$\frac{A_2}{A_1} = \mathrm{e}^{-\pi}$$

[4 marks]

16 (c) (iii) Given that

$$\frac{A_{n+1}}{A_n} = \mathrm{e}^{-\pi}$$

show that the exact value of the total area enclosed between the curve and the x-axis is

$$\frac{1+\mathrm{e}^{\pi}}{2(\mathrm{e}^{\pi}-1)}$$

[4 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
16(a)	Chooses an appropriate technique to differentiate accept any evidence of product rule or quotient rule	3.1a	M1	$\frac{dy}{dx}=-e^{-x}(\sin x+\cos x)+e^{-x}(\cos x-\sin x)$ $=-2e^{-x}\sin x$
	Differentiates fully correctly	1.1b	A1	
	Obtains fully correct simplified answer.	1.1b	A1	
16(b)	Uses their result from (a) in the form of $Be^{-x}\sin x$ showing an understanding of the fundamental theorem of calculus Condone missing constant.	3.1a	M1	$\int(-2e^{-x}\sin x)dx=e^{-x}(\sin x+\cos x)+k$ $\therefore\int(e^{-x}\sin x)dx=-\frac{1}{2}e^{-x}(\sin x+\cos x)+c$
	Obtains $\frac{1}{B}e^{-x}(\sin x+\cos x)$	2.1	R1F	
16 (c)(i)	Writes the area as $\frac{1}{B}\left[e^{-x}(\sin x+\cos x)\right]_0^{\pi}$ Condone missing limits	3.1a	M1	$\int_0^{\pi}(e^{-x}\sin x)dx=-\frac{1}{2}\left[e^{-x}(\sin x+\cos x)\right]_0^{\pi}$ $=-\frac{1}{2}[e^{-\pi}(\sin\pi+\cos\pi)$ $-e^{0}(\sin 0+\cos 0)]$ $=\frac{e^{-\pi}+1}{2}$
	Deduces correct limits and substitutes correctly	2.2a	A1	
	Obtains correct exact value from correct answer in part(b) CSO	1.1b	A1	
16 (c)(ii)	Substitutes correct limits for A_2 Into their $\frac{1}{B}\left[e^{-x}(\sin x+\cos x)\right]_0^{\pi}$ Or Writes $A_2=\pm\int_{\pi}^{2\pi}(e^{-x}\sin x)dx$ and uses the substitution $u=x-\pi$	1.1a	M1	$\int_{\pi}^{2\pi}(e^{-x}\sin x)dx=-\frac{1}{2}\left[e^{-x}(\sin x+\cos x)\right]_{\pi}^{2\pi}$ $=-\frac{e^{-\pi}+1}{2}e^{-\pi}$ Area $=\frac{e^{-\pi}+1}{2}e^{-\pi}$ $\frac{A_2}{A_1}=\frac{\frac{e^{-\pi}+1}{2}e^{-\pi}}{\frac{e^{-\pi}+1}{2}}=e^{-\pi}$
	Obtains correct exact area for $\pm A_2$ CSO Or Makes complete substitution $A_2=\pm\int_0^{\pi}\left(e^{-(u+\pi)}\sin(u+\pi)\right)du$	1.1b	A1	

	Forms required ratio using their exact A1 and A2, may be unsimplified Or Extracts factor of $\mathrm{e}^{-\pi}$ and uses $\sin(u+\pi) = -\sin u$ To obtain $A_2 = -\mathrm{e}^{-\pi}\int_0^{\pi} \mathrm{e}^{-u}\sin u\,du$	1.1a	M1	
	Completes rigorous argument, with correct limits and negatives handled correctly CSO	2.1	R1	
16 (c)(iii)	Deduces that the areas form a geometric series Accept any indication of this being geometric series	2.2a	B1	$\frac{a}{1-r} = \frac{\mathrm{e}^{-\pi}+1}{2} \times \frac{1}{1-\mathrm{e}^{-\pi}}$ $= \frac{1+\mathrm{e}^{\pi}}{2(\mathrm{e}^{\pi}-1)}$
	Uses $\frac{A_1}{1-e^{-\pi}}$	3.1a	M1	
	Obtains a value for the geometric series first term using their (c)(i)	1.1a	B1F	
	Completes rigorous argument to achieve required result in correct form. CSO AG	2.1	R1	
	Total		**16**	

Video solution:

https://youtu.be/RNgva0ZnNlk

June 2019 Question 7 Paper 3

7 (a) Express $\dfrac{4x+3}{(x-1)^2}$ in the form $\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}$

[3 marks]

7 (b) Show that

$$\int_3^4 \frac{4x+3}{(x-1)^2}\,\mathrm{d}x = p + \ln q$$

where p and q are rational numbers.

[5 marks]

Arancha Ruiz

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
7(a)	Forms $4x+3\equiv A(x-1)+B$	1.1b	B1	$\frac{4x+3}{(x-1)^2}\equiv\frac{A}{x-1}+\frac{B}{(x-1)^2}$ $4x+3\equiv A(x-1)+B$
	Uses substitution or comparison of coefficients to find their A or B (must have degree of LHS = degree of RHS)	1.1a	M1	Let x = 1 hence B = 7 Let x = 0 then 3 = B – A and hence A = 4
	Obtains correct A and B	1.1b	A1	A = 4 and B = 7
7(b)	Integrates their expression, at least one term correct	3.1a	M1	$\int_3^4\left(\frac{4}{x-1}+\frac{7}{(x-1)^2}\right)dx$ $=\left[4\ln(x-1)-\frac{7}{x-1}\right]_3^4$ $=\left[4\ln 3-\frac{7}{3}\right]-\left[4\ln 2-\frac{7}{2}\right]$ $=4\ln\frac{3}{2}+\frac{7}{6}$ $=\ln\frac{3^4}{2^4}+\frac{7}{6}$ $=\ln\frac{81}{16}+\frac{7}{6}$
	Integrates their expression fully correctly Must be of the form $A\ln(x-1)-\frac{B}{x-1}$ **OE** **FT** their A and B	1.1b	A1F	
	Substitutes limits correctly into their integrated expression	1.1a	M1	
	Uses at least one law of logs correctly	1.1a	M1	
	Completes argument to obtain **correct exact answer** in correct form or stating $p=\frac{7}{6}$ and $q=\frac{81}{16}$ No subsequent incorrect working	2.1	R1	
	Total		**8**	

June 2018 Question 7 Paper 3

7 (a) Express $\dfrac{4x+3}{(x-1)^2}$ in the form $\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}$

[3 marks]

7 (b) Show that

$$\int_3^4 \frac{4x+3}{(x-1)^2}\,dx = p + \ln q$$

where p and q are rational numbers.

[5 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
7(a)	Forms $4x+3 \equiv A(x-1)+B$	1.1b	B1	$\frac{4x+3}{(x-1)^2} \equiv \frac{A}{x-1}+\frac{B}{(x-1)^2}$ $4x+3 \equiv A(x-1)+B$
	Uses substitution or comparison of coefficients to find their A or B (must have degree of LHS = degree of RHS)	1.1a	M1	Let x = 1 hence B = 7 Let x = 0 then 3 = B – A and hence A = 4
	Obtains correct A and B	1.1b	A1	A = 4 and B = 7
7(b)	Integrates their expression, at least one term correct	3.1a	M1	$\int_3^4 \left(\frac{4}{x-1}+\frac{7}{(x-1)^2}\right)dx$
	Integrates their expression fully correctly Must be of the form $A\ln(x-1)-\frac{B}{x-1}$ **OE** **FT** their A and B	1.1b	A1F	$= \left[4\ln(x-1)-\frac{7}{x-1}\right]_3^4$ $=\left[4\ln 3-\frac{7}{3}\right] - \left[4\ln 2-\frac{7}{2}\right]$ $=4\ln\frac{3}{2}+\frac{7}{6}$
	Substitutes limits correctly into their integrated expression	1.1a	M1	$=\ln\frac{3^4}{2^4}+\frac{7}{6}$
	Uses at least one law of logs correctly	1.1a	M1	$=\ln\frac{81}{16}+\frac{7}{6}$
	Completes argument to obtain **correct exact answer** in correct form or stating $p = \frac{7}{6}$ and $q = \frac{81}{16}$ No subsequent incorrect working	2.1	R1	
	Total		**8**	

Ch 11 Parametric equations

June 2022 Question 1 Paper 1

1 A curve is defined by the parametric equations

$$x = \cos\theta \quad \text{and} \quad y = \sin\theta \qquad \text{where } 0 \le \theta \le 2\pi$$

Which of the options shown below is a Cartesian equation for this curve?

Circle your answer.

[1 mark]

$\frac{y}{x} = \tan\theta$ $\qquad x^2 + y^2 = 1$ $\qquad x^2 - y^2 = 1$ $\qquad x^2y^2 = 1$

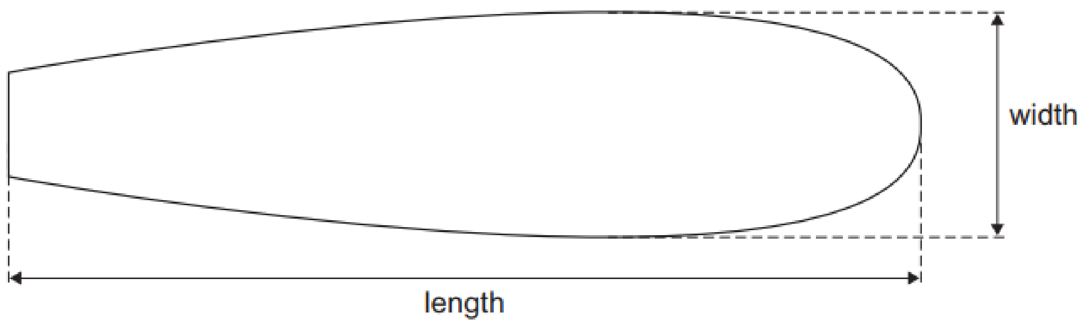
ANSWER

Q	Marking instructions	AO	Marks	Typical solution
1	Circles the correct answer	1.2	B1	$x^2 + y^2 = 1$
	Question 1 Total		**1**	

June 2022 Question 6 Paper 3

6 A design for a surfboard is shown in **Figure 1**.

Figure 1



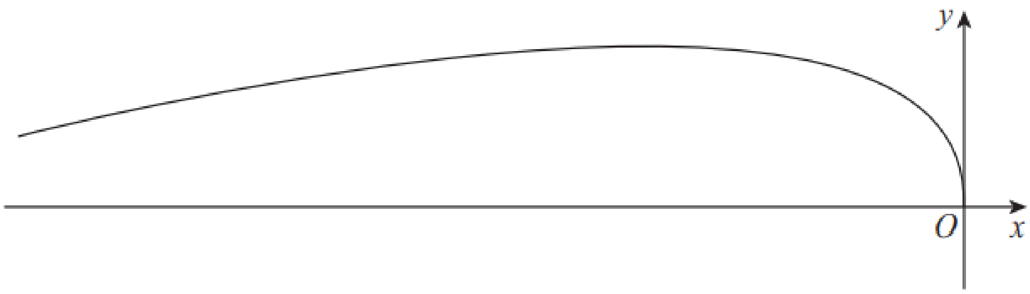
The curve of the **top half** of the surfboard can be modelled by the parametric equations

$$x = -2t^2$$

$$y = 9t - 0.7t^2$$

for $0 \leq t \leq 9.5$ as shown in **Figure 2**, where x and y are measured in centimetres.

Figure 2



6 (a) Find the length of the surfboard.

[2 marks]

6 (b) (i) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t.

[3 marks]

6 (b) (ii) Hence, show that the width of the surfboard is approximately one third of its length.

[4 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Substitutes t = 9.5 into $x = -2t^2$ or $2t^2$	3.4	M1	$t = 9.5 \Rightarrow x = -2 \times 9.5^2 = -180.5$ Length = 180.5 cm
	Obtains 180.5 Condone incorrect or missing units ISW	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)(i)	Obtains 9 – 1.4t or –4t OE Ignore labels	1.1b	B1	$\frac{dy}{dt} = 9 - 1.4t$ $\frac{dx}{dt} = -4t$
	Uses chain rule to obtain $\frac{dy}{dx}$ Condone sign error	3.1a	M1	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$
	Obtains a correct expression Do not ISW	1.1b	A1	$\frac{dy}{dx} = \frac{9 - 1.4t}{-4t}$
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)(ii)	Equates their $\frac{dy}{dx}$ or their $\frac{dy}{dt}$ or their numerator of their $\frac{dy}{dx}$ to 0 PI by correct t from correct $\frac{dy}{dx}$	3.1a	M1	$\frac{9-1.4t}{-4t}=0$ $t=\frac{45}{7}=6.43$ $y=9\times6.43-0.7\times(6.43)^2$ $=28.9$ Width of surfboard = 58 cm $180.5\div3=60.2\approx58$ Hence the width is approximately one third of the length
	Obtains correct value for t ACF eg t = 6.4 or $\frac{9}{1.4}$ Must come from correct $\frac{dy}{dx}$	1.1b	A1	
	Substitutes their value for t into the model for y and obtains a value for y provided $0 < t < 9.5$	3.4	M1	
	Compares correct width and correct length and $\frac{1}{3}$ or 3 with a correct concluding statement OE CSO Allow 180 for length	3.2a	R1	
	Subtotal		**4**	

Video solution:

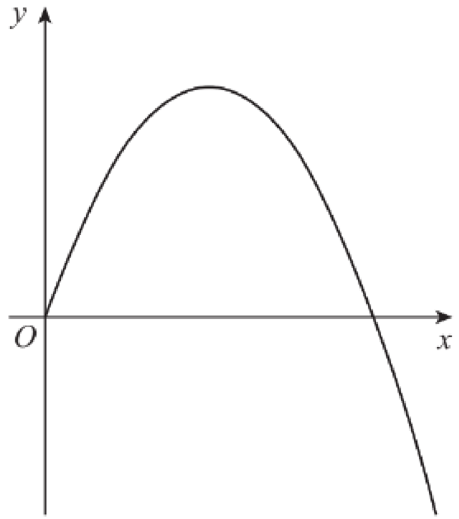
https://youtu.be/YwuaSzct-Gc?feature=shared

November 2021 Question 14 Paper 1

14 The curve C is defined for $t \geq 0$ by the parametric equations

$$x = t^2 + t \quad \text{and} \quad y = 4t^2 - t^3$$

C is shown in the diagram below.



14 (a) Find the gradient of C at the point where it intersects the positive x-axis.

[5 marks]

14 (b) (i) The area A enclosed between C and the x-axis is given by

$$A = \int_0^b y \, \mathrm{d}x$$

Find the value of b.

[1 mark]

14 (b) (ii) Use the substitution $y = 4t^2 - t^3$ to show that

$$A = \int_0^4 (4t^2 + 7t^3 - 2t^4) \, \mathrm{d}t$$

[3 marks]

14 (b) (iii) Find the value of A.

[1 mark]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
14(a)	Uses y = 0 to obtain a non-zero value of t	3.1a	M1	$y=0 \Rightarrow 4t^2-t^3=0$ $t=0\, or\, 4$ $\frac{dy}{dt}=8t-3t^2$ $\frac{dx}{dt}=2t+1$ $t=4 \Rightarrow \frac{dy}{dx}=-\frac{16}{9}$
	Obtains $\frac{dy}{dt}=8t-3t^2$ or $\frac{dy}{dt}=-16$	1.1b	B1	
	Obtains $\frac{dx}{dt}=2t+1$ or $\frac{dx}{dt}=9$	1.1b	B1	
	Uses $\text{their } \frac{dy}{dt} \div \text{their } \frac{dx}{dt}=\frac{dy}{dx}$ and their non-zero value of t to find a numerical expression or value for $\frac{dy}{dx}$	3.1a	M1	
	Obtains $-\frac{16}{9}$ OE	1.1b	A1	
	Subtotal		**5**	

Q	Marking instructions	AO	Mark	Typical solution
14(b)(i)	Deduces b = 20 (FT t^2+t for their value of t)	2.2a	B1F	b = 20
	Subtotal		**1**	

Q	Marking instructions	AO	Mark	Typical solution
14(b)(ii)	Substitutes their $dx=(2t+1)dt$	3.1a	M1	$\frac{dx}{dt}=2t+1 \Rightarrow dx=(2t+1)dt$ $A=\int_0^{20} y\,dx$ $=\int_0^4 (4t^2-t^3)(2t+1)dt$ $=\int_0^4 8t^3+4t^2-2t^4-t^3dt$ $=\int_0^4 4t^2+7t^3-2t^4dt$
	Completes correct substitution for y and dx Condone incorrect or omission of limits.	1.1b	A1F	
	Completes rigorous argument, to show given result. t = 4 when x = 20 must be justified either here or in part (b)(i)	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Mark	Typical solution
14(b)(iii)	Evaluates A = 1856/15 or AWRT 124	1.1b	B1	$A = \frac{1856}{15}$
	Subtotal		**1**	

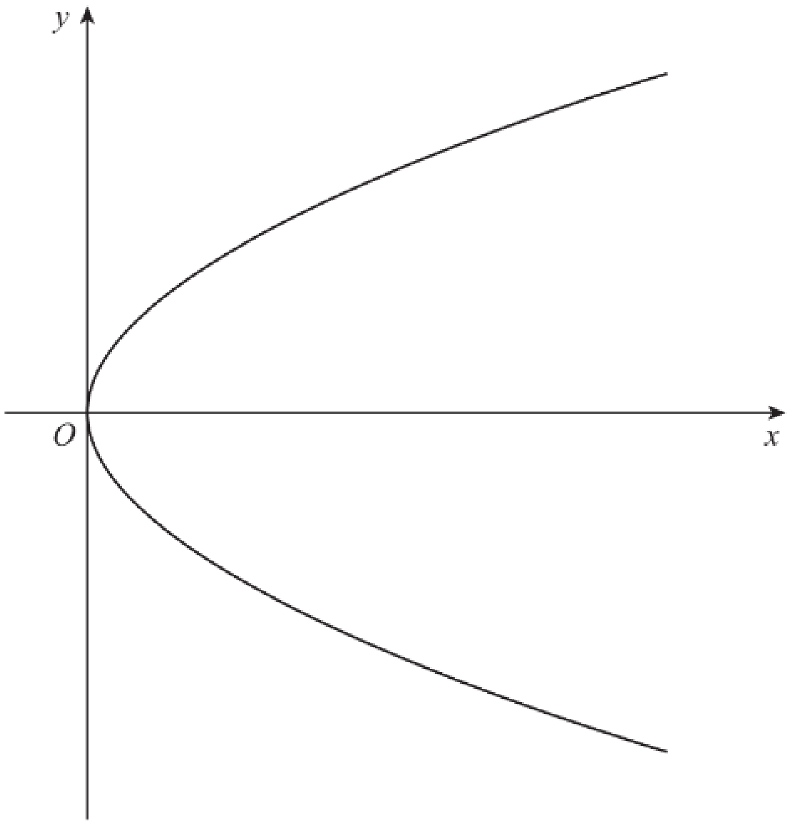
November 2020 Question 8 Paper 2

8 The curve defined by the parametric equations

$$x = t^2 \quad \text{and} \quad y = 2t \qquad -\sqrt{2} \le t \le \sqrt{2}$$

is shown in **Figure 1** below.

Figure 1



8 (a) Find a Cartesian equation of the curve in the form $y^2 = f(x)$

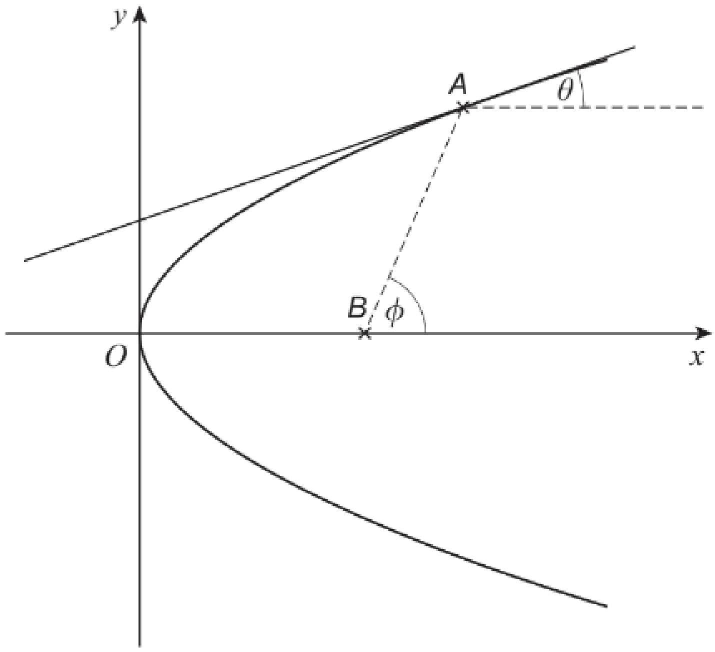
[2 marks]

8 (b) The point A lies on the curve where $t = a$

The tangent to the curve at A is at an angle θ to a line through A parallel to the x-axis.

The point B has coordinates (1, 0)

The line AB is at an angle ϕ to the x-axis.



8 (b) (i) By considering the gradient of the curve, show that

$$\tan\theta = \frac{1}{a}$$

[3 marks]

8 (b) (ii) Find $\tan\phi$ in terms of a.

[2 marks]

8 (b) (iii) Show that $\tan 2\theta = \tan\phi$

[3 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
8(a)	Eliminates t	1.1a	M1	$\frac{y}{2} = t, \quad x = \frac{y^2}{4}$ $y^2 = 4x$
	Writes the Cartesian equation in the required form	1.1b	A1	
	Subtotal		**2**	
8(b)(i)	Differentiates both $\frac{dx}{dt} = 2t$, $\frac{dy}{dt} = 2$ with at least one correct Or Differentiates their $y^2 = 4x$ to obtain $y\frac{dy}{dx} = A$ Or rearranges and differentiates $y = 2\sqrt{x}$ and obtains $\frac{dy}{dx} = Ax^{-\frac{1}{2}}$ OE	3.1a	M1	$\frac{dx}{dt} = 2t, \quad \frac{dy}{dt} = 2$ $\frac{dy}{dx} = \frac{2}{2a} = \frac{1}{a}$ The gradient of a line is equal to the tangent of the angle between the line and the horizontal hence $\tan\theta = \frac{1}{a}$
	Obtains correct $\frac{dy}{dx}$ at $t = a$	1.1b	A1	
	Explains that the gradient of a line is the tangent of the angle between the line and the horizontal or shows on right-angled triangle on diagram and links to $\tan\theta$ **and** concludes $\tan\theta = \frac{1}{a}$	2.4	E1	
	Subtotal		**3**	
8(b)(ii)	Uses formula for gradient of straight line with points A and B Must have $a^2 - 1$ or $1 - a^2$ in denominator	1.1a	M1	$\tan\phi = \frac{2a - 0}{a^2 - 1}$ $= \frac{2a}{a^2 - 1}$
	Obtains correct $\tan\phi$ OE	1.1b	A1	
	Subtotal		**2**	

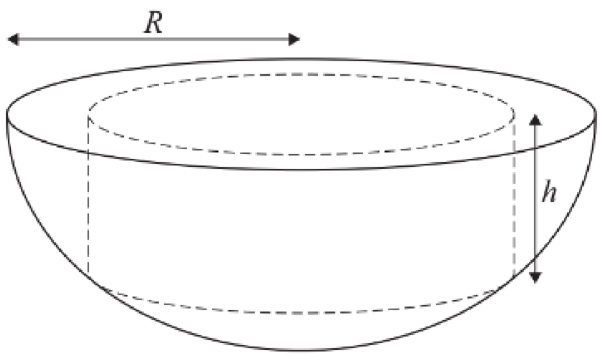
8(b)(iii)	States double angle formula for $\tan 2\theta$	1.2	B1	$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$ $= \dfrac{2\times\frac{1}{a}}{1-\left(\frac{1}{a}\right)^2}$ $= \dfrac{2a}{a^2-1}$ $= \tan\phi$
	Substitutes $\tan\theta = \dfrac{1}{a}$ into their $\tan 2\theta = \dfrac{2\tan\theta}{1\pm\tan^2\theta}$	1.1a	M1	
	Simplifies and completes argument to show required result	2.1	R1	
	Subtotal		**3**	
	Question Total		**10**	

November 2020 Question 9 Paper 2

9 A cylinder is to be cut out of the circular face of a solid hemisphere.

The cylinder and the hemisphere have the same axis of symmetry.

The cylinder has height h and the hemisphere has a radius of R.



9 (a) Show that the volume, V, of the cylinder is given by

$$V = \pi R^2 h - \pi h^3$$

[3 marks]

9 (b) Find the maximum volume of the cylinder in terms of R.

Fully justify your answer.

[7 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Identifies and clearly defines variable for radius of cylinder. Can be shown on diagram or can be implied by use in $V = \pi r^2 h$	2.5	B1	Radius of cylinder = r $h^2 + r^2 = R^2$ $V = \pi r^2 h$
	Uses Pythagoras to connect h, r and R	3.1a	M1	
	Eliminates the radius variable to form an expression for the volume of the cylinder in terms of h, completing argument to show given result. Condone undefined r	2.1	R1	$V = \pi\left(R^2 - h^2\right)h$ $= \pi R^2 h - \pi h^3$
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)	Differentiates the expression for volume w.r.t. h with at least one term correct.	3.1a	M1	$\frac{dV}{dh} = \pi R^2 - 3\pi h^2$
	Obtains correct $\frac{dV}{dh}$	1.1b	A1	For maximum volume $\frac{dV}{dh} = 0$
	Explains that their derivative w.r.t h equals zero for a maximum or stationary point	2.4	E1	$\Rightarrow R^2 - 3h^2 = 0$ $h^2 = \frac{R^2}{3} \Rightarrow h = \frac{R}{\sqrt{3}}$
	Equates volume derivative w.r.t. h to zero and correctly obtains a value for h in terms of R	1.1a	M1	

	Substitutes their h, in terms of R, from derivative w.r.t. h into volume formula.	1.1a	M1	Hence volume $V = \pi R^2 \frac{R}{\sqrt{3}} - \pi\left(\frac{R}{\sqrt{3}}\right)$ $= \frac{2\sqrt{3}\pi R^3}{9}$
	Obtains the correct max volume in the form $kR^3 - pR^3$ or better	3.2a	A1	
	Justifies correct volume in the form $kR^3 - pR^3$ or better form is the maximum eg: • V = 0 when h=0 or R and V>0 in between. • Sketches shape of graph passing through the origin with (min on negative side) and max on positive side • Obtains $\frac{d^2V}{dh^2} = -6\pi h$ <0 NB R1 can be awarded even if E1 is not awarded.	2.1	R1	$\frac{d^2V}{dh^2} = -6\pi h$ When $h = \frac{R}{\sqrt{3}}$ $\frac{d^2V}{dh^2} < 0$ Therefore maximum

Video solution:

https://youtu.be/xIm9q8Q_Ccg

June 2019 Question 15 Paper 1

15 (a) At time t hours **after a high tide**, the height, h metres, of the tide and the velocity, v knots, of the tidal flow can be modelled using the parametric equations

$$v = 4 - \left(\frac{2t}{3} - 2\right)^2$$

$$h = 3 - 2\sqrt[3]{t - 3}$$

High tides and low tides occur alternately when the velocity of the tidal flow is zero.

A high tide occurs at 2 am.

15 (a) (i) Use the model to find the height of this high tide.

[1 mark]

15 (a) (ii) Find the time of the first **low** tide after 2 am.

[3 marks]

15 (a) (iii) Find the height of this low tide.

[1 mark]

15 (b) Use the model to find the height of the tide when it is flowing with maximum velocity.

[3 marks]

15 (c) Comment on the validity of the model.

[2 marks]

ANSWER

Q	Marking Instructions	AO	Mark	Typical Solution
15 (a)(i)	Uses model with *t*=0 to find correct value of *h* *AWRT 5.9*	AO3.4	B1	5.88metres
15 (a)(ii)	Uses *v*=0 to set up quadratic equation for ***t***	AO3.4	M1	$0 = 4 - (\frac{2t}{3} - 2)^2$
	Obtains t = 6 PI correct answer	AO1.1b	A1	t = 6 8 am
	Interprets their lowest positive solution correctly NMS can score 3	AO3.2a	A1F	
15 (a)(iii)	Obtains correct h for their positive t provided h < 5.88 Accept 0.12 If given to more decimal places AWFW 0.115 to 0.116 FT their t allow negative h	AO3.4	B1F	0.12m
15(b)	Identifies t=3 (for maximum velocity)	AO3.1b	B1	*t* = 3
	Substitutes their t into model for h PI their answer	3.4	M1	$h = 3 - 2\sqrt[3]{3-3}$ *h* = 3 metres
	Finds correct height **with units**	3.2a	A1	
15(c)	Explains that the validity of the model is limited by time.	3.5b	B1	The model breaks down after one cycle of the tide.
	Explains that the height continues to decrease after 6 hours (or after the first cycle or first low tide). Or explains there are no other times when v=0	3.5a	B1	After 6 hours the model shows the height continues to decrease.
	Total		**10**	

June 2018 Question 5 Paper 1

5 A curve is defined by the parametric equations

$$x = 4 \times 2^{-t} + 3$$

$$y = 3 \times 2^{t} - 5$$

5 (a) Show that $\frac{dy}{dx} = -\frac{3}{4} \times 2^{2t}$

[3 marks]

5 (b) Find the Cartesian equation of the curve in the form $xy + ax + by = c$, where a, b and c are integers.

[3 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
5(a)	Differentiates 2^t or 2^{-t} to obtain $\pm A\ln 2\times 2^{\pm t}$	AO1.1a	M1	
	Obtains $\dfrac{dy}{dt}=(\pm A\ln 2)2^t$ and $\dfrac{dx}{dt}=(\pm B\ln 2)2^{-t}$	AO1.1b	A1	$\dfrac{dy}{dt}=(3\ln 2)2^t$ $\dfrac{dx}{dt}=(-4\ln 2)2^{-t}$
	Uses chain rule with correct $\dfrac{dy}{dt}$ and $\dfrac{dx}{dt}$ and completes rigorous argument to obtain fully correct printed answer	AO2.1	R1	$\dfrac{dy}{dx}=\dfrac{(3\ln 2)2^t}{(-4\ln 2)2^{-t}}$ $=-\dfrac{3}{4}\times 2^{2t}$
(b)	Rearranges to write 2^{-t} in terms of x or 2^t in terms of y Or Writes given expression in terms of t	AO3.1a	M1	$2^t=\dfrac{y+5}{3}$ $2^{-t}=\dfrac{x-3}{4}$
	Eliminates t Or Compares coefficients PI by a=5 or b=-3	AO1.1a	M1	$1=\left(\dfrac{y+5}{3}\right)\left(\dfrac{x-3}{4}\right)$ $12=xy+5x-3y-15$ $xy+5x-3y=27$
	Completes rigorous argument to obtain correct values of a, b and c and write the Cartesian equation in the required form ISW	AO2.1	R1	**ALT** $xy+ax+by=(4\times 2^{-t}+3)(3\times 2^t-5)+a(4\times 2^{-t}+3)+b(3\times 2^t-5)$ $=12-15+(4a-20)2^{-t}+(3b+9)2^t+3a-5b$ $a=5, b=-3$ $xy+5x-3y=-3+15+15$ $=27$
	Total		**6**	

Ch 12 Vectors

Ch 13 Differential equations

June 2019 Question 5 Paper 2

5 Solve the differential equation

$$\frac{\mathrm{d}t}{\mathrm{d}x} = \frac{\ln x}{x^2 t} \qquad \text{for } x > 0$$

given $x = 1$ when $t = 2$

Write your answer in the form $t^2 = \mathrm{f}(x)$

[7 marks]

ANSWER

Q	Marking instructions	AO	Mark	Typical solution
5	Separates the variables – one side correct Condone missing integral signs **PI** by correct integration	3.1a	M1	$\int \frac{1}{x^2}\ln x \, dx = \int t \, dt$
	Integrates their $\int t \, dt$ correctly	1.1b	A1F	$\int t \, dt = \frac{t^2}{2} + c$
	Obtains $u' = \frac{1}{x}$ and $v = -\frac{1}{x}$ **OE**	1.1b	B1	$u = \ln x$ $u' = \frac{1}{x}$
	Integrates $\int \frac{1}{x^2}\ln x \, dx$ Substitutes their u, u', v and v' into the correct formula for integration by parts Condone sign errors in formula	1.1a	M1	$v' = x^{-2}$ $v = -x^{-1}$ $-\frac{1}{x}\ln x - \int \frac{1}{x}\left(-x^{-1}\right) dx$ $-\frac{1}{x}\ln x + \int \frac{1}{x^2} dx$
	Obtains $-\frac{1}{x}\ln x - \frac{1}{x}$	1.1b	A1	$-\frac{1}{x}\ln x - \frac{1}{x}$
	Substitutes $t = 2$ and $x = 1$ into their integrated equation to find their $+c$	1.1a	M1	$-\frac{1}{x}\ln x - \frac{1}{x} = \frac{t^2}{2} + c$
	Substitutes $t = 2$ and $x = 1$ into their integrated equation to find their $+c$	1.1a	M1	$-\frac{1}{x}\ln x - \frac{1}{x} = \frac{t^2}{2} + c$
	Obtains correct solution must have t^2 = …. **ACF**	2.5	A1	$t = 2, x = 1 \Rightarrow -1 = 2 + c$ $c = -3$ $t^2 = 6 - 2\left(\frac{1 + \ln x}{x}\right)$
	Total		**7**	

June 2018 Question 5 Paper 2

5 Solve the differential equation

$$\frac{dt}{dx} = \frac{\ln x}{x^2 t} \qquad \text{for } x > 0$$

given $x = 1$ when $t = 2$

Write your answer in the form $t^2 = f(x)$

[7 marks]

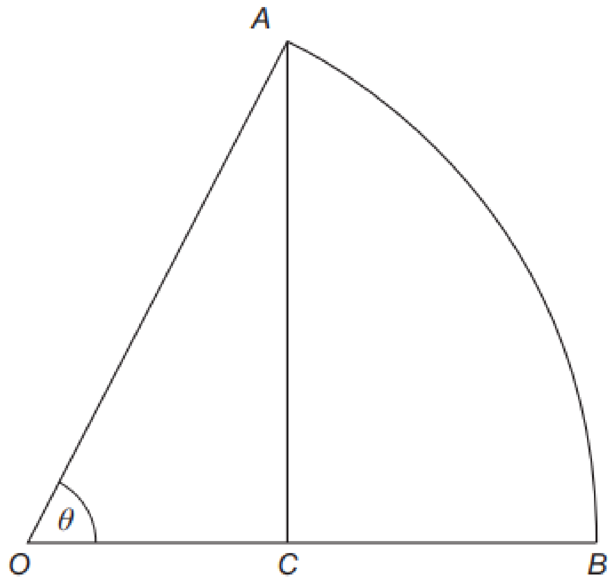
ANSWER

Q	Marking instructions	AO	Mark	Typical solution
5	Separates the variables – one side correct Condone missing integral signs **PI** by correct integration	3.1a	M1	$\int \frac{1}{x^2}\ln x\,dx = \int t\,dt$
	Integrates their $\int t\,dt$ correctly	1.1b	A1F	$\int t\,dt = \frac{t^2}{2} + c$
	Obtains $u' = \frac{1}{x}$ and $v = -\frac{1}{x}$ **OE**	1.1b	B1	$u = \ln x$ $u' = \frac{1}{x}$
	Integrates $\int \frac{1}{x^2}\ln x\,dx$ Substitutes their u, u', v and v' into the correct formula for integration by parts Condone sign errors in formula	1.1a	M1	$v' = x^{-2}$ $v = -x^{-1}$ $-\frac{1}{x}\ln x - \int \frac{1}{x}\left(-x^{-1}\right)dx$ $-\frac{1}{x}\ln x + \int \frac{1}{x^2}dx$
	Obtains $-\frac{1}{x}\ln x - \frac{1}{x}$	1.1b	A1	$-\frac{1}{x}\ln x - \frac{1}{x}$
	Substitutes $t = 2$ and $x = 1$ into their integrated equation to find their $+c$	1.1a	M1	$-\frac{1}{x}\ln x - \frac{1}{x} = \frac{t^2}{2} + c$
	Obtains correct solution must have t^2 = …. **ACF**	2.5	A1	$t = 2, x = 1 \Rightarrow -1 = 2 + c$ $c = -3$ $t^2 = 6 - 2\left(\frac{1+\ln x}{x}\right)$
	Total		**7**	

Ch 14 Numerical methods

June 2022 Question 10 Paper 1

10 The diagram shows a sector of a circle OAB.



The point C lies on OB such that AC is perpendicular to OB.

Angle AOB is θ radians.

10 (a) Given the area of the triangle OAC is half the area of the sector OAB, show that

$$\theta = \sin 2\theta$$

[4 marks]

10 (b) Use a suitable **change of sign** to show that a solution to the equation

$$\theta = \sin 2\theta$$

lies in the interval given by $\theta \in \left[\frac{\pi}{5}, \frac{2\pi}{5}\right]$

[2 marks]

10 (c) The Newton-Raphson method is used to find an approximate solution to the equation

$$\theta = \sin 2\theta$$

10 (c) (i) Using $\theta_1 = \frac{\pi}{5}$ as a first approximation for θ apply the Newton-Raphson method twice to find the value of θ_3

Give your answer to three decimal places.

[3 marks]

10 (c) (ii) Explain how a more accurate approximation for θ can be found using the Newton-Raphson method.

[1 mark]

10 (c) (iii) Explain why using $\theta_1 = \frac{\pi}{6}$ as a first approximation in the Newton-Raphson method does not lead to a solution for θ.

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
10(a)	Recalls or uses the area of sector = $\frac{1}{2}r^2\theta$ r can be any letter or OA or OB or any consistent value throughout	1.2	B1	Area of sector = $\frac{1}{2}r^2\theta$ Area of triangle = $\frac{1}{2}ab\sin C$ Hence $\frac{1}{2}ab\sin C = \left(\frac{1}{2}\right)\frac{1}{2}r^2\theta$ $\frac{1}{2}r^2\sin\theta\cos\theta = \frac{1}{2}\left(\frac{1}{2}r^2\theta\right)$ $2\sin\theta\cos\theta = \theta$ $\theta = \sin 2\theta$
	Forms an equation relating the area of the triangle OAC and sector using $\frac{1}{2}bh = k\frac{1}{2}r^2\theta$ where $k > 0$	3.1a	M1	
	Deduces area of triangle is $\frac{1}{2}r\cos\theta \times r\sin\theta$ OE Must use trigonometry for height and base	2.2a	B1	
	Completes reasoned argument with clear use of double angle identity to show that $\theta = \sin 2\theta$ or $\sin 2\theta = \theta$	2.1	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)	Rearranges to obtain $\theta - \sin 2\theta = 0$ or $\sin 2\theta - \theta = 0$ (which may be seen in conclusion) **and** evaluates $\theta - \sin 2\theta$ or $\sin 2\theta - \theta$ at $\frac{\pi}{5}$(0.6284) and $\frac{2\pi}{5}$(1.257) Evaluates using any two other appropriate values inside the interval but either side of root.	1.1a	M1	$\theta = \sin 2\theta \Rightarrow \theta - \sin 2\theta = 0$ Let $f(\theta) = \theta - \sin 2\theta$ $f\left(\frac{\pi}{5}\right) = -0.3227... < 0$ $f\left(\frac{2\pi}{5}\right) = 0.6688... > 0$ Hence solution lies between $\frac{\pi}{5}$ and $\frac{2\pi}{5}$
	Completes reasoned argument with reference to change of sign and evidence of correct evaluation accepting values rounded or truncated to 1 sf Must refer to $\frac{\pi}{5}$ and $\frac{2\pi}{5}$ in the conclusion	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(c)(i)	Differentiates $\sin 2\theta$ to obtain $2\cos 2\theta$ OE PI by correct θ_2 or θ_3 PI by sight of $2\cos\frac{2\pi}{5}$	1.1b	B1	$f(\theta) = \theta - \sin 2\theta$ $f'(\theta) = 1 - 2\cos 2\theta$ $\theta_{n+1} = \theta_n - \frac{\theta_n - \sin 2\theta_n}{1 - 2\cos 2\theta_n}$ $\theta_2 = 1.4732575...$ $\theta_3 = 1.0413241...$ $\theta_3 = 1.041$
	Obtains a correct expression for $\theta_n - \frac{\theta_n - \sin 2\theta_n}{1 - 2\cos 2\theta_n}$ Accept use of ANS or $\frac{\pi}{5}$ Condone missing or incorrect subscript PI by correct θ_2 or θ_3 AWRT θ_2 1.473	1.1a	M1	
	Obtains correct θ_3 AWRT θ_3 1.041	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
10(c)(ii)	Explains that more iterations could be used Accept keep on using Newton Raphson, keep re-iterating	2.4	E1	Use more iterations
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
10(c)(iii)	States that $f'\left(\frac{\pi}{6}\right)=0$	2.4	E1	$f'\left(\frac{\pi}{6}\right)=0$ The value is on a stationary point
	Explains a general reason for the Newton Raphson iteration not to converge to a particular root Accept only • too close to a stationary point • the value is on a stationary point • the tangent does not cross the x-axis • it converges to a different root • the formula is undefined Accept equivalents to these five bullet points only	2.4	E1	
	Subtotal		**2**	

	Question 10 Total		**12**	

November 2021 Question 7 Paper 1

7 The equation $x^2 = x^3 + x - 3$ has a single solution, $x = \alpha$

7 (a) By considering a suitable change of sign, show that α lies between 1.5 and 1.6

[2 marks]

7 (b) Show that the equation $x^2 = x^3 + x - 3$ can be rearranged into the form

$$x^2 = x - 1 + \frac{3}{x}$$

[2 marks]

7 (c) Use the iterative formula

$$x_{n+1} = \sqrt{x_n - 1 + \frac{3}{x_n}}$$

with $x_1 = 1.5$, to find x_2, x_3 and x_4, giving your answers to four decimal places.

[2 marks]

7 (d) Hence, deduce an interval of width 0.001 in which α lies.

[1 mark]

7 (e) After several hours the water has stopped dripping.

Give **two** reasons why the amount of water in the bucket is not as much as the answer found in part **(d)**.

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
7(a)	Rearranges to the form $\mathrm{f}(x)=0$ and evaluates $\mathrm{f}(x)$ at least once in the interval [1.5,1.6]	1.1a	M1	$x^2=x^3+x-3$ $\Rightarrow x^3-x^2+x-3=0$ $\mathrm{f}(x)=x^3-x^2+x-3$ $\mathrm{f}(1.5)=-0.375<0$ $\mathrm{f}(1.6)=0.136>0$ Hence α lies between 1.5 and 1.6
	Completes argument with correct evaluation either side of root and reference to change of sign	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)	Isolates x^3 or divides by x and cancels terms Eg $x=x^2+1-\frac{3}{x}$ OE Condone one slip in cancelling or one sign error	1.1a	M1	$x^2=x^3+x-3$ $x^3=x^2-x+3$ $x^2=x-1+\frac{3}{x}$
	Completes argument to show the given result. Three terms need not be in the given order	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7(c)	Obtains any one correct value to at least three decimal places, ignoring labels.	1.1a	M1	$x_2=1.5811$ $x_3=1.5743$ $x_4=1.5748$
	Obtains x_2, x_3 and x_4 correct to four decimal places or better If no labels only accept answers in clearly the correct order with no extras seen beyond x_4	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Mark	Typical solution
7(d)	Uses sum to infinity formula with their value of A substituted	3.4	M1	$S_\infty = \frac{30}{1-0.98}$ $= 1500$ 1.5 + 4 = 5.5 litres
	Obtains their correct value of sum to infinity	1.1b	A1F	
	Obtains 5.5 litres **CAO** Accept answer in litres or millilitres	3.2a	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Mark	Typical solution
7(e)	Explains that the model used assumes the drips continue indefinitely which is unrealistic	3.5b	E1	The sum to infinity was used but this assumes there are infinite drips, but they have stopped
	States a relevant environmental factor eg water has evaporated or wind affected water level or water consumed by animals	3.5a	E1	Water will evaporate over several hours
	Subtotal		**2**	

November 2020 Question 2 Paper 1

2 A student is searching for a solution to the equation $f(x) = 0$

He correctly evaluates

$$f(-1) = -1 \text{ and } f(1) = 1$$

and concludes that there must be a root between -1 and 1 due to the change of sign.

Select the function $f(x)$ for which the conclusion is **incorrect**.

Circle your answer.

[1 mark]

$f(x) = \frac{1}{x}$ $\quad$ $f(x) = x$ $\quad$ $f(x) = x^3$ $\quad$ $f(x) = \frac{2x+1}{x+2}$

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
2	Circles the correct answer	2.3	R1	$f(x) = \frac{1}{x}$
	Total		**1**	

November 2020 Question 14 Paper 1

14 The function f is defined by

$$f(x) = 3^x\sqrt{x} - 1 \qquad \text{where } x \geq 0$$

14 (a) $f(x) = 0$ has a single solution at the point $x = \alpha$

By considering a suitable change of sign, show that α lies between 0 and 1

[2 marks]

14 (b) (i) Show that

$$f'(x) = \frac{3^x(1 + x\ln 9)}{2\sqrt{x}}$$

[3 marks]

14 (b) (ii) Use the Newton–Raphson method with $x_1 = 1$ to find x_3, an approximation for α.

Give your answer to five decimal places.

[2 marks]

14 (b) (iii) Explain why the Newton–Raphson method fails to find α with $x_1 = 0$

[2 marks]

ANSWER

Q	Marking instructions	AO	Marks	Typical solution
14(a)	Evaluates $f(0) = -1$ and $f(1) = 2$ or Evaluates two other suitable appropriate values correct to 1 sig fig	1.1a	M1	$f(0) = -1 < 0$ $f(1) = 3 - 1 = 2 > 0$ Change of sign implies root therefore α is between 0 and 1
	Completes argument correctly stating $f(0) < 0$ and $f(1) > 0$ and concludes that $0 < \alpha < 1$	2.1	R1	
	Subtotal		**2**	
14(b)(i)	Uses product rule to obtain an expression of the form $Ax^{\frac{1}{2}}(3^x) + Bx^{-\frac{1}{2}}(3^x)$ A and /or B can be positive or negative	3.1a	M1	$f'(x) = x^{\frac{1}{2}}(3^x)\ln 3 + \frac{1}{2}x^{-\frac{1}{2}}(3^x)$ $= 3^x\left(\ln 3\sqrt{x} + \frac{1}{2\sqrt{x}}\right)$
	Obtains fully correct $f'(x)$	1.1b	A1	$= 3^x\left(\frac{2x\ln 3}{2\sqrt{x}} + \frac{1}{2\sqrt{x}}\right)$
	Completes convincing argument with no slips to show the required result. AG	2.1	R1	$= 3^x\left(\frac{x\ln 9}{2\sqrt{x}} + \frac{1}{2\sqrt{x}}\right)$ $= 3^x\left(\frac{1 + x\ln 9}{2\sqrt{x}}\right)$
	Subtotal		**3**	
14(b)(ii)	Forms correct Newton-Raphson expression PI by correct value of $x_2$or x_3 stated to at least 3 decimal places	1.1a	M1	$x_{n+1} = x_n - \dfrac{(3^{x_n}\sqrt{x_n} - 1)}{\frac{3^{x_n}(1 + x_n\ln 9)}{2\sqrt{x_n}}}$
	Obtains the correct value of x_3 Must be stated to five decimal places	1.1b	A1	$x_{n+1} = x_n - \dfrac{2\sqrt{x_n}\,(3^{x_n}\sqrt{x_n} - 1)}{3^{x_n}(1 + x_n\ln 9)}$ x_2= 0.5829716.. x_3= 0.4246536.. $x_3 \approx$ 0.42465
	Subtotal		**2**	

14(b)(iii)	Explains that convergence is impossible Must use the word convergence or convergent	2.4	E1	Convergence is impossible as all values of x_n would equal 0
	Explains that the tangent at $x = 0$ is vertical or Explains all values of x_n would equal 0 or Demonstrates that several values of x_n would be 0	2.4	E1	
	Subtotal		**2**	
	Question Total		**9**	

Video solution:

https://youtu.be/jgHcMp714FU

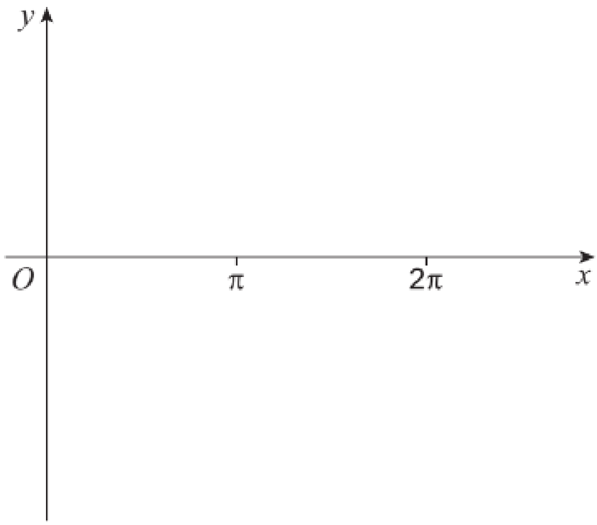
June 2019 Question 7 Paper 1

7 (a) By sketching the graphs of $y = \frac{1}{x}$ and $y = \sec 2x$ on the axes below, show that the equation

$$\frac{1}{x} = \sec 2x$$

has exactly one solution for $x > 0$

[3 marks]



7 (b) By considering a suitable change of sign, show that the solution to the equation lies between 0.4 and 0.6

[2 marks]

7 (c) Show that the equation can be rearranged to give

$$x = \frac{1}{2}\cos^{-1} x$$

[2 marks]

7 (d) (i) Use the iterative formula

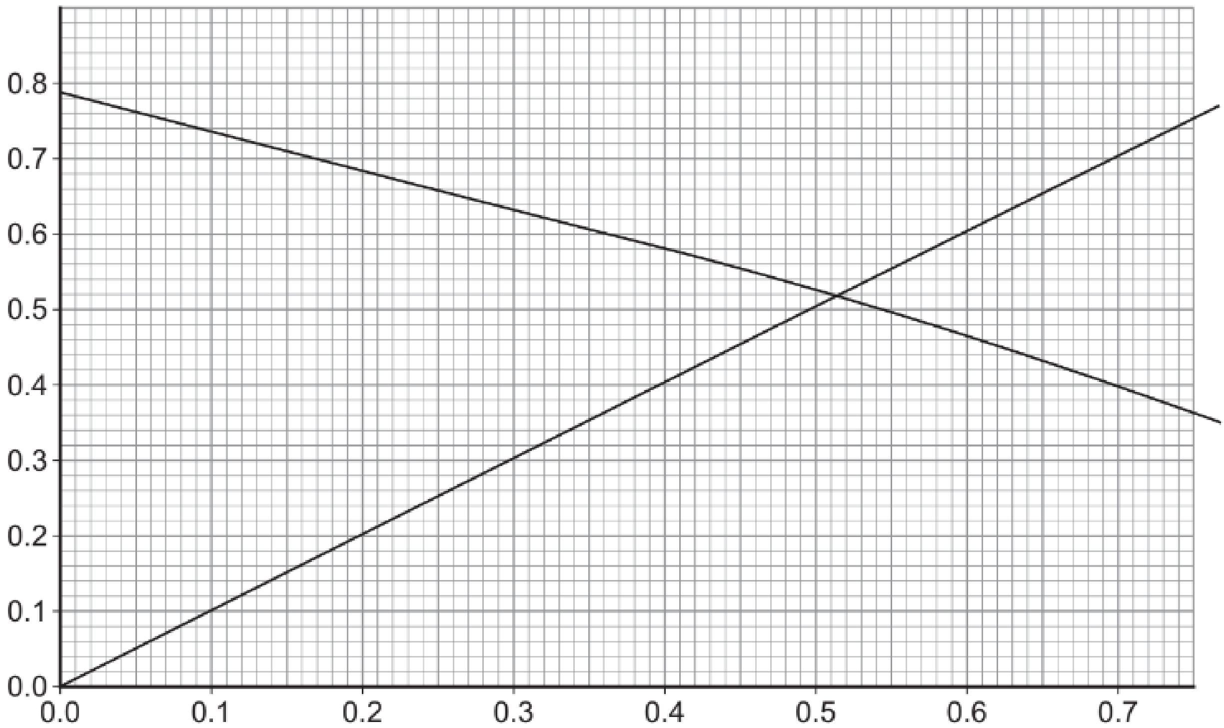
$$x_{n+1} = \frac{1}{2}\cos^{-1} x_n$$

with $x_1 = 0.4$, to find x_2, x_3 and x_4, giving your answers to four decimal places.

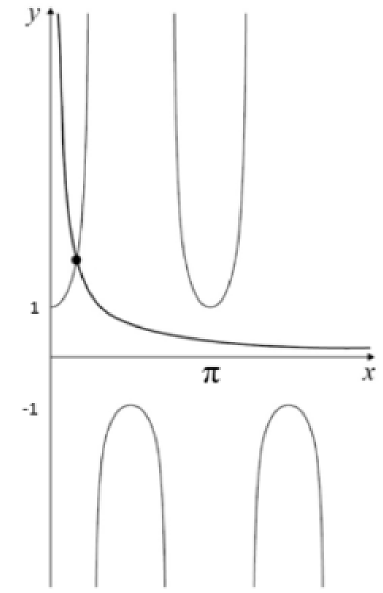
[2 marks]

7 (d) (ii) On the graph below, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2, x_3 and x_4.

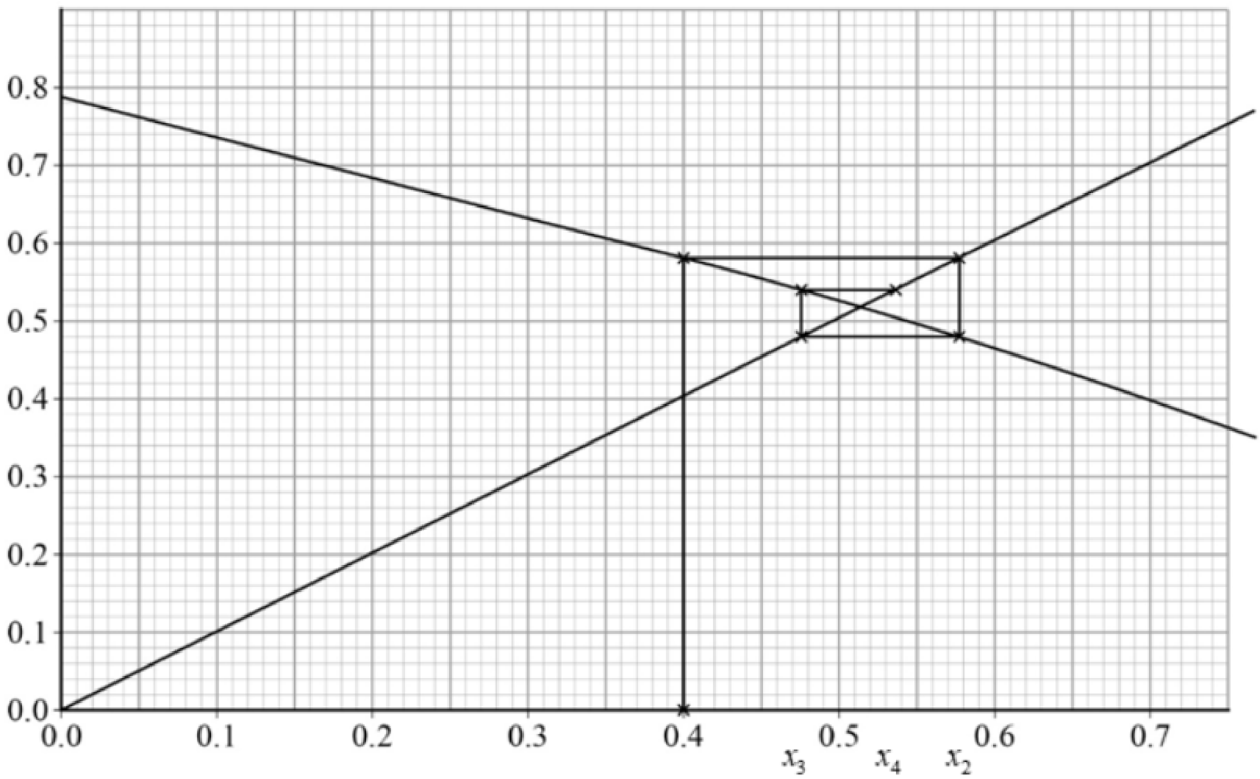
[2 marks]



ANSWER

Q	Marking instructions	AO	Mark	Typical solution
7(a)	Sketches graph of $y=\frac{1}{x}$ Must not cross axes Correct asymptotes	1.2	B1	
	Sketches graph of $y=\sec kx$ up to first asymptote	1.1a	M1	
	Draws fully correct graphs in **first quadrant,** intersecting at one point with sec 2x up to asymptote at x=$\frac{5\pi}{4}$. Ignore fourth quadrant/negative y. Condone missing labels on y-axis.	1.1b	A1	
7(b)	Rearranges to the form $\mathrm{f}(x)=0$ and evaluates $\mathrm{f}(x)$ at 0.4 and 0.6 Can evaluate at two values either side of the root 0.515 in the interval [0.4,0.6]	1.1a	M1	$\frac{1}{x}=\sec 2x \Rightarrow \frac{1}{x}-\sec 2x=0$ $\mathrm{f}(x)=\frac{1}{x}-\sec 2x$ $\mathrm{f}(0.4)=1.06..>0$ $\mathrm{f}(0.6)=-1.09..<0$
	Completes rigorous argument with any reference to change of sign. Must see evidence of correct evaluation accepting values correct to 1 sf. If function notation used it must be defined.	2.1	R1	Hence the solution lies between 0.4 and 0.6
7(c)	Uses $\sec 2x=\frac{1}{\cos 2x}$ to obtain a correct equation in $\cos 2x$ eg, $\frac{1}{x}=\frac{1}{\cos 2x}$ or $1=\frac{x}{\cos 2x}$	1.1a	M1	$\frac{1}{x}=\sec 2x$ $\frac{1}{x}=\frac{1}{\cos 2x}$ $x=\cos 2x$ $2x=\cos^{-1}x$ $x=\frac{1}{2}\cos^{-1}x$
	Completes rearrangement Must see $\cos^{-1}x=2x$ before given answer	2.1	R1	

7 (d)(i)	Obtains any one correct value to at least 3 decimal places, ignoring labels.	1.1a	M1	$x_2 = 0.5796$ $x_3 = 0.4763$ $x_4 = 0.5372$
	Obtains x_2, x_3 and x_4 correct to 4 decimal places If no labels only accept the three correct answers in the correct order with no extras seen beyond x_4 **CAO**	1.1b	A1	
7 (d)(ii)	Draws correct cobweb diagram Condone missing vertical line x = 0.4	1.1a	M1	See diagram below
	Shows positions of x_2, x_3 and x_4 with clear indication of positioning on x-axis not on y=x	1.1b	A1	
	Total		**11**	

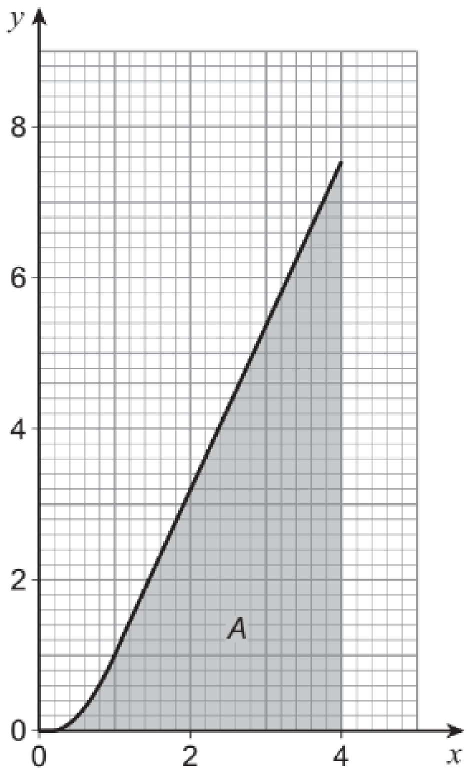


Video solution:

https://youtu.be/GBS-ddqDc3c

June 2019 Question 14 Paper 1

14 The graph of $y = \dfrac{2x^3}{x^2+1}$ is shown for $0 \leq x \leq 4$



Caroline is attempting to approximate the shaded area, *A*, under the curve using the trapezium rule by splitting the area into n trapezia.

14 (a) When $n = 4$

14 (a) (i) State the number of ordinates that Caroline uses.

[1 mark]

14 (a) (ii) Calculate the area that Caroline should obtain using this method.

Give your answer correct to two decimal places.

[3 marks]

14 (b) Show that the exact area of *A* is

$$16 - \ln 17$$

Fully justify your answer.

[5 marks]

14 (c) Explain what would happen to Caroline's answer to part **(a)(ii)** as $n \to \infty$

[1 mark]

ANSWER

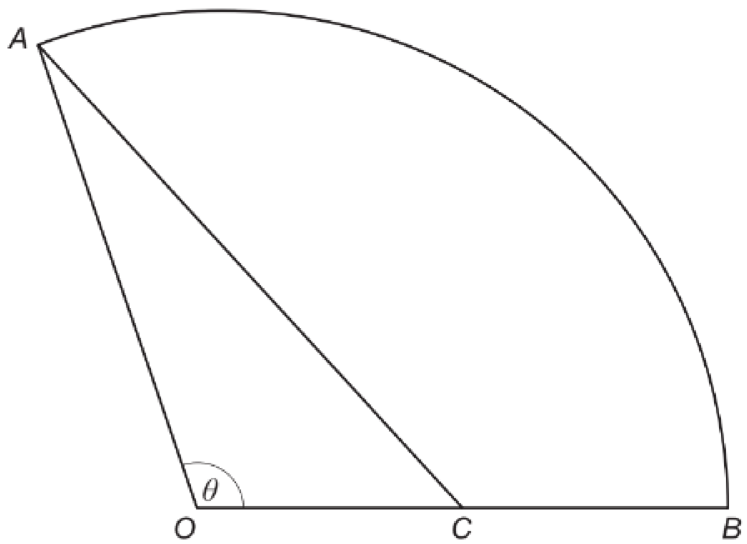
Q	Marking instructions	AO	Mark	Typical solution
14 (a)(i)	States correct number of ordinates	1.2	B1	5
14 (a)(ii)	Obtains at least 4 correct y values (condone 7.5… for y_4) and correct h	1.1b	B1	x: 0, 1, 2, 3, 4; y: 0, 1, 3.2, 5.4, 7.52941 $Area = \frac{1}{2}\times 1\times(0+7.529+2(1+3.2+5.4))$ $=13.36$
	Substitutes their y values into trapezium rule with correct number of strips. Condone missing 0. May see working on graph 0.5+2.1+4.3+6.4647. (Exact value 1136/85)	1.1a	M1	
	Obtains correct area. NMS correct answer award full marks **CAO**	1.1b	A1	
14(b)	Selects substitution $u = x^2 + 1$ or $u = x^2$ and obtains $\frac{du}{dx} = 2x$ or writes the integrand in the form $Ax + \frac{Bx}{x^2+1}$	3.1a	M1	$u = x^2 + 1 \Rightarrow \frac{du}{dx} = 2x$ $\int_1^{17} \frac{2x^3}{u} \times \frac{1}{2x} du$ $= \int_1^{17} \frac{x^2}{u} du$ $= \int_1^{17} \frac{u-1}{u} du$ $= \int_1^{17} 1 - \frac{1}{u} du$ $= [u - \ln u]_1^{17}$ $= (17 - \ln 17) - (1 - \ln 1)$ $= 16 - \ln 17$
	Obtains $\int_1^{17} 1 - \frac{1}{u} du$ OE Or $\int_0^4 2x - \frac{2x}{x^2+1} dx$ Ignore limits	1.1b	A1	
	Integrates their expression obtaining an ln term correctly.	1.1a	M1	
	Obtains fully correct integral $u - \ln u$ or $x^2 - \ln(x^2+1)$ Condone missing limits.	1.1b	A1	
	Completes fully correct argument Substituting correct limits for their method to show the correct required result with correct notation with **AG**	2.1	R1	
14(c)	Explains that as n increases the approximation found will tend to the value of $\int_0^4 \frac{2x^3}{x^2+1} dx$ OE	2.4	E1	Area $\to 16 - \ln 17$

June 2018 Question 8 Paper 1

8 The diagram shows a sector of a circle *OAB*.

C is the midpoint of *OB*.

Angle *AOB* is θ radians.



8 (a) Given that the area of the triangle *OAC* is equal to one quarter of the area of the sector *OAB*, show that $\theta = 2\sin\theta$

[4 marks]

8 (b) Use the Newton-Raphson method with $\theta_1 = \pi$, to find θ_3 as an approximation for θ. Give your answer correct to five decimal places.

[3 marks]

8 (c) Given that $\theta = 1.89549$ to five decimal places, find an estimate for the percentage error in the approximation found in part **(b)**.

[1 mark]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)	Uses $A=\frac{1}{2}ab\sin C$ for triangle OAC or OAB PI by equation	AO1.2	B1	$\frac{1}{2}r\times\frac{r}{2}\sin\theta=\frac{1}{4}\left(\frac{1}{2}r^2\theta\right)$ $\Rightarrow\frac{r^2}{4}\sin\theta=\frac{1}{8}r^2\theta$ $\Rightarrow 2r^2\sin\theta=r^2\theta$ $\Rightarrow 2\sin\theta=\theta$ **AG**
	Forms an equation relating the area of OAC and ABC in the form $Ar^2\sin\theta=Br^2\theta$	AO3.1a	M1	
	Obtains fully correct equation ACF	AO1.1b	A1	
	Simplifies to obtain required equation, only award if all working correct with rigorous argument.	AO2.1	R1	
(b)	Rearranges to the form $f(\theta)=0$ PI by correct θ_2 or θ_3	AO1.1a	M1	$f(\theta)=\theta-2\sin\theta=0$ $\theta_{n+1}=\theta_n-\frac{\theta_n-2\sin\theta_n}{1-2\cos\theta_n}$ $\theta_2=2.094395...$ $\theta_3=1.913222...$ $\theta_3=1.91322\ (5\text{ d.p.})$
	Differentiates their $f(\theta)$ or uses calculator PI correct θ_2 or θ_3	AO1.1b	A1	
	Obtains correct θ_3	AO1.1b	A1	
(c)	Obtains percentage error for θ_3 AWRT 0.94%	AO3.2b	B1	0.935%
	Total		**8**	

June 2018 Question 11 Paper 1

11 The daily world production of oil can be modelled using

$$V = 10 + 100\left(\frac{t}{30}\right)^3 - 50\left(\frac{t}{30}\right)^4$$

where V is volume of oil in millions of barrels, and t is time in years since 1 January 1980.

11 (a) (i) The model is used to predict the time, T, when oil production will fall to zero.

Show that T satisfies the equation

$$T = \sqrt[3]{60T^2 + \frac{162\,000}{T}}$$

[3 marks]

11 (a) (ii) Use the iterative formula $T_{n+1} = \sqrt[3]{60{T_n}^2 + \frac{162\,000}{T_n}}$, with $T_0 = 38$, to find the values of T_1, T_2, and T_3, giving your answers to three decimal places.

[2 marks]

11 (a) (iii) Explain the relevance of using $T_0 = 38$

[1 mark]

11 (b) From 1 January 1980 the daily use of oil by one technologically developing country can be modelled as

$$V = 4.5 \times 1.063^t$$

Use the models to show that the country's use of oil and the world production of oil will be equal during the year 2029.

[4 marks]

ANSWER

Q	Marking Instructions	AO	Marks	Typical Solution
11(a)(i)	Uses model to form equation with V=0	AO3.4	M1	$\therefore 10+100\left(\frac{T}{30}\right)^3-50\left(\frac{T}{30}\right)^4=0$
	Rearranges to isolate T^4 term	AO1.1a	M1	
	Completes rigorous and convincing argument to clearly show the required result. Need to see evidence of division by T to isolate T^3 term Must be an equation throughout **AG**	AO2.1	R1	$\Rightarrow 50\left(\frac{T}{30}\right)^4=10+100\left(\frac{T}{30}\right)^3$ $\Rightarrow \frac{T^4}{16200}=10+\frac{T^3}{270}$ $\Rightarrow \frac{T^3}{16200}=\frac{10}{T}+\frac{T^2}{270}$ $\Rightarrow T=\sqrt[3]{\frac{162000}{T}+60T^2}$
11(a)(ii)	Calculates T_1 (44.96345…..)	AO1.1a	M1	T_1=44.963
	Calculates T_2 and T_3 (49.98742….) Condone greater than 3dp (53.50407….)	AO1.1b	A1	T_2 = 49.987 T_3 = 53.504
11(a)(iii)	Explains 38 in context	AO3.2a	B1	38 represents current year 2018
11(b)	Translates 2029 into t=49	AO3.3	B1	$10+100\left(\frac{t}{30}\right)^3-50\left(\frac{t}{30}\right)^4=4.5\times1.063^t$ $\Rightarrow t=49.009$ 1980 + 49 = 2029 Therefore use of oil and production of oil will be equal in the year 2029
	Uses models to set up equation or evaluate both models at one value of t	AO3.4	M1	
	Obtains correct values for **both** models for two appropriate values of t. $t\in[49,50]$ eg t=49 and t=50 t =49 gives: 89.89 and 89.81 t =50 gives: 87.16 and 95.47 Or Solves equation using any method to obtain AWFW 49.009 to 49.01	AO1.1b	A1	
	Explains that the use of oil and the production of oil are equal when t = 49.009 Or Uses a change of sign argument OE to explain that the value of each model for two appropriate values of t shows that the production of oil and the use of oil are the same for $t\in(49,50)$	AO2.4	E1	
	Total		**10**	